UNIVERSITÀ DEGLI STUDI DI MILANO

FACOLTÀ DI SCIENZE POLITICHE, ECONOMICHE E SOCIALI

CORSO DI LAUREA MAGISTRALE IN

Data Science and Economics

'GAMES WITH INCOMPLETE INFORMATION: AN APPLICATION TO THE BANKING SECTOR’

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Key Definitions

- Assymetric Information
- Moral Hazard
- Adverse Selection
- Signaling
- Screening

- ***Asymmetric information*** arises when ***one party has more information*** about a transaction than the other. In a situation where one party has more information while the other has less, the party with more information may seek to exploit this advantage, either due to the other party's lack of awareness or inability to recognize the incomplete information provided. As a result of this situation, there is a disruption of information symmetry between those with greater access to information and those with less access to information.

→ Akerlof (1970) The Market for "Lemons" was a pioneering paper that discuss assymetric information.

- ***Signaling*** is a mechanism that may arise in the market as ***a response to the problems induced by asymmetric information***. It consists in the possibility for the informed party to take actions in order to convey signals to the uninformed party. Concerning the subject of this thesis, we can think of signaling as a ***mechanism that serves to build trust and reduce uncertainty for the uninformed party***.

→ Spence`s Labour Market Signaling Model (1973) : University degree as a signal

- ***Screening*** is another mechanism that can emerge in the market to overcome the asymmetric information problems. It can be described as the strategic gathering and utilization of information by one party to evaluate or discover hidden attributes of another party, and this action is taken ***prior to entering into an agreement or transaction*** where one party has limited information about the quality or characteristics of the other party. The ***purpose of screening is to mitigate adverse selection.***

- ***Adverse selection*** results when one party makes a decision based on limited or incorrect information, which leads to an undesirable result. ***Moral hazard*** is when an individual takes more risks because he knows that he is protected due to another individual bearing the cost of those risks.

Suppose that solidly learning econometrics increases the student's ability, but a grade of A is not enough to show that he solidly learned the material. To signal his newly acquired ability, the student must also take "Time Series," which he cannot pass without a solid understanding of econometrics. "Time Series" might be useless in itself, but if it did not exist, the students would not be able to show he had learned basic econometrics.

-Eric Rasmusen

Equilibrium Concepts

		Timing	
		Simultaneous	Sequential
Information	Complete	Nash	SPE
	Incomplete	Bayesian Nash	Perfect Bayesian

Dynamic Games with Incomplete Information

- The presence of private information naturally leads informed parties to either communicate or use this information towards their own objectives, and prompts uninformed parties to make efforts to learn this information and respond accordingly. Therefore, many incomplete information games are inherently dynamic in nature (Gibbons, 1992).

