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'GAMES WITH INCOMPLETE INFORMATION: AN APPLICATION TO THE BANKING SECTOR'

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Abstract

The aim of this thesis is to use the structure provided by Bayesian Games to analyze the interaction between a bank and its users. Bayesian games are fundamental in game theory: they provide an insightful framework for understanding strategic decision making in contexts of incomplete information. This work explores the application of “signaling games” in the implementation of mobile banking applications. It shows how banks strategically signal their credibility and reliability through the design, user interface and features of the app, thereby influencing customer perception and trust. It also explores the impact of Bayesian games on optimizing user experience, improving security protocols, and adapting services based on customers` preferences towards risk.

Keywords: Signaling Games, Bayesian Games, Asymmetric Information, Mobile Banking Apps, Intuitive criterion

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Introduction

Applied game theory has contributed significantly to our understanding of strategic interactions in scenarios where one agent has access to more accurate information than his opponents. Asymmetric information arises when one party has more information about a transaction than the other. In a situation where one party has more information while the other has less, the party with more information may seek to exploit this advantage, either due to the other party's lack of awareness or inability to recognize the incomplete information provided. As a result of this situation, there is a disruption of information symmetry between those with greater access to information and those with less access to information (Yay, Yay, and Yılmaz, 2001: 69). In this work, we deal with strategic environments (or games) with incomplete (or asymmetric) information. Meaningly, some payoff relevant parameters are unknown to some of the players. One side has information that the other side does not have. First of all, there are at least two parties (sides) since it is a strategic environment. For instance, if we consider a market environment, we can think of buyers and sellers. A seller may be aware of the quality of a product while the buyer may not. If consumers are unaware of the quality of a product they are considering purchasing, some sellers may attempt to sell a low quality product at the price of a high quality one. If consumers are aware of the high risk of purchasing a subpar product, they will be unwilling to pay a high price for goods of uncertain quality. In this scenario, firms producing high-quality goods will not want to sell their products at a price close to production costs (Dennis and Perloff, 1994). In the housing market, for instance, a home seller knows the defects of the property, unlike the buyer. Hidden characteristics and hidden actions are two types of asymmetric information. A hidden characteristic is a feature known by one party but unknown to the other, regarding an agent or thing (Tumay, 2009: 108). For example, a company manager may use a corporate jet for personal use without the knowledge of the owners (Perloff, 2015: 624). Asymmetric information arises due to the unequal information held by the parties involved in a market transaction. Buyers and sellers do not have the same information about the price, quality, or other aspects of a good or service. Normally, there is enough market information available to efficiently produce and purchase goods and services. However, in some cases, incomplete information makes it difficult to distinguish between honest and dishonest sellers or buyers (Mavlanova & Benbunan-Fich, 2010). Unlike a competitive market with perfect information where consumers can obtain high quality goods that maximize their benefits, if there are firms with information that consumers do not possess, these firms may tend to sell relatively inferior quality goods. Asymmetric information leads to opportunistic behavior, where one party takes advantage of the other where it is possible. Due to asymmetric information, such opportunistic behavior results in market failures and hinders many desired features of competitive markets (Perloff, 2015: 624).

If the seller of a used car shares all information about the car, there wouldn`t be a problem of information asymmetry (Nikolaou et al. 2013: 617). However, the seller has an incentive to exaggerate the quality of the car in order to make more money from the sale. Similarly to this, sellers tend to hide negative information about the car and only share positive information. The buyer knows this and cannot trust the seller even if the seller shares all the information. This means that the buyer will try to pay less even if the car is really good. The buyer who cannot distinguish between a good used car, a “cherry”, and a used car with bad quality, a "lemon" (terms introduced by Akerlof in 1970), will be able to offer the same price for both, not a different price. Basically, this means that the seller of a good car has no incentive to sell it (Akerlof, 1970). If the market is full of vehicles that have unpredictable quality, buyers that face an information asymmetry will watch out lemons masked as cherries. Sellers who know the true condition of their cars exploit this extra information, leading to lower prices for reliable vehicles (cherry) and driving them off the market. This scenario that exemplifies adverse selection leaves buyers with suboptimal choices wherein hidden information leads market disequilibrium. Moreover, the consequences of adverse selection are observable in other markets.

In the context of health insurance, healthy individuals may forego due to higher premiums or lower deductibles, leaving the pool vulnerable towards the high demand of unhealthy individuals (Grossman, 2017). Similarly, credit markets can be plagued by borrowers concealing unfavorable financial information, leading to bad debt and financial instability (Stiglitz & Weiss, 1981).

The problem of moral hazard refers to the tendency of one party to alter their behavior in ways that may be costly for the other party after a contract is signed (Pindyck, 2018: 658). A party not observed in their actions can influence the probability or magnitude of a payment associated with an event (Tumay, 2009: 113). Moral hazard arises when individuals engage in risk sharing under conditions where their privately taken actions affect the probability distribution of the outcome (Hart & Holmstrom, 1987). This situation is common in insurance, labor contracting, and the delegation of decision making. The problem is an asymmetry of information among individuals that results because individual actions cannot be observed and hence contracted upon (Tumay, 2009). A natural remedy to the problem is to invest resources into monitoring of actions and use this information in the contract. However, full observation of actions is almost impossible if not costly (Hart & Holmstrom, 1987). When one party is fully insured and cannot be accurately monitored by a limited-information insurance company, the insured party may take a measure that increases the likelihood of an accident or injury. For example, if my house is fully insured against theft, I might be less likely to lock the doors when I leave it and may choose not to install an alarm system. The possibility of opportunistic change in behavior due to being insured leads to moral hazard (Pindyck, 2018). In general, moral hazard arises when an unobserved agent affects the outcomes and costs among parties. As another example, if I have comprehensive health insurance, I may visit the doctor more frequently with respect to the case in which my coverage is limited. If the insurance providers can monitor the behavior of the insured, they may charge higher fees to those who make more demands (Grossman, 2017).

If we think about homeowner is exploiting the insurance, this sense of security may lead to riskier behavior (i.e., he/she neglecting home maintenance). This shift in incentives is called moral hazard and has been analyzed by Arrow

(1971). The information asymmetry after the contract is formed, that is, the insurer's inability to perfectly observe homeowner actions, incentivizes being risk lover and potentially endangers the insurer's financial stability. From this perspective, moral hazard is a situation where a contract between two parties leads one party to change their behavior in a way that harms the welfare of the other party (Tumay, 2009)

Moral hazard is a major challenge in a wide variety of situations and also applies to problems where a worker operates below his capacity when employers are unable to monitor his behavior. According to the Shapiro-Stiglitz Model, employers grapple with workers giving less effort after securing neat jobs. In a similar way, moral hazard can affect the health care system, where insured patients may demand unnecessary medical interventions (Tumay, 2009).

Signaling is a mechanism that may arise in the market as a response to the problems induced by asymmetric information. It consists in the possibility for the informed party to take actions in order to convey signals to the uninformed party. Concerning the subject of this thesis, we can think of signaling as a mechanism that serves to build trust and reduce uncertainty for the uninformed party. In a job market setting, for instance, this could involve costly investments like university degrees/certifications and/or actions that only high quality individuals would take (Spence, 1973). Signaling can mitigate adverse selection and influence market dynamics, leading to better decisions and more efficient outcomes in situations where information isn't fully available or shared. Let's turn back the car market and suppose that a car dealer wants to get rid of the adverse selection problem. The seller, guided by Spence (1973)'s concept of signaling, embarks on a strategic mission. In particular, he can show rigorous independent evaluations and provide verifiable and/or extended warranties. These signals act as warning lights and separate the real gems (cherries) from the market's cheap imitations (lemons). Analogously, educational credentials and professional certifications act as signals in labor markets, helping employers identify qualified candidates (Spence, 1973). Similarly, brands and product reputations serve as signals in consumer markets, guiding purchase decisions (Dawar and Parker, 1994).

Screening is another mechanism that can emerge in the market to overcome the asymmetric information problems. It can be described as the strategic gathering and utilization of information by one party to evaluate or discover hidden attributes or characteristics of another party, and this action is taken prior to entering into an agreement or transaction where one party has limited information about the quality or characteristics of the other party. Another saying is screening is a mechanism that arises to solve the problem of precontractual information asymmetries. The purpose of screening is to mitigate adverse selection, a situation where the party with more information selects options that benefit them because of their asymmetric knowledge. Differently from signaling, with screening it is the less informed party that implements mechanisms or tools to extract this hidden information from the other party, aiming to distinguish between different types. This process enables the less informed party to make more rational decisions or to design contracts that match their risk tolerance. Screening mechanisms are widely used in various economic contexts, particularly in insurance, labor markets, and principal-agent relationships, to manage asymmetric information and optimize outcomes for both parties to a transaction or agreement. Let`s consider Spence's (1973) labor market. Imagine a firm faces numerous resumes, each claiming extraordinary talent and experience. The company is aware of this challenge and may employ the multi-step screening process described in detail by Rothschild and Stiglitz (1976). Skills assessments, tough interviews and work simulations act as filters, separating the truly qualified candidates from the others. This strategic process ensures that the organization uncovers hidden talent and builds a skilled and productive workforce. Screening mechanisms contribute to the efficient allocation of resources across different disciplines. As another example, in financial markets, creditworthiness assessments and credit scores help lenders identify viable borrowers (Avery et al., 1996).

Asymmetric Information in the Banking Sector

Typically, information asymmetry in credit markets implies that the difference in available information between certain investors and others translates into higher returns for those who are well informed. This idea is summarized by the statement "A positive relationship exists between information asymmetry and stock returns" (p. 109) by Goel et al. (2021). Although this result is legit, in reality it is bit more complex.

To begin with, information isn't always readily accessible. According to Sciubba (2005), "acquiring valuable information is costly" (p. 354). These costs might be financial, for instance, expensive subscriptions or research charges, or timely manner like wasting valuable hours processing data. To properly transform knowledge into profits, the prospective return must be greater than the overall cost of acquisition. Moreover, not all data is created equal and misinterpreted or false signals can occur when data is noisy or incomplete. Even high quality data may not be used instantly, demanding further analysis about investment decisions. As Stigler (1961) pointed out in "The Economics of Information", the value of information does not come from its existence, but also its transferability to effective action. Moreover, the market efficiency plays a crucial role in the lifespan of any information advantage. Highly efficient markets by Fama (1970)[1] are described as rapidly incorporating new information into asset prices, quickly eroding any temporary advantage gained by informed investors. In such markets, sustained profitability depends on continually accessing fresh and actionable information against the market's greedy need for extra knowledge (Delcey, 2019). Last but not least, the human element adds extra layer of complexity. Investors often make decisions based not only on their own information, but also on the observed behavior of other investors. This phenomenon, explained by Bikhchandani, Hirshleifer, and Welch (1992) in "Information Cascades in Financial Markets," can lead to herding behavior (discussed in many papers and books that analyze behavioral finance), where

[1] Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. The Journal of Finance, 25(2), 383-417.

a simple price movement triggers a domino effect, with uninformed investors mimicking the actions of perceived "informed" players. These cascades can enhance false signals and create irrational price movements, further obscuring and/or masking the relationship between information and returns. The regulatory landscape adds a crucial dimension. Insider trading and other forms of market manipulation, as highlighted by the Securities and Exchange Commission, can distort the information landscape, unfairly benefiting uninformed investors. In addition, a strong regulatory framework is needed to ensure a fair and efficient market in which information advantage is translated into fair compensation for genuine insight.

