# Johnson Noise Interpolation of Boltzmann's Constant $k_B$ and Absolute Zero $T_0$

Bhaskar Mookerji Charles Herder

8.13 Experimental Physics I MIT Department of Physics

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### Outline

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  - Johnson Noise Dependence on R: Boltzmann's Constant k<sub>B</sub>
  - Interpolation of Absolute Zero T<sub>0</sub> (°C)

### What is Johnson noise?

- Random thermal motion  $\rightarrow$  instantaneous voltage,  $\langle V \rangle = 0$
- Minimum mean-square voltage from resistive impedance
- Magnitude of random noise input  $J_+$ :

$$\langle V^2 \rangle = \int_0^\infty d\omega J_+(\omega) \,.$$
 (1)

## From equipartition theorem:

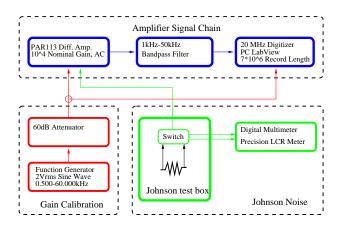
$$J_{+}(\omega) = \frac{2}{\pi} \frac{\hbar \omega}{e^{\hbar \omega / k_{B}T} - 1} R(\omega) \stackrel{\hbar \omega \ll k_{B}T}{\longrightarrow} (2/\pi) k_{B}TR(\omega)$$
 (2)



Network impedance, signal gain — Nyquist's Theorem

$$\operatorname{Re}(R_0||Z_C) = R(\omega) = \frac{R_0}{1 + (\omega R_0 C)^2}$$
 (3)

$$\langle V^2 \rangle = \frac{2k_BT}{\pi} \int_0^\infty d\omega R(\omega) |Y(\omega)|^2 \tag{4}$$



Calibrate measurement chain, measure RMS voltage across R and short circuit (varying R and temperature T).

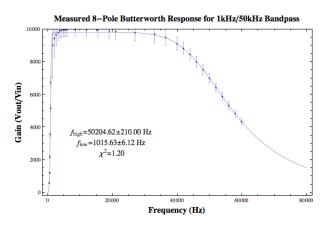
### Parameters:

- Background voltage noise from amplifier and electrical interference: Adds in quadrature to Johnson noise:  $V^2 = V_R^2 V_S^2$
- Shielding: Aluminum foil on test box (both grounded), BNC cables
- Parasitic capacitance: Coax cables, amplifier input impedance, alligator clip/resistor junction

Random/Systematic Error Accounting

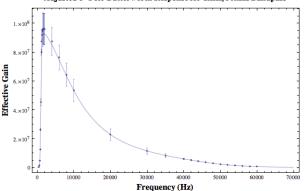
## Approximately,

- Voltage 0.01mV—0.05mV
- Bandpass 0.1dB
- Capacitance 0.005 nF
- Resistance 0.05 kΩ
- Temperature 0.5K
- Integration Error 1% Deviation

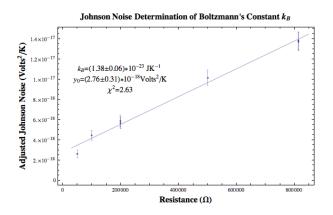


$$Y(f) = \frac{A_0}{\sqrt{1 + \left(\frac{f_h}{f}\right)^8} \sqrt{1 + \left(\frac{f}{f_c}\right)^8}} \qquad \sigma_g^2 = 4 \left| g(f) \right|^2 \left(\frac{\sigma_{\text{in}}^2}{V_{\text{in}}^2} + \frac{\sigma_{\text{out}}^2}{V_{\text{out}}^2}\right) + \sigma_{\text{BP}}^2$$

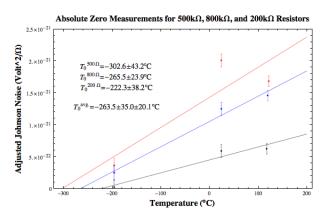
#### Adjusted 8-Pole Butterworth Response for 1kHz/50kHz Bandpass



$$R(f) = rac{R_0}{1 + (2\pi f R_0 C)^2}$$
  $\sigma_g^2 = 4 rac{g^2}{1 + (2\pi f R_0 C)^2} \left( rac{\sigma_{g^2}^2}{g^4} + rac{\sigma_R^2}{R^2} + rac{\sigma_C^2}{V_C^2} 
ight)$ 



$$\sigma_{\langle V^2 
angle}^2 = 4 \left( V_R^2 \sigma_{V_R}^2 + V_S^2 \sigma_{V_S}^2 
ight) \quad \sigma_{rac{\langle V^2 
angle}{4GT}}^2 = rac{\langle V^2 
angle}{4GT} \left( rac{\sigma_{\langle V^2 
angle}^2}{\langle V^2 
angle^2} + rac{\sigma_G^2}{T^2} + rac{\sigma_G^2}{G^2} 
ight)$$



$$\sigma_{rac{\langle V^2 
angle}{4GR}}^2 = rac{\langle V^2 
angle}{4GR} \left( rac{\sigma_{\langle V^2 
angle}^2}{\langle V^2 
angle^2} + rac{\sigma_R^2}{R^2} + rac{\sigma_G^2}{G^2} 
ight)$$

## Summary

- Validity of Nyquist's Theorem, Butterworth response confirmed
- Theoretical values:  $k_B = 1.3806504(24) \cdot 10^{-23} J \cdot K^{-1}$ ,  $T_0 = -273.15 K$
- Measured vales:  $k_B = 1.37617 \pm 0.06421 \cdot 10^{-23} J \cdot K^{-1}$ ,  $T_0 = -263.5 \pm 35.0 \pm 20.1 K$
- Problems: Measuring capacitance, temperature

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