Probability ⇒

Coin Toss ⇒ $\{H, T\} = 1/2$

Dice Throw ⇒ $\{1, 2, 3, 4, 5, 6\} = 1/6$

2-Dice Throws = $\{(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)\} = 9/36 = 1/4$

$P(A) + P(B) = 1$

$= 1, 1$

Conditional Probability :)

2-events A & B $\Rightarrow$ $P(A) \neq 0$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

3-children $\Rightarrow$

All being boys

i) If we know nothing

ii) If we know two ~~eld~~ eldest are boys.

iii) " " " at least two of them are boys

$$S = \{(B,B,B)\ (B,G,B),\ (BBG),\ (GBB),\ (GGB),\ (G,G,G),\ (BGG),\ (GBG)\} = 8$$

$$A = \{BBB\} = 1/8$$

$$C = \{BBB, BBG\} = 2/8$$

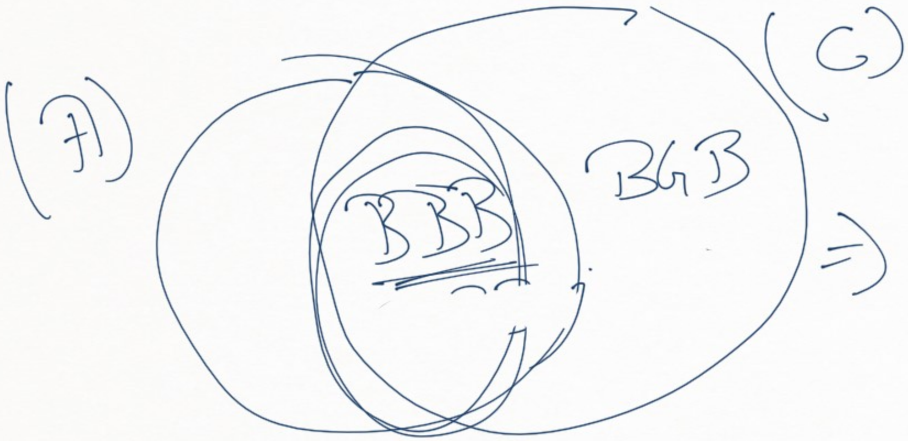
$$D = \{\ \} = 4/8$$

$$P(A) = 1/8,\quad P(C) = 2/8,\quad P(D) = 4/8$$

$$P(A \cap C) = 1/8 \quad P(A \cap D) = 1/8$$

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = 1/8 \times \frac{8}{2} = 1/2$$

$$P(A|D) = \frac{P(A \cap D)}{P(D)} = 1/8 \times \frac{8}{4} = 1/4$$



$\Rightarrow 1 = \underline{(BBB)}$ $(A \cap C)$ $\quad P(A \cap C) = 1/8$

$A = \{1, 3, 4\}$ $\quad B = \{2, 3, 5\} \Rightarrow$ $(A \cup B) = \{1, 3, 4, 2, 3, 5\}$

$(A \cap B) = \{3\}$

$(A - B) = \{1, 4\}$

$(B - A) = \{2, 5\}$

Sum is 8 = $P(A) = \{(2,6)\ (4,4)\ ,\ (3,5),\ (5,3),\ (6,2)\}$

$= 5/36$

First dice shows 3 = $P(B) = 1/6$

$P(A \cap B) = 1/36$ $\Rightarrow P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{1}{36} \times \dfrac{6}{1} = 1/6$