Therefore, the relationship between information asymmetry and returns is complex, even though it is still a strong force in the financial markets. Investors must have an extensive awareness of the cost, quality, and usability of financial information in addition to knowledge of market dynamics and the regulatory environment to navigate the complex world of financial data and optimize their investment strategies.

Another type of asymmetric information in credit markets is that between credit suppliers (banks) and credit seekers (individuals and firms). Typically, among the individuals seeking credit, there are both honest and dishonest customers. Therefore, credit suppliers, due to having asymmetric information about the riskiness of credit seekers or failing to realize that the provided information is incomplete, will face an adverse selection problem (i.e., the situation where credit is loaned to the problematic customer). As discussed by Erdoğan (2008), extending credit to the problematic customers will affect the probability of repayment. As a result, banks are forced to reduce the amount of loans they provide by tightening their credit allocation due to this situation. Moreover, even if banks are able to distinguish honest customers from dishonest ones, there is no guarantee that the customer who receives the loan will use it for the purpose stated in the application. If the credit is used in a different manner after being obtained, this is explained as a moral hazard, and as a result, the probability of repayment to the bank decreases due to the loans being used in riskier projects (Erdoğan, 2008). Adverse selection occurs when a bank lacks the relevant information on a customer prior to entering into a loan arrangement. Inadequate knowledge regarding the customer's risk level and payback probability yields in poor selection (Okuyan, 2014: 700). Moreover, banks that don’t have full details about clients seeking loans will hike interest rates and/or impose more aggressive requirements on loan contracts in order to account for the presence of dishonest customers with a reduced likelihood of payback. As the interest rate and other terms of the loan change, customer behavior will also change. Raising the interest rate reduces the acceptance rate of loans by honest borrowers. High interest rates encourage customers to select riskier projects that have a lower likelihood of success but offer higher returns if successful. In a market with perfect and costless information, the bank would precisely determine all actions the customer would take (which could affect loan repayment). However, the bank cannot directly control all of the customer's actions. For instance, a customer may accept payment at a high-interest rate due to knowing that the probability of repaying his loan is less likely, thereby the customer wants the loan (Aras & Müslümov, 2004: 57).

Therefore, it will offer loan contracts with terms that incentivize borrowers to take measures in the bank's interest and attract honest customers (Stiglitz and Weiss, 1981: 394). However, those preventive measures are not sufficient over time (Erdoğan, 2008). As a result of the strict terms of these contracts, honest clients may stop requesting credit and withdraw from the market.

Cont`d on Notion of Risk

Let's consider the signaling mechanism. The sender has private information about his type, specifically whether he is the type of customers with a risky project or portfolio or the type with a trustworthy project or portfolio. Conversely, the receiver lacks direct knowledge of the specific type of the sender, but does have information about the general prevalence of types within the customer base.

Within this setting, the "Sender" can send a message to signal his type, which the "Receiver" observes. Subsequently, the "Receiver" formulates his actions based on the beliefs constructed about the probable type of the "Sender". These actions are determined by the receiver's beliefs about the likelihood of encountering either a customer with a risky project/portfolio or a customer with a trustworthy project/portfolio. This framework simulates the interaction between a financial institution, functioning as the "Receiver," and customers with distinct project or portfolio types, conveying vital information through their actions or signals. In the case of lending, banks don't always know in advance how likely it is that the borrower will fail and not pay back the money (Ślązak, 2011: 83).

Borrowers may signal their level of risk to lenders by offering particular loan conditions, such as collateral or interest rates, which help the lenders to discriminate among the different types. This approach is effective when borrowers are similar in most regards except for the likelihood of defaulting on the loan (Bester, 1985). Then, Smart (2000) came up with an interesting point. When we consider how likely the borrowers fear risks, calculations become more complex. It's possible that borrowers with different risk aversion levels might all end up getting the same kind of loan deal (Finkelstein & McGarry, 2006).

To illustrate, less risky borrowers who are also heavily risk-averse may not be too interested in putting down more collateral to prove that they are low risk. This is because their willingness to offer collateral is tied to how scared they are of risks (Cohen & Einav, 2007; Barseghyan et al., 2011). As their fear of risks goes up, their readiness to offer more collateral to prove their safety decreases. So, even though offering less collateral reduces their risk, it also means they end up paying higher interest rates.

In simpler terms, when borrowers are both cautious about risks and less likely to fail, they might not be up for offering lots of collateral just to prove they're trustworthy. This changes how the whole system of showing risk works when it comes to getting loans.

Dynamic Games with Incomplete Informatıon

The presence of private information naturally leads informed parties to either communicate or use this information towards their own objectives, and prompts uninformed parties to make efforts to learn this information and respond accordingly. Therefore, many incomplete information games are inherently dynamic in nature (Gibbons, 1992: 143).

Perfect Bayesian Equilibrium

In a signaling game, an information set can either fall on the path that leads to the equilibrium or lie off that path. The path determined by a specific equilibrium comprises all the decision points reached by following the strategies laid out by that equilibrium. An information set is considered on the equilibrium path if it's reached with a chance of occurrence when the equilibrium strategies are in play, while it's off the path if it's reached with zero probability. Some Nash equilibria result in unlikely plays for information sets off the equilibrium path. The perfect Bayesian equilibrium (PBE) refines the Nash equilibrium concept by ensuring that players cannot threaten to use strategies that are always inferior at any off-path information set. In addition to strategy profiles, PBE demands that players hold beliefs. These beliefs involve a player's understanding of which decision point within an information set will be reached and are represented as a probability distribution encompassing the decision points within that information set (Carroll & Grosu, 2011).

The theoretical framework of the concept of Perfect Bayesian Equilibrium (PBE) can be explained through two different approaches. Firstly, Perfect Bayesian Equilibrium was developed to strengthen the concept of Bayesian Nash equilibrium. Incomplete information static games often have multiple Bayesian Nash equilibria. When dynamic elements are added to these games, some of the equilibria may involve unreliable threats. The solution to this problem involves eliminating all unreasonable Bayesian Nash equilibria. In incomplete information dynamic games, the more general concept of a continuation game (where players can choose a strategy from a continuous strategy set) replaces the subgame perfect equilibrium concept. This is because the continuation game can start not only from a singleton intersection forming its own information set, but also from any information set. Therefore, if players' strategies form a perfect Bayesian equilibrium in the continuation game, this ensures that the strategies constitute a Bayesian Nash equilibrium in the original dynamic game (Bierman and Fernandez, 1998: 325; Gibbons, 1992: 174).

Secondly, Perfect Bayesian Equilibrium strengthens or advances the concept of subgame perfect Nash equilibrium. Subgame perfection assumes that at every stage of the game, players who know their position exactly take actions that are the best responses to the strategies undertaken by other players. Perfect Bayesian Equilibrium, on the other hand, assumes that at every stage of the game, players who do not know their exact position or the intersection in which they find themselves take actions that are the best response to the strategies of others at each stage of the game. In other words, in subgame perfection, player i's strategy is the best response to the strategies of the others at any point in the game where i makes a move. In Perfect Bayesian Equilibrium, player i's strategy is the best response to the strategies of the others in any information set in which i makes a move (Hargreaves and Varoufakis, 2005: 97). Due to this theoretical groundwork, Perfect Bayesian Equilibrium is also referred to as Bayesian subgame perfect Nash equilibrium, perfect Bayesian Nash equilibrium, or simply Bayesian perfection (Carmichael, 2005: 171).

One of the main conditions encompassed by Perfect Bayesian Equilibrium is that a player, by observing the actions of another player with a known preference (best responses), should rationally update their beliefs according to Bayes' theorem. Bayes' theorem, in essence, seeks to determine the probability of one event (e.g., event L) occurring given that another event (e.g., event A) is known to have occurred (Rouder & Morey, 2018). This updated probability, under the condition of event L happening, is referred to as the conditional probability of event A, denoted as Prob (A|L). The probabilities obtained through Bayesian updating are termed as posterior probabilities or beliefs (Romp, 2005: 53).

Perfect Bayesian Equilibrium conditions

- Optimization (Rational Optimization in Game Theory):

In line with Aumann et al. (2008), a PBE requires players to optimize their strategies by choosing actions that maximize their expected payoffs given their beliefs and the information available at each decision point.

- Belief Consistency (Rational Expectations):

Following the rational expectations framework discussed by Aumann and Dreze (2008), a PBE assumes that players' beliefs are consistent with available information and are updated according to the principles of Bayes' theorem. Players take in new information in a logical and rational way.

- Sequential Rationality (Optimal Decision-Making):

As emphasized by Aumann et al. (2008), PBE demands that players exhibit sequential rationality. This means that at each stage of the game, players' chosen actions should be optimal responses to their beliefs, ensuring strategic foresight and consistent decision-making throughout the game. In order to apply sequential rationality in a setting where players do not observe the types of other players, we need to extend the notion of sequential rationality applied to games with complete information to games with incomplete information. In particular, this means that each player must maximize his expected utility given his own beliefs about the other player types. More precisely, given any set of information where a player is asked to make a move, he must choose the strategy that maximizes his expected utility if he assumes that all other players will do the same, and if he has his own beliefs about the other players' types.

- Consistency of Beliefs (Consistent Rationality):

Extending the discussion in Aumann et al. (2008), the PBE conditions include consistency of beliefs across different information sets. Players' expectations and beliefs, given their strategies, should not only be individually rational but also collectively consistent, ensuring a coherent and believable narrative of the unfolding game.

These conditions collectively establish a robust framework for a Perfect Bayesian Equilibrium, aligning with the principles of optimization, belief consistency, sequential rationality, and the overall coherence of players' strategic interactions as discussed in Aumann et al. (2008).

Condition 1: Belief Formation in Information Sets:

In every information set, the player making a move must hold a belief about which point in the information set of the game has been reached. The belief formed for a non-singleton information set is a probability distribution defined for the points in that information set. For a singleton information set, the player's belief is a single decision point, making the probability of being at that point one. This ensures that the player precisely knows their location in the game tree (Gibbons, 1992: 177).

