UNIVERSITÄT OSNABRÜCK

Wireless Medium Access Technologies

Performance Evaluation

Mobile Communication, WS 2014/2015, Kap.3.3

Prof. Dr. Nils Aschenbruck

Bernoulli Random Variables

- Bernoulli trial
  - only two possible outcomes (success or failure)
    - $p + q = 1$

- Bernoulli sequence
  - n trials independent
  - probability of success or failure do not change

- one parameter p
  - probability for success is p
  - probability for failure is $q = 1 - p$

- Bernoulli sequence of n trials with success probability p

- Binomial Random Variable
  - counts the number k of successes in the n trials
  - result of X can assume values 0,1,2,..,n

- the probability of k successes in n trial with probability p of success:

$$b(k;n,p)=\binom{n}{k}p^k(1-p)^{n-k}$$

Example:

A call center is connected via ten ISDN basic rate interfaces (20 communication lines). The lines operate independently. The probability that any particular line is in use is 0.6.
What is the probability that 10 or more lines are in operation?

Geometric Random Variables

- a sequence of Bernoulli trials is continued until the first success

- X is the random variable that counts the number of trials before the first success

- the probability of success after k unsuccessful trials is

$$p(k) = (1-p)^k p$$

Poisson Random Variables

- If n is large and p is small a Binomial random variable

$$b(k;n,p) = \binom{n}{k} p^k (1-p)^{n-k}$$

can be approximated with a Poisson random variable

$$p(k;\alpha) = e^{-\alpha} \frac{\alpha^k}{k!} \qquad \alpha > 0, \quad k = 0,1,2,\ldots$$

- some examples of events that can be modeled as Poisson distributions:
  - the number of cars that pass through a certain point on a road during a given period of time.
  - the number of spelling mistakes a secretary makes while typing a single page.
  - the number of phone calls at a call center per minute.
  - the number of times a web server is accessed per minute.
  - the number of roadkill found per unit length of road.
  - …

Poisson Random Variables (2)

$$p(k;\lambda) = e^{-\lambda}\frac{\lambda^k}{k!} \qquad \lambda > 0, \quad k = 0,1,2,\ldots$$

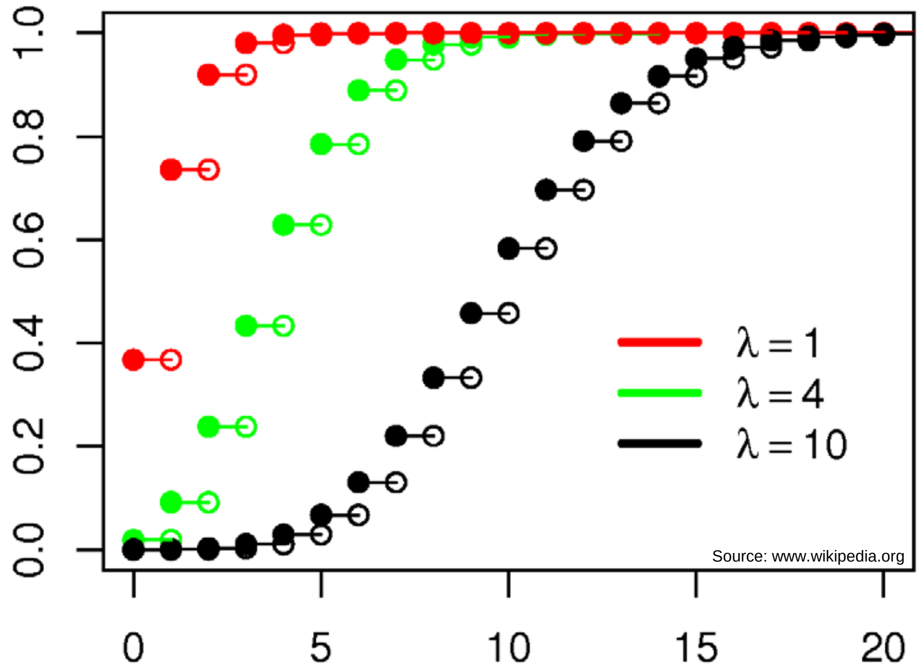
- mean

$$E[X] = \lambda$$

- variance

$$Var[X] = \lambda$$

cdf



Exponential Random Variables

- pdf

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

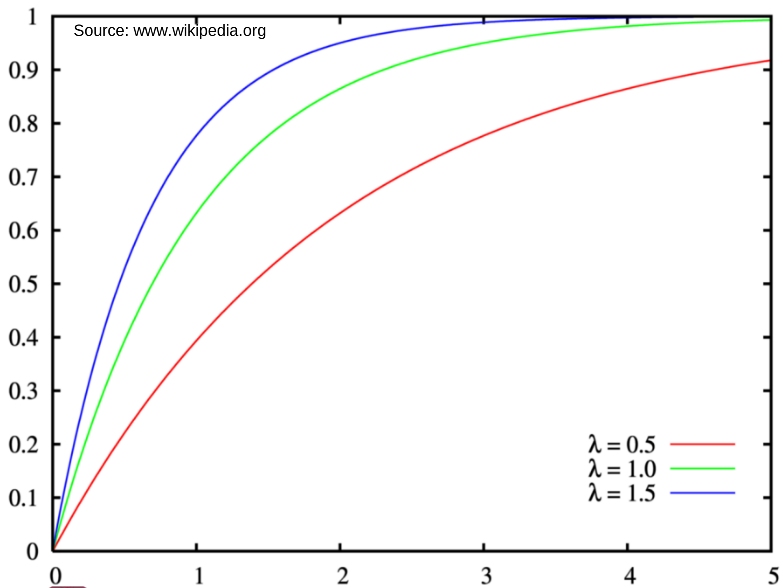
- cdf

$$F(x;\lambda) = \begin{cases} 1-e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

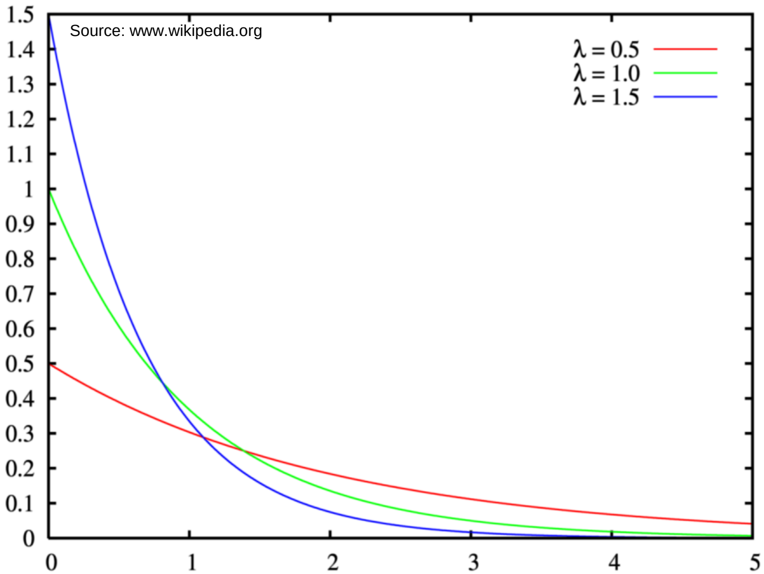
- exponential distribution is the only continuous distribution with the memoryless property
  - remembering the time since the last event does not help in predicting the time till the next

- some examples of events that can be modeled as exponential distributions:
  - the time until you have your next car accident;
  - the time until a radioactive particle decays, or the time between beeps of a geiger counter;
  - the number of dice rolls needed until you roll a six 11 times in a row;

cdf



pdf



In random access networks, each station – to some extent – is **free to broadcast its frames any time** … with the **risk of other stations transmitting at the same time**.

Here, the performance strongly depends on the load and on the load distribution across the stations attached to the network:

- **Under heavy load,** the network may **collapse** completely as a result of **collisions and follow-up collisions**.
- **Network collapse cannot be observed** if the complete load is generated **by one station only** … with all others just listening.

With “pure aloha”, stations enjoy **unrestricted medium access any time.** **Collisions** are detected by **timeout** (missing Ack).

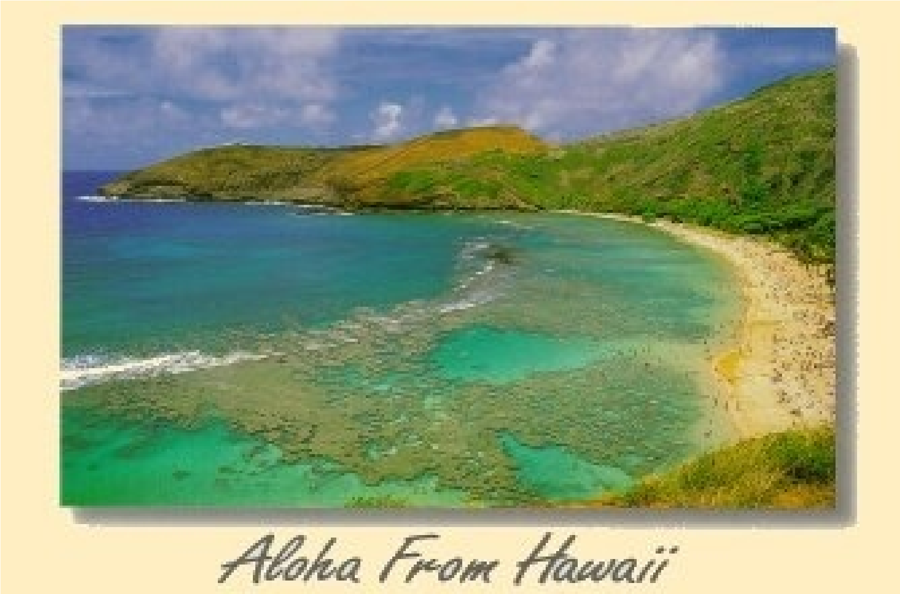
Aloha: Start a transmission any time you like.

