

# Klein-Nishina Formula and Total Compton Scattering Determination

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8.13 Experimental Physics I  
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## Outline

### 1 Introduction

- Compton Scattering
- Klein-Nishina and QFT

### 2 Experimental

- Signal Chain and Scattering Geometry

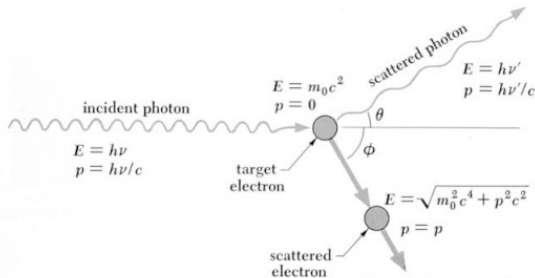
### 3 Results and Error Analysis

- Random/Systematic Error Accounting
- Compton Profile and Features
- Linear Attenuation from PVT, PP, and PC
- Total Cross Section

### 4 Fermi Energy and Free Electron Gas Model

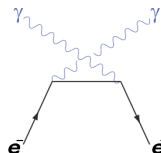
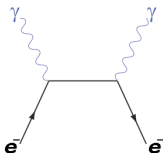
- Theory: “Simple” Metal (Al) as a Free Electron Gas
- Problems and Possible Results

## Interaction picture:



$$\frac{\omega'}{\omega} = \frac{1}{1 + \frac{\omega}{m_e} (1 - \cos \theta)} \quad (1)$$

## Sum of possible events:

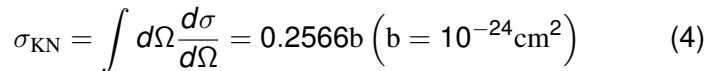


## S-Matrix scattering and Feynman Rules:

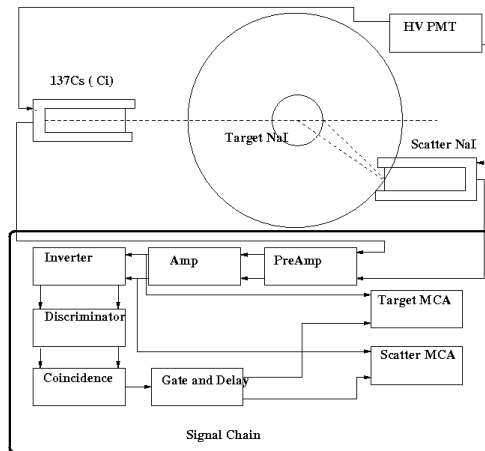
$$\begin{aligned}
 \sigma &= (2\pi)^2 \frac{\epsilon\omega}{|p_\mu k^\mu|} \mathbf{S}_f \mathbf{S}_i \delta(p' + k' - p - k) |\langle f | M | i \rangle|^2 \\
 &= \left( \frac{e^2}{4\pi} \right)^2 \frac{1}{2\kappa\epsilon'\omega'} \int d\Gamma \delta(p' + k' - p - k) X
 \end{aligned} \tag{2}$$

$\beta = 0, \gamma = 1, k = m\omega, k' = m\omega' \ (c = 1 = \hbar)$ :

$$\begin{aligned}
 \sigma &\xrightarrow{\text{Magic}} \frac{d\sigma}{d\Omega} = \frac{1}{4} r_0^2 \left( \frac{\omega'}{\omega} \right)^2 \left( \frac{\omega}{\omega'} + \frac{\omega'}{\omega} - 2 + 4|\hat{\mathbf{e}} \cdot \hat{\mathbf{e}}'|^2 \right) \\
 &\sim \frac{1}{2} r_0^2 \left( \frac{\omega'}{\omega} \right)^2 \left( \frac{\omega}{\omega'} + \frac{\omega'}{\omega} - \sin^2 \theta \right)
 \end{aligned} \tag{3}$$



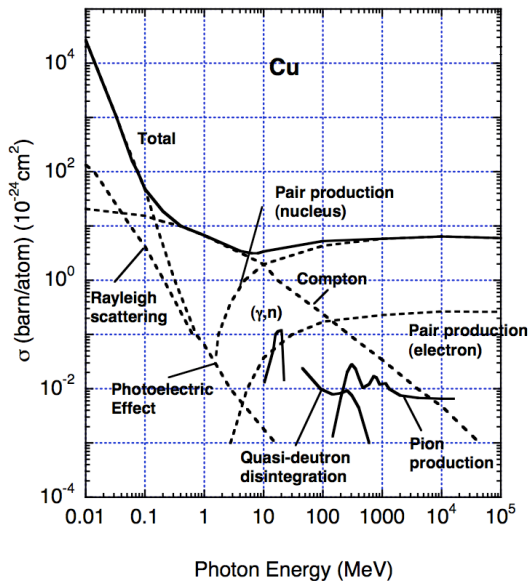
$$\sigma_T = \int d\Omega [\theta] r_0^2 \frac{1 + \cos^2 \theta}{2} = \frac{8\pi}{3} r_0^2 = 0.662b \quad (5)$$



$$\cos \theta' = \frac{R \cos \theta - r}{\sqrt{R^2 + r^2 - 2Rr \cos \theta}} \quad (6)$$