Condition 2: Sequentially Rational Strategies:

Considering their beliefs, players' strategies must be sequentially rational. In other words, at each information set, the player's action and the subsequent strategy should be optimal given the player's belief about this information set and the anticipated future strategies of other players. The "following strategy" here refers to a comprehensive action plan covering every possible contingency that might arise after reaching the given information set (Gibbons, 1992: 177).

Condition 3: Beliefs Determined by Bayes Rule and Equilibrium Strategies:

In information sets along the equilibrium path, beliefs are determined by the Bayes rule and the equilibrium strategies of players (Gibbons, 1992: 178).

Condition 4: Beliefs Determined by Bayes Rule and Equilibrium Strategies Upon Determining the Equilibrium Path:

When identifying the equilibrium path, beliefs must be determined by the Bayes rule and, if possible, the equilibrium strategies of players (Gibbons, 1992: 180).

These conditions collectively establish a comprehensive framework for a Perfect Bayesian Equilibrium, emphasizing belief formation, rationality in strategies, and the consistent determination of beliefs in line with equilibrium strategies and Bayes' rule, as discussed by Gibbons (1992).

Definition: A ***perfect Bayesian equilibrium*** consists of a strategy profile resulting from optimization for players and, particularly, a belief system consistent with probability rules, notably Bayes' theorem. The strategies and beliefs that constitute a perfect Bayesian equilibrium must satisfy Conditions 1, 2, 3, and 4 (Gibbons, 1992: 178; Bierman and Fernandez, 1998: 328).

In some economic applications, such as signaling games and cheap talk games, the first three conditions are sufficient to define a Perfect Bayesian Equilibrium. However, in richer economic applications, additional conditions are needed to eliminate unreasonable equilibria. Different scholars have used different definitions for a Perfect Bayesian Equilibrium. All these definitions include the first three conditions; many include the fourth condition, and some impose additional conditions (Gibbons, 1992: 179).

In game theory, a Perfect Bayesian Equilibrium (PBE) is a crucial concept that involves a strategy profile $s = (s_1, s_2, ..., s_N)$ for N players and a belief system over all information set nodes.

First of all, each player's strategies must specify the best possible actions, considering the strategies of other players and their own beliefs. This condition mirrors the best response condition in the standard definition of the Nash equilibrium, but it is applied to settings with incomplete information. In these settings, players must find the optimal actions based on their beliefs about the types of their opponents. Lately, the players' beliefs must be consistent with Bayes' rule whenever possible. This condition emphasizes that beliefs must adhere to Bayes' rule. This rule must be followed not only when players form beliefs along the equilibrium path but also off-the-equilibrium path.

In summary, a PBE is characterized by optimal strategies based on players' beliefs and the consistency of these beliefs with Bayes' rule. These elements highlight the strategic decision-making process in game theory, where players must consider both their own and others' strategies and beliefs.

Bierman and Fernandez (1998: 329) draw attention to a crucial point regarding the process of obtaining a Perfect Bayesian Equilibrium. In many economic analyses, an "algorithm" is employed to find equilibrium. For instance, in a perfectly competitive market, the algorithm used to find the equilibrium price involves plotting supply and demand curves and determining the intersection point. Alternatively, an inference algorithm is often used to find the subgame perfect Nash equilibrium of a perfectly informed game. However, there isn't a well-structured algorithm for finding a Perfect Bayesian Equilibrium. The general procedure for finding a Perfect Bayesian Equilibrium consists of three steps: (1) proposing a set of possible strategies and beliefs, (2) verifying if the proposed strategies meet the conditions of the definition, and (3) checking if the beliefs meet the conditions of the definition. Implementing this procedure is often time-consuming due to the potentially large number of possible strategies and beliefs.

As games become more complex, equilibrium concepts also become richer, with stronger equilibrium concepts replacing weaker ones in more sophisticated games.

In the context of incomplete information dynamic games, Kreps and Wilson (1982) introduced the concept of Sequential Equilibrium (SE) as a new and more advanced expansion of the Perfect Bayesian Equilibrium. This equilibrium concept is slightly stronger than the Perfect Bayesian Equilibrium concerning the consistency of the solution (Romp, 2005: 46). However, in many economic applications, it has been demonstrated that Sequential Equilibrium and Perfect Bayesian Equilibrium are very similar and, in fact, entirely equivalent. In the framework of incomplete information and at most two-stage games or games where each player has at most two types, there is complete equivalence between the two equilibrium concepts (Fudenberg and Tirole, 1991). While Sequential Equilibrium is commonly defined as an equilibrium concept in extensive form games, the literature has reached a consensus that it is a broadly applicable equilibrium concept. Nevertheless, in the context of incomplete information dynamic games, trembling hand perfect equilibrium (Selten, 1975) and proper equilibrium (Myerson, 1978) have been developed as principal extensions, often treated as equilibrium concepts defined in normal form, but they are also found applicable to extensive-form games (i.e., Fudenberg and Tirole, 1991). All these equilibria, developed in either normal or extensive form, represent stronger forms of the Perfect Bayesian Equilibrium in incomplete information dynamic games.

Signaling Games

Perfect Bayesian Equilibrium is the standard solution concept for signaling games. These games involve players who possess asymmetric information about the unknown parameters of the game. Typically, in such games, one player has complete information about an unknown parameter, while others can only estimate it with a certain probability. For instance, only the job applicant may know his own skills, while the employer may not be aware of this. Moreover, a salesman may be aware of the quality of the vehicles/goods they sell, but buyers may not. In such situations, information is one sided (Yılmaz, 2016: 238).

Spence (1973) was a pioneer in investigating and applying the “signaling games” into economics through his signaling model of the labor market. Signaling models have found broader applications in the literature. Spence, Akerlof, and Stiglitz were awarded the Nobel Prize in Economic Sciences in 2001 for their work on asymmetric information in markets.[2]

Generally speaking, the simplest signaling game involves a two-stage dynamic game with two players (a sender and a receiver) where one party is less informed than the other. In signaling games, one player possesses private information that can affect the payoffs of both players, with this information typically referred to as the player's type (Dutta, 1999: 387). Signals are actions undertaken by players to send information, representing the effort a player makes to communicate among them (Carmichael, 2005: 164). Typically, the timing of a signaling game is as follows (Gibbons, 1992: 183):

1. Nature, according to a probability distribution $p(t_i)$, selects a type ti for the sender from a set of possible types $T = \{t_1, ..., t_L\}$ (where $p(t_i) > 0$ for each i and

[2]Further details can be reached via official website: https://www.nobelprize.org/prizes/economic-sciences/2001/press-release/

$p(t_1) + ... + p(t_L) = 1$). The probability distribution is assumed to be general information, and the set of types are assumed to be finite (Montet and Serra, 2003: 176).

2. The sender observes t_i and then chooses a message m_j from a set of possible messages $M = \{m_1, ..., m_j\}$. In other words, the sender's strategy is a function defined from the type set to the message set (Yılmaz, 2016: 239). In the simplest setting, the set of messages is assumed to be finite.

3. The receiver observes m_j (but cannot observe t_i) and then chooses an action a_k from a set of possible actions $A = \{a_1, ..., a_k\}$. That is, the receiver's strategy is a function defined from the message set to the action set (Yılmaz, 2016: 240). In the simplest setting, the set of actions is assumed to be finite.

4. The receiver and the sender receive payoffs: $U_S(t_i, m_j, a_k)$ and $U_R(t_i, m_j, a_k)$.

In a signaling game, the action undertaken by the player with private information and who makes the initial move (the sender) is interpreted as a "signal" or "message". After the first player's action has realized, the second player, who is unaware of the first player's type but observes this action, takes a "move" or a "response." The conflict in signaling games arises from the information asymmetry between these two players, where the sender controls the information, and the receiver controls the action. In signaling games, the set of possible messages is contingent on the nature's choice of sender type, and the potential actions depend on the sender's chosen signal (Montet and Serra, 2003: 176-177). In contrast, as we previously discussed, games where the more informed player can only act after the less informed player are referred to as screening games. In signaling games, the more informed player moves first. (Bierman and Fernandez, 1998: 336).

The components of signaling games find explication through illustrative examples in various economic applications. One is found in the Labor Market Signaling Model proposed by Spence (1973). In this model, the worker assumes the role of the sender, while the employer in the market is the receiver. The worker's type corresponds to his ability related to productivity, the message encapsulates the worker's choice of education, and the action denotes the wage paid by the employer.

Another application is evident in the Corporate Investment and Capital Market Model formulated by Myers and Majluf (1984). In this context, the sender represents a company seeking capital for a new project and the receiver is a potential investor. The player's type pertains to the profitability of the company's existing assets, the message signifies the firm's offer of shares in return for financing, and the action involves the investor's decision to provide financial support or not. These examples highlight the versatility of signaling games in capturing strategic interactions in economic contexts.

The applicability of signaling games extends to diverse economic disciplines, as illustrated by the Monetary Policy Model introduced by Vickers (1986). Here, the Federal Reserve (FED) acts as the sender, while the employer market acts as the receiver. For instance, the type of the FED represents its willingness to accept inflation in the initial phase in order to increase employment.

Additionally, the utilization of signaling games in the context of digital commerce[3] platforms unpacks a subtle interplay of strategic interactions between discerning buyers and sellers. In navigating this virtual terrain, sellers deal with the major task as communicating effectively the quality of their goods or service, relying on strategic actions like adding detailed product descriptions, adding detailed pictures (high resolution etc.) and incentivized positive customer reviews as essential signaling mechanisms[4]. Building trust in online transactions assumed vital importance, prompting sellers in order to strategically signal their reliability via the accumulated positive feedback and/or displaying verified mark. Furthermore, sellers competently impose discounts and promotions as signals to transfer appropriate information about the desirability of their goods and services in in order to influence on buyer decision-

[3] Smith, J., & Johnson, A. (2021). "Strategic Signaling in E-commerce: An Empirical Analysis." Journal of Online Business Research, 25(3), 112-130

[4] Wang, L., & Chen, H. (2020). "Signaling Trustworthiness in Online Marketplaces: A Game-Theoretic Approach." International Journal of Electronic Commerce, 15(4), 245-263.

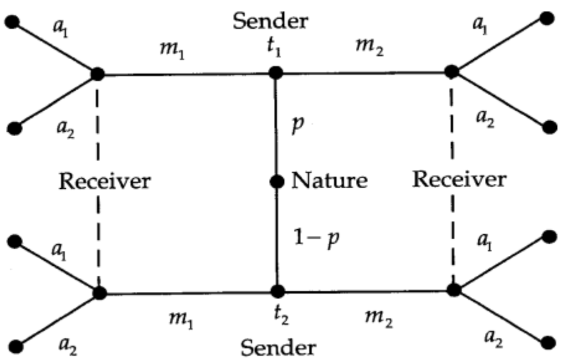
making process. Also, faster (safe and sounds indeed) delivery also constitutes a strategically crucial signal, indeed impacting the buyers decision in the e-commerce area. Moreover, returning the product in the worst case scenario, can be seen as the implementation of accommodating refund policies[5].