Advantages of Pure Aloha:

- **Simple mechanism, easy to implement**
- **Illustration of general random access network characteristics**
- **Easy to analyze**

Disadvantage of Pure Aloha:

- **Low maximum throughput**



Assumptions for our analysis:

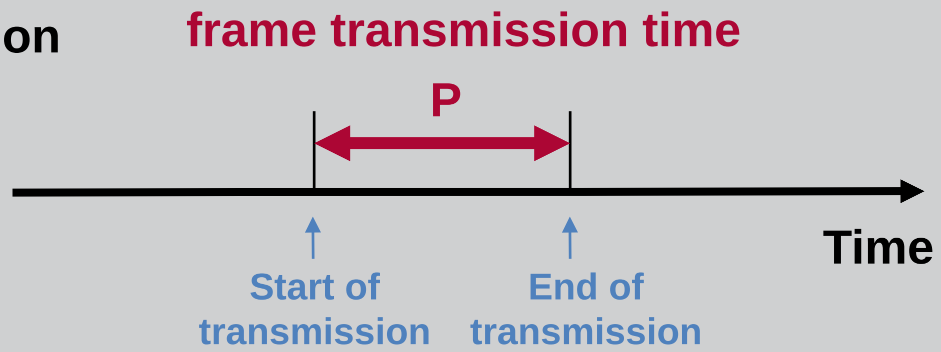
- **Error-free channel** (frames lost by collisions only)
- **Retransmission in case of missing ACK** after random backoff time
- **Fixed frame length P** (in seconds, “P” like “packet”)
- **Complete arrival process** (incl. retransmissions) **Poisson Arrivals**
  (Model of infinite user population with very small individual arrival rates)

We define throughput S and offered load G as follows:

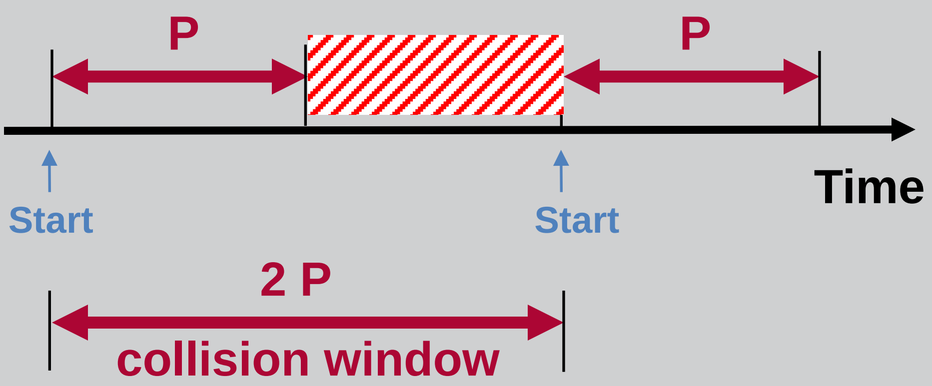
S: Average number of **successful transmissions per frame transmission time *P***
G: Average number of **attempted transmissions per frame transmission time *P***

Now consider the behavior of a **reference station**:

Transmission by the reference station



Transmission by other station



All transmission attempts starting during the collision window result in a collision

Throughput versus Offered Load for Aloha

Obviously, S may be calculated as

S = G · *P* {successful transmission}
S = G · *P* {0 arrivals in interval of length 2P}

Due to the **Poisson assumption** we find:

$$S = G \cdot e^{-2G}$$

$$P\{\text{k arrivals in T seconds}\} = \frac{(\lambda T)^k e^{-\lambda T}}{k!}, k = 0, 1, 2, \ldots$$

$$P\{\text{0 arrivals in T seconds}\} = \frac{(\lambda T)^0 e^{-\lambda T}}{0!} = e^{-\lambda T}$$

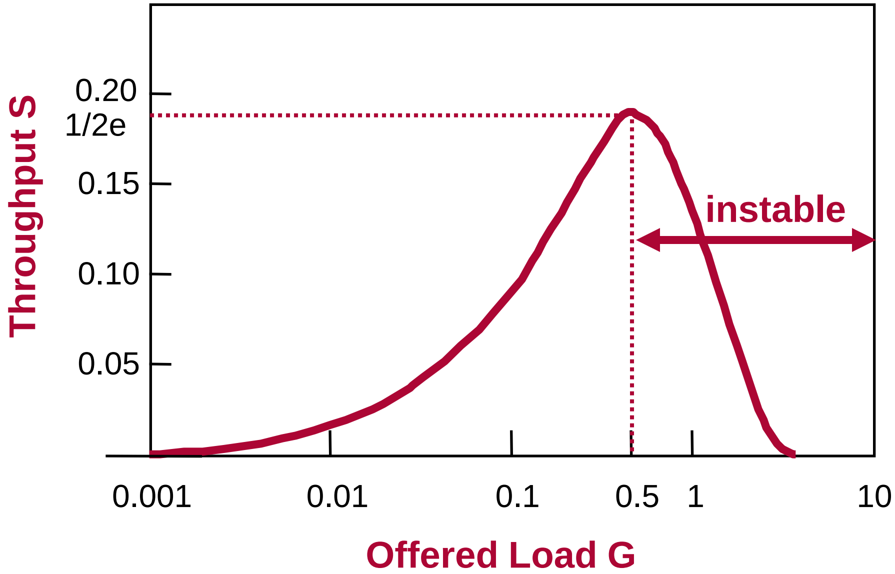
λ T = average number of arrivals in interval of length T

2 G = average number of arrivals in interval of length 2P

It may easily be shown that the **maximum throughput is reached for G = 0.5**.

In this case: **S = 1/2e ≈ 0.184** < 20 %

Note:
In the **instable area**
- **additional load** results in
- **throughput degradation**



Obviously:

- Pure Aloha is **instable for G > 0.5.**
- The **throughput** for Pure Aloha is **< 20 %.**

Slotted Aloha **improves the performance by** segmenting the time into

- **slots of fixed length P**
- exactly **equal to the frame transmission time**

Idea: **Delay frame transmissions** until the start of the next slot

Result: **Frames either overlap completely or not at all**, i.e. the collision window is reduced by 50 %.

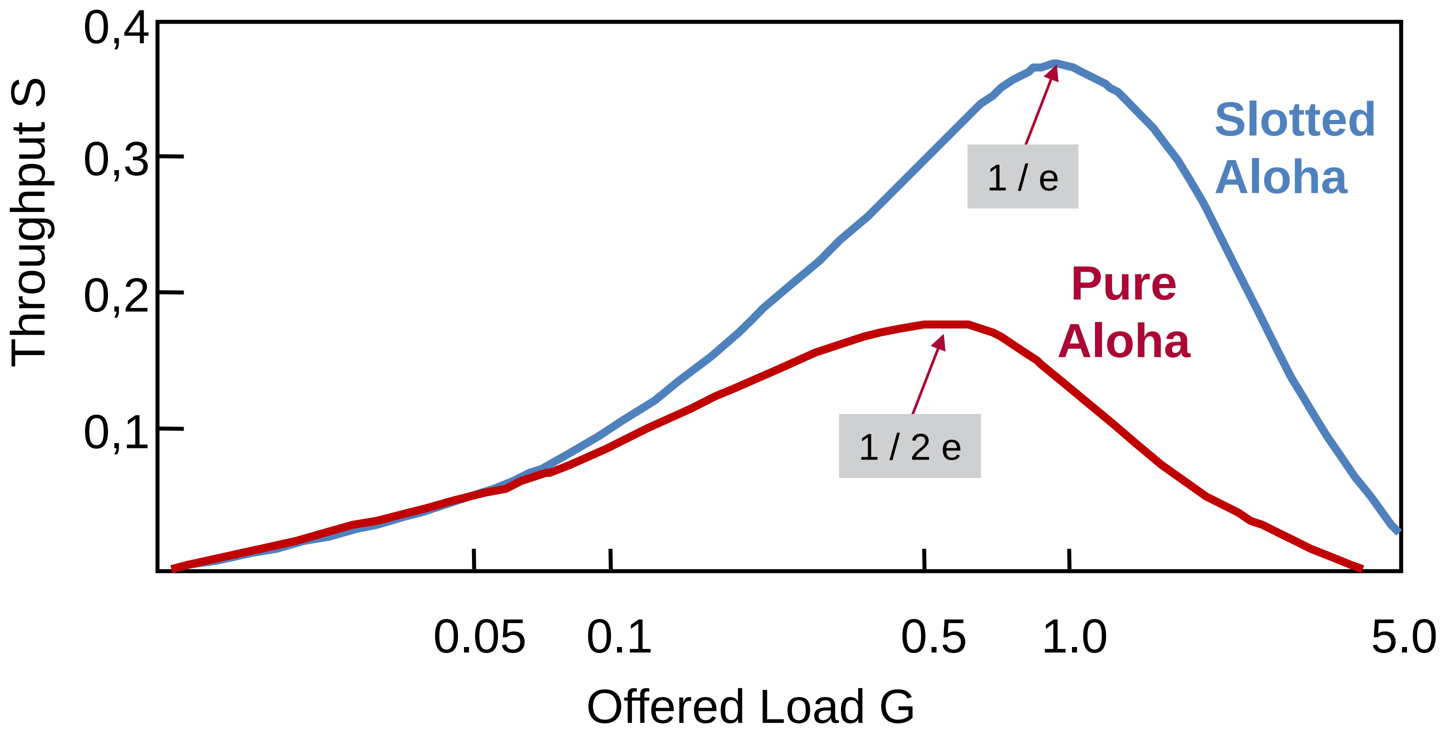
Now the throughput S may be calculated as

S = G · *P* {successful transmission}
S = G · *P* {0 arrivals in interval of length P}

instead of 2P

Because of Poisson arrivals: $S = G \cdot e^{-G}$ (instead of $S = G \cdot e^{-2G}$)

The maximum throughput is $S = 1/e \approx 0.368$ (for $G = 1$).



Slotted Aloha achieves **higher throughput** and **(slightly) improved stability**.

Up to now we assumed an infinite number of users.

Now: **Analysis for finite population.**

Idea: The transmissions of each station are modeled as a sequence of **independent Bernoulli trials**.

Note:
- **Backoff** after collisions is not considered
- For $M \to \infty$ the distribution of a sum of M independent Bernoulli random variables approaches a Poisson distribution.

For **S_i:** Probability of **station i transmitting a frame successfully in any slot**.

G_i: Probability of **station i trying to transmit a frame in any slot**.

Then we find

$$S_j = G_j \prod_{i=1; i \neq j}^{M} (1 - G_i)$$

Station j tries to transmit, all other stations don't try.

For symmetric load this may be simplified with $S_i = S/M$ and $G_i = G/M$ to

$$S = G\left(1 - G/M\right)^{M-1}.$$

With $\lim_{n\to\infty}\left[1+\frac{x}{n}\right]^n = e^x$ we find for $M \to \infty$: $\underbrace{S = G \cdot e^{-G}}$

The formula already known for Slotted Aloha

The throughput $S = G\,(1-G/M)^{M-1}$ reaches its maximum for $G = 1$,

i.e. we find: $S_{max} = (1 - 1/M)^{M-1}$.

