CASA Colloquium, February 15, 2023, Eindhoven University of Technology

A numerical modeling technique designed for error insight, with application to 1D shallow-water flow simulations

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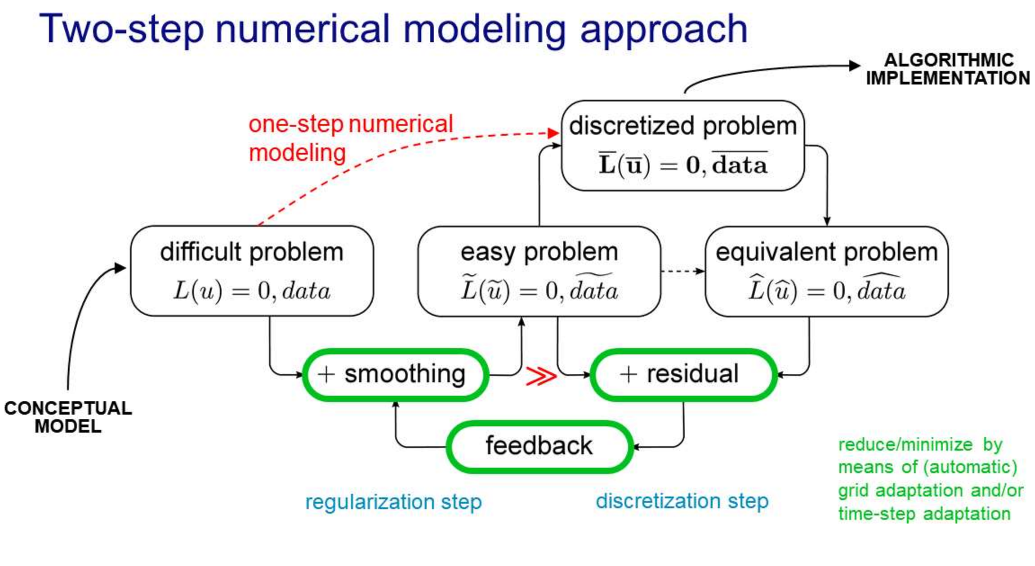
When designing a numerical implementation (discretization and solution method) of a physical model (equations, boundary and initial conditions, geometry, parameters), there are many aspects that one must pay attention to. Examples of properties that are sought are accuracy, efficiency, robustness, flexibility and suitability for modern hardware (parallelizability), but also ease of (software) development and (software) maintenance.

In this presentation we focus on reliability. Can a numerical technique be developed that, next to the solution, provides insight in the numerical modeling error in that solution? We have constructed such a method by means of a two-step modeling approach, cf. the Figure below: in the first step physics-based smoothing errors are introduced to transform a problem that is numerically difficult into one that is easy. Solving the easy problem numerically in the second step should then be relatively straightforward. The idea is that the physically interpretable smoothing errors of the first step form the dominant numerical errors and that the effect of discretization errors in the second step can be ignored. This requires, besides a proper feedback of the discretization errors into smoothing coefficients, a determination of the discretization errors. A serious complication is that not the discretization errors in the model equations are required, but their effect on the numerical solution. To be able to assess this effect we have constructed a discretization method designed for error analysis [1]. We remark that the smoothing/regularization of difficult problems in order to make them easy to solve numerically is a 73-year-old idea [2] and still applied, e.g., in LES.

We have developed the two-step modeling approach for the 1D shallow-water equations. It has been applied in a number of simulations, including pipe flow. The method turns out to come with several benefits, such as efficiency and robustness. We will present an overview of the details of this development and the obtained results.



1. Borsboom, M. (2002), A finite volume method designed for error analysis, *Finite Volumes for Complex Applications III*, R. Herbin and D. Kröner (eds.), Hermes Penton Science, 705—712.
2. Von Neumann, J. and R. D. Richtmyer (1950), A method for the numerical calculation of hydrodynamic shocks, *J. Appl. Phys.* **21**(3), 232—237.