Klein-Nishina Formula and Total Compton Scattering Determination

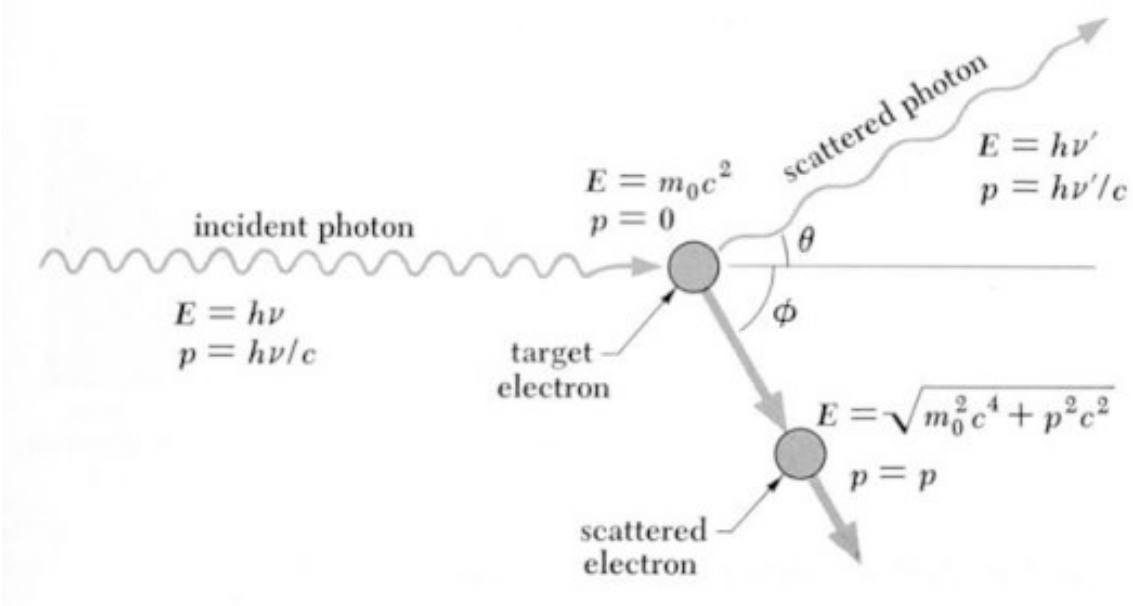
Bhaskar Mookerji Charles Herder

8.13 Experimental Physics I
MIT Department of Physics

3 December 2007

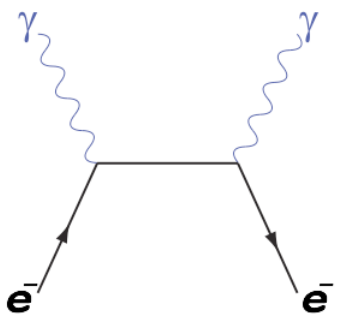
Outline

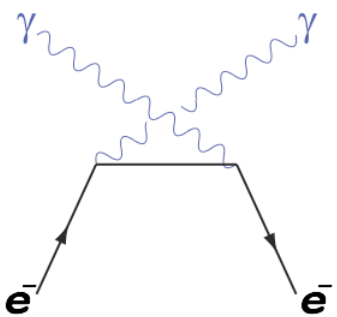
Interaction picture:



$$\frac{\omega'}{\omega} = \frac{1}{1 + \frac{\omega}{m_e}(1 - \cos\theta)} \quad (1)$$

Sum of possible events:



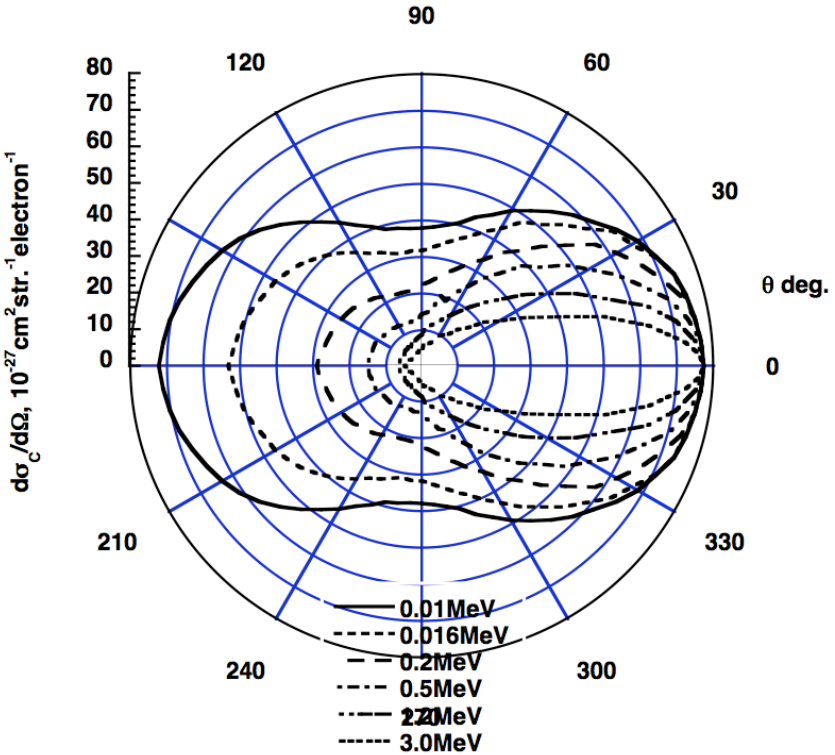


S-Matrix scattering and Feynman Rules:

$$\begin{aligned}\sigma &= (2\pi)^2 \frac{\epsilon\omega}{|p_\mu k^\mu|}\mathbf{S}_f\mathbf{S}_i\delta\left(p' + k' - p - k\right) |\left\langle f\right| M \left|i\right\rangle|^2 \\ &= \left(\frac{e^2}{4\pi}\right)^2 \frac{1}{2\kappa\epsilon'\omega'}\int d\Gamma\delta\left(p' + k' - p - k\right) X\end{aligned} \tag{2}$$

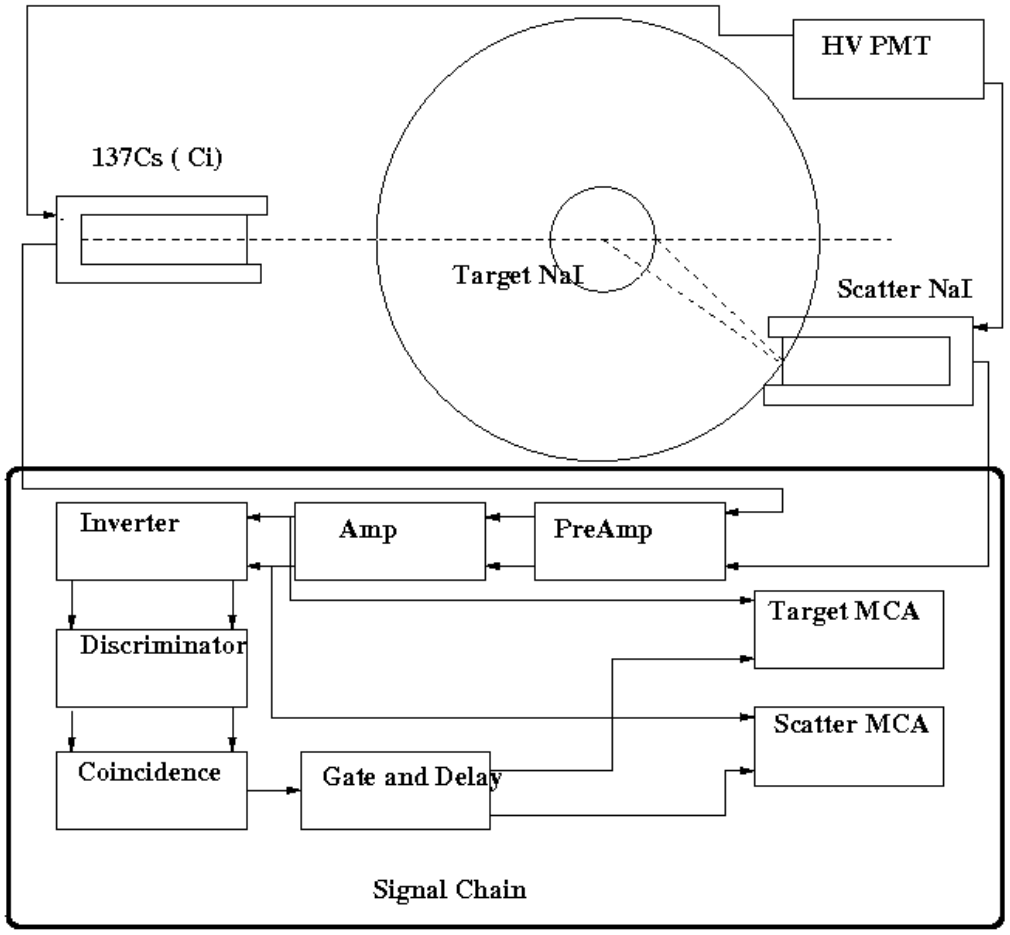
$\beta = 0$, $\gamma = 1$, $k = m\omega$, $k' = m\omega'$ ($c = 1 = \hbar$):