Poisson Arrivals

M	1	2	5	10	20	100	∞
S_{max}	1	0.5	0.410	0.387	0.377	0.370	1/e

≈ 0.368

3.3.3. CSMA

If the **propagation delay is small when compared to the frame transmission time**, then the network performance may be massively improved by **avoiding collisions with ongoing transmissions**.

If a station ready to transmit **detects a busy channel** (i.e., senses a carrier) then different strategies may be applied:

- **Nonpersistent CSMA**
  If the channel is sensed busy, **a backoff algorithm** is applied
  (sleep for randomly chosen time, then use carrier sense again)
- **1-persistent CSMA**
  Keep on sensing until the channel goes idle;
  Then transmit the frame with probability 1.
- **p-persistent CSMA**
  Like 1-persistent CSMA, but **start transmission with probability p only**.
  With probability (1 – p) wait 2 x τ with τ end-to-end-propagation delay;
  After waiting, check for carrier and – if channel still idle – start transmission with probability p….

Slotted versions of CSMA (Slotted CSMA) have also been proposed.

Assumptions for our analysis:

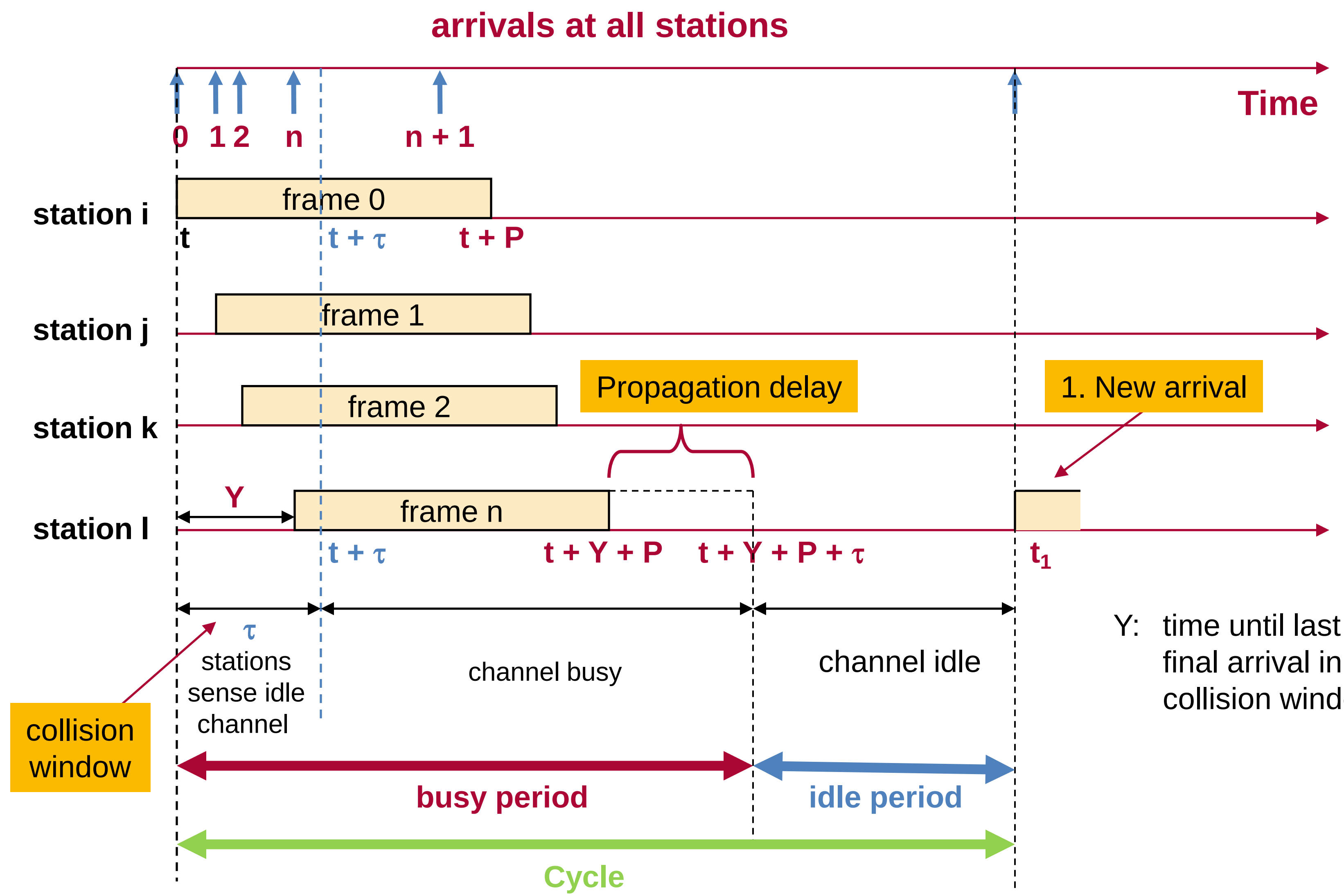
- **Infinite user population**; arrival process: Poisson ("old" and "new" frames).
- The propagation delay between any 2 stations is τ (τ = One-way end-end-delay); results in **overestimation of collision rate**.
- All **frames** have **identical length** and **transmission time P**.
- At any point in time, each station has **at most one frame ready for transmission**.
- "**Carrier Sensing**" is **instantaneous** (no delays between receiving and transmitting).
- The **channel is noiseless** (transmission failure always due to collision).
- **Overlap of any fraction** of 2 frames results in **destructive interference** (frames lost).

Note: Assuming an infinite number of users yields reasonable results for more than ≈ 20 stations

In CSMA systems, **collisions can only happen during the collision window,** i.e., up to τ after the start of transmission.

The sequence of transmission attempts and idle times results in a **cyclic behavior**, which is the basis for an easy to follow analysis.

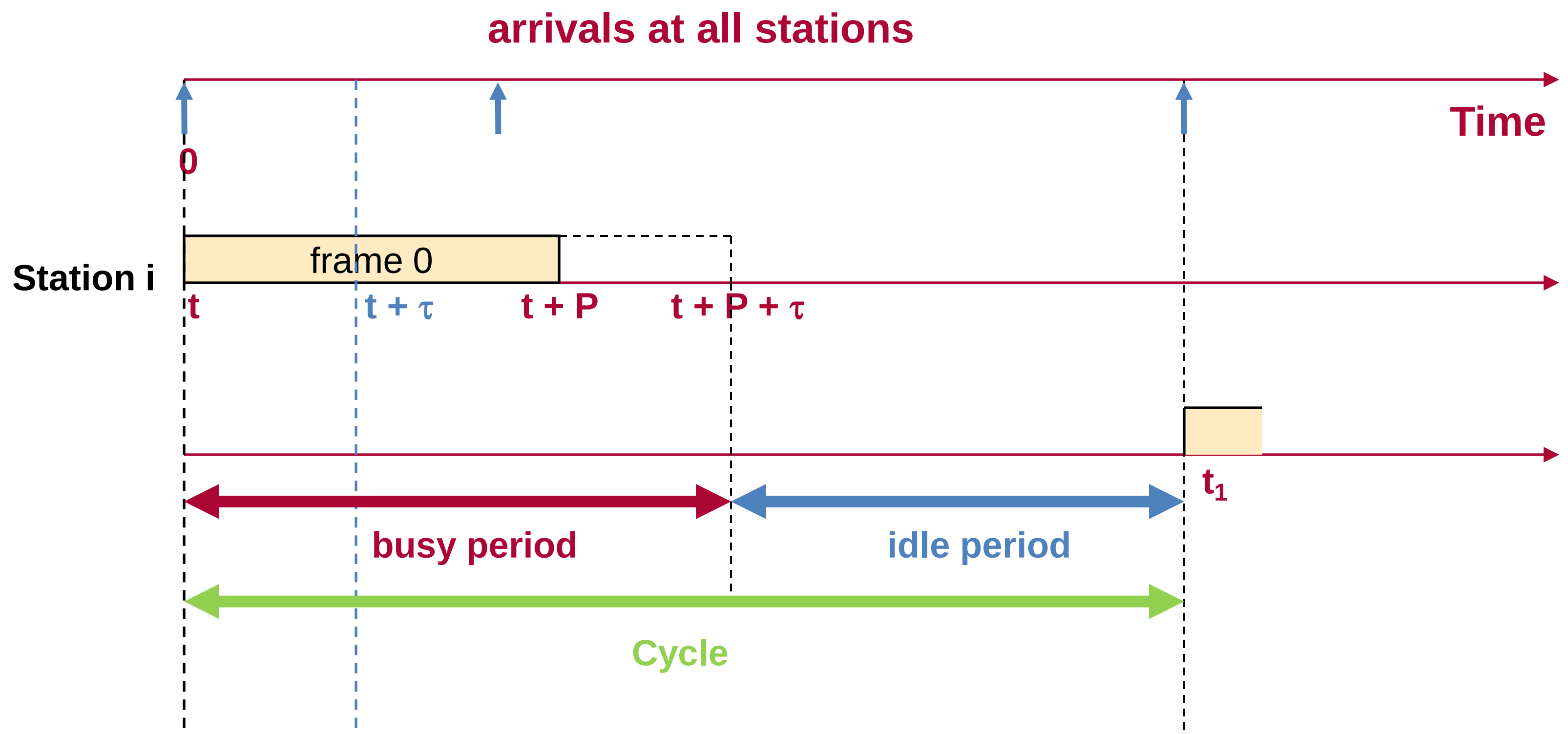
Cycle containing an unsuccessful Busy Period (nonpersistant CSMA)



Y: time until last and final arrival in the collision window

Details may be found in: L. Kleinrock, F.A. Tobagi “Packet Switching in Radio Channels: Part I: Carrier Sense Multiple Access Modes and Their Throughput-Delay Characteristics,” IEEE Trans. on Commun., COM-33, No. 12, Dec. 1975, pp. 1400-1416

Cycle containing a successful Busy Period (nonpersistant CSMA)



Note:

The successful busy period is shorter than the unsuccessful one.

Details may be found in: L. Kleinrock, F.A. Tobagi "Packet Switching in Radio Channels: Part I: Carrier Sense Multiple Access Modes and Their Throughput-Delay Characteristics," IEEE Trans. on Commun., COM-33, No. 12, Dec. 1975, pp. 1400-1416

Let

U denote the **time** during a cycle when the channel is **used without collision**,

I denote the average length in time of the **idle period**,

B denote the average length in time of the **busy period**.

