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CORSO DI LAUREA MAGISTRALE IN

Data Science and Economics

**'GAMES WITH INCOMPLETE  
INFORMATION: AN APPLICATION TO  
THE BANKING SECTOR'**

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# Main concepts

- Asymmetric Information
- Adverse Selection
- Signaling

- ***Asymmetric information*** arises when ***one party has more information*** about a transaction than the other.
- ***Adverse selection*** arises when the informed individual's trading decisions depend on her private information in a manner that adversely affects uninformed market participants. This may lead to inefficiencies.
- ***Signaling*** is a mechanism that may arise in the market as a response to the inefficiencies induced by asymmetric information. It consists in the possibility for the informed party to take actions in order to convey signals to the uninformed party.

- The market for “lemons” (Akerlof, 1970): pioneering work on adverse selection.
- The labor market signaling model (Spence, 1973): pioneering work on signaling.
- Signaling models have found many applications in economics.
- Spence, Akerlof, and Stiglitz were awarded the Nobel Prize in Economic Sciences in 2001 for their work on asymmetric information in markets.

# Thesis

- The thesis analyses the previous concepts in a game-theoretical perspective and applies them to a banking sector problem.
- Banks (more informed party) offer mobile applications to customers (less informed party).
- Signaling emerges as a ***mechanism that serves to build trust and reduce uncertainty for the uninformed party.***

# Bayesian Games

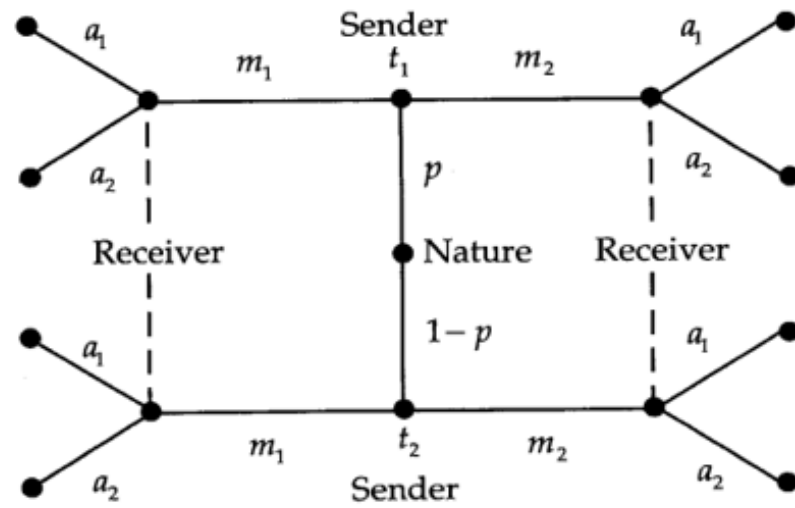
Games with incomplete information: some player doesn't know the characteristics (i.e. the preferences) of some other player.

Solution concept: **Perfect Bayesian Equilibrium (PBE)**.

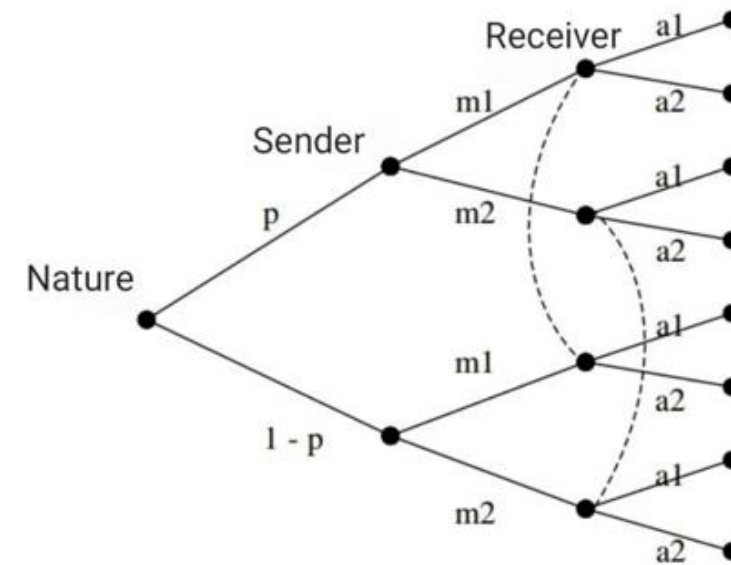
A PBE is a set of ***strategies*** and ***beliefs*** for every player such that

- the strategies are sequentially rational given the players' beliefs;
- the players update their beliefs via Bayes rule **whenever possible** (some moves may only arise off-the-equilibrium path of the game tree. The probability of arriving there is 0. In these cases, players may have any possible beliefs).

Gibbons (1992):



Dutta (1999):



# Designing a Signaling Game: The case of MBAs

- The banking sector problem is modeled as a game above.
- There are two types of **banks** selected (stochastically) by nature as **Trustworthy** and **Untrustworthy**, who benefit from the customers' usage of their mobile application.
- The type is known by the bank but not by the customers.
- The bank can **signal** its type by offering a certain level of encryption for the customers' sensitive data. Encryption is costly.
- **Customers** observe the signal and decide whether to use the mobile application or delete it.