$$\begin{aligned}\sigma \xrightarrow{\text{Magic}} \frac{d\sigma}{d\Omega} &= \frac{1}{4}r_0^2\left(\frac{\omega'}{\omega}\right)^2\left(\frac{\omega}{\omega'} + \frac{\omega'}{\omega} - 2 + 4|\hat{e}\cdot\hat{e}'|^2\right) \\ &\sim \frac{1}{2}r_0^2\left(\frac{\omega'}{\omega}\right)^2\left(\frac{\omega}{\omega'} + \frac{\omega'}{\omega} - \sin^2\theta\right)\end{aligned} \tag{3}$$



$$\sigma_{\mathrm{KN}} = \int d\Omega \frac{d\sigma}{d\Omega} = 0.2566\mathrm{b} \left(\mathrm{b} = 10^{-24}\mathrm{cm}^2\right) \tag{4}$$

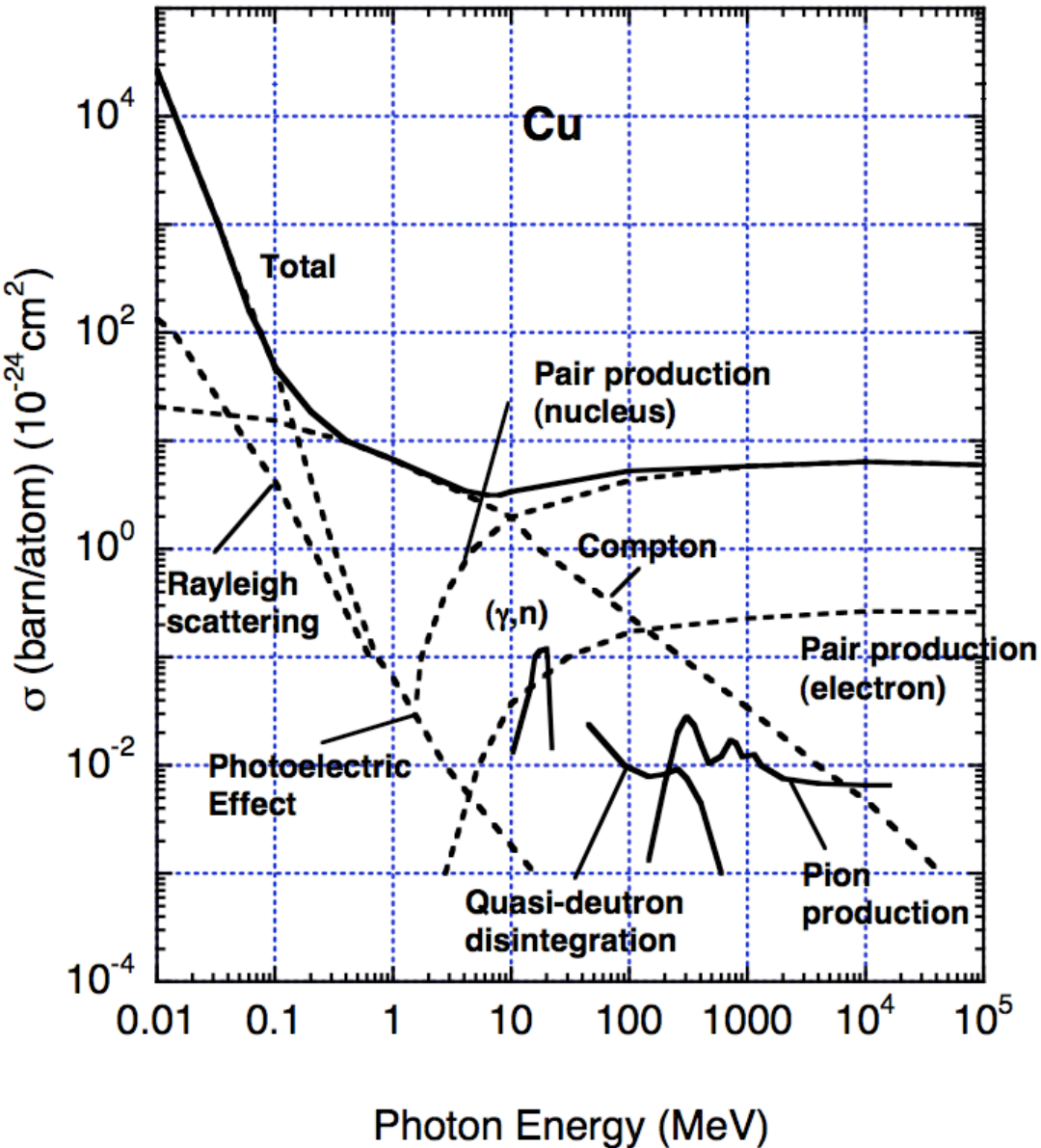
$$\sigma_{\mathrm{T}} = \int d\Omega \left[\theta\right] r_0^2 \frac{1+\cos^2\theta}{2} = \frac{8\pi}{3} r_0^2 = 0.662b \tag{5}$$