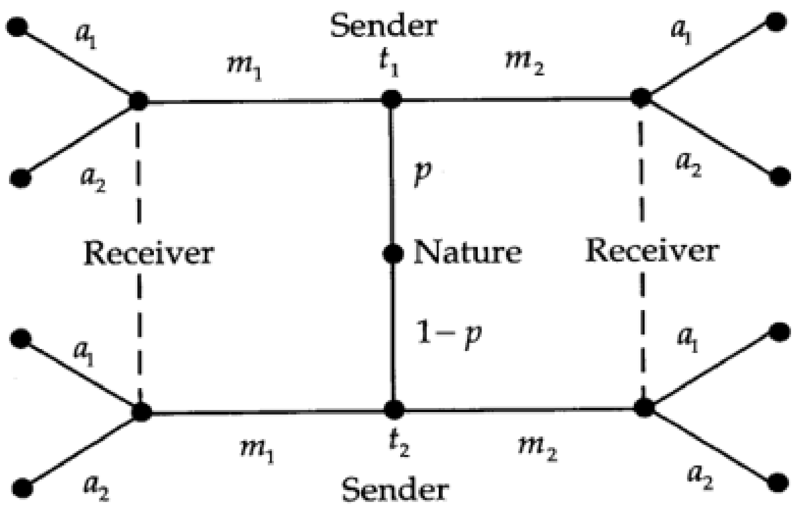
Perfect Bayesian Equilibrium

- It is an equilibrium concept that uses Bayesian updating to describe player behavior in dynamic games with incomplete information.
- The “perfect” part is a nod to subgame perfect equilibrium. The “Bayesian” part is a nod to the need to update beliefs.
- A perfect Bayesian equilibrium (PBE) is a set of ***strategies and beliefs*** such that the strategies are sequentially rational given the players’ beliefs **and the players update their beliefs via Bayes rule wherever possible**.
- “***wherever possible***” denotes some moves may only arise off-the-equilibrium path of game tree. The probability of arriving at these is 0, and thus the denominator of Bayes’ rule is 0, and so the belief is undefined. In these cases, we must consider *all* possible beliefs that a player could adopt. This makes solving for PBE substantially more complicated than other types of equilibrias.

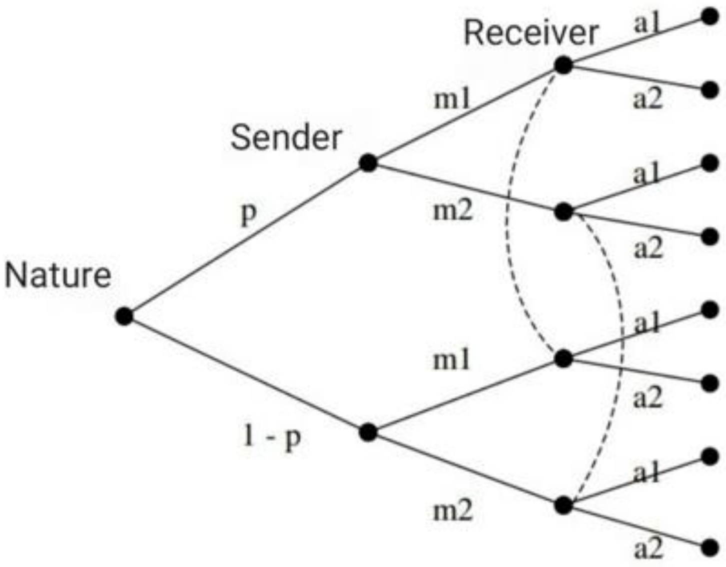
Signaling Games

- Spence (1973) was a pioneer in investigating and applying the "signaling games" into economics through his signaling model of the labor market. Signaling models have found broader applications in the literature. Spence, Akerlof, and Stiglitz were awarded the Nobel Prize in Economic Sciences in 2001 for their work on asymmetric information in markets.

Gibbons (1992):



Dutta(1999):



Typically, the ***timing of a signaling game*** is as follows (Gibbons, 1992: 183):

- 1. Nature draws a type $t_i \in T = \{t_1, ..., t_L\}$ for the Sender according to the probability distribution $p(t_i)$, where $p(t_i) > 0$ and $p(t_1) + \ldots + p(t_L) = 1$. (set of types are assumed to be finite.)

- 2. The sender observes t_i and then chooses a message m_j from a set of possible messages $M = \{m_1, ..., m_j\}$. The set of messages is assumed to be finite. The sender's strategy is a function defined from the type set to the message set.

- 3. The receiver observes m_j (but cannot observe t_i) and then chooses an action a_k from a set of possible actions $A = \{a_1, ..., a_k\}$. The set of actions is assumed to be finite. Additionally, the receiver's strategy is a function defined from the message set to the action set.

- 4. The receiver and the sender receive payoffs: $U_S(t_i, m_j, a_k)$ and $U_R(t_i, m_j, a_k)$.

→ Everything about the game is common knowledge, other than nature's choice of type t .

Designing a Signaling Game: The case of MBAs

- The problem of asymmetric information in the banking sector is modeled using the standard signaling mechanism in a dynamic game with incomplete information. The game features two types of banks selected (stochastically) by nature as Trustworthy and Untrustworthy, who benefit from the customers’ usage of their mobile application. The type is known by the bank but not by the customers. The bank can signal its type by offering a certain level of encryption for the customers’ sensitive data. Customers observe the signal and decide whether to use the mobile application or delete it.