Getting 8 given first throw is 3 $\Rightarrow$ $\dfrac{A \cap B}{B}$

A = Sum 8 , $P(A) = 5/36$

B = First dice 3, $P(B) = 1/6$

$P(A|B) = \dfrac{1}{36} \times \dfrac{6}{1} = 1/6$

$A \Rightarrow$ First one is $3 \Rightarrow \frac{1}{6}$

$B =$ Sum $8 \Rightarrow \frac{1}{36}$

$A \cap B = \frac{1}{36}$ $\quad P(B|A) = \frac{1}{6}$

Bayes Theorem $\Rightarrow$ A can happen only if B happened

$$P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}$$

$P(X|Y=0) \Rightarrow P(X|Y=1)$ → Prior knowledge

$P(Y=0|X) \Rightarrow P(Y=1|X)$

Prediction

$P(X|Y) \Rightarrow P(Y|X)$

$$P(Y|X) = \frac{P(X|Y) \cdot P(Y)}{P(X)}$$

$P(X|Y=0)$ $P(X|Y=1)$

$P(Y=0|X)$ $P(Y=1|X)$

Color → X → A

Fruit_Name → Y → B

Red Apple

Yellow Banana

Green Orange

$P(X|Y) \Rightarrow$ Training Data

$P(Y|X) \Rightarrow$

$P(A) =$

$P(B) =$

$P(A|B) =$

$P(B|A) =$

$P(Y = \text{Yes}) = 9/14$

$P(Y = \text{No}) = 5/14$

$P(X = \text{Sunny} \mid Y = \text{yes}) =$

$P(X = \text{Sunny} \mid Y = \text{No}) =$

$P(A|B) \Rightarrow P(B|A)$

A can occur only if n mutualy exclusive events occur

$\boxed{B_1, B_2, B_3 \ldots B_n}$

$P(A|B_1), P(A|B_2), P(A|B_3) \ldots P(A|B_n)$

$$P(B_i|A) = \frac{P(B_i) \cdot P(A|B_i)}{P(A)}$$

$$P(B_i/A) = \frac{P(A|B_i).P(B_i)}{P(A)}$$

A depends on $B_1 - - - B_n$

$$\left\{ \begin{array}{l} B_1 \Rightarrow \\ P(A|B_1) = \end{array} \right.$$

$*$ $P(AB) = P(A).P(B/A)$

$P(B_1 A) = P(B_1).P(A/B_1)$

$$P(A) = P(B_1 A + B_2 A + \cdots B_n A)$$

$$= P(B_1 A) + P(B_2 A) + \cdots + P(B_n A)$$

$$= \boxed{P(B_1).P(A/B_1)} + P(B_2).P(A/B_2) + \cdots + P(B_n).P(B_n/A)$$

$$= \sum_{i=1}^{n} P(B_i).P(A/B_i)$$

$$P(B_i|A) = \frac{P(A|B_i).P(B_i)}{\sum_{i=1}^{n} P(B_i).P(A/B_i)}$$

3-Boxes $\Rightarrow$ $\underset{I}{4W\,3R}$, $\underset{II}{3W\,7R}$, $\underset{III}{2W\,3R}$

Choose Box at Random, Ball is White, Find Probabilit that the box II

3-Boxes ⇒ 4W3R ⇒ I, 3W7R ⇒ II, 2W3R ⇒ III

We choose Random Box and drawn ball is white.

$B_1 \ldots B_n$ as Box selection

A ⇒ The ball being white

(52)

$P(B_1) = P(B_2) = P(B_3) = 1/3$

$P(A|B_1) = 4/7$, $P(A|B_2) = \frac{3}{10}$, $P(A|B_3) = \frac{2}{5}$

$$P(B_2|A) = \frac{P(B_2).P(A|B_2)}{\sum_{i=1}^{3} P(B_i).P(A|B_i)} = \frac{1/3 \times \frac{3}{10}}{P(B_1) \times P(A|B_1) + P(B_2) \times P(A|B_2) + P(B_3).P(A|B_3)}$$

$$= \frac{1/3 \times \frac{3}{10}}{1/3 \times \frac{4}{7} + 1/3 \times \frac{3}{10} + 1/3 \times \frac{2}{5}}$$

$$= \frac{1}{10} \times \frac{210}{89} = \frac{21}{89}$$

$$\frac{4}{21} + \frac{1}{10} + \frac{2}{5} = \frac{40 + 21 + 28}{210} = \frac{89}{210}$$

$P(X \mid Y = Yes) \quad P(X \mid Y = No)$

$P(X_i \mid Y = Yes) \quad P(X_i \mid Y = No) \Rightarrow P(Y = Yes \mid X_i)$

$P(Y = No \mid X_i)$