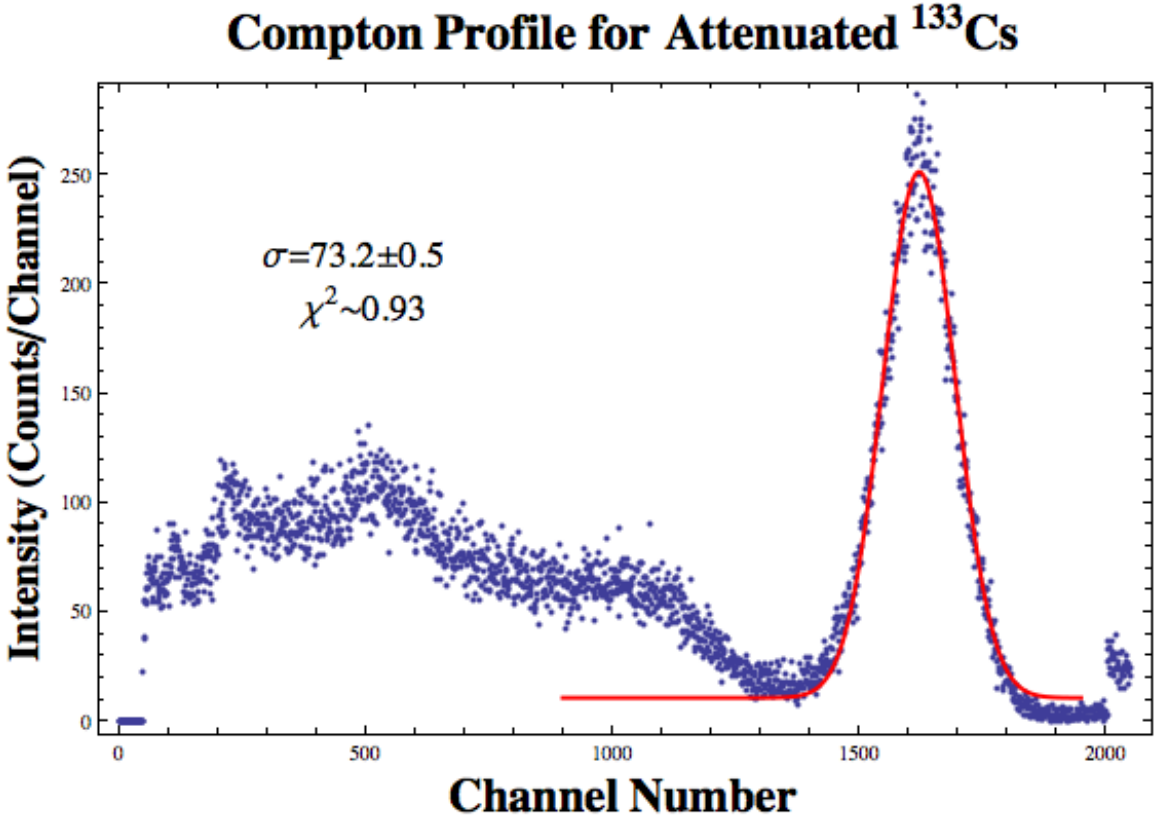


$$\cos\theta' = \frac{R\cos\theta - r}{\sqrt{R^2 + r^2 - 2Rr\cos\theta}} \tag{6}$$