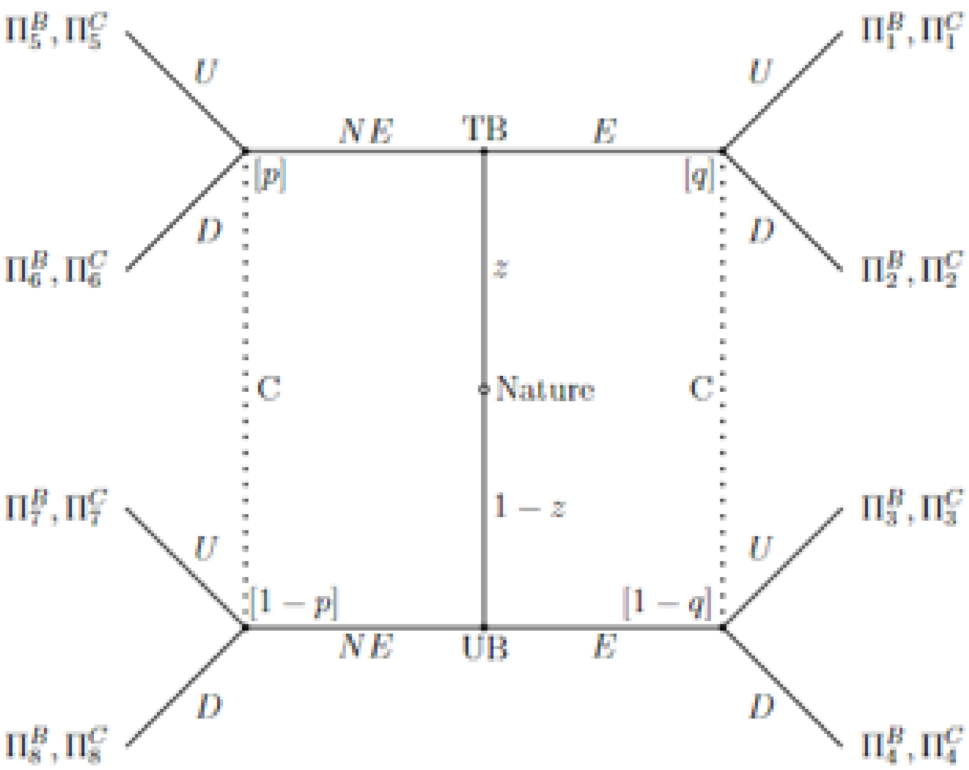


Figure: Illustration of the game

Designing a Signaling Game: The case of MBAs

- The **players** are the Bank and the Customer.

- The Bank can be of two types, Trustworthy (TB) or Untrustworthy (UB). It can send two possible messages to signal its type: Encryption or No Encryption. For simplicity, we are generalizing superior security measures by saying Encryption, and less security measures by saying No Encryption.

- The customer does not know the bank’s type but can observe the signal. Then, he decides whether to keep using the mobile banking app or not.

Actions:

- 1. Bank's actions: No Encryption (NE) or Encryption (E).

- 2. Customer's actions: keep Using the app (U) or Delete it (D).

The **payoffs** are defined as follows.

The ***Bank will obtain revenue R*** whenever the customer keeps using the app, and ***zero revenue if the customer deletes the app***. Encryption entails positive costs, which are different for the two types of banks. Costs are lower for the trustworthy bank. We denote the cost for the trustworthy bank with C_{TB} and the cost for the untrustworthy bank with C_{UB}.

