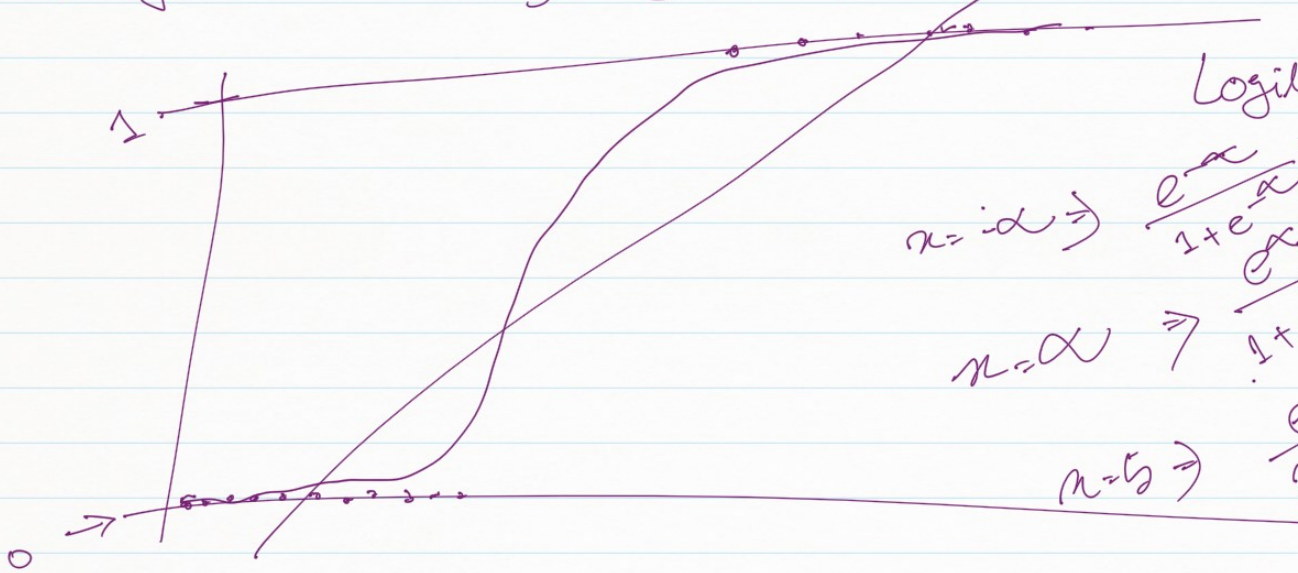
Classification ⇒ Linear methods ⇒ $h_\theta(x) = \theta^T X$