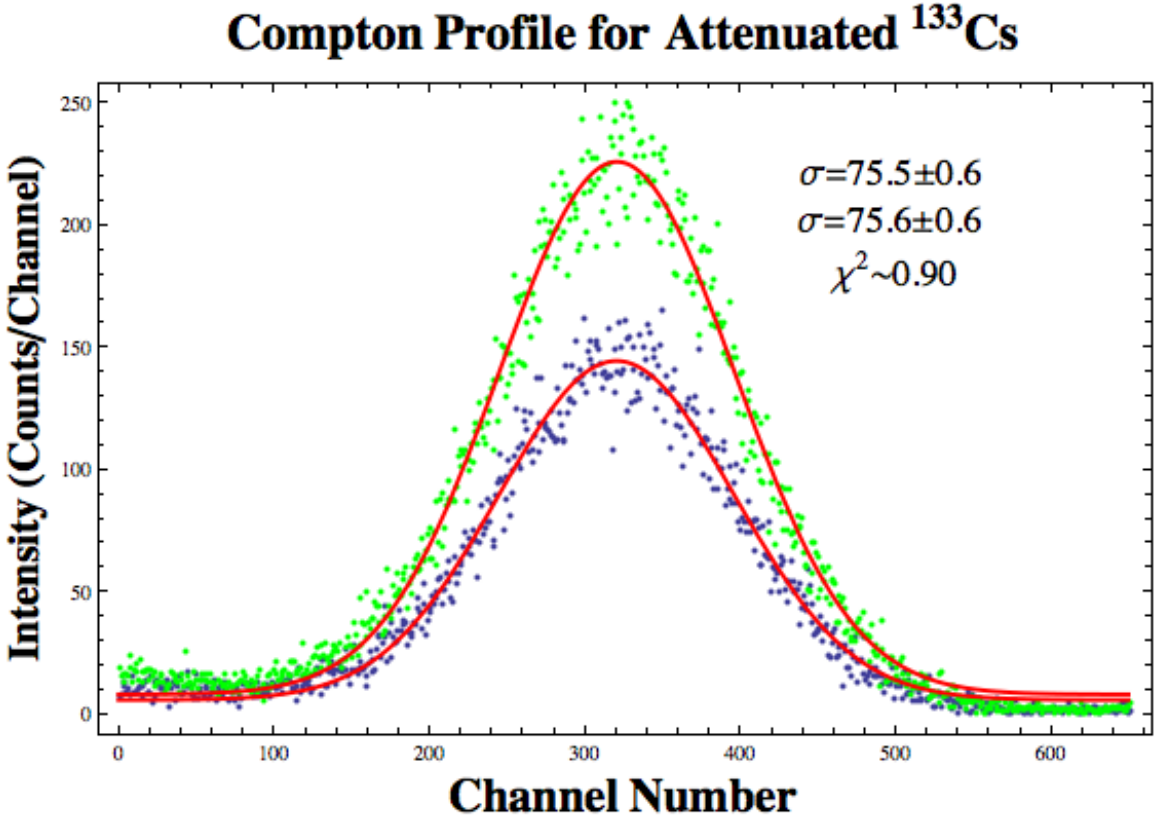
- Gaussian line broadening (Poisson with large mean)
- Spread of beam profile ($\Delta\theta \sim 8°$)
- Fermi motion of target electron (Lorentzian line broadening)
- Efficiency in NaI scintillator
- Mean free path of photon: Scattering Geometry (θ' vs. θ)
- Integration Error