The bank obtains revenue 3 if the customer uses the app.

Encryption is costly:  
 $C_{TB} = 1$ ,  $C_{UB} = 4$

The customer gets:

0 if he deletes the app

payoff  $> 0$  if uses the app  
of a TB

payoff  $< 0$  if uses the app  
of an UB

$z = 0.6$

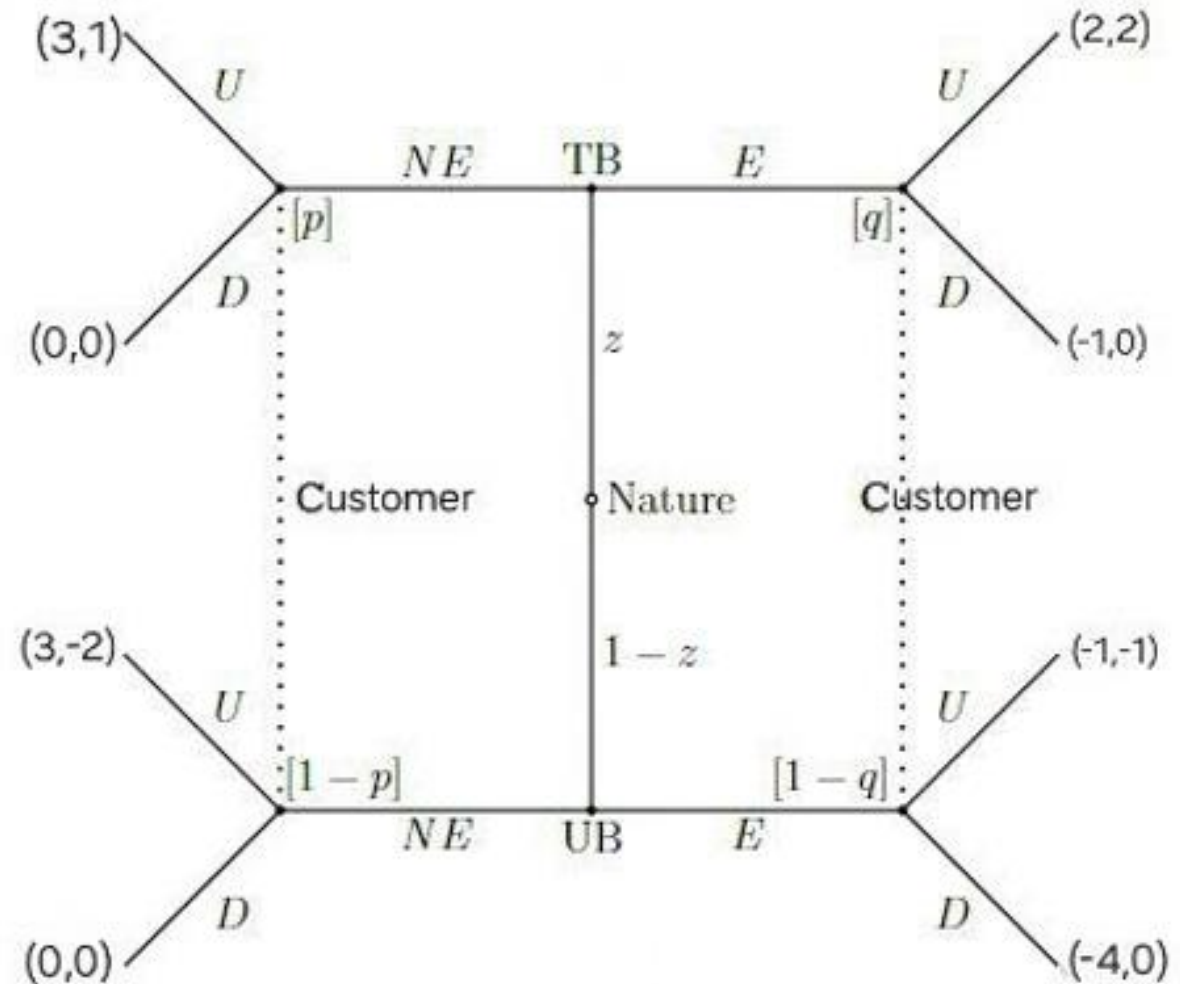


Figure: Illustration of the game

# Pooling PBE: (NE, NE, D, D)

- (NE, NE): both the TB and the UB choose No Encryption
- The Customer's beliefs after the No Encryption signal must be computed *via Bayes' rule*,

which gives  $p = \frac{(0.6)x1}{(0.6)x1 + (0.4)x1} = 0.6$ .

