Johnson Noise Interpolation of Boltzmann's Constant k_B and Absolute Zero T_0

Bhaskar Mookerji Charles Herder

8.13 Experimental Physics I
MIT Department of Physics

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Outline

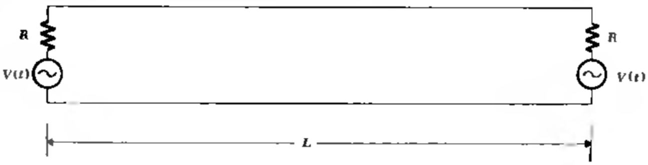
What is Johnson noise?

- Random thermal motion $\rightarrow$ instantaneous voltage, $\langle V \rangle = 0$
- Minimum mean-square voltage from resistive impedance
- Magnitude of random noise input J_+:

$$\langle V^2 \rangle = \int_0^\infty d\omega J_+ (\omega) \,. \tag{1}$$

From equipartition theorem:

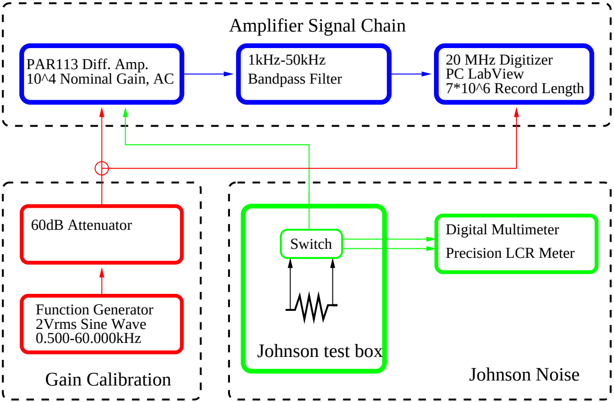
$$J_+(\omega) = \frac{2}{\pi}\frac{\hbar\omega}{e^{\hbar\omega/k_BT}-1}R(\omega) \xrightarrow{\hbar\omega \ll k_BT} (2/\pi)\,k_BTR(\omega) \qquad (2)$$



Network impedance, signal gain $\longrightarrow$ Nyquist's Theorem

$$\mathrm{Re}\left(R_0 || Z_C\right) = R(\omega) = \frac{R_0}{1+(\omega R_0 C)^2} \qquad (3)$$

$$\langle V^2 \rangle = \frac{2k_BT}{\pi}\int_0^\infty d\omega R(\omega)\left|Y(\omega)\right|^2 \qquad (4)$$



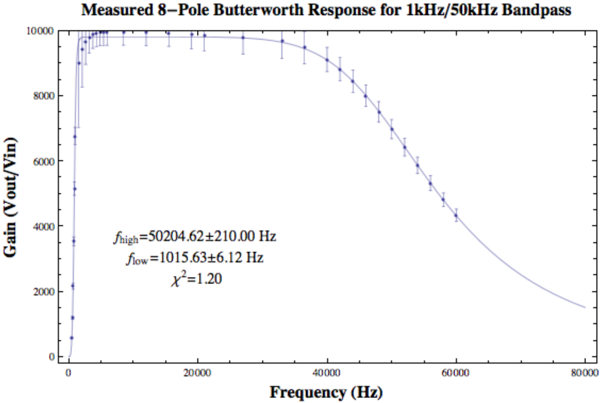
Calibrate measurement chain, measure RMS voltage across R and short circuit (varying R and temperature T).

Parameters:

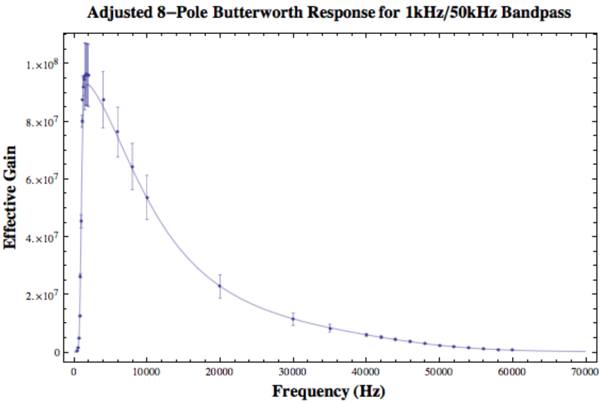
- Background voltage noise from amplifier and electrical interference: Adds in quadrature to Johnson noise: $V^2 = V_R^2 - V_S^2$
- Shielding: Aluminum foil on test box (both grounded), BNC cables
- Parasitic capacitance: Coax cables, amplifier input impedance, alligator clip/resistor junction