Possible photon-electron interaction cross-sections

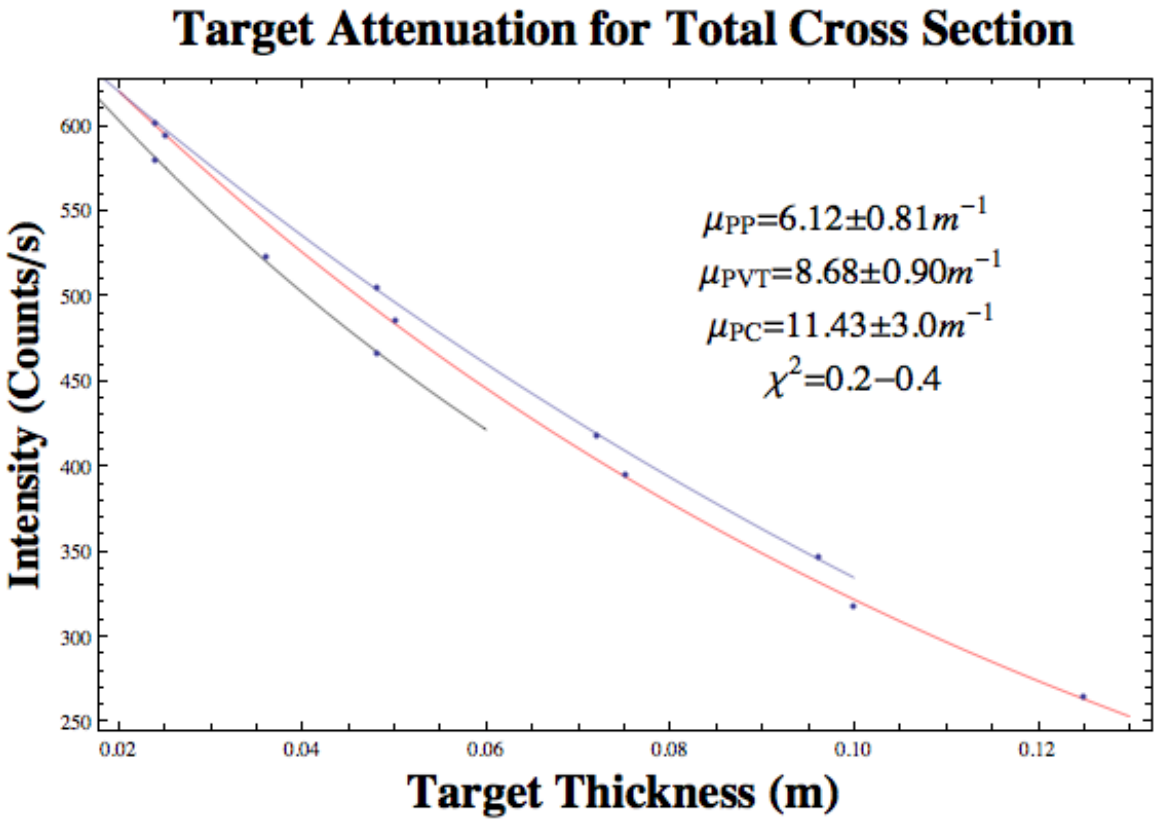




$$I(n) = \frac{I_0}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n-n_0)^2}{2\sigma^2}} \tag{7}$$



$$I(n) = \frac{I_0}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n-n_0)^2}{2\sigma^2}} \tag{8}$$



$$I(x) = I_0 e^{-\mu x} \qquad \mu = n_e \sigma_{\mathrm{KN}} = \rho \frac{NZ}{A} \sigma_{\mathrm{KN}} \tag{9}$$

Table: Total electron cross-section.

Target	μ (m^{-1})	n_e (cm^{-3})	σ (b$= 10^{-24} cm^2$)
PP $(C_3H_6)_n$	6.12 ± 0.81	5.11×10^{23}	0.12 ± 0.02
PVT	8.68 ± 0.90	3.37×10^{23}	0.25 ± 0.03
PC $(C_{16}H_{14}O_3)_n$	11.43 ± 3.00	3.81×10^{23}	0.28 ± 0.04