Let`s now focus on the payoffs of the customer. The customer will always get 0 if he chooses D, since he deletes the app. Otherwise, if he chooses to keep using the app, the ordering of payoffs is the following:

- If the bank is Trustworthy and chooses Encryption, it is safe and sound for the customer to use the app. Payoff is $\pi^C_1 > 0$.
- If the bank is Trustworthy and chooses No Encryption, it is not that risky for the customer to use the app, given the bank's type. Payoff is $\pi^C_5 > 0$.
- If the bank is Untrustworthy and chooses Encryption (that is, it tries to mimic the other type), the outcome is pretty bad for the customer. Payoff is $\pi^C_3 < 0$.
- If the bank is Untrustworthy and chooses No Encryption, the outcome is the worst one for the customer. Payoff is $\pi^C_7 < 0$.

We have that ordering of payoffs: $\boldsymbol{\pi^C_1 > \pi^C_5 > 0 > \pi^C_3 > \pi^C_7}$

For concreteness, we assumed
$R = 3$, $C_{TB} = 1$, $C_{UB} = 4$,

and assigned

$\pi^C{}_1 = 2$, $\pi^C{}_5 = 1$, $\pi^C{}_3 = -1$, $\pi^C{}_7 = -2$

by considering

$\pi^C{}_1 > \pi^C{}_5 > 0 > \pi^C{}_3 > \pi^C{}_7$

Thus, the game becomes the following:

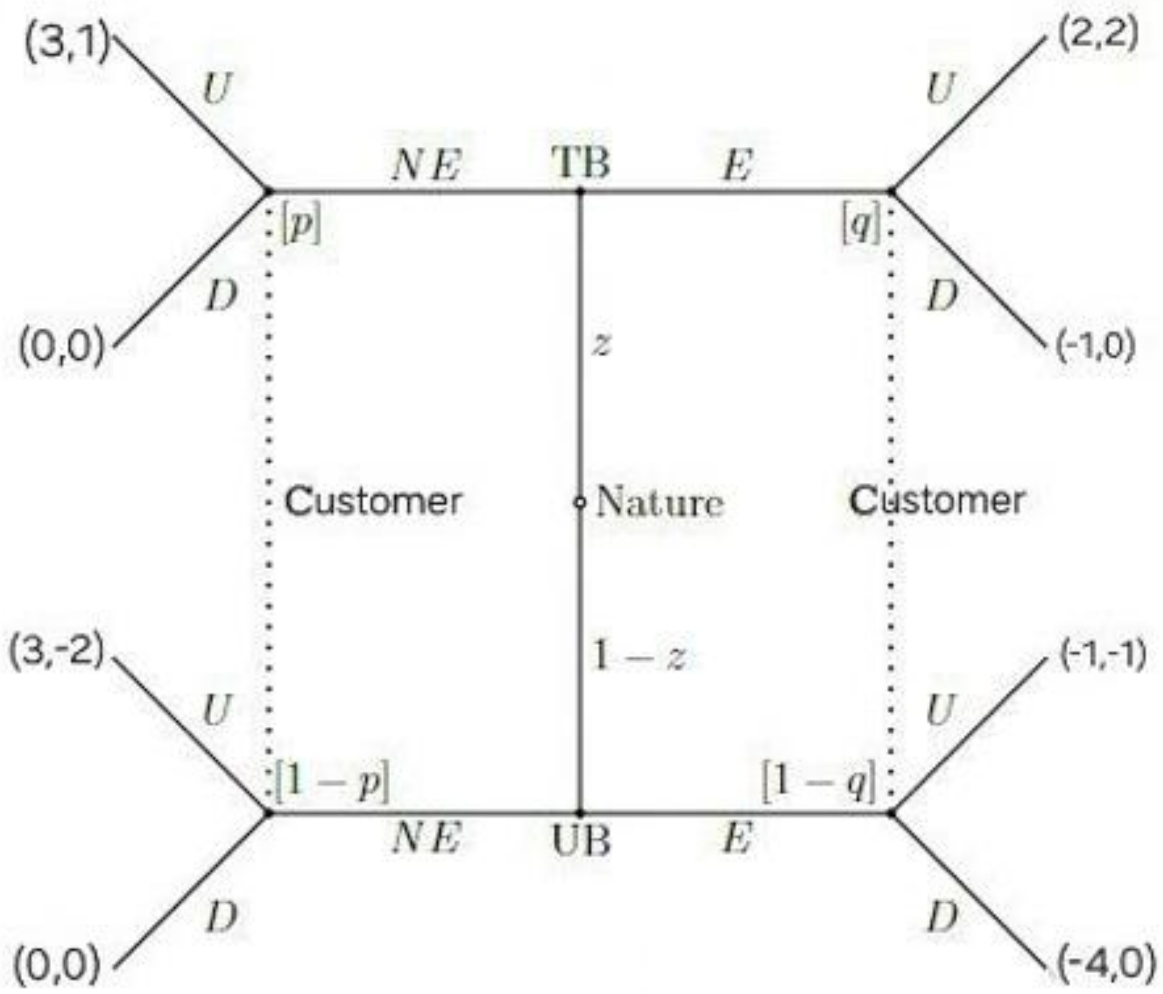


Figure: Illustration of the game

Four Tentative PBEs in pure strategies :

- **(E, NE): TB chooses Encryption, UB chooses No Encryption.**
- **(NE, E): TB chooses No Encryption, UB chooses Encryption.**
- **(E, E): both the TB and the UB choose Encryption.**
- **(NE, NE): both the TB and the UB choose No Encryption.**

We can start analyzing this game by using the game theoretical tools offered by signaling games:

- First, let's ***assume that z = 0.6 (that is, the prior probability that the bank is Trustworthy is 60%).***

- Let's analyze the Perfect Bayesian Equilibria of the game, starting with the Separating ones.

1. (E, NE): TB chooses Encryption, UB chooses No Encryption.

Since beliefs must be derived via Bayes rule, the Costumer's beliefs are given by p = 0 and q = 1. So, if the Customer uses the app after observing E he gets 2, otherwise he gets 0. Accordingly, ***the customer should use the app after the Encryption signal***. On the other hand, ***whenever the customer observes No Encryption, he will play D (***delete the app since the payoff is higher than that if he keeps using it 0 > -2). The optimal choice for the customer is therefore (D, U).

Now, we must check whether the Bank is best responding by playing (E, NE) against (D, U).

- If the Trustworthy Bank type plays E, it knows the customer will play U. The payoff will be 2. If the Trustworthy Bank type deviates by playing NE, it knows the customer will play D. The payoff will be 0. Thus, ***playing E is a Best Response for the Trustworthy Bank*** against (D, U): the ***Trustworthy Bank will implement Encryption as superior security measures.***
- If the Untrustworthy Bank type plays NE, it knows the customer will play D so the payoff will be 0. What if the Untrustworthy Bank type decides to deviate by playing E instead NE? Since the customer can only observe Encryption (E) or No Encryption (NE) as signals, the customer believes that the Untrustworthy Bank type will play NE while the Trustworthy Bank type will play E. So, if the Untrustworthy Bank type were to deviate and play E, the customer being then playing U, it would get -1. Since 0 > -1**,** the ***deviation would not be profitable. So, NE is a best response for the Untrustworthy Bank*** against (D, U).

We can conclude that ***[(E, NE, D, U), p = 0, q = 1] is a Separating PBE.***

2) (NE, E): TB chooses No Encryption, UB chooses Encryption.

----> There is ***no Separating PBE*** of this kind.

3) (E, E): both the TB and the UB choose Encryption.

----> There is ***no Pooling PBE*** where both types play Encryption.

4) (NE, NE): both the TB and the UB choose No Encryption.

- In this case, we have that the Customer's beliefs after the No Encryption signal must be computed ***via Bayes' rule***, which gives p = $\frac{(0.6)x1}{(0.6)x1+(0.4)x1}$ = 0.6. On the other hand, **there is no consistency requirement for beliefs after the Encryption signal**, that is, ***on q***, as the corresponding information set ***is off the equilibrium path***. So, **p must be equal to 0.6 and 1-p is equal to 0.4**. Given these beliefs, the Customer's expected payoffs are:

π_C(U | NE) = (0.6)x(1) + (0.4)x(-2) = - 0.2

π_C(D | NE) = (0.6)x(0) + (0.4)x(0) = 0

Since π_C(D | NE) > π_C(U | NE), the ***Customer plays D when he does not see the Encryption signal***.