Then the throughput can be expressed as:

$$S = \frac{\overline{U}}{\overline{I} + \overline{B}} = \frac{\text{time used for successful transmission}}{\text{total time}}$$

Determining the average values finally yields:

$$S = \frac{G \bullet e^{-aG}}{G(1+2a) + e^{-aG}} \text{ with } a = \tau / P$$

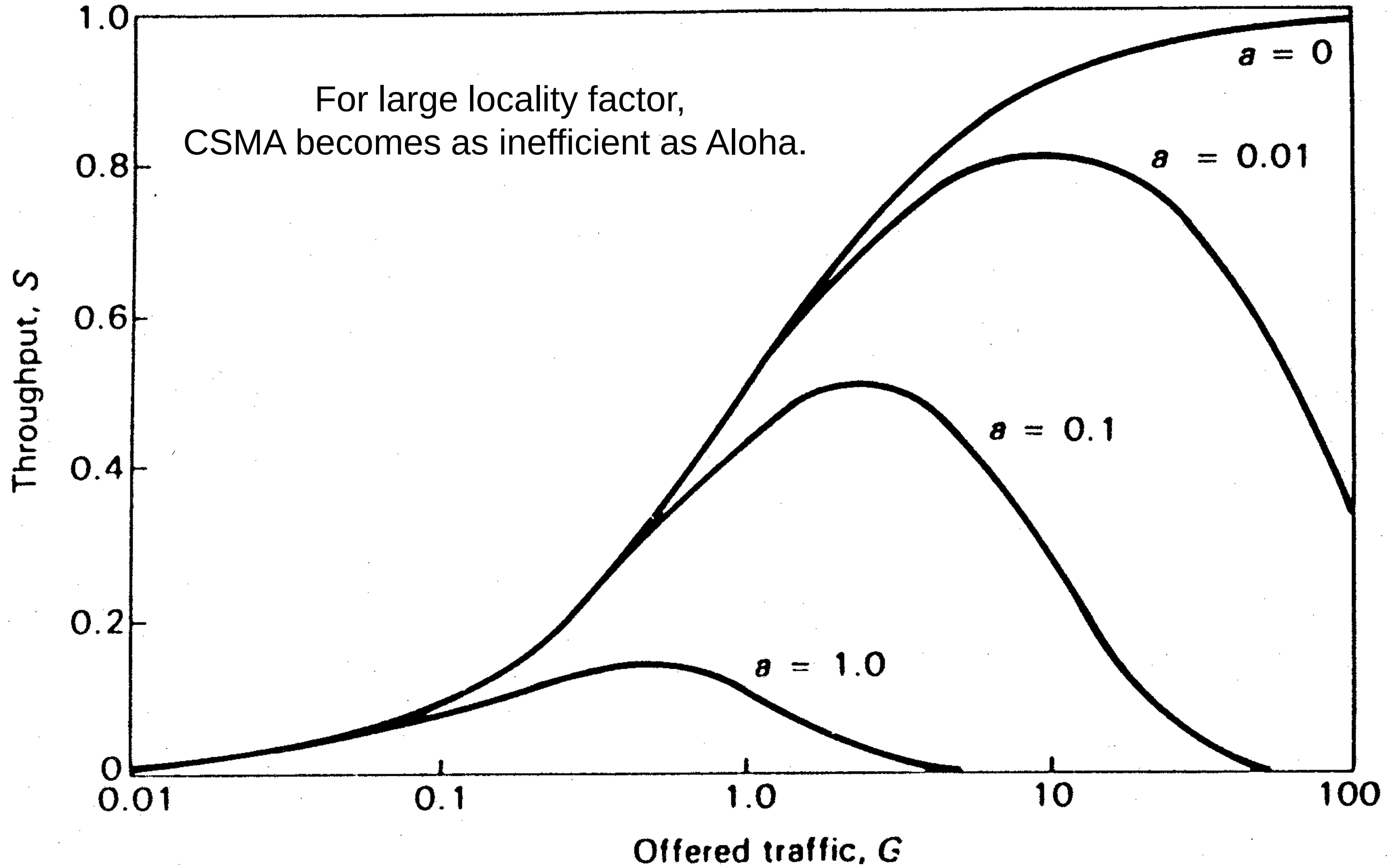
L. Kleinrock, F.A. Tobagi
"Packet Switching in Radio Channels: Part I: Carrier Sense Multiple Access Modes and Their Throughput-Delay Characteristics,"
IEEE Trans. on Commun., COM-33, No. 12, Dec. 1975, pp. 1400-1416

The Locality Factor

The "locality factor" is defined as

$a = \tau / (\overline{X}/R)$ = max. propagation delay / average frame TX_time

Throughput for Nonpersistent CSMA



Source:
J.L. Hammond, P.J.P. O'Reilly,
"Performance Analysis of Local Computer Networks," Addison-Wesley, 1986

Other variations of CSMA

For the other variations of CSMA, the throughput may also be expressed by formulae:

Slotted Nonpersistent CSMA:

$$S = \frac{a\,G \bullet e^{-aG}}{1 - e^{-aG} + a} \text{ with } a = \tau / P$$

1-persistent CSMA:

$$S = \frac{G\left[1 + G + aG\left(1 + G + aG/2\right)\right]e^{-G(1+2a)}}{G(1+2a) - \left(1 - e^{-aG}\right) + (1 + aG)e^{-G(1+a)}} \text{ with } a = \tau / P$$

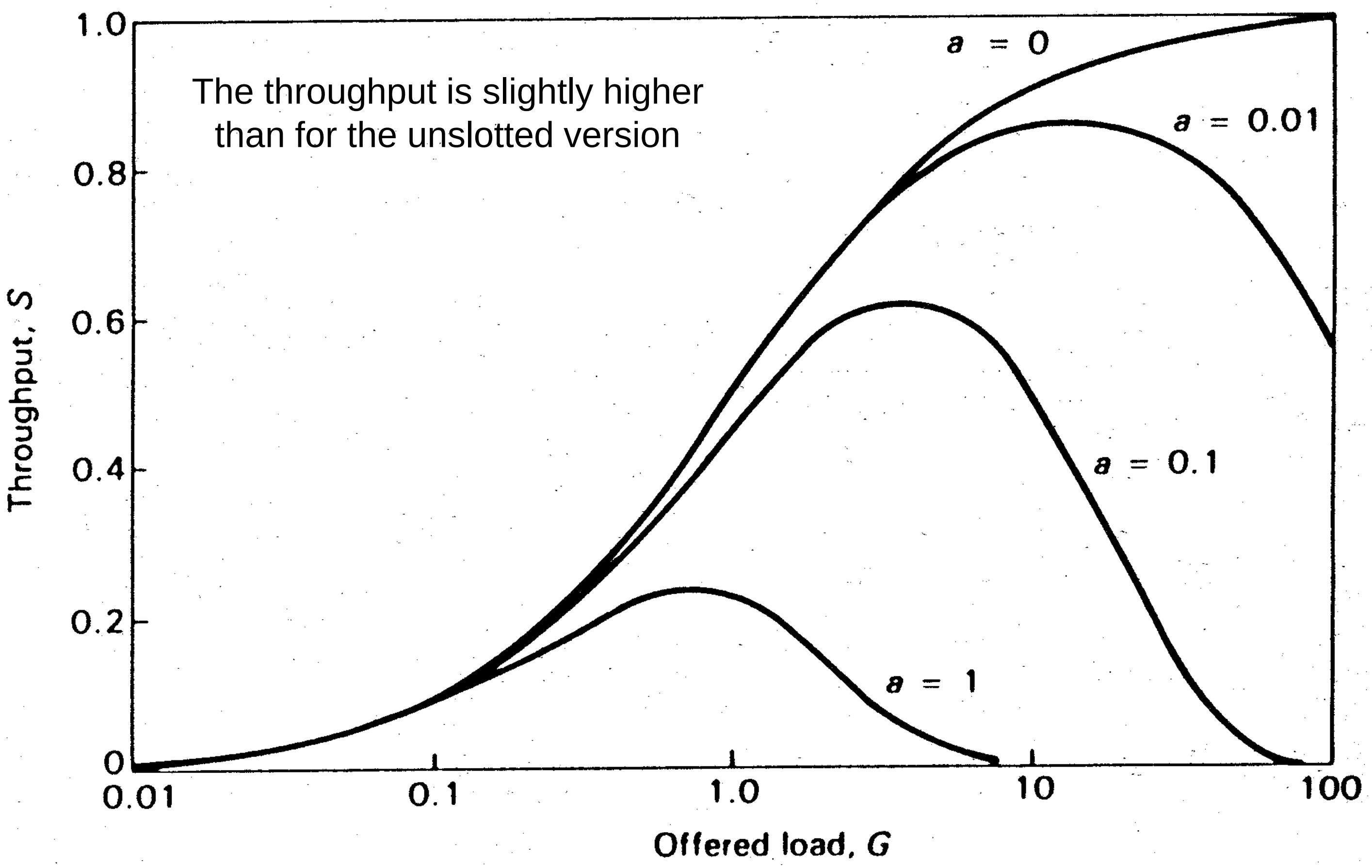
Slotted 1-persistent CSMA:

$$S = \frac{G\,e^{-G(1+a)}\left[1 + a - e^{-aG}\right]}{(1+a)\left(1 - e^{-aG}\right) + a\,e^{-G(1+a)}} \text{ with } a = \tau / P$$

Details may be found in:

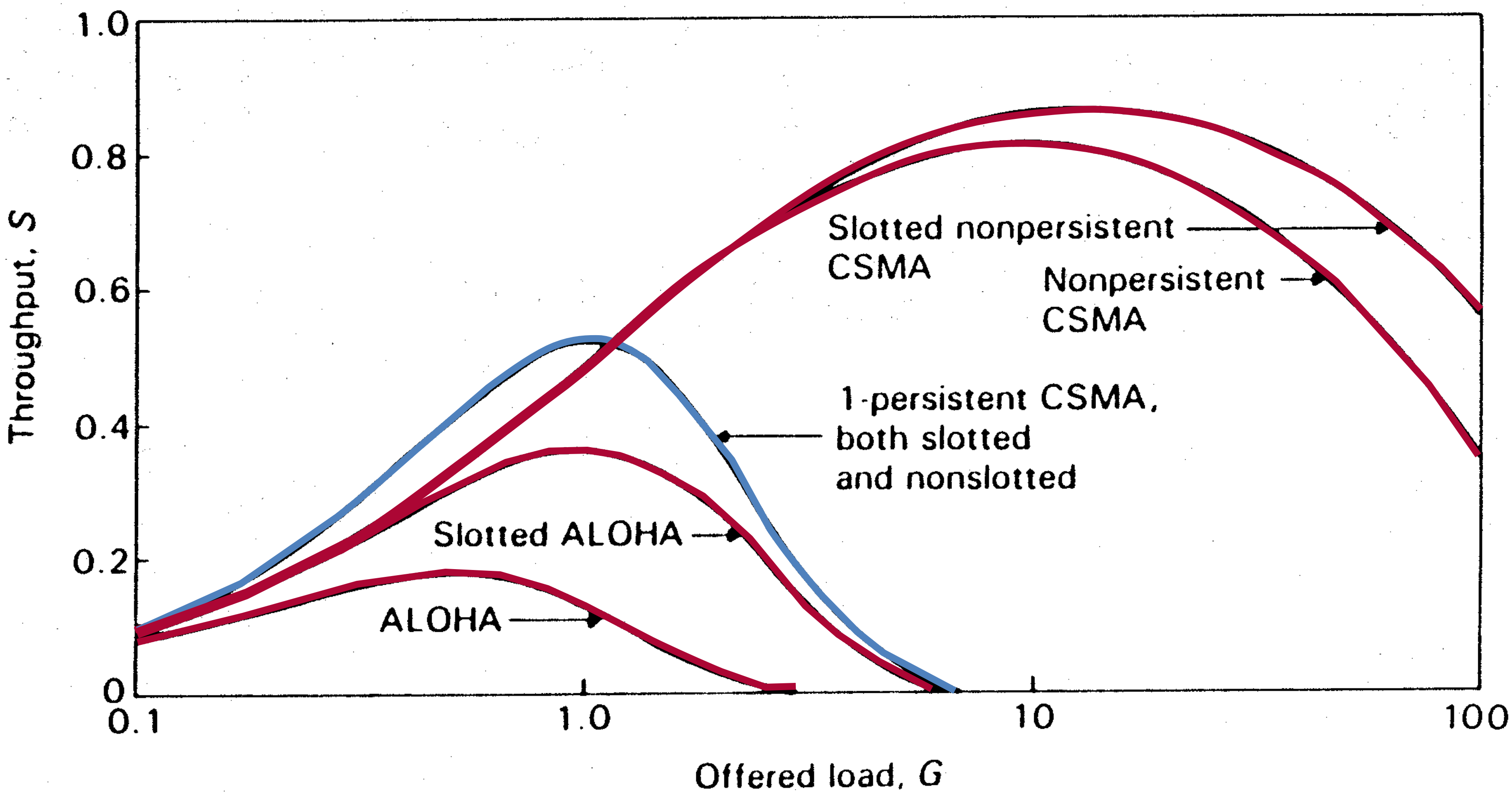
L. Kleinrock, F.A. Tobagi
“Packet Switching in Radio Channels: Part I: Carrier Sense Multiple Access Modes and Their Throughput-Delay Characteristics,” IEEE Trans. on Commun., COM-33, No. 12, Dec. 1975, pp. 1400-1416

Throughput for Slotted Nonpersistent CSMA



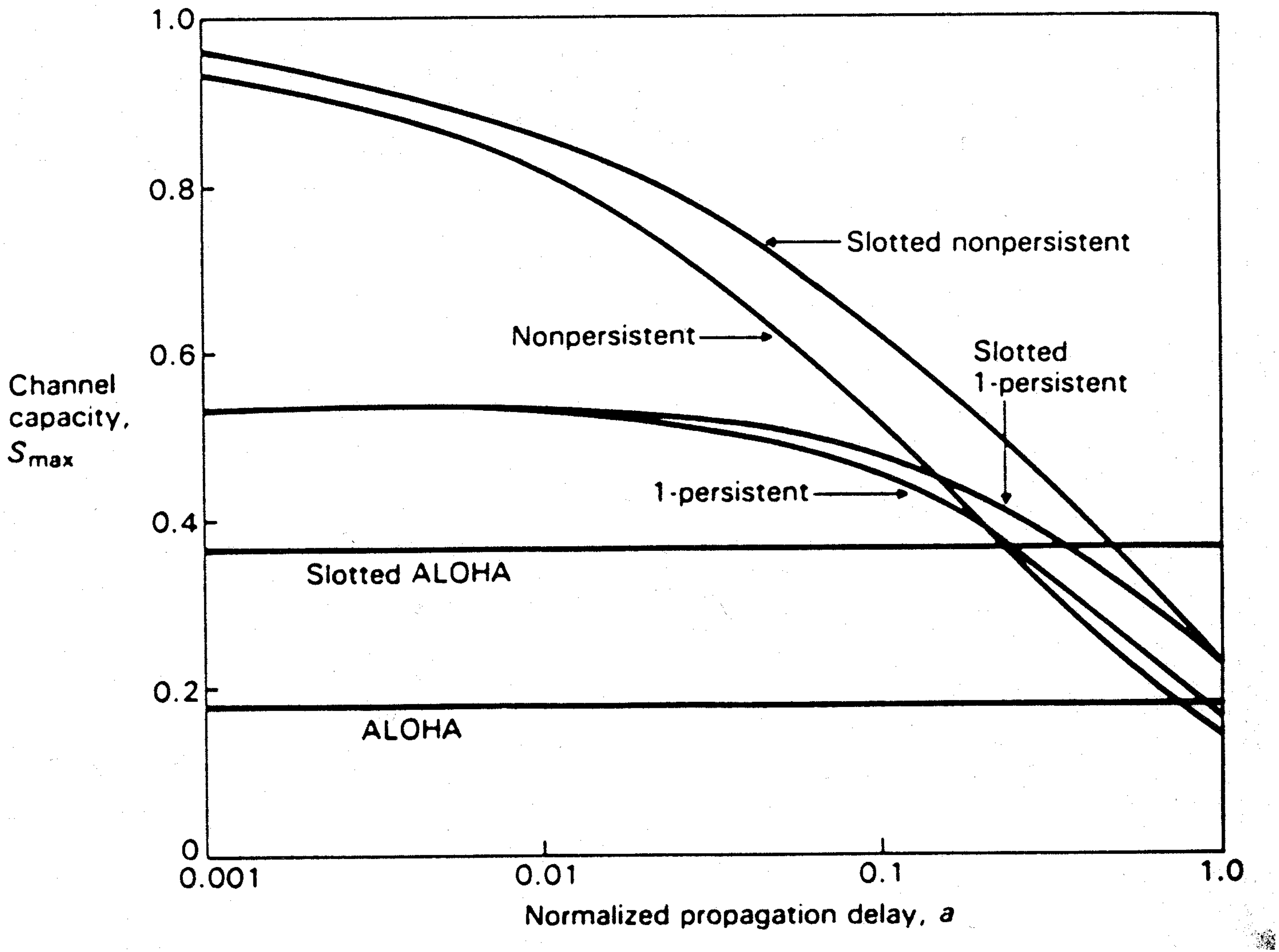
Source:
J.L. Hammond, P.J.P. O'Reilly,
“Performance Analysis of Local Computer Networks,” Addison-Wesley, 1986

Comparison of Aloha, Slotted Aloha and CSMA for a = 0.01



Source:
J.L. Hammond, P.J.P. O'Reilly,
“Performance Analysis of Local Computer Networks,” Addison-Wesley, 1986

Maximum Throughput versus Locality Factor a



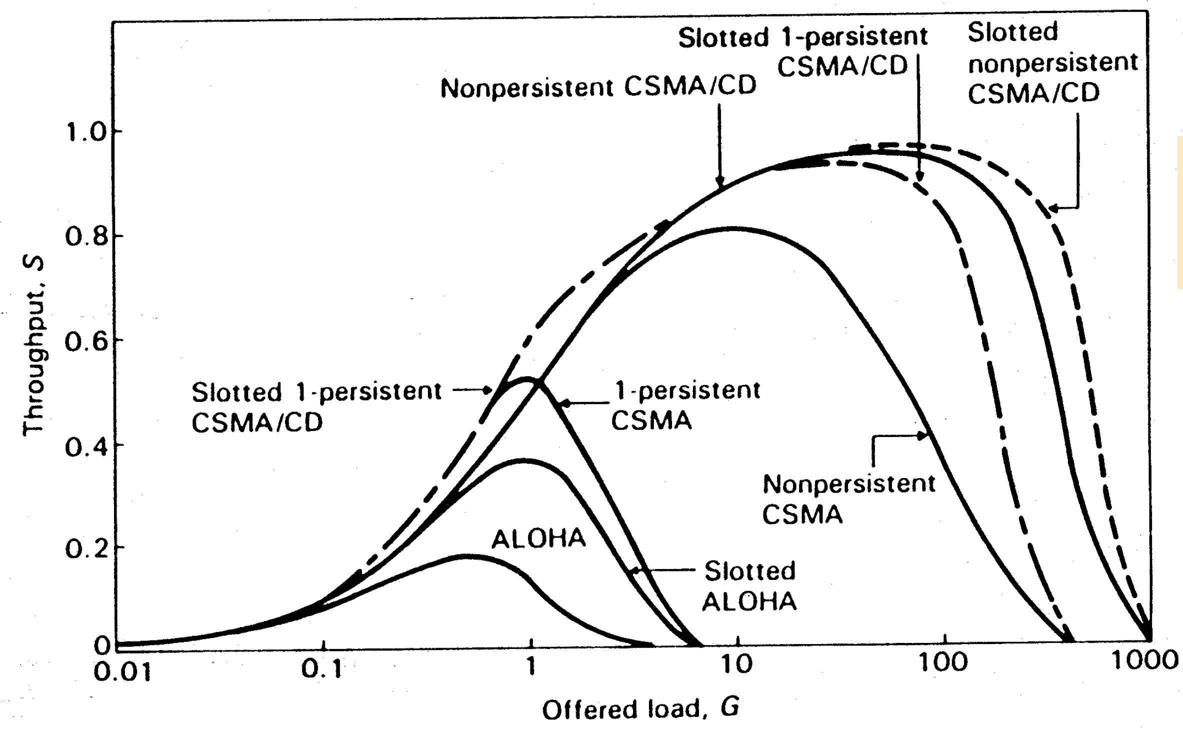
Source:
J.L. Hammond, P.J.P. O'Reilly,
“Performance Analysis of Local Computer Networks,” Addison-Wesley, 1986

3.3.4. CSMA/CD

When using CSMA with collision detection, the transmission is aborted as soon as a collision is detected.

For identical assumptions, an analysis similar to the analysis of CSMA yields complex formulae describing the throughput.