Depiction of Signaling Games

Consider the case in which T = $\{t_1,t_2\}$, M = $\{m_1,m_2\}$, A = $\{a_1,a_2\}$ and prob$\{t_1\}$ = p.

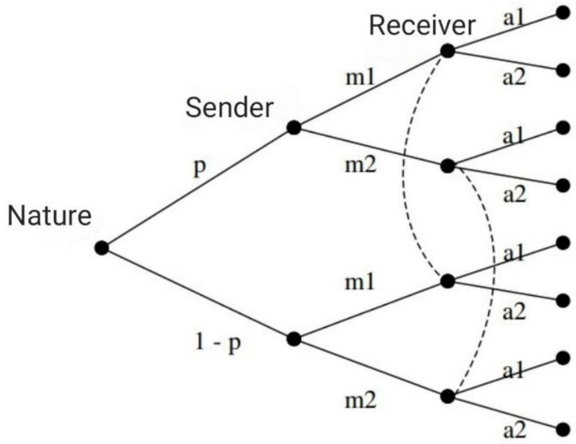
Signaling games are usually represented by means of the following game trees.

Gibbons (1992):



[5] Garcia, M., & Kim, S. (2019). "Navigating Trust in Virtual Transactions: An Investigation into Signaling Strategies." Journal of Digital Marketing Strategies, 12(2), 87-104.

Dutta(1999):



"In a signaling game, a pure strategy for the sender is a function that specifies which message will be chosen depending on each possible nature's choice, denoted as $m(t_i)$. For the receiver, a pure strategy is a function that specifies which action will be chosen based on each message the sender could send, denoted as $a(m_j)$. In the commonly presented form of this game, each player has four pure strategies (Gibbons, 1992: 185-186).

Sender's Strategy 1: If nature selects t_1, play m_1; if nature selects t_2, play m_1.

Sender's Strategy 2: If nature selects t_1, play m_1; if nature selects t_2, play m_2.

Sender's Strategy 3: If nature selects t_1, play m_2; if nature selects t_2, play m_1.

Sender's Strategy 4: If nature selects t_1, play m_2; if nature selects t_2, play m_2.

Receiver's Strategy 1: If sender chooses m_1, play a_1; if sender chooses m_2, play a_1.

Receiver's Strategy 2: If sender chooses m_1, play a_1; if sender chooses m_2, play a_2.

Receiver's Strategy 3: If sender chooses m_1, play a_2; if sender chooses m_2, play a_1.

Receiver's Strategy 4: If sender chooses m_1, play a_2; if sender chooses m_2, play a_2.

In signaling games, the messages of the sender (signals) may or may not reveal this player's type. For the sender's type to be revealed in an equilibrium, observing the sender's equilibrium message (m) must be equivalent to observing the sender's type (t). This condition implies a one-to-one relationship in the sender's strategy: for type t and t', with messages m(t) and m(t'), if $t \neq t'$, then $m(t) \neq m(t')$. In other words, the sender chooses a different action for each type. If the sender's message (m) is one-to-one and the sender chooses a different action for each type, the equilibrium is termed separating or fully revealing.

If (m) is a constant function, in other words, if each type leads to the same action, the equilibrium is termed pooling. On the other hand, if the sender's strategy is not one-to-one, meaning in a model with more than two types, some types lead to different actions while others lead to the same actions, this equilibrium is termed semi-pooling or hybrid. This equilibrium could also be called partially revealing, partially pooling, or partially separating. These strategies resemble mixed strategies. For instance, m_1 is played in type t_1, but there is a probability distribution between m_1 and m_2 for type t_2 (Gibbons, 1992: 186).

When the equilibrium is not separating, the receiver cannot make a fully or accurately informed inference from the signal regarding the sender's type (Montet & Serra, 2003: 176). Within this framework, in the commonly presented signaling game model, the sender's 1st and 4th strategies, sending the same message for each type, are pooling strategies, while the sender's 2nd and 3rd strategies, sending a different message for each type, are separating strategies."

The solution concept typically considered in a signaling game is Perfect Bayesian Equilibrium and it is defined by the conditions it must satisfy. In this thesis, in order to simplify the discussion, the focus will be on equilibria in pure strategies.

The first condition developed for Perfect Bayesian Equilibrium is irrelevant for the sender. This is because when selecting their message, the sender possesses complete knowledge of the game's entire history, resulting in the message selection occurring within a singleton information set (there exists such an information set for each possible nature's choice). Conversely, the receiver, after observing the sender's message, chooses an action without knowing the sender's type. Therefore, the receiver's choice emerges within an information set that contains multiple decision nodes. Such an information set exists for each message the sender could choose, and each of these information sets has one node for each type nature could choose.

Signal Condition 1 for the receiver is expressed as follows:

Signal Condition 1: After observing any message m_j from the set of messages M, the receiver must possess a belief about which types could have led to the transmission of m_j. If this belief is represented by a probability distribution $p(t_i \mid m_j)$ over the set of types, for each t_i in the set of types,

$$p(t_i \mid m_j) \geq 0, \text{ and } \sum_{t_i \in T} p(t_i \mid m_j) = 1.$$

Based on the sender's message and the receiver's belief, the receiver's optimal action can be determined. In this context, Signal Condition 2 for the receiver is expressed as follows:

Signal Condition 2 (Receiver): Within the belief framework of $p(t_i \mid m_j)$ concerning which types could have led to the message m_j , for each m_j in the set of messages M, the receiver's action $a^*(m_j)$ must maximize the receiver's expected payoff. In other words, $a^*(m_j)$ must solve the following maximization problem:

$$\max_{a_k \in \mathbf{A}} \sum_{t_i \in \mathbf{T}} p(t_i \mid m_j) \; U_R(t_i, m_j, a_k)$$

Signal Condition 2 also applies to the sender, but the sender possesses complete information (and therefore an inconsequential belief) and acts only at the beginning of the game. Thus, Condition 2 for the sender demands optimality of the sender's strategy within the framework of the receiver's strategy and is expressed as follows:

Signal Condition 2 (Sender): If the receiver's strategy is known to be $a^*(m_j)$, for each t_i in the set of types `T`, the sender's message $m^*(t_i)$ must maximize the sender's payoff. In other words, $m^*(t_i)$ must solve the following maximization problem:

$$\max_{m_j \in \mathbf{M}} U_S (t_i, m_j, a^*(m_j))$$

If the sender's strategy is $m^*(t_i)$, let T_j be the set of types that send the message m_j. In other words, if $m^*(t_i) = m_j$, t_i is an element of T_j. If T_j is a set that contains elements, the information set corresponding to m_j is on the equilibrium path. If T_j is an empty set, m_j will not be sent by any type, and thus, the information set corresponding to this message will be off the equilibrium path. For messages on the equilibrium path, Signal Condition 3 is expressed as follows:

Signal Condition 3: For each m_j in the set of messages M, if there exists a t_i in the set of types `$T_j$` such that $m^*(t_i) = m_j$, the receiver's belief in the information set corresponding to mj must be derived from the Bayes' rule and the sender's strategy.

$$p(t_i \mid m_j) = \frac{p(t_i)}{\sum p(t_i)}$$

$t_i \in T_i$

In a signaling game, a pure strategy perfect Bayesian equilibrium consists of a pair of strategies emerging as $m^*(t_i)$ and $a^*(m_j)$ that satisfy all signal conditions, along with a belief represented by $p(t_i \mid m_j)$. When the sender's strategy is pooling or separating, the perfect Bayesian equilibrium is termed pooling or separating, respectively (Gibbons, 1992: 185-188).

Dutta (1999: 387) has defined the perfect Bayesian equilibrium in a signaling game with two types in a different manner. Let Θ and μ denote the 2 types, and let p represent the probability as the prior probability regarding type Θ. A knowledgeable player (Player 2) sends a signal, denoted as s, to the other player (Player 1). Upon receiving this signal, Player 1 assumes an action denoted as t. In this scenario, Player 2's type determines each player's payoff in conjunction with Player 2's signal and Player 1's action. Consequently, the payoffs for Player 1 and Player 2 will be as follows:

If Player 2 is a player with type Θ, Player 2's payoff is $\pi_2(s, t, \Theta)$, and Player 1's payoff is $\pi_1(s, t, \Theta)$. If Player 2 is a player with type μ, Player 2's payoff is $\pi_2(s, t, \mu)$, and Player 1's payoff is $\pi_1(s, t, \mu)$.

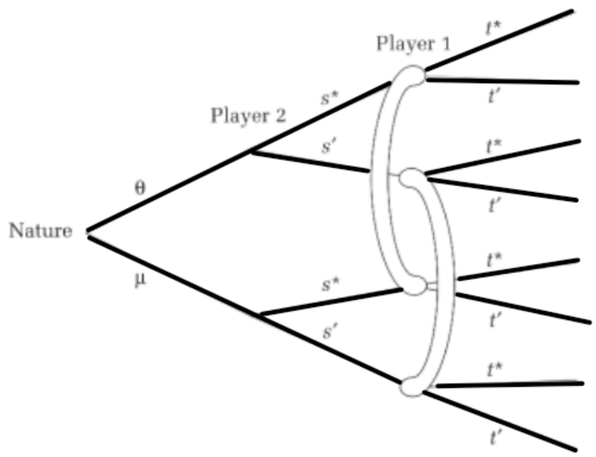


Figure: Dutta (1999 : 388)

In the Extensive form of this signaling game, Player 2's strategy depending on type consists of a signal pair (e.g., s* for type Θ and s′ for type μ). Player 1's strategy is also an action choice dependent on the signal (e.g., denoted as t(s) for the action selected after receiving a signal s). Since this action is chosen after observing Player 2's signal, at this point, Player 1 holds an updated estimate of a prior probability distribution, which is the revised belief, represented as p(s). In this game, a perfect Bayesian equilibrium (PBE) is expressed as follows:

Best response of each type of player 2:

π_2 (s*, t(s*), Θ) ≥ π_2 (s, t(s), Θ), ∀s

π_2 (s', t(s'), μ) ≥ π_2 (s, t(s), μ), ∀s

where π_2 (s, t(s), Θ) represents the payoff function encompassing π_2 (s*, t(s*), Θ) and π_2 (s', t(s'), Θ).

Best response of player 1 $\forall s$

$$p(s)\, \pi_1 (s, t(s), \Theta) + (1 - p(s))\, \pi_1 (s, t(s), \mu) \geq$$

$$p(s)\, \pi_1 (s, t, \Theta) + (1 - p(s))\, \pi_1 (s, t, \mu),\ \forall t$$

Correct estimate revision, in either of two scenarios.