Given that the customer will play D, is playing NE a best response for both types of the Bank?

- If the Untrustworthy Bank plays NE, it gets payoff 0. If it deviates by playing E, it will get either -1 or -4. In this case, there is ***no incentive to deviate for the Untrustworthy Bank type***.

- If the **Trustworthy Bank** type plays NE, it gets payoff 0. If it deviates by playing E, it will get either 2 or -1. Thus, ***its choice will depend on the Customer's choice in the information set after Encryption***, which, in turn, ***depends on his beliefs q***. In particular, the TB type will not have incentive to deviate if the Customer will choose to Delete the app there.

- Thus, ***let's consider the off-path behavior of the Customer*** and compute his ***expected payoffs after signal E***. We have

$\pi_C(U \mid E) = (q)x(2) + (1-q)x(-1) = 3q - 1,$

$\pi_C(D \mid E) = (q)x(0) + (1-p)x(0) = 0.$

What are the values of q which sustain a choice for the Customer such that the Bank's strategy (NE, NE) is part of a Pooling PBE? That is, such that the Trustworthy Bank type does not have incentive to deviate? ***What off-path beliefs make the TB deviation irrational?***

---> The customer will be willing to Delete the app after observing signal E whenever

$$\pi_C(D \mid E) \geq \pi_C(U \mid E) \Leftrightarrow 0 \geq 3q - 1 \Leftrightarrow \mathbf{q \leq \frac{1}{3}}$$

- Since there are no consistency requirements for q, we can impose this requirement which ensures the optimality of the UB's strategy.

Therefore, ***we can conclude that [(NE, NE, D, D), p = 0.6, q ≤ $\frac{1}{3}$] is a Pooling PBE.***

We found 2 PBEs so far:

- **(E, NE): TB chooses Encryption, UB chooses No Encryption. ✓**

- **(NE, E): TB chooses No Encryption, UB chooses Encryption. ✗**

- **(E, E): both the TB and the UB choose Encryption. ✗**

- **(NE, NE): both the TB and the UB choose No Encryption. ✓**

- ***Recall that we assumed that z = 0.6 initially***. This sustains the choice of the Customer of Deleting the app after observing the No Encryption signal. In fact, ***the Customer is going to choose D as long as z is below some given threshold.*** To find this threshold, let's compute the Customer's expected payoffs as function of z:

$\pi_C(U \mid NE) = (z)x(1) + (1 - z)x(-2) = 3z - 2$

$\pi_C(D \mid NE) = (z)x(0) + (1 - z)x(0) = 0$

- Thus, the ***threshold for z which sustains the Pooling Equilibrium we found on the previous page*** is given by

$$\pi_C(U \mid NE) \leq \pi_C(D \mid NE) \Leftrightarrow \boldsymbol{z \leq \frac{2}{3}}$$

What if $z \geq 2/3$?

- If z is above the threshold 2/3, the ***Customer will choose to Use the app even if it observes only the No Encryption signal***. This is because the probability that he will meet a Trustworthy Bank is sufficiently high, more than 66%. In this case, the Pooling Equilibrium are **[(NE, NE, U, ϕ), p = z, q]** where ϕ can be either U or D according to the value of q. In particular, we have seen before that

- $\pi_C(D \mid E) \geq \pi_C(U \mid E) \Leftrightarrow 0 \geq 3q - 1 \Leftrightarrow q \leq \frac{1}{3}$ (We already discussed)

- Thus, if $z \geq \frac{2}{3}$, the Pooling PBE are:

- **[(NE, NE, U, D), p = z, q ≤ $\frac{1}{3}$]** and **[(NE, NE, U, U), p = z, q ≥ $\frac{1}{3}$].**

Intuitive Criterion

- Let's go back to the case in which z = 0.6 (in general, z ≤ $\frac{2}{3}$), and consider the Pooling PBE