Logistic Regression ⇒

Apply a Logit Function

Sigmoid Function ⇒

Target variable ⇒ 2 classes ⇒ 0, 1



Logit Function ⇒ $\dfrac{1}{1+e^{-x}}$

$x = -\infty \Rightarrow \dfrac{e^{-\infty}}{1+e^{-\infty}} \cong 0$

$x = \infty \Rightarrow \dfrac{e^{\infty}}{1+e^{\infty}} \cong 1$

$x = 5 \Rightarrow \dfrac{e^5}{1+e^5} = 0.73$

$\Rightarrow \dfrac{1}{1+\dfrac{1}{e^x}}$

$\Rightarrow \dfrac{e^x}{1+e^x}$

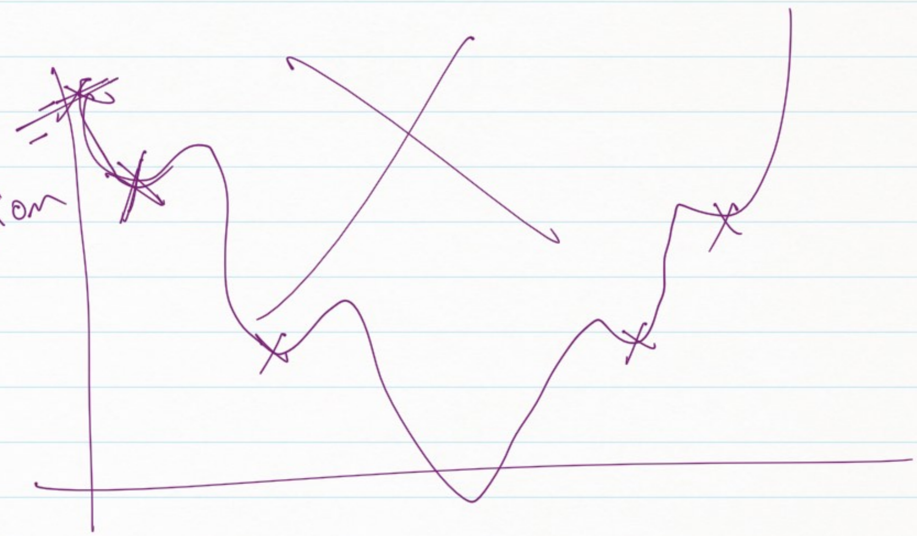
$h_\theta(x) = \theta^T X$

$$\text{logit}(h_\theta(x)) = \frac{e^{\theta^T x}}{1+e^{\theta^T x}}$$

$$= \frac{e^{\theta_0+\theta_1 x_1+ \dots + \theta_n x_n}}{1+e^{\theta_0+\theta_1 x_1+ \dots + \theta_n x_n}}$$

$$\frac{1}{2m}\sum(y_i - \hat{y})^2 \Rightarrow$$

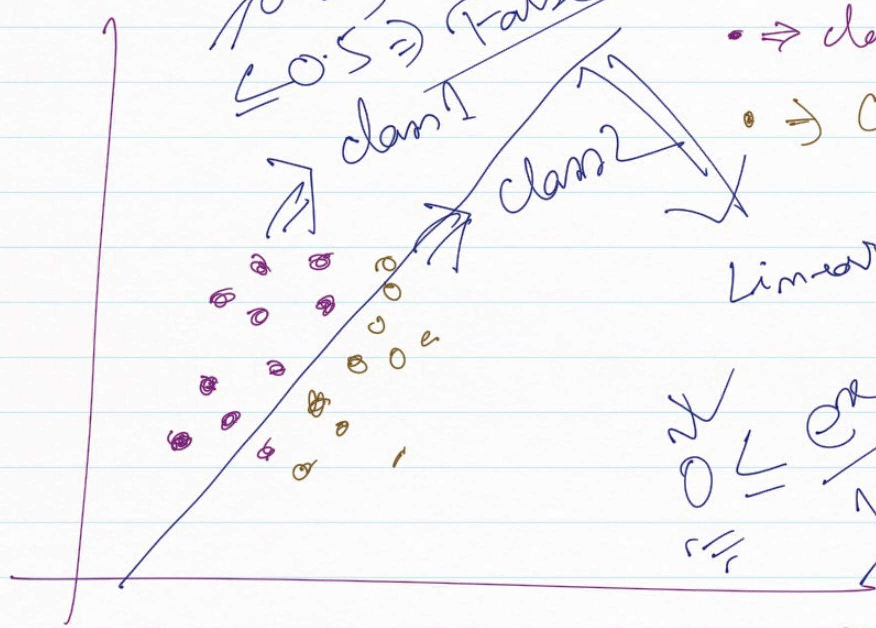
Error function graph



Probability ⇒ $\underline{0 \text{ to } 1}$ ⇒ Probability = $\theta^T X$ ⇒ Single value (X) ⇒ $\frac{e^x}{1+e^x}$

> 0.5 ⇒ class 1, true

≤ 0.5 ⇒ False



• ⇒ class 1

• ⇒ class 2

Linear for Logistic Regression

$$0 \le \frac{e^x}{1+e^x} \le 1$$

Asymptotic Fn ⇒ $\left.\begin{array}{l} x \Rightarrow 0 \\ x \Rightarrow 1 \end{array}\right\}$ ⇒ Asymptotic

Logit Fn = $\dfrac{e^{h_\theta(x)}}{1+e^{h_\theta(x)}}$

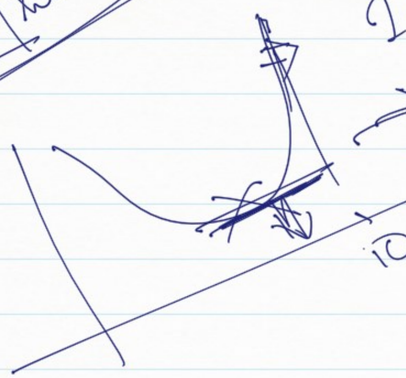
$$= \frac{e^{\theta^T X}}{1+e^{\theta^T X}}$$

$$= \frac{e^{(\theta_0+\theta_1 x_1 + \cdots + \theta_n x_n)}}{1+e^{(\theta_0+\theta_1 x_1 + \cdots + \theta_n x_n)}}$$