Scenario 1: $s^* \neq s'$

$$p(s) = \begin{cases} 1 \text{ if } s = s* \\ 0 \text{ if } s = s' \\ \text{Any ratio otherwise} \end{cases}$$

Scenario 2: $s^* = s'$

$$p(s) = \begin{cases} p & \text{if } s = s* \text{ or } s = s' \\ \text{Any ratio} & \text{otherwise} \end{cases}$$

Briefly, when Player 1 anticipates distinct signals (s* being different from s′), he is certain about encountering Player 2's type 1 upon receiving the s* signal. In essence, it establishes p(s) = 1. Conversely, when Player 1 gets the s′ signal, he is sure to interact with Player 2's second type, that is, p(s) = 0. If Player 1 observes identical signals (s* = s′), inferring the types from receiving that signal becomes elusive. Put differently, p(s) = p. Upon receiving an unforeseen signal (if s doesn't match s* or s′), Player 1 might infer an error made by Player 2 (intending to transmit either the s* or s′ signals) or a change in his decision. Further elucidation in the latter section (scenario 2) may encompass diverse explanations. However, this segment emphasizes the acceptance of any sound reasoning and the validity of any updated predictions. A perfect Bayesian equilibrium, distinguishing types by different signal transmissions, stands as a separating equilibrium, while an equilibrium where types convey indistinguishable signals constitutes a pooling equilibrium. Based on accurate predictions for both types, it implies the first player's need to select one of the t(s) actions to maximize their expected payoffs. Different updated estimations (p(s)'s) lead to entirely distinct optimal responses. It provides the standard optimal response condition for the informed player (Player 2) in each type (Dutta, 1999: 387-389), creating varying optimal reactions depending on diverse revised estimations.

We already have shown the depictions of Signaling Games with two different game trees. We will focus on Gibbon's depiction[6] and we will implement his illustration to our model.

An example

We first show how a PBE is derived in the next example. The Sender can be of two types, each one having two possible signals, L and R; after observing a signal, the Receiver will choose between u and d.

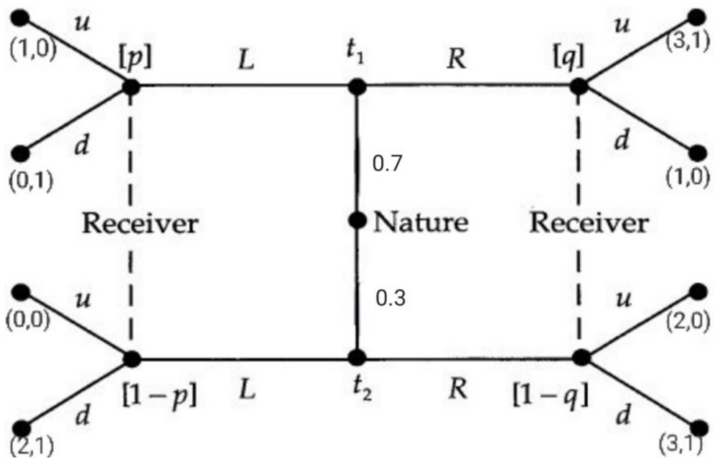


Figure: Example[7] with different payoffs and Nature draw

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[6] Gibbons (1992: 185)
[7] Gibbons (1992: 189)

A PBE in this game is given by a vector [S_1, S_2, S_R^L, S_R^R, p, q], where:

- S_1 is the strategy of sender's type t_1,
- S_2 is the strategy of sender's type t_2,
- S_R^L is the receiver's strategy after having observed signal L,
- S_R^R is the receiver's strategy after having observed signal R.

Are there any Separating and/or Pooling Equilibria?

Separating Equilibria can be of 2 kinds:

1) **S_1 = R** and **S_2 = L**: type t_1 plays R, type t_2 plays L;

2) **S_1 = L** and **S_2 = R**: type t_1 plays L, type t_2 plays R.

Pooling Equilibria can be of 2 kinds:

3) **S_1 = S_2 = L**: both types t_1 and t_2 play L;

4) **S_1 = S_2 = R**: both types t_1 and t_2 play R.

We will start looking for Separating Equilibria of the first kind.

1. Separating Equilibria where t_1 plays R, t_2 plays L: (R, L)

According to Bayes Rule, q must be equal to 1 while p must be equal to 0, since being on the top-left and bottom-right decision nodes cannot occur. Given this, the Receiver's optimal strategy is playing ***u*** when t_1 played R and playing ***d*** when t_2 played L. Is the Sender`s strategy (R, L) a best response to this behavior by the Receiver? If t_1 chooses to play L, he would get 0 instead of 3. So, playing R is a best response for t_1. If t_2 chooses playing R he would still get 2, so he does not have incentive to deviate. So, also t_2 is best responding by playing L. We can conclude that **[(R, L, d, u), p = 0, q = 1]** is a Separating Equilibrium.

2. Seperating Equilibria where t_1 plays L, t_2 plays R: (L, R)

According to Bayes Rule, q must be equal to 0 and p must be equal to 1. Given this, the Receiver's optimal strategy is playing ***d*** both when t_1 played L and when t_2 played R. Is the Receiver`s strategy (L, R) a best response to this? If t_1 chooses to play R, he would get 1 as a payoff instead of 0. Therefore, t_1 is not best responding. Note that L is dominated by R for t_1 since playing R gives him always a higher payoff. Then, there is need to investigate for t_2 and we can conclude that there are no equilibria of this kind.

3. Pooling Equilibria where both types choose playing L: (L, L)

First, q is a free parameter now but p = 0.7 has to be transferred from the Nature's move. Receiver's optimal choice, in the information set following signal L, is d. If he chooses u, he will always get 0, while if he chooses to play d he will always get 1, independently from Nature's draw. Like we discussed above p = 0.7 from Nature. By choosing L, t_1 gets 0. However, If he were to play R, he would get 1 or 3 depending on Receiver's strategy. In any case, the payoff of t_1 if he plays R is always greater than 0. Therefore, t_1 is not best responding and there is no such pooling equilibrium.

4. Pooling Equilibria where both types choose playing R: (R, R)

First of all, q is 0.70 and p is a free parameter for this case. By playing u after signal R, the Receiver's expected payoff would be:

$\pi_R(u \mid R) = 1 \times (0.7) + 0 \times (0.3) = 0.7.$

By playing d after signal R, the Receiver's expected payoff would be:

$\pi_R(d \mid R) = 0 \times (0.7) + 1 \times (0.3) = 0.3.$

Obviously, playing u for Receiver is in expected terms better than playing d. So, he will play u in such information set. On the other hand, in his left information set, regardless of the value of p, the Receiver gets always 1 by playing d, while playing u he always gets 0. So, d is his optimal choice after L. Is (R, R) a best response for the Sender? By playing R, t_1 gets 3. If he were to play L, he would end up with 0. Definitely, t_1 is better off by playing R and we can conclude that t_1 is best responding. By playing R, t_2 gets 2. If he chooses playing L, he would end up with the payoff 2. Since he is indifferent between playing R or L, we can conclude that t_2 is also best responding. Moreover, there are no constraint restrictions on p, which is a completely free parameter. It follows that all of the four conditions of PBE are satisfied. We can conclude that **[(R, R, d, u), p, q = 0.7] where p ∈ [0,1]** are Pooling PBE.

As observed in this example, finding a perfect Bayesian equilibrium in signaling games initially involves proposing a strategy combination. Subsequently, the beliefs induced by these strategies are derived, and then one must verify whether the proposed strategies are optimal given the players' beliefs. This approach is applicable to determine a PBE in all incomplete information games (Carmichael, 2005: 164). To eliminate unreasonable perfect Bayesian equilibria arising in signaling games (other than Sequential Equilibrium and Trembling Hand Perfect Equilibrium), some concepts such as the Intuitive Criterion (Cho and Kreps, 1987) and the D1 Criterion (Banks and Sobel, 1987) have been developed.

Example of Signaling Games in e-commerce or online marketplaces[8]

Imagine a company launching a new product, the intrinsic quality of which is not known by consumers. The product can be either good (G) or bad (B). Assuming a prior belief of 60% for the good product, the company faces the challenge of convincing a representative consumer to buy it. To move in signaling setting, the signal is given by the company's advertising intensity, and the consumer's purchase decision (to buy or not to buy) depends on his evaluation of the good. There are 2 possible advertising strategies: an Advertisement (A) campaign, which is costly (the cost is given by $c > 0$), and a costless No Advertisement (NA) strategy. The catch is that the consumer only observes the chosen intensity of advertisement, not the product's true quality. This asymmetry of information fuels the strategic gameplay. For the consumer, the stakes are assumed as follows:

- buying a good product yields a payoff equal to +1,
- buying a bad product induces a loss equal to -1,
- not buying the product guarantees a payoff equal to 0.

However, the product's quality reveals itself upon purchase, impacting the company's future revenue. Selling a good product to the consumer translates to a higher revenue of R for the company, symbolizing future sales potential. But if the product is bad and the consumer discovers it, the company salvages a relatively little revenue of r. The company must strategically signal quality through advertising to affect the consumer's decision, possibly increasing its own revenue stream. It is assumed that $0 < r < c < R$.

The elements of the game can be summarized as follows.

[8] Example is modified version of the MIT OpenCourseWare resource page 347-349 https://ocw.mit.edu/courses/14-12-economic-applications-of-game-theory-fall-2012/resources/mit14_12f12_chapter16/

Players:

1. **Company:** has a product to sell and chooses the advertisement level.
2. **Representative Consumer:** decides whether to buy the product based on the observed advertisement level.

Information:

- The company knows the quality of the product (either Good or Bad) but not the consumer's knowledge of the product's quality.
- The consumer observes the advertisement level but doesn't know the quality of the product.

Actions:

1. **Company's Actions:** Advertise (A) or Not (NA).
2. **Consumer's Actions:** Buy the product (B) or Do not Buy it (DB).

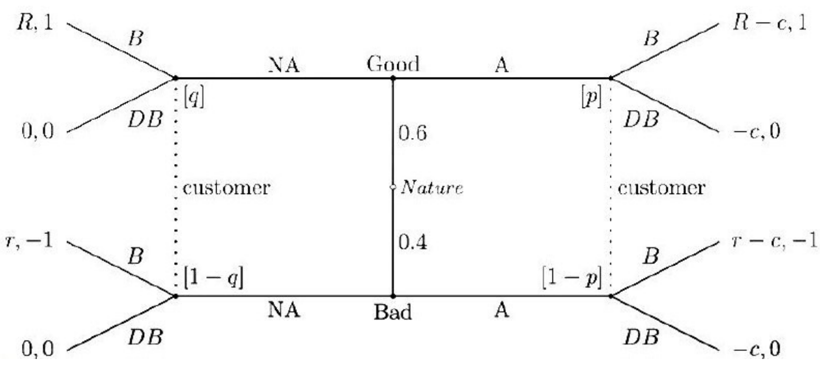
Payoffs:

- If the consumer purchases a Good product his payoff is +1, if he buys a Bad product his payoff is -1, if he does not buy the product his payoff is 0.
- If the consumer buys a Good product, the company gets a high revenue R; if the consumer buys a Bad product, the company gets a small revenue r. If the consumer doesn't buy the product, the revenue is 0. Advertisement is costly for the company.