- **[(NE, NE, D, D), p = z, q ≤ $\frac{1}{3}$].**

- Note that q is the probability that the Customer assigns to the fact that a deviation to Encryption comes from the Untrustworthy Bank type. Thus, in this pooling equilibrium, ***the Customer***, after observing a deviation to Encryption, ***must believe that such a deviation comes with larger probability from the Untrustworthy Bank type. Does this seem reasonable?*** If the ***UB type deviates***, it is surely ***strictly worse off*** than in equilibrium, independently of the resulting Customer's choice (it gets either -1 or -4, while in equilibrium it gets 0). ***On the other hand, the Trustworthy bank type can get a payoff larger than the equilibrium one*** (2 instead of 0) by deviating to playing E.

- The **Intuitive Criterion** has been introduced by Cho and Kreps (1987) as an ***equilibrium refinement for signaling games***, which helps to ***get rid of unreasonable equilibria***. Firstly, by restricting possible deviations to those types of agents which could obtain higher utility levels by deviating to off-the-equilibrium messages. Secondly, by considering in this subset the types for which the ***off-the-equilibrium message is not equilibrium dominated***.

- In general, ***equilibrium refinement techniques are ways of reducing the set of equilibria based on rationality principles***. Many refinement techniques ***are based on restricting players' beliefs off-the-equilibrium path, by requiring off-equilibrium beliefs to be reasonable in some sense.***

- Intuitively, a PBE can be eliminated if there is some type of player who wants to deviate even though he is not sure what the belief of some other player is. ***The deviating player is only sure that the other player will not think that he is a type for which the deviation is an equilibrium-dominated action.***

Let's apply the Intuitive Criterion in the above example.

- After observing a deviation to the Encryption signal, the Customer must assign probability 1 to the fact that this deviation comes from the Trustworthy Bank type, since for the Untrustworthy Bank type the deviation is equilibrium dominated.

Thus, it ***must be q = 1***. It follows that

- $\pi_C(U \mid E) = 2 \geq \pi_C(D \mid E) = 0$,

- and hence the Customer will choose U in his second information set (after E). Given this, the Trustworthy Bank type has incentive to deviate from NE to E, since it gets 0 if it plays NE and gets 2 if it plays E.

- ***Therefore, the Intuitive Criterion eliminates the Pooling PBE of the game, and only the Separating Equilibrium survives.***

- We can conclude that the MBAs game has a unique reasonable PBE, which is the Separating equilibrium **[(E, NE, D, U), p = 0, q = 1]**, in which the two types of Banks perfectly reveal themselves through their signals.

Conclusion

We obtained some remarkable results:

- The ***model has a Separating Perfect Bayesian equilibrium*** in which the two types of banks adopt different strategies, thereby allowing their type to be perfectly identified by their signals. ***The trustworthy type chooses to offer encryption, while the untrustworthy type does not, and the Customer uses the mobile app whenever he observes encryption and deletes it otherwise***. Thus, the signaling mechanism allows the trustworthy banks to distinguish itself from the untrustworthy one and increases efficiency of the market outcome.
- In fact, ***without*** the possibility of ***signaling***, the ***customer would always delete the app***, given a sufficiently high probability of encountering an untrustworthy bank, ***and every agent would get zero revenue***(Would ***not be Pareto optimal***).

- ***With signaling, the trustworthy bank pays some positive cost to signal its type*** by offering encryption, but the ***revenue it gets*** from the fact that the customer uses its mobile app is ***higher***, and the ***customer also derives a positive payoff*** from using the app. This represents a *Pareto improvement.*

Future Works

- The model we have presented is highly stylized but allows us to derive clear predictions for the agents’ equilibrium behavior. Of course, the analysis could be extended in several directions. For instance, one could consider more types of banks, expand the strategy sets, or change the payoff functions relaxing the assumptions made on preferences. This could yield further insights about the strategic behavior of the agents.

Referances

- https://gametheory101.com/courses/game-theory-101/perfect-bayesian-equilibrium/
- https://cramton.umd.edu/econ703