The Customer's expected payoffs are:

$$\pi_C(U \mid NE) = (0.6)x(1) + (0.4)x(-2) = -0.2$$

$$\pi_C(D \mid NE) = (0.6)x(0) + (0.4)x(0) = 0$$

→ *the customer deletes the app after the Non Encryption signal.*

# Pooling PBE: (NE, NE, D, D)

- If the Untrustworthy Bank plays NE, it gets payoff 0. If it deviates by playing E, it will get either -1 or -4

→ *the UB always chooses NE.*

- If the **Trustworthy Bank** type plays NE, it gets payoff 0. If it deviates by playing E, it will get either 2 or -1. Thus, *its choice will depend on the Customer's choice in the information set after Encryption*, which, in turn, *depends on his beliefs  $q$* . In particular, the TB type will not have incentive to deviate if the Customer will choose to Delete the app there.

# Pooling PBE: (NE, NE, D, D)

- There is no consistency requirement for customers' beliefs after the Encryption signal (q), as the corresponding information set is off the equilibrium path.
- What are the values of q which sustain a choice for the Customer such that the Bank's strategy (NE, NE) is part of a Pooling PBE? That is, such that the Trustworthy Bank type does not have incentive to deviate?

The customer will be willing to Delete the app after observing signal E whenever

$$\pi_C(D \mid E) \geq \pi_C(U \mid E) \iff q \leq \frac{1}{3}$$

→ ***(NE, NE, D, D),  $p = 0.6$ ,  $q \leq \frac{1}{3}$  is a (pooling) PBE.***

# Separating PBE: (E, NE, U, D)

- (E, NE): TB chooses Encryption, UB chooses No Encryption.

Since beliefs must be derived via Bayes rule, the customer's beliefs are given by  $p = 0$  and  $q = 1$ .

- After observing E, if the Customer uses the app he gets 2, otherwise he gets 0  
→ *the customer uses the app after the Encryption signal.*
- After observing NE, if the Customer uses the app he gets -2, otherwise he gets 0  
→ *the customer deletes the app after the No Encryption signal.*
- E is optimal for the TB, NE is optimal for the UB → (E, NE, U, D),  $p=0$ ,  $q=1$  is a (separating) PBE.

# Intuitive Criterion

$$[(NE, NE, D, D), p = z, q \leq \frac{1}{3}].$$

- $q$  is the probability that the Customer assigns to the fact that a deviation to Encryption comes from the Untrustworthy Bank type.
- In this equilibrium, the Customer, after observing a deviation to Encryption, must believe that such a deviation comes with larger probability from the Untrustworthy Bank type.
- Does this seem reasonable?
- If the UB type deviates to E, it is always strictly worse off than in equilibrium, independently of the resulting customer's choice (it gets either -1 or -4, while in equilibrium it gets 0).
- If the TB type deviates to E, it can get a payoff larger than the equilibrium one (2 instead of 0).

# Intuitive Criterion

- The **Intuitive Criterion** has been introduced by Cho and Kreps (1987) as an equilibrium refinement for signaling games, which helps to get rid of unreasonable equilibria.
- It puts restrictions on players' beliefs off-the-equilibrium path.
- Intuitively, a PBE can be eliminated if there is some type of player who wants to deviate even though he is not sure what the belief of some other player is. The deviating player is only sure that the other player will not think that he is a type for which the deviation is an equilibrium-dominated action.

- After observing a deviation to the Encryption signal, the Customer must assign probability 1 to the fact that this deviation comes from the Trustworthy Bank type.
  - $\pi_C(U \mid E) = 2 \geq \pi_C(D \mid E) = 0 \rightarrow$  the customer uses the app after the Encryption signal.
  - Then, the TB prefers E to NE (it gets 2 instead of 0).
- $\rightarrow$  the Intuitive Criterion eliminates the Pooling PBE of the game.

**The MBAs game has a unique reasonable PBE, which is the Separating equilibrium [(E, NE, D, U),  $p = 0$ ,  $q = 1$ ], in which the two types of Banks perfectly reveal themselves through their signals.**



# Conclusion

- We applied the classical signaling game structure to a banking sector problem.
- We show that the model has a *Separating Perfect Bayesian equilibrium* in which the two types of banks adopt different strategies, thereby allowing their type to be perfectly identified by their signals.
- **The trustworthy type chooses to offer encryption, while the untrustworthy type does not, and the customer uses the mobile app whenever he observes encryption and deletes it otherwise.**
- The signaling mechanism allows the trustworthy banks to distinguish itself from the untrustworthy one and increases efficiency of the market outcome.

# Conclusion

- Without the possibility of signaling, the customer would always delete the app, given a sufficiently high probability of encountering an untrustworthy bank, and every agent would get zero revenue.
- With signaling, the trustworthy bank pays some positive cost to signal its type by offering encryption, but the revenue it gets from the fact that the customer uses its mobile app is higher, and the customer also derives a positive payoff from using the app. This represents a Pareto improvement.
- The analysis could be extended in several directions. For instance, one could consider more types of banks, expand the strategy sets, or change the payoff functions relaxing the assumptions made on preferences. This could yield further insights about the strategic behavior of the agents.