Strategy:

- The company decides its advertising level based on the quality of the product, aiming to signal its quality to the consumer indirectly through the advertisement intensity.
- The consumer will decide whether to buy the product according to the signal received.

The game can be illustrated by the following tree:



The company has two actions, A and NA. A strategy for the company maps each type to the action set.

The customer has also two actions, B and DB. Hence, the possible strategies are four, {(B, B), (DB, DB), (B, DB), (DB, B)}, since a strategy maps each information set to the corresponding set of actions, and the customer has two information sets, the one on the left hand side of the game tree after the choice NA from the company and the one on the right hand side of the game tree after the choice A of the company. Therefore, a strategy for the customer is a tuple specifying what to do at each information set.

Let's analyze the PBE of the game, starting from the Separating ones.

1. **(A, NA): the Company with good product plays Advertise (A) and the Company with bad product plays No Advertise (NA).**

First, we know that the consumer must assign probability 1 to the Good product after observing Advertisement, and probability 1 to the Bad product after observing No Advertisement. Thus, his beliefs will be given by $p = 1$ and $q = 0$. After signal A, if he buys the product, he will get 1, otherwise, he gets 0. Hence, he should Buy the product when he observes Advertise (A). Whenever the customer observes the Company playing No Advertisement (NA), his optimal strategy is Do not Buy (DB), since this guarantees 0 while buying the product he gets -1. Thus, the optimal strategy for the customer is (DB, B).

Now, we should check whether the firm is best responding by playing (A, NA) against (DB, B). If **the Company with the product being of good quality** plays A, the company knows the customer will play B. The payoff will be $R - c$ which is greater than 0, since $R > c$ (as denoted in the description of the game). It follows that the Company with good product will Advertise. So, A is a Best Response for the Company with good product towards (DB, B). We are done with the good firm who is, in fact, best responding.

If **the Company with bad product** plays NA, the company knows the customer will play DB and the payoff will be 0. What if the Company with bad product decides to Advertise instead? Since the customer can only observe Advertisement (A) or No Advertisement (NA) as signals, the customer believes that the Company with the product being of bad quality will not advertise and the Company with the product being of good quality will advertise. Suppose the Company with the product being of bad quality were to deviate and advertises. Since the customer is playing B after Advertisement, the Company with bad product can mimic the good company and Advertise. So, we must check the payoff that the Company with bad product gets if it plays A, whether this deviation is profitable or not. Since this payoff is r – c, which is < 0**,** NA is a best response against (DB, B) for the Company with bad product. We conclude that **[(A, NA, DB, B), p = 1, q = 0] is a Separating PBE.**

2. **(NA, A): the Company with good product plays No Advertise (NA) and the Company with bad product plays Advertise (A)**

This is the second possible strategy in a Separating PBE. Given (NA, A), we have that the customer's beliefs will be given by p = 0 and q = 1 according to Bayesian updating, so that the customer knows in which decision nodes he is. If the customer observes an Advertisement, he is going to believe that the Advertisement is coming from the Company with bad product, and he will play DB**.** If the customer observes that there is No Advertisement, he is going to believe that the No Advertisement is coming from the Company with good product and so he will play B**.** Given those beliefs (p = 0 and q = 1), the optimal strategy for the customer is (B, DB).

If the **Company with good product** does not deviate and plays NA, its payoff will be R. However, if it deviates its payoff would be -c since the customer has a prior belief that the Advertisement comes from the Company with bad product. So, NA is best response for the Company with good product against (B, DB).

If the **Company with bad product** does not deviate and plays A, its payoff will be -c. However, if it deviates its payoff would be r since the customer has a prior belief that the No Advertisement comes from the Company with good product. In this scenario, deviation is more profitable (r > -c). So, playing A is not a best response against (B, DB) for the Company with bad product. Meaningly, we can conclude that (NA, A) is not part of a Separating PBE since it fails to satisfy Bayesian Requirement for the Company with bad product.

Let's analyze now the Pooling Equilibria of the game.

3. **(A, A): Both the Company with good product and the Company with bad product play Advertise (A)**

Whenever the customer faces an Advertisement, he cannot distinguish between the Company with a good and a bad product. We must impose the Bayesian updating in this case. So, p will be:

p = $\frac{(0.6)x1}{(0.6)x1+(0.4)x1}$ = 0.6.

We do not have restrictions on q because it represents the probability distribution over the decision nodes in the information set which is off the equilibrium path. According to the definition of PBE, q should be updated whenever it is possible but here it is not possible (Bayes rule in this case does not give a well-defined value). So, p must be equal to 0.6 and 1-p is equal 0.4. Given these beliefs, the customer's expected payoffs are:

π_c(B | A) = (0.6)x1 + (0.4)x(-1) = 0.2,

π_c(DB | A) = (0.6)x0 + (0.4)x(0) = 0.

In expected terms, by playing Buy, the customer will be better off when he sees an Advertisement though he cannot distinguish whether the Company has bad product or good product, since π_c(B | A) > Eπ_c(DB | A).

Given that the customer will play B after observing A, is playing A a best response for both Company types?

If the **Company with good product** chooses playing A, it knows it will get R - c. If it deviates by playing NA, it may get more than R – c. In fact, it can get R if the customer plays B, which happens whenever q is high enough. The company may also get less than R – c: it can get 0 if the customer plays DB, which happens whenever q below some threshold. So, we cannot conclude whether playing A is the best response for the company yet.

If the **Company with bad product** chooses playing A, it knows it will get r - c. If it deviates by playing NA, it will get either r or 0. Since r - c is less than 0, whatever the customer does on the left hand side of the game tree, the

Company with bad product prefers to deviate playing NA. Thus, we can conclude that we do not have a Pooling PBE where both Company types play A.

4. **(NA, NA): Both the Company with good product and the Company with bad product play No Advertise (NA)**

In this case, by Bayes' rule we have q = $\frac{(0.6)x1}{(0.6)x1+(0.4)x1}$ = 0.6, while p is free because it appears in the information set which is off the equilibrium path (there are no restrictions coming from the definition of PBE).

So, q must be equal to 0.6 and 1-q is equal to 0.4. Given these beliefs, the Customer's expected payoffs are:

π_c(B | NA) = (0.6)x1 + (0.4)x(-1) = 0.2

π_c(DB | NA) = (0.6)x0 + (0.4)x(0) = 0.

Since π_c(B | NA) > π_c(DB | NA), the customer plays Buy when he does not see an Advertisement (NA).

Given that the customer will play B after NA, is playing NA a best response for both Company types?

The **Company with good product** playing NA will get payoff of R. If it deviates by playing A, it will get either R - c or -c. Since R – c < R and -c << R, the Company with good product is best responding by playing NA (there is no profitable deviation).

The **Company with bad product** will get a payoff of r. If it deviates by playing A, it will get either r - c or -c. Since r – c < R and -c << R, the Company with the product being of bad quality is also best responding by playing NA (there is no profitable deviation). We can conclude that the Company is best responding against the customer playing B.

However, we must calculate p to denote Pooling PBE correctly.

Given p, the Customer's expected payoffs after A are:

$\pi_c(B \mid A) = (p)x1 + (1-p)x(-1) = 2p - 1$

$\pi_c(DB \mid A) = (p)x0 + (1-p)x(0) = 0$

→ if $2p - 1 \geq 0$, the Customer will play B, if $0 \geq 2p - 1$, the Customer will play DB.

There might be some mixed strategies PBE as well. However, as we mentioned above, we will ignore mixed strategy equilibria for simplicity, and we will focus on pure strategies. We have

$2p-1 \geq 0 \Leftrightarrow p \geq 0.5$.

We can conclude that

- **[(NA, NA, B, B), $p \geq 0.5$, $q = 0.6$]**
- **[(NA, NA, B, DB), $p \leq 0.5$, $q = 0.6$]**

are Pooling PBE of the game.

Designing a Signaling Game: The case of MBAs

Nowadays, mobile phones and smart devices are crucial gadgets of our daily life. While the number of available apps continues to grow, app usage and mobile phone prevalence are still multiplying and show no sign of drawback, indeed. Moreover, mobile applications offer various conveniences such as fast payments, and their popularity leads to an increase in online payments made via smartphones. However, these applications also lead to increased concerns about data security (Carroll and Grosu, 2011) and lead to a sensitive interaction between the bank's signaling of security measures and the risk-averse customer's decision to utilize this service. In the remaining of this thesis, I will try to examine this strategic interaction in depth, exploring the compatibility between trust and convenience in the context of Mobile Banking Applications (MBAs) use.

Mobile banking has emerged as an effective way for enhancing financial inclusion and increasing cashless transactions. However, it is prone to various types of cyber frauds (Beju et al., 2023). When developing MBAs, security is the essential thing to take care of. Users expect financial institutions' responsibility because they often entrust them with everything they have. And to build that trust, it is essential to keep users' data and money safe (Beju et al., 2023).

Modern applications store any user's information, server metadata, sensitive files, and basic settings. And all this happens even if you build regular apps, but some applications also provide offline access to the user's data. In this case, the amount of local data grows exponentially over time. Generally, the default tools we use on daily basis do not take advantage of the advanced security practices and we are subject to data vulnerability (Ciriani et al., 2023).

The simple solution is to encrypt the database and eliminate even a slight chance of attack. Encryption is a key part of data security. It transforms information into a format that can't be understood without a special key, providing a strong layer of protection (Ciriani et al., 2023). This is especially important when data is being sent over the internet or stored digitally. So, its main role is to safeguard sensitive data from threats such as cyber attackers (hackers) and identity thieves (Di Vimercati et al., 2007). These individuals often target businesses, aiming to steal valuable information like customer details and financial records. As data is a valuable resource, it's crucial for organizations to encrypt their data, whether it's being stored or transferred.

The process of encryption involves using a cryptographic key, a special string of text that converts the data into code (ciphertext). The security level of the encryption depends on the length of this key. To prevent unauthorized use of the key, it's important to manage it properly. This includes steps like tracking its lifecycle, storing it securely, and restricting who can access and use it (Singh & Gilhotra, 2011).

Without encryption, sensitive data is at risk. For example, if a server storing credit card numbers is physically accessed, those numbers could be stolen. Similarly, if a website is hacked, malicious software could capture any credit card numbers stored in its database. Since data breaches are quite common, encryption became a necessity (Goodman, M., 2015).