Linear Regression

$$\frac{1}{2m}\sum (h_\theta(x_i) - \hat{y}_i)^2$$

$$= \frac{1}{2m}\left\{ (h_\theta(x_0)-\hat{y}_0)^2 + (h_\theta(x_1)-\hat{y}_1)^2 \right\}$$



$\Rightarrow$ Derivatives

Error Function For Logistic Regression

Cost Function / $J(\theta) \Rightarrow -y \log \hat{y} - (1-y) \log (1-\hat{y})$

$\Rightarrow -y \log (P(y_i)) - (1-y_i) \log (1 - P(y_i)) \Rightarrow \simeq 0$

$\Rightarrow 0$

$\log (1 - P(y_i))$

$= y = 1$

$P(y_i) = 1 \Rightarrow -\log 1 = 0$

$P(y_i) = 0 \Rightarrow -\log(0) = \infty$

$P(y_i) = 0 \Rightarrow$

$P(y_i) \Rightarrow 1 =$

$y = 0$

$y = 1$

$y = 0 \Rightarrow$

$P(y_i) = 0 \Rightarrow -\log 1 \Rightarrow 0$

$P(y_i) = 1 \quad -\log 0 \Rightarrow \infty$

$\rightarrow 0.001$

$y=0$ & $P(y_i) < 0.5$

$y=1 \Rightarrow P(y_i) > 0.5$

Logistic Regression $\Rightarrow$ Outputs Probability $\Rightarrow$ Decision Boundary.

$y=1 \mid P(y_i) = 1$

$-\log(1) = 0$

$$\text{Log Loss} \Rightarrow -\frac{1}{N}\sum_{i=1}^{n} \log(P(y_i))$$

$P(y_i) = 1$

$\log(1-1) = \alpha$

$\log(0) = \alpha$

$\Rightarrow$ Class $= 0 \Rightarrow -y \log(P(y_i)) \Rightarrow 0$

Class $= 1 \Rightarrow -(1-y)\log(1-P(y_i)) \Rightarrow 0$

$y=0 \mid$ Class $=0$

$\cancel{-y \log(P(y_i))} - (1-y)\log(1-P(y_i))$

$= -\log(1-P(y_i))$

$\Rightarrow P(y_i) \Rightarrow 0$

$-\log(1) \Rightarrow 0$

R^2 ⇒ Confusion Matrix ⇒ $10 \Rightarrow 7T,\ 3F$ → 5FN, 1TN

4T, 6F → 2TP 2FP $= \frac{2}{4} = \frac{1}{2}$

P \ A	True	False
True	TP	FP
False	FN	TN

$\frac{2}{3} = 67\%$

$\Rightarrow \frac{TP + TN}{TP + FP + FN + TN}$ ⇒ Accuracy

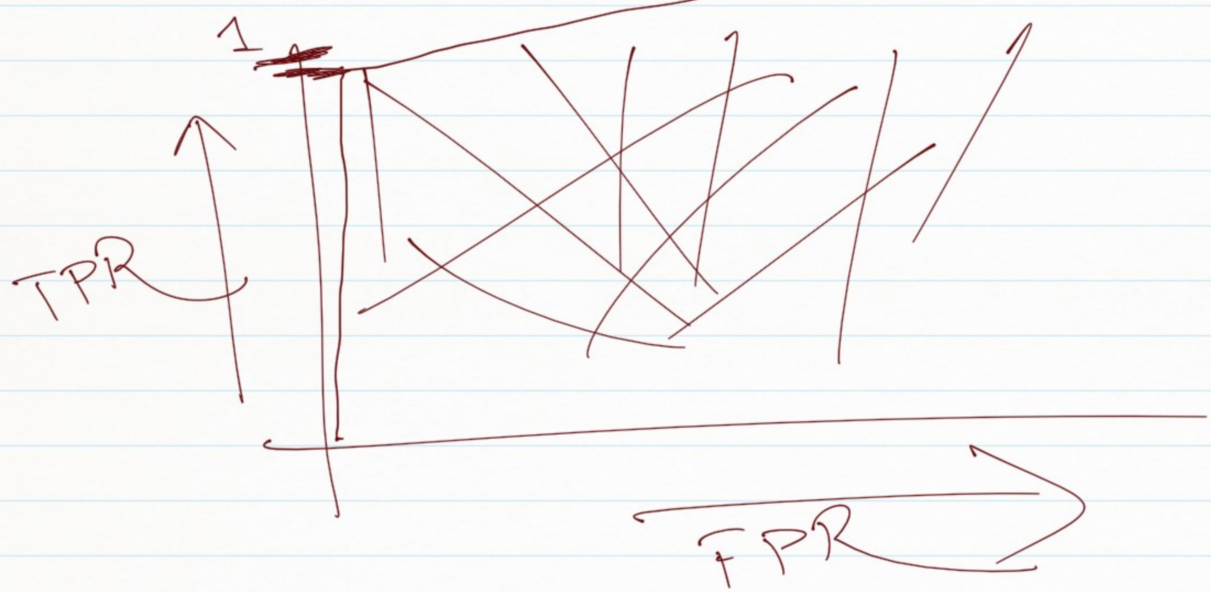
Specificity ⇒ $\frac{FP}{FP + TN}$ ⇒ False Positive Rate