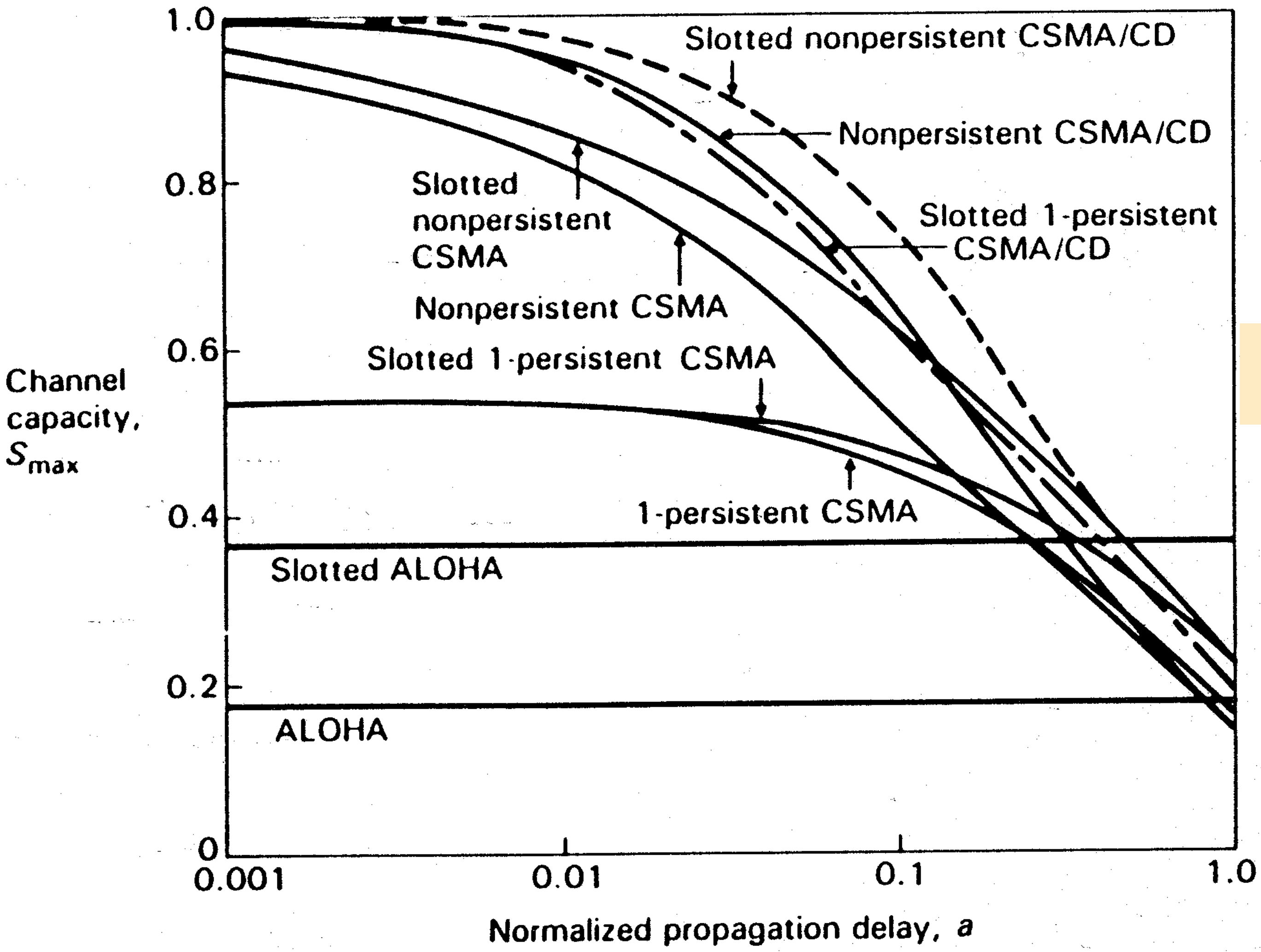
For a = 0.01 the graphical representation is as follows:



CSMA/CD with binary exponential backoff

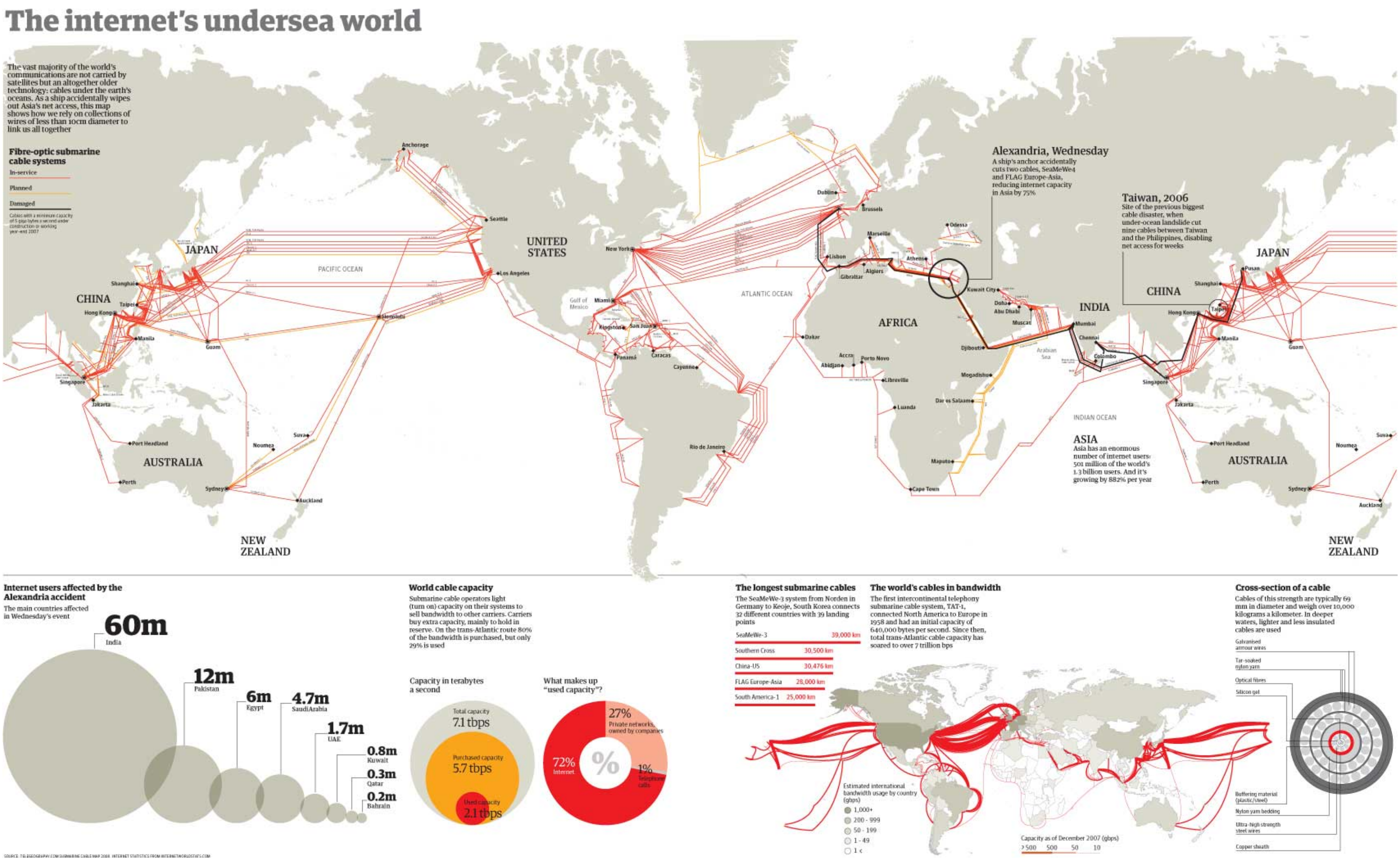
Source:
J.L. Hammond, P.J.P. O'Reilly,
“Performance Analysis of Local Computer Networks,” Addison-Wesley, 1986

Maximum Throughput versus Locality Factor a



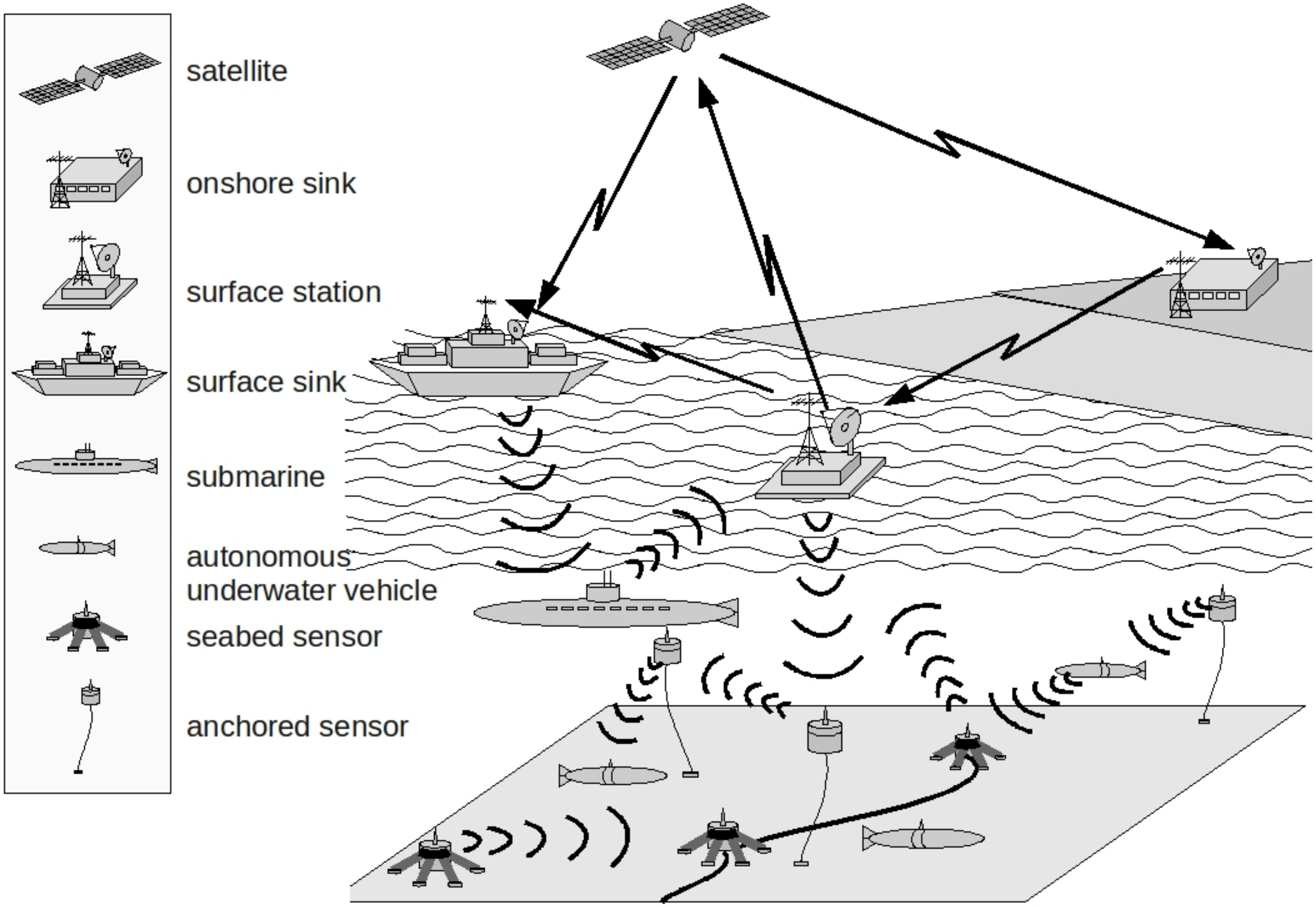
For very large locality factor, "CD" does not help any more.