Approximately,

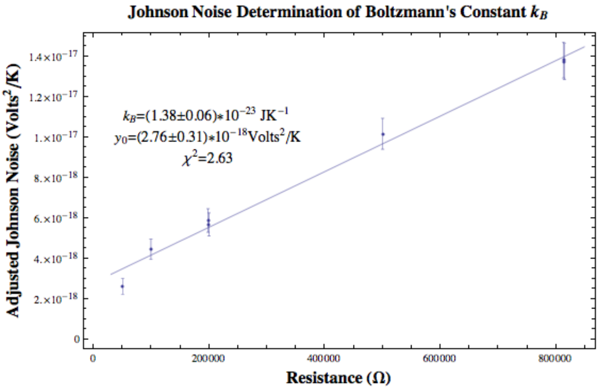
- Voltage 0.01mV—0.05mV
- Bandpass 0.1dB
- Capacitance 0.005 nF
- Resistance 0.05 kΩ
- Temperature 0.5K
- Integration Error 1% Deviation



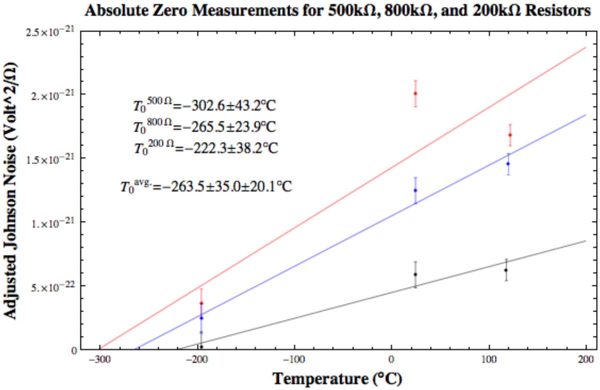
$$Y(f) = \frac{A_0}{\sqrt{1+\left(\frac{f_h}{f}\right)^8}\sqrt{1+\left(\frac{f}{f_C}\right)^8}} \qquad \sigma_g^2 = 4\,|g(f)|^2\left(\frac{\sigma_{\mathrm{in}}^2}{V_{\mathrm{in}}^2}+\frac{\sigma_{\mathrm{out}}^2}{V_{\mathrm{out}}^2}\right)+\sigma_{\mathrm{BP}}^2$$



$$R(f) = \frac{R_0}{1+(2\pi f R_0 C)^2} \qquad \sigma_g^2 = 4\frac{g^2}{1+(2\pi f R_0 C)^2}\left(\frac{\sigma_{g^2}^2}{g^4} + \frac{\sigma_R^2}{R^2} + \frac{\sigma_C^2}{V_C^2}\right)$$



$$\sigma^2_{\langle V^2\rangle} = 4\left(V_R^2\sigma^2_{V_R} + V_S^2\sigma^2_{V_S}\right) \qquad \sigma^2_{\frac{\langle V^2\rangle}{4GT}} = \frac{\langle V^2\rangle}{4GT}\left(\frac{\sigma^2_{\langle V^2\rangle}}{\langle V^2\rangle^2} + \frac{\sigma_T^2}{T^2} + \frac{\sigma_G^2}{G^2}\right)$$



$$\sigma^2_{\frac{\langle V^2\rangle}{4GR}} = \frac{\langle V^2\rangle}{4GR}\left(\frac{\sigma^2_{\langle V^2\rangle}}{\langle V^2\rangle^2} + \frac{\sigma_R^2}{R^2} + \frac{\sigma_G^2}{G^2}\right)$$

Summary

- Validity of Nyquist's Theorem, Butterworth response confirmed
- Theoretical values: $k_B = 1.3806504(24) \cdot 10^{-23} J \cdot K^{-1}$, $T_0 = -273.15K$
- Measured vales: $k_B = 1.37617 \pm 0.06421 \cdot 10^{-23} J \cdot K^{-1}$, $T_0 = -263.5 \pm 35.0 \pm 20.1K$
- Problems: Measuring capacitance, temperature

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