Sensitivity ⇒ $\frac{TP}{TP + FN}$ ⇒ Recall → True positive Rate.

Precision ⇒ $\frac{TP}{TP + FP}$ ⇒ Ratio of True predictions over total true predictions

F1-Score ⇒ $2 \times \frac{\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}$

ROC $\Rightarrow$ FPR / TPR $\Rightarrow$ ROC AUC Score

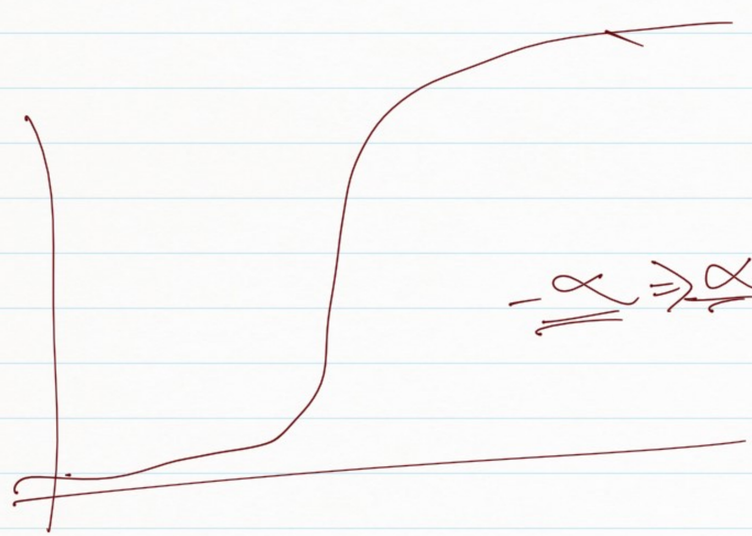


Binary Classification $= \{0, 1\}, \{True, False\}$

Multinomial Classification = Target ⇒ Discrete

$\{0, 1, 2\}$

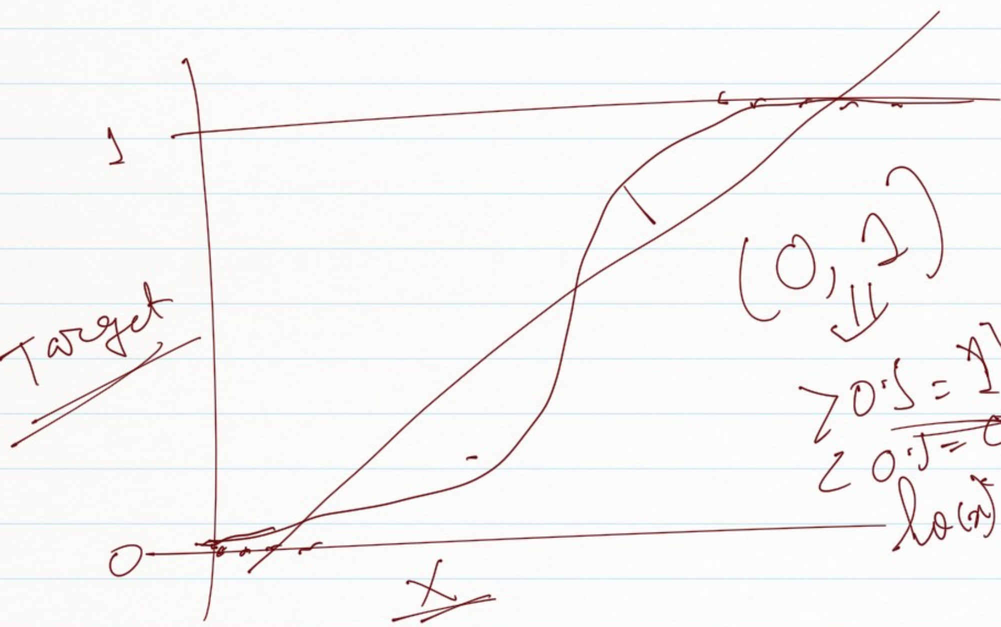
$\{A, A+, B, B+, C\}$



$-\infty \Rightarrow \infty \Rightarrow \{0, 1\}$ $h_\theta(x) \Rightarrow logit$ $\frac{1}{1+e^{-h_\theta(x)}}$