Source:
J.L. Hammond, P.J.P. O'Reilly,
"Performance Analysis of Local Computer Networks," Addison-Wesley, 1986



Guardian.co.uk: The internet's undersea world

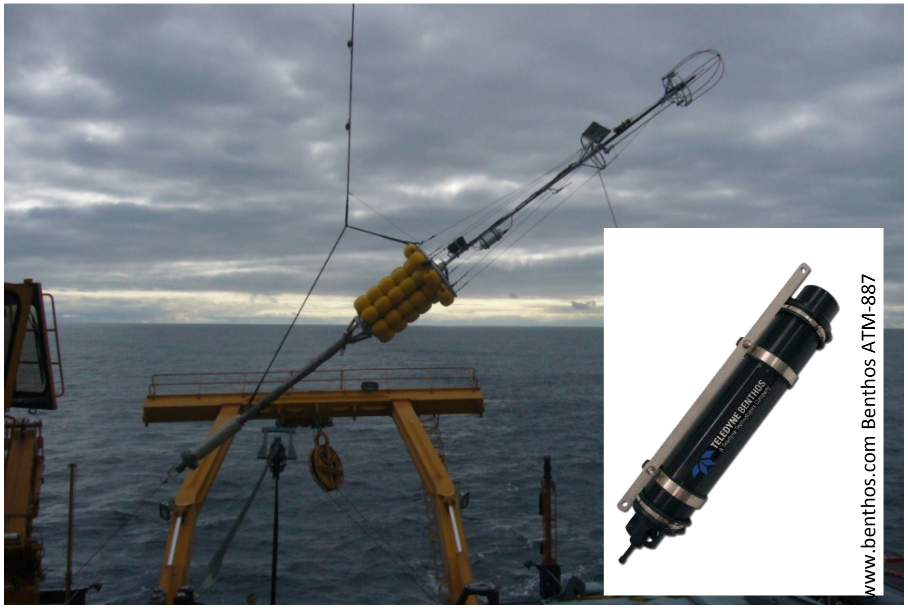
Underwater Acoustic Networks



J. Bauer: Diplomarbeit: Adaption und simulative Evaluation terrestrischer ad-hoc Routing-Protokolle für akustische Unterwassernetzwerke
(Diploma Thesis: Adaption and simulative evaluation of terrestrial ad-hoc routing protocols for underwater acoustic networks)

Underwater
- Long ranges possible
- Data rate depends on frequency (low rates)
- Signals travel with speed of sound

Terrestrial
- Short ranges due to sound level limits
- Low data rates
- Signals travel with speed of sound



www.mediaculture-online.de

Underwater Acoustic Networks suffer from major **disadvantages** compared to terrestrial radio networks:
- **low data rates** because of the high attenuation of high frequency signals
- **high bit error rates** because of omnipresent and strong noise
- very **slow signal propagation**

High bit error rates

Bit Errors and the Bit Error Rate (BER)

Transmitted bit sequence
1 0 1 0 1 0 1 0 1 0

Received bit sequence
1 1 1 0 1 1 1 0 0 0

Bit Error Rate 0.3

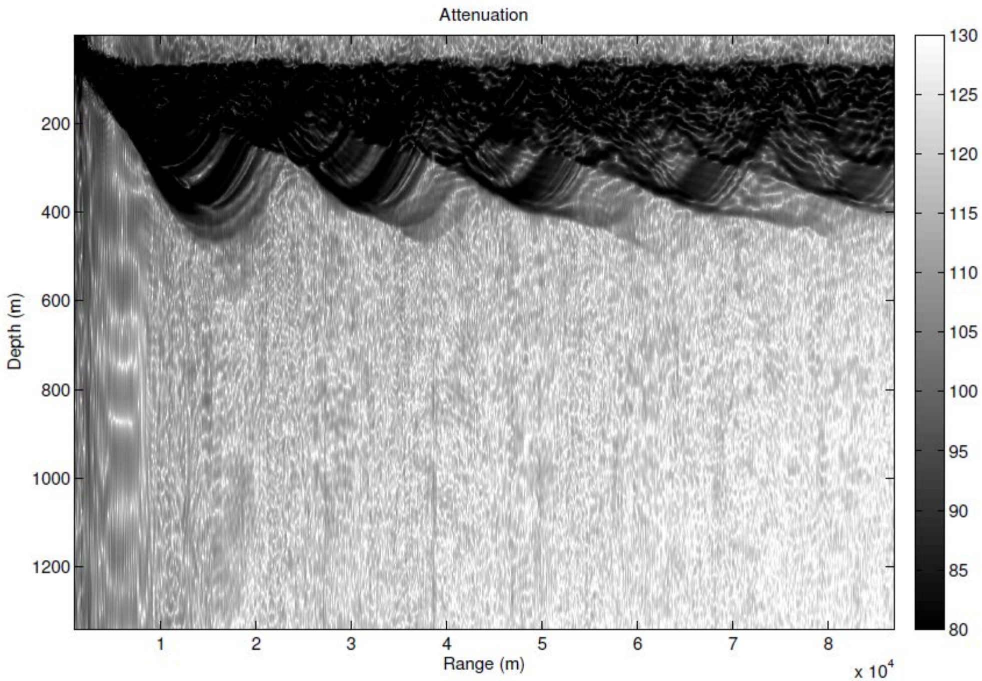
The BER is affected by

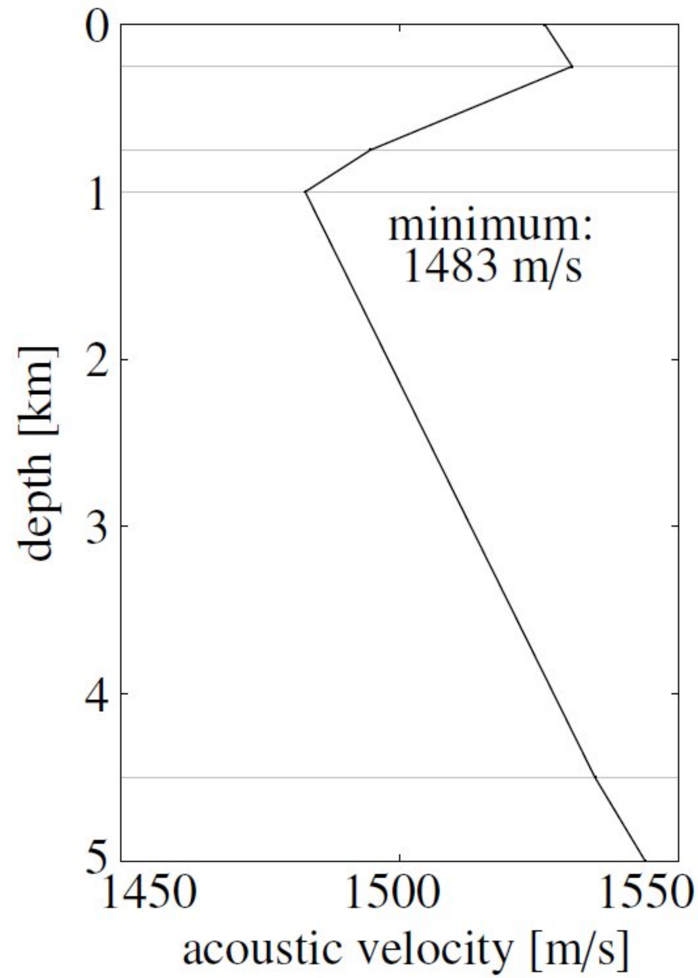
- transmission channel noise
- distortion
- attenuation
- multipath fading

Slow signal propagation

Underwater acoustic signal propagation follows the same physical rules, but the medium is quite different!

- Slow signal propagation
  - 1400-1500 m/s
- Propagation speed depends on water temperature, salinity and pressure





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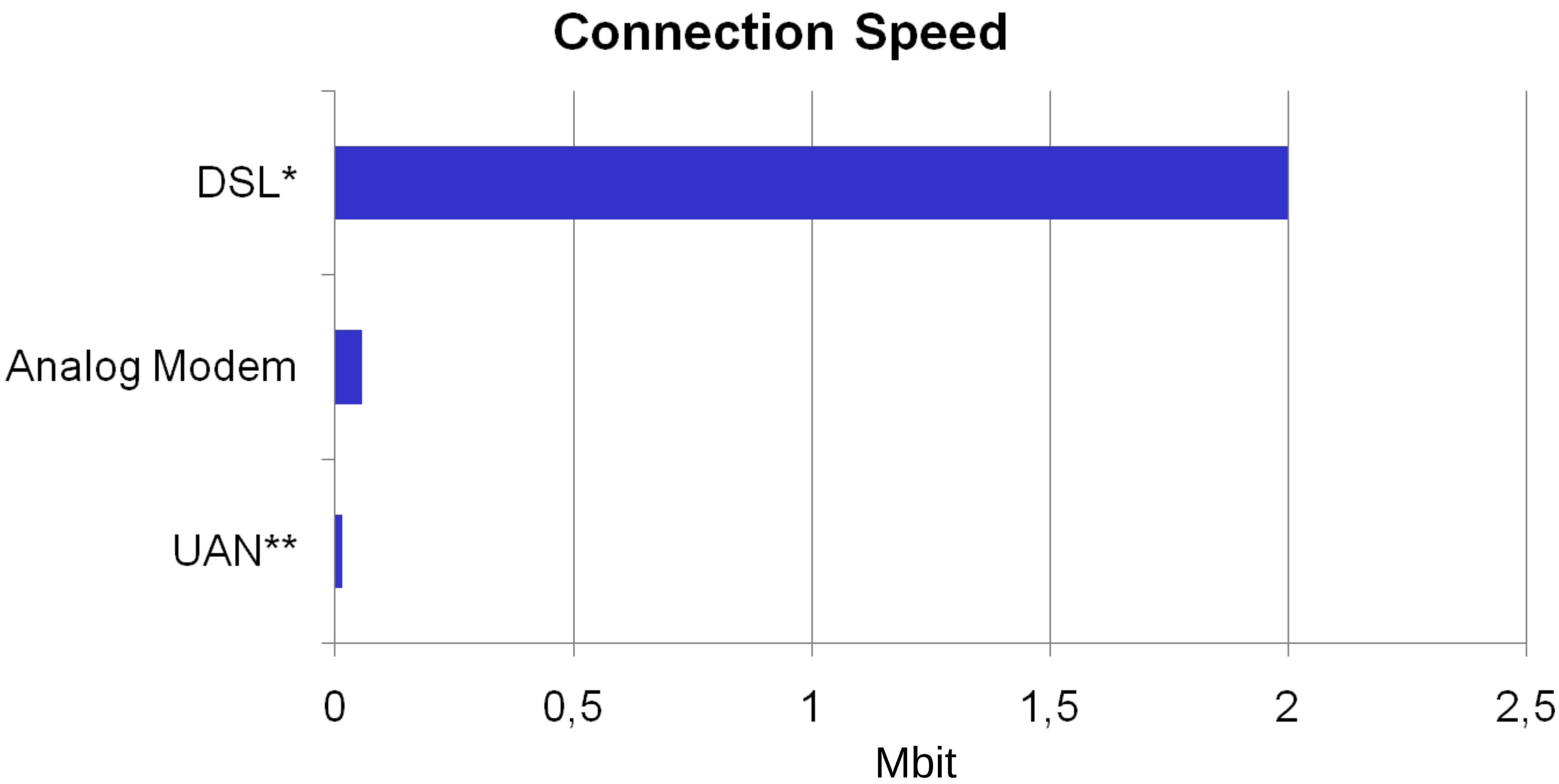