When a bank clearly communicates its commitment to data security through explicit signals, such as highlighting the use of encryption (like a banner or a prompt on the mobile app denoting its security measures), it tries to induce trust among its customers. However, misinterpretations of these signals can lead some consequences. If the message denoting security measures taken is unclear and ambiguous, customers may perceive the service as less secure than signaled. As a result, potentially, it may cause lack of trust and induce the customer to seek alternative choices or the deletion of the mobile banking app.

Risk-averse customers prioritize data security to a significant degree. While they care about the convenience offered by banking apps, this is dominated by the perception of security. This creates a situation where customers weigh the convenience of the app against their concerns about data breaches and the CIA (Confidentiality, Integrity, Availability) triad[9]. Misinterpretations of signals play a crucial role in this process. Although security is vital for risk-averse customers, convenience remains an important factor for risk-neutral and risk-lover customers. In fact, they may be willing to tolerate some level of perceived risk, as the app offers significant convenience benefits.

We can represent the interaction between the bank and the customer via the following game:

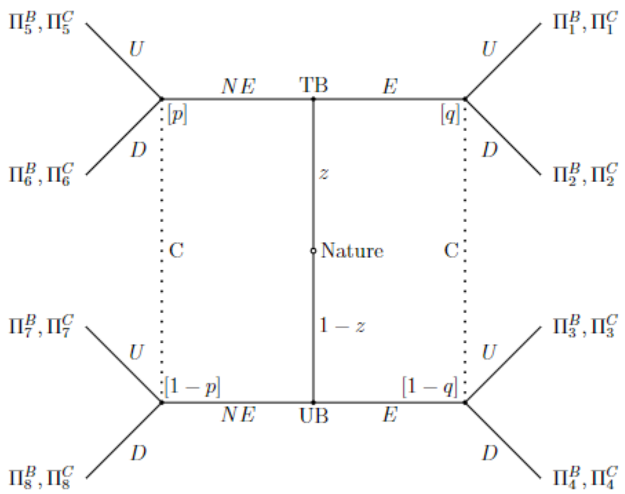


Figure: Illustration of the game

[9] Confidentiality refers to protecting information from unauthorized access. Integrity means data are robust, complete, and have not been modified by an unauthorized user/attacker. Availability means data are accessible when we need them.

The **players** are the Bank and the Customer.

The Bank can be of two types, Trustworthy (TB) or Untrustworthy (UB). It can send two possible messages to signal its type: Encryption or No Encryption. For simplicity, we are generalizing superior security measures by saying Encryption, and less security measures by saying No Encryption.

The customer does not know the bank's type but can observe the signal. Then, he decides whether to keep using the mobile banking app or not.

Actions:

1. Bank's actions: No Encryption (NE) or Encryption (E).

2. Customer's actions: keep Using the app (U) or Delete it (D).

The **payoffs** are defined as follows.

The Bank will obtain revenue R whenever the customer keeps using the app, and zero revenue if the customer deletes the app. Encryption entails positive costs, which are different for the two types of banks. Costs are lower for the trustworthy bank. We denote the cost for the trustworthy bank with C_{TB} and the cost for the untrustworthy bank with C_{UB}.

Let`s now focus on the payoffs of the customer. The customer will always get 0 if he chooses D, since he deletes the app. Otherwise, if he chooses to keep using the app, the ordering of payoffs is the following:

- If the bank is Trustworthy and chooses Encryption, it is safe and sound for the customer to use the app. Payoff is $\pi^C{}_1 > 0$.
- If the bank is Trustworthy and chooses No Encryption, it is not that risky for the customer to use the app, given the bank's type. Payoff is $\pi^C{}_5 > 0$.
- If the bank is Untrustworthy and chooses Encryption (that is, it tries to mimic the other type), the outcome is pretty bad for the customer. Payoff is $\pi^C{}_3 < 0$.

- If the bank is Untrustworthy and chooses No Encryption, the outcome is the worst one for the customer. Payoff is $\pi^{C}_{7} < 0$.

We have that $\pi^{C}_{1} > \pi^{C}_{5} > 0 > \pi^{C}_{3} > \pi^{C}_{7}$.

Thus, when the signal is Encryption, we have:

- $(\pi^{B}_{1}, \pi^{C}_{1}) = (R - C_{TB}, \pi^{C}_{1})$,
- $(\pi^{B}_{2}, \pi^{C}_{2}) = (- C_{TB}, 0)$,
- $(\pi^{B}_{3}, \pi^{C}_{3}) = (R - C_{UB}, \pi^{C}_{3})$,
- $(\pi^{B}_{4}, \pi^{C}_{4}) = (- C_{UB}, 0)$.

While when the signal is No Encryption, we have:

- $(\pi^{B}_{5}, \pi^{C}_{5}) = (R, \pi^{C}_{5})$,
- $(\pi^{B}_{6}, \pi^{C}_{6}) = (0, 0)$,
- $(\pi^{B}_{7}, \pi^{C}_{7}) = (R, \pi^{C}_{7})$,
- $(\pi^{B}_{8}, \pi^{C}_{8}) = (0, 0)$.

For concreteness, we assume

$R = 3$, $C_{TB} = 1$, $C_{UB} = 4$, $\pi^{C}_{1} = 2$, $\pi^{C}_{5} = 1$, $\pi^{C}_{3} = -1$, and $\pi^{C}_{7} = -2$.

Thus, the game becomes the following:

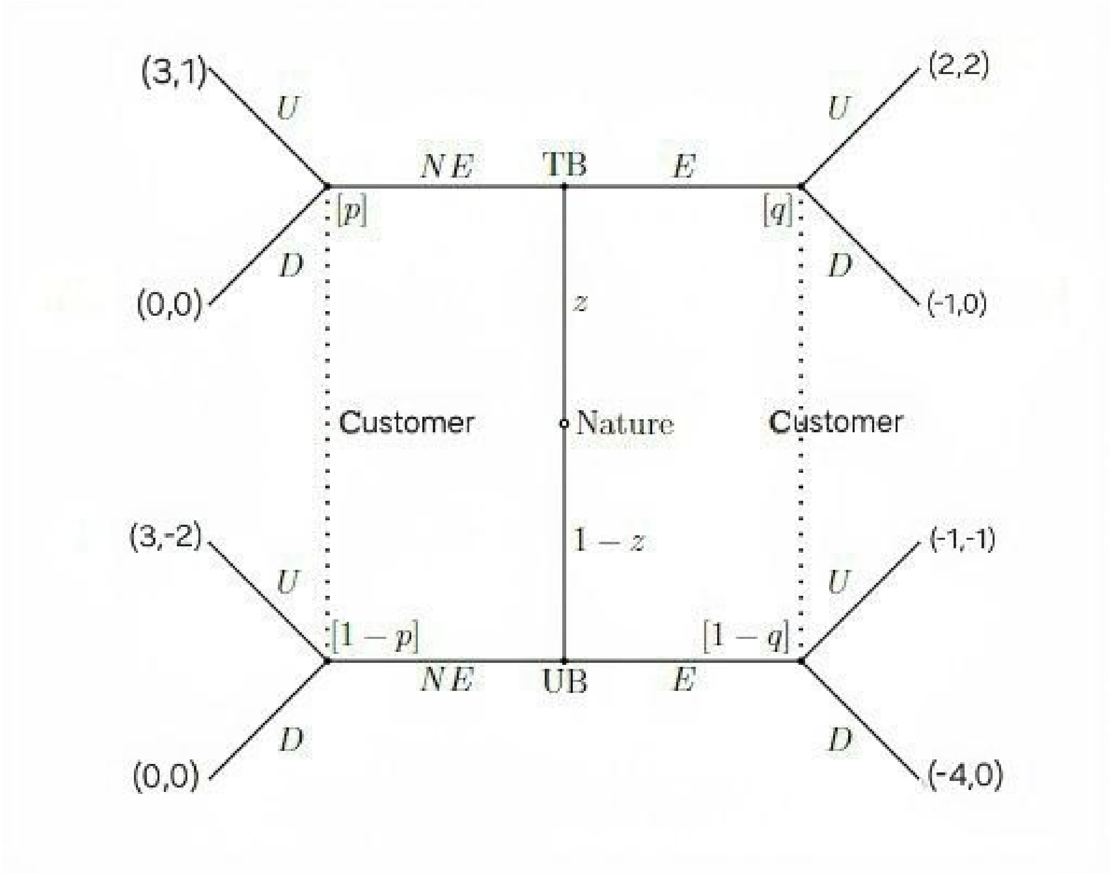


Figure: Illustration of the game

First, let's assume that z = 0.6 (that is, the prior probability that the bank is Trustworthy is 60%).

Let's analyze the Perfect Bayesian Equilibria of the game, starting with the Separating ones.

1. (E, NE): TB chooses Encryption, UB chooses No Encryption.

Since beliefs must be derived via Bayes rule, the Costumer's beliefs are given by p = 0 and q = 1. So, if the Customer uses the app after observing E he gets 2, otherwise he gets 0. Accordingly, he should use the app after the Encryption signal. On the other hand, whenever the customer observes No Encryption, he will delete the app since the payoff is higher than that if he keeps using it (0 > -2). The optimal choice for the customer is therefore (D, U).

Now, we must check whether the Bank is best responding by playing (E, NE) against (D, U). If the Trustworthy Bank type plays E, it knows the customer will play U. The payoff will be 2. If the Trustworthy Bank type plays NE, it knows the customer will play D. The payoff will be 0. Thus, playing E is a Best Response for the Trustworthy Bank towards (D, U): the Trustworthy Bank will implement Encryption as superior security measures.

If the Untrustworthy Bank type plays NE, it knows the customer will play D so the payoff will be 0. What if the Untrustworthy Bank type decides to deviate by playing E instead NE? Since the customer can only observe Encryption (E) or No Encryption (NE) as signals, the customer believes that the Untrustworthy Bank type will play NE while the Trustworthy Bank type will play E. So, if the Untrustworthy Bank type were to deviate and play E, the customer being then playing U, it would get -1. Since 0 > -1**,** the deviation would not be profitable. So, NE is a best response for the Untrustworthy Bank against (D, U).

We can conclude that **[(E, NE, D, U), p = 0, q = 1] is a Separating PBE.**

2. (NE, E): TB chooses No Encryption, UB chooses Encryption.

This is the second tentative strategy profile for being a Separating PBE. Beliefs are given by p = 1 and q = 0 according to Bayesian updating, so that the Customer knows in which decision node he is at each information set. If the customer observes signal E, he is going to believe that this is coming from the Untrustworthy Bank type, and he will play D (since 0 > -1). If the customer observes signal NE, he is going to believe that the signal is coming from Trustworthy Bank type, and he will play U (since 1 > 0). Given beliefs p = 1 and q = 0, the optimal strategy for the Customer is (U, D).

If the Trustworthy Bank type does not deviate and plays NE, its payoff is 3. However, if it deviates its payoff would be -1 since the Customer believes that the signal E comes from the Untrustworthy Bank type. So, NE is a best response for the Trustworthy Bank type against (U, D).