Linear models $\Rightarrow \underline{h_\theta(x) = \theta^T X} \Rightarrow X^T \theta.$

$= \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \dots + \theta_n x_n$



Sigmoid $\Rightarrow \frac{1}{1+e^{-x}}$

$(0, 1)$

$\Downarrow$

$> 0.5 = 1$

$< 0.5 = 0$

Logit $\Rightarrow \frac{1}{1+e^{-h_\theta(x)}}$

$h_\theta(x) \Rightarrow -\infty \Rightarrow \frac{1}{1+e^{\infty}} \simeq 0$

$\Rightarrow \infty = \frac{1}{1+e^{-\infty}} \simeq 1$

True False

$\{0, 1\}$

A B

$A, B, C \Rightarrow P(A) + P(B) + P(C) = 1$

$0 \leq P(E) \leq 1$

Coin Toss = $\{H, T\}$

$\frac{1}{2} + \frac{1}{2} = 1$

$\{HH, HT, TH, TT\}$

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$

$x_i \Rightarrow K$ - Classes

$\sum_{c=1}^{K} Pr(x_{ic}) = 1$

$K \Rightarrow$ Baseline 1 class

$(K-1)$ Classes

$P(A) + P(B) + P(C) = 1$

$P(B)\left(1 + \text{logit of class } A + \text{logit of class } C\right)$

$P(B) = \frac{1}{1 + \ldots}$

K^{th} class fixed

$P(>0.5)$ A else 0

$A \Rightarrow$ Logistic Reg model.

$B =$ "

$C =$ "

$\frac{P(A)}{P(B)} = \frac{1}{1 + e^{-h_\theta(x)}}$

$P(A) = P(B) \times \text{logit}$

$\frac{P(C)}{P(B)} = \frac{1}{1 + e^{-h_\theta(x)}}$

$P(C) = P(B) \times \text{logit}$

Baye's Theorem

Probability & Conditional Probability ⇒

Probability ⇒ Chance of something probability

Dice throw ⇒ $\frac{1}{6} \times 6 = 1$ $\{HH, TT, HT, TH\}$

$\{1, 2, 3, 4, 5, 6\}$
outcomes.

Conditional Probability ⇒ $P(A) \Rightarrow 1/2$

$P(B) \Rightarrow 1/2$

P(Both faces will be head) = $1/4$

$P(A) = \frac{P}{}$

6 Balls $\Rightarrow$ 3 Red, 3 Black

We are picking balls

Probability of 2 balls being black from 2 picks $\Rightarrow$ when first one is already black

1-ball $\Rightarrow \frac{3}{6} = \frac{1}{2}$

2-balls $\Rightarrow \frac{3}{6} \cdot \frac{2}{5}$

$P(A) = 3/6$

$P(B) = 2/5$

$P(A|B) = 3/6 \times 2/5 = 1/5$

$P(B|A) =$

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

$$P(A|B) \cdot P(B)$$

$$P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}$$

100 ⇒ 40 B Alarms (⇒)
⇒ 30 Bought Seats =
⇒ 20 Bought Both

$P(A) = P(\text{Buying Alarms}) = \frac{40}{100} = 0.4$

$P(B) = P(\text{Buying Seats}) = \frac{30}{100} = 0.3$

$P(A \cap B) = 0.4 \times 0.3$

$= 0.12$

$P(B|A) = \frac{0.12 \times 0.3}{0.4}$

$= 0.09$

$Pr(Y \mid X = x) \Rightarrow \cancel{(X)}$

$P(\text{Playing} = \text{Yes}) \Rightarrow$ } $\Rightarrow$ (Ac. 1 Sun. 1 Win?)

$P(\text{Humidity} = \text{High}) \Rightarrow$

$$\frac{P(A \mid B) = X}{P(B \mid A)} = \text{Baye's Formula.}$$