* Slowest DSL connection speed available from Telekom (16.Jan.2011)
** Teledyn Benthos ATM-887

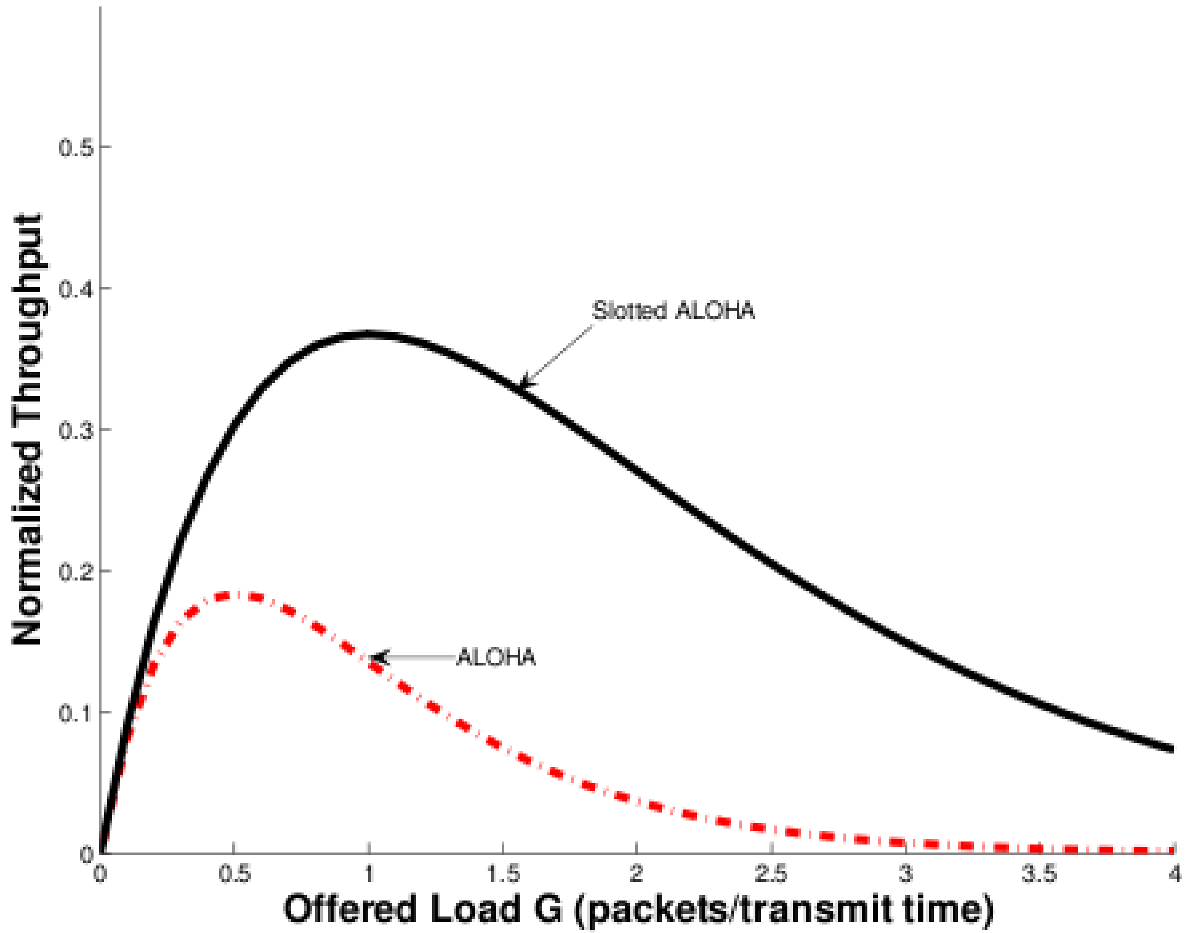

Figure 3: Classical throughput analysis for ALOHA.

Source: "Understanding Spatio-Temporal Uncertainty in Medium Access with ALOHA Protocols", Syed et al., Proc. WUWNET, 2007

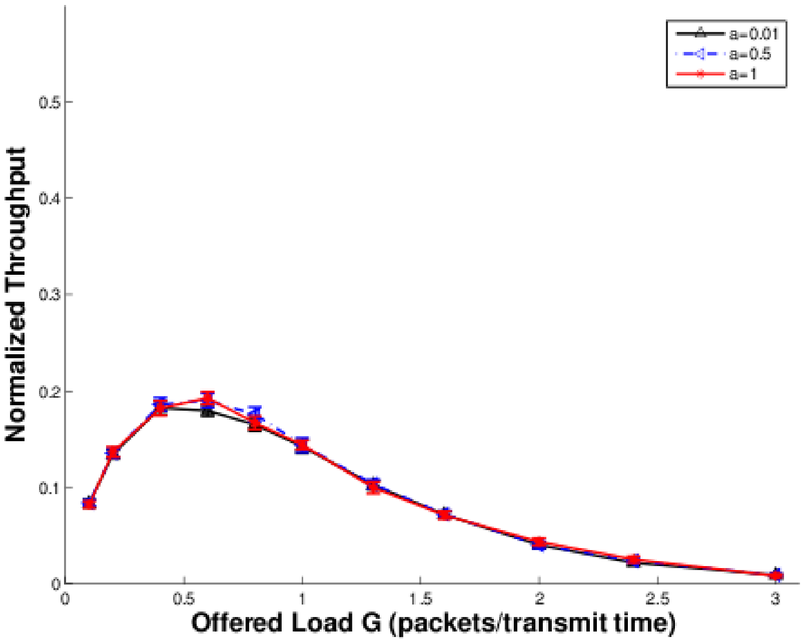

Figure 4: Throughput of pure ALOHA is not affected by propagation delay.

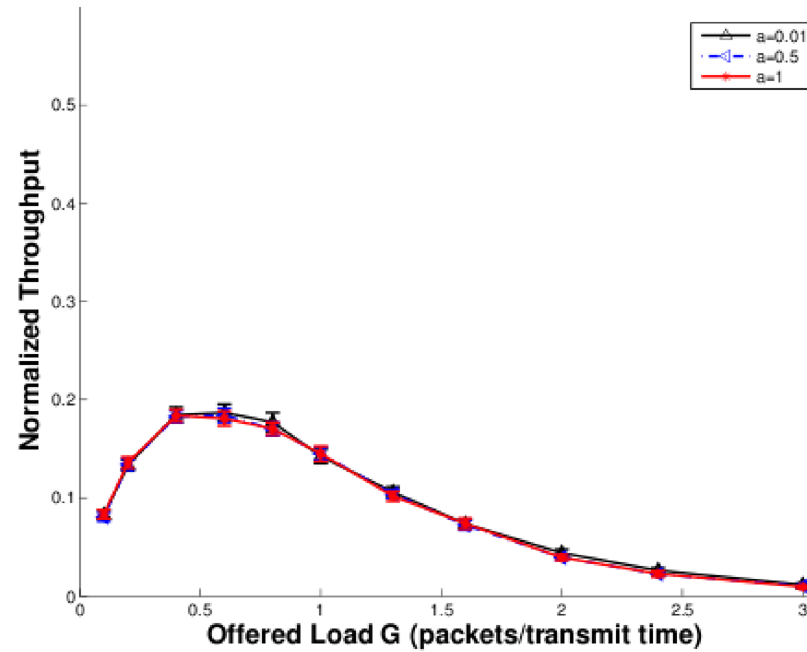

Figure 5: Throughput of slotted ALOHA degrades with *any* propagation latency.

Source: "Understanding Spatio-Temporal Uncertainty in Medium Access with ALOHA Protocols", Syed et al., Proc. WUWNET, 2007

.. a simple mathematical analysis as well as simulations to show the performance of Aloha and Slotted Aloha in the acoustic environment. Our results show that long **propagation delay of acoustic signals prohibits the coordination among nodes** (e.g., slotted methods), and thus, it does not yield any performance gain. ..

Gerla et al. "Analysis of Aloha Protocols for Underwater Acoustic Sensor Networks", Proc. of WUWNET 2006

Efficiency estimation of CSMA/CA in an UAN

Assumptions:
- **125 byte** packets and a data rate of **5000 bit/s**.
- interference range of **20 km** and a signal propagation speed of **1400 m/s**.

⇒ SIFS ~**14.3 s**

⇒ DIFS of **14.5 s** **(**DIFS > SIFS)

- ACK is just **10 byte**.

Time to send 1000bit

1000bit / 5000 bit/s + 14.5 s + 80 bit / 5000 bit/s + 14.3 s ≈ 29.2 s

Efficiency: #data bits sent / (total time * data rate)

= 1000 bit / (29.2 s*5000 bit/s) ≈ 0.007

⇒ **less than 1%** of the channel capacity is available for data.

⇒ **effective data rate** of only **34 bit/s**

- not considered the overhead of higher layer protocols
- not considered the signal-propagation time to the receiver