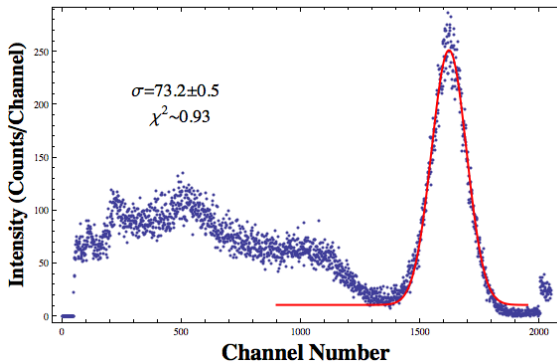
- Gaussian line broadening (Poisson with large mean)
- Spread of beam profile ( $\Delta\theta \sim 8^\circ$ )
- Fermi motion of target electron (Lorentzian line broadening)
- Efficiency in NaI scintillator
- Mean free path of photon: Scattering Geometry ( $\theta'$  vs.  $\theta$ )
- Integration Error

# Possible photon-electron interaction cross-sections

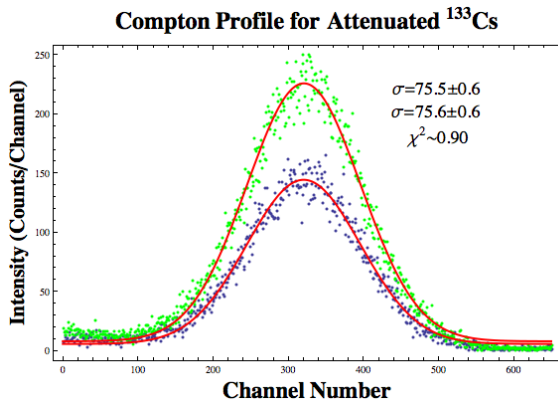




### Compton Profile for Attenuated $^{133}\text{Cs}$

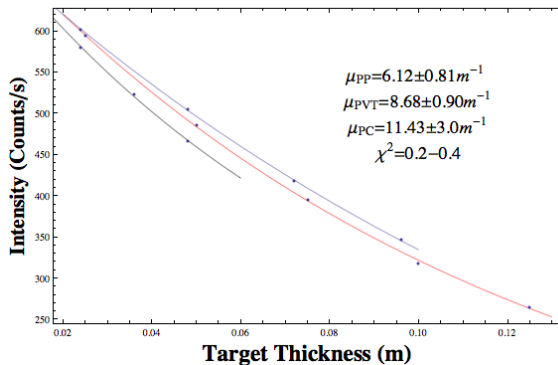


$$I(n) = \frac{I_0}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n-n_0)^2}{2\sigma^2}} \quad (7)$$



$$I(n) = \frac{I_0}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n-n_0)^2}{2\sigma^2}} \quad (8)$$

### Target Attenuation for Total Cross Section



$$I(x) = I_0 e^{-\mu x} \quad \mu = n_e \sigma_{\text{KN}} = \rho \frac{NZ}{A} \sigma_{\text{KN}} \quad (9)$$

Table: Total electron cross-section.

Target	$\mu$ ( $\text{m}^{-1}$ )	$n_e$ ( $\text{cm}^{-3}$ )	$\sigma$ ( $\text{b} = 10^{-24} \text{cm}^2$ )
PP ( $\text{C}_3\text{H}_6$ ) <sub>n</sub>	$6.12 \pm 0.81$	$5.11 \times 10^{23}$	$0.12 \pm 0.02$
PVT	$8.68 \pm 0.90$	$3.37 \times 10^{23}$	$0.25 \pm 0.03$
PC ( $\text{C}_{16}\text{H}_{14}\text{O}_3$ ) <sub>n</sub>	$11.43 \pm 3.00$	$3.81 \times 10^{23}$	$0.28 \pm 0.04$

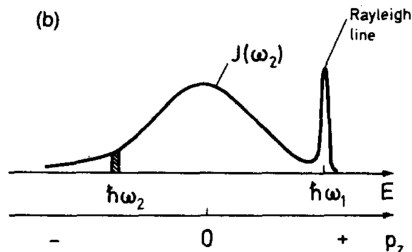
## Summary

- QFT origins and assumptions for KN cross section
- Characterized Compton Profile
- Measured total cross-section to  $0.25 \pm 0.03b$ , in excellent agreement with theoretical  $0.2566b$ .

General idea: Energy spectrum into momentum scale for Al foil.

$$\frac{p_z}{m_e c} = \frac{\omega_1 - \omega_2 + \hbar \omega_1 \omega_2 (1 - \cos \theta) / m_0 c^2}{(\omega_1 + \omega_2 - 2\omega_1 \omega_2 \cos \theta)^{1/2}} \quad (10)$$

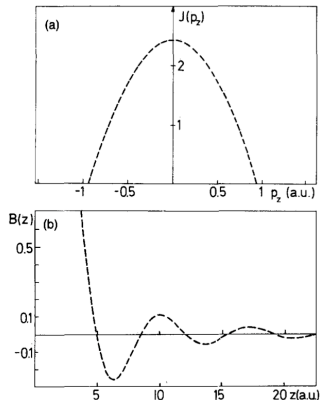
$$J(p_z) = \int d^2 p_{xy} n(\mathbf{p}) = \frac{3N}{4p_F^3} (p_F^2 - p_z^2) \quad (11)$$



$$B(z) = F[J(z)] = \frac{3N}{(p_F z)^2} \left( \frac{\sin p_F z}{p_F z} - \cos p_F z \right) \quad (12)$$

## Problems:

- $^{137}\text{Cs}$  is too an high-energy  $\gamma$ -source.
- $^{133}\text{Ba}$  can calibrate, but low activity.
- Resolution is currently too low.



## Acknowledgements

- Charles Herder
- JLab staff: For being helpful despite knowing more than we do.
- E.M. Purcell: You can't spell NMR without “enema”+“r”.