If the Untrustworthy Bank type does not deviate and plays E, its payoff is -4. However, if it deviates its payoff would be 3 since the customer believes that the signal NE comes from the Trustworthy Bank type. In this scenario, deviation is profitable ($\pi^B_7 > \pi^B_4$). So, for the Untrustworthy Bank type it is not optimal to play E. Hence, there is no Separating PBE of this kind.

Let us now consider Pooling PBE.

3. (E, E): both the TB and the UB choose Encryption.

Whenever the Customer faces signal E, he cannot distinguish between the Trustworthy Bank type and the Untrustworthy Bank type. We will perform the Bayesian updating in this case. So, q will be:

$q = \frac{(0.6)x1}{(0.6)x1+(0.4)x1} = 0.6$

On the other hand, p is not derived from Bayes rule because it is relative to the information set which is off the equilibrium path. According to the definition of PBE, believes should be updated via Bayes rule whenever it is possible but here it is not possible (the probability of NE for both the Trustworthy Bank type and the Untrustworthy Bank type is 0). So, q must be equal to 0.6 and 1-q is equal to 0.4. Given these beliefs, the Customer's expected payoffs after E are:

$\pi_C(U \mid E) = (0.6)x(2) + (0.4)x(-1) = 0.8$

$\pi_C(D \mid E) = (0.6)x(0) + (0.4)x(0) = 0$

In expected terms, by Using the app (U), the Customer will be better off when he observes Encryption, though he cannot distinguish between the Trustworthy Bank or Untrustworthy Bank type, since $\pi_C(U \mid E) > \pi_C(D \mid E)$.

Given that the customer will play U after E, is playing E a best response for both Bank types?

If the Trustworthy Bank type chooses to play E, it gets 2. If it deviates by playing NE, it will get either 3 or 0 depending on the Customer`s choice (which, in turn, depends on his belief p). So, we cannot conclude that playing E is a best response for the Trustworthy Bank type yet. If the Untrustworthy Bank type chooses to play E, it gets -1. If it deviates by playing NE, it will get either 3 or 0 depending on the Customer's choice. No matter what this choice is, the Untrustworthy Bank type will prefer to deviate and play NE instead of E. We can conclude that there are no Pooling PBE where both types play Encryption.

4. (NE, NE): both the TB and the UB choose No Encryption.

In this case, we have that the Customer's beliefs after the No Encryption signal must be computed via Bayes' rule, which gives p = $\frac{(0.6)x1}{(0.6)x1+(0.4)x1}$ = 0.6. On the other hand, there is no consistency requirement for beliefs after the Encryption signal, that is, on q, as the corresponding information set is off the equilibrium path. So, p must be equal to 0.6 and 1-p is equal to 0.4. Given these beliefs, the Customer's expected payoffs are:

$\pi_C(U \mid NE) = (0.6)x(1) + (0.4)x(-2) = -0.2$

$\pi_C(D \mid NE) = (0.6)x(0) + (0.4)x(0) = 0.$

Since $\pi_C(D \mid NE) > \pi_C(U \mid NE)$, the Customer plays D when he does not see the Encryption signal.

Given that the customer will play D, is playing NE a best response for both types of the Bank?

If the Untrustworthy Bank plays NE, it gets payoff 0. If it deviates by playing E, it will get either -1 or -4. In this case, there is no incentive to deviate for the Untrustworthy Bank type.

If the Trustworthy Bank type plays NE, it gets payoff 0. If it deviates by playing E, it will get either 2 or -1. Thus, its choice will depend on the Customer's choice in the information set after Encryption, which, in turn, depends on his beliefs q. In particular, the TB type will not have incentive to deviate if the Customer will choose to Delete the app there.

Thus, let's consider the off-path behavior of the Customer and compute his expected payoffs after signal E. We have

$\pi_C(U \mid E) = (q)x(2) + (1\text{-}q)x(\text{-}1) = 3q - 1$,

$\pi_C(D \mid E) = (q)x(0) + (1\text{-}p)x(0) = 0$.

What are the values of q which sustain a choice for the Customer such that the Bank's strategy (NE, NE) is part of a Pooling PBE? That is, such that the Trustworthy Bank type does not have incentive to deviate? What off-path beliefs make the TB deviation irrational?

The customer will be willing to Delete the app after observing signal E whenever

$\pi_C(D \mid E) \geq \pi_C(U \mid E) \Leftrightarrow 0 \geq 3q - 1 \Leftrightarrow q \leq \frac{1}{3}$.

Since there are no consistency requirements for q, we can impose this requirement which ensures the optimality of the UB's strategy. Therefore, we can conclude that **[(NE, NE, D, D), p = 0.6, q ≤ $\frac{1}{3}$] is a Pooling PBE.**

Recall that we assumed that z = 0.6. This sustains the choice of the Customer of Deleting the app after observing the No Encryption signal. In fact, the Customer is going to choose D as long as z is below some given threshold. To find this threshold, let's compute the Customer's expected payoffs as function of z:

$\pi_C(U \mid NE) = (z)x(1) + (1 - z)x(-2) = 3z – 2$,

$\pi_C(D \mid NE) = (z)x(0) + (1 - z)x(0) = 0.$

Thus, the threshold for z which sustains the above Pooling Equilibrium is given by

$\pi_C(U \mid NE) \leq \pi_C(D \mid NE) \Leftrightarrow z \leq \frac{2}{3}.$

If z is above the threshold 2/3, the Customer will choose to Use the app even if it observes only the No Encryption signal. This is because the probability that he will meet a Trustworthy Bank is sufficiently high, more than 66%. In this case, the Pooling Equilibrium are **[(NE, NE, U, ϕ), p = z, q]** where ϕ can be either U or D according to the value of q. In particular, we have seen above that

$\pi_C(D \mid E) \geq \pi_C(U \mid E) \Leftrightarrow 0 \geq 3q - 1 \Leftrightarrow q \leq \frac{1}{3}.$

Thus, if $z \geq \frac{2}{3}$, the Pooling PBE are:

[(NE, NE, U, D), p = z, q ≤ $\frac{1}{3}$] and **[(NE, NE, U, U), p = z, q ≥ $\frac{1}{3}$].**

Intuitive Criterion

Let's go back to the case in which z = 0.6 (in general, $z \leq \frac{2}{3}$), and consider the Pooling PBE

[(NE, NE, D, D), p = z, q ≤ $\frac{1}{3}$].

Note that q is the probability that the Customer assigns to the fact that a deviation to Encryption comes from the Untrustworthy Bank type. Thus, in this pooling equilibrium, the Customer, after observing a deviation to Encryption, must believe that such a deviation comes with larger probability from the Untrustworthy Bank type. Does this seem reasonable? If the UB type deviates, it is surely strictly worse off than in equilibrium, independently of the resulting

Customer's choice (it gets either -1 or -4, while in equilibrium it gets 0). On the other hand, the Trustworthy bank type can get a payoff larger than the equilibrium one (2 instead of 0) by deviating to E.

The **Intuitive Criterion** has been introduced by Cho and Kreps (1987) as an equilibrium refinement for signaling games, which helps to get rid of unreasonable equilibria. Firstly, by restricting possible deviations to those types of agents which could obtain higher utility levels by deviating to off-the-equilibrium messages. Secondly, by considering in this subset the types for which the off-the-equilibrium message is not equilibrium dominated.

In general, equilibrium refinement techniques are ways of reducing the set of equilibria based on rationality principles. Many refinement techniques are based on restricting players' beliefs off-the-equilibrium path, by requiring off-equilibrium beliefs to be reasonable in some sense.

Intuitively, a PBE can be eliminated if there is some type of player who wants to deviate even though he is not sure what the belief of some other player is. The deviating player is only sure that the other player will not think that he is a type for which the deviation is an equilibrium-dominated action.

Let's apply the Intuitive Criterion in the above example.

After observing a deviation to the Encryption signal, the Customer must assign probability 1 to the fact that this deviation comes from the Trustworthy Bank type, since for the Untrustworthy Bank type the deviation is equilibrium dominated. Thus, it must be q = 1. It follows that

$\pi_C(U \mid E) = 2 \geq \pi_C(D \mid E) = 0$,

and hence the Customer will choose U in his second information set (after E). Given this, the Trustworthy Bank type has incentive to deviate from NE to E, since it gets 0 if it plays NE and gets 2 if it plays E.

Therefore, the Intuitive Criterion eliminates the Pooling PBE of the game, and only the Separating Equilibrium survives.

We can conclude that the MBAs game has a unique reasonable PBE, which is the Separating equilibrium **[(E, NE, D, U), p = 0, q = 1]**, in which the two types of Banks perfectly reveal themselves through their signals.

Conclusion

Bayesian games delve into the strategic interactions between rational agents in an environment with imperfect information. Throughout this thesis, we have investigated the application of Bayesian games in mobile banking app development. By signaling reliability, banks create a good environment for users to engage confidently with their mobile banking apps. As a result, it often leads to increased adoption rates and customer loyalty. Furthermore, banks dynamically adjust their offerings, security protocols, and user interfaces based on the data gathered from user interactions. This dynamic approach ensures that mobile banking apps remain relevant and responsive to the changing needs, concerns and expectations of users.

In this thesis, the problem of asymmetric information in the banking sector is modeled using the standard signaling mechanism in a dynamic game with incomplete information. The game features two types of banks selected (stochastically) by nature as Trustworthy and Untrustworthy, who benefit from the customers' usage of their mobile application. The type is known by the bank but not by the customers. The bank can signal its type by offering a certain level of encryption for the customers' sensitive data. Customers observe the signal and decide whether to use the mobile application or delete it. Using the game theoretical apparatus offered by signaling games to model such a problem, we obtained some remarkable results.

We found that the model has a separating perfect Bayesian equilibrium in which the two types of banks adopt different strategies, i.e. different signals, thereby allowing their type to be perfectly identified by their chosen strategy. The trustworthy type chooses to offer encryption, while the untrustworthy type does not, and the Customer uses the mobile app whenever he observes encryption and deletes it otherwise. Thus, the signaling mechanism allows the trustworthy banks to distinguish itself from the untrustworthy one and increases efficiency of the market outcome.

In fact, without the possibility of signaling, the customer would always delete the app, given a sufficiently high probability of encountering an untrustworthy bank, and every agent would get zero revenue. With signaling, the trustworthy bank pays some positive cost to signal its type by offering encryption, but the revenue it gets from the fact that the customer uses its mobile app is higher, and the customer also derives a positive payoff from using the app. This represents a Pareto improvement.

The model we have presented is highly stylized but allows us to derive clear predictions for the agents' equilibrium behavior. Of course, the analysis could be extended in several directions. For instance, one could consider more types of banks, expand the strategy sets, or change the payoff functions relaxing the assumptions made on preferences. This could yield further insights about the strategic behavior of the agents.

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