Homework:

(1) Describe the EM algorithm for estimating a Gaussian mixture model. Run GMMexample.m and discuss the effectiveness of the algorithm by comparing the true model and results.

(2) Modify GMMexample.m to estimate a Gaussian mixture model based on 1000 data points data samples drawn from

$$\begin{aligned} p(\mathbf{x}) \quad &= \tfrac{1}{8\pi} \exp\left(-\tfrac{(x_1-2)^2}{2}\right) \exp\left(-\tfrac{(x_2-2)^2}{2}\right) \\ &\tfrac{3}{8\pi} \exp\left(-\tfrac{(x_1-3)^2}{2}\right) \exp\left(-\tfrac{(x_2+1)^2}{2}\right) \end{aligned}$$

(3) Suppose that we are given a data set consisting of points $x_{i,j}$ from two classes respectively, where $j = 1, 2$, denotes class label, and i denotes the data index. (a) Determine the class label for a new data point $x = 1.5$ using a probabilistic neural network, with the Gaussian function as window function and $\sigma = 1$. (b) How do you find the classification decision boundary of the probabilistic neural network used in (a)? The data set is as follows:

Class 1: {1, 2, 0.5}
Class 2: {2, 3, 3.5}

(4) Run pnn2D.m and describe the results given by the figures, in relation to the data set.

(5) Given $\boldsymbol{w} = [1,\ 2]^{\mathrm{T}}$, $w_0 = 0.5$, find $g(x) = \boldsymbol{x}^{\mathrm{T}} + w_0$ for (a) $\boldsymbol{x} = [-1,\ 2]^{\mathrm{T}}$; (b) $\boldsymbol{x} = [-1,\ 1]^{\mathrm{T}}$ and (c) $\boldsymbol{x} = [1,\ -1]^{\mathrm{T}}$ respectively. Plot $g(\boldsymbol{x}) = 0$. Indicate two half planes with $g(\boldsymbol{x}) > 0$, and $g(\boldsymbol{x}) < 0$.

(6) Describe procedure of Fisher Linear discriminant method. Run FisherEx.m several times and explain the results.