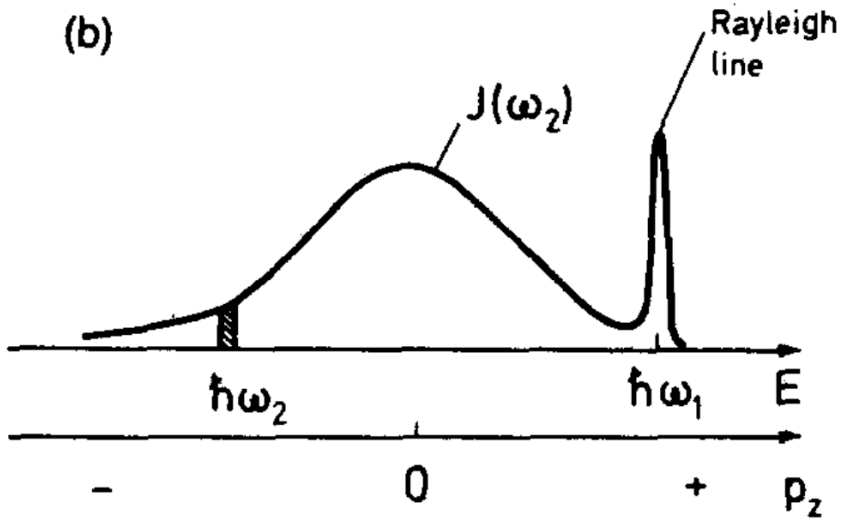
Summary

- QFT origins and assumptions for KN cross section
- Characterized Compton Profile
- Measured total cross-section to 0.25 ± 0.03b, in excellent agreement with theoretical 0.2566b.

General idea: Energy spectrum into momentum scale for Al foil.

$$\frac{p_z}{m_e c} = \frac{\omega_1 - \omega_2 + \hbar\omega_1\omega_2\left(1 - \cos\theta\right)/m_0c^2}{\left(\omega_1 + \omega_2 - 2\omega_1\omega_2\cos\theta\right)^{1/2}} \tag{10}$$

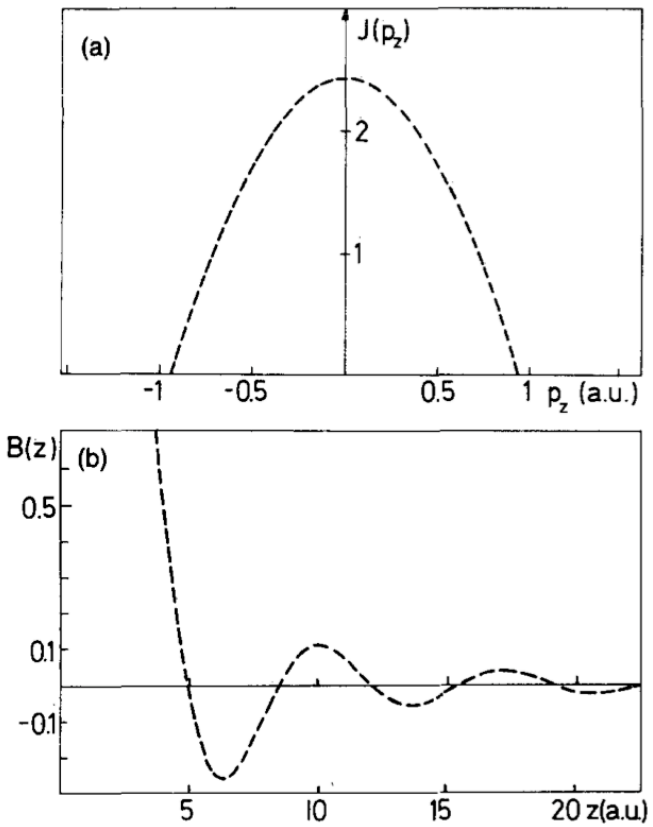
$$J\left(p_z\right) = \int d^2p_{xy}\, n\left(\mathbf{p}\right) = \frac{3N}{4p_F^3}\left(p_F^2 - p_z^2\right) \tag{11}$$



$$B\left(z\right) = F\left[J\left(z\right)\right] = \frac{3N}{\left(p_F z\right)^2}\left(\frac{\sin p_F z}{p_F z} - \cos p_F z\right) \tag{12}$$

Problems:

- ^{137}Cs is too an high-energy γ-source.
- ^{133}Ba can calibrate, but low activity.
- Resolution is currently too low.



Acknowledgements

- Charles Herder
- JLab staff: For being helpful despite knowing more than we do.
- E.M. Purcell: You can't spell NMR without "enema"+"r".