Question 7.1 a

* remarks: % is the modulo operator
// is the integer division operator

- coordinates to index:

$$\boxed{\text{index} = X_2 L_1 + X_1}$$

- index to coordinates:

$$\boxed{\begin{aligned} X_1 &= \text{index} \,\%\, L_1 \\ X_2 &= \text{index} \,//\, L_1 \end{aligned}}$$

Question 7.2 a

- coordinates to index:

For 4-dimensions:

$$\begin{aligned} \text{index} &= X_4 L_3 L_2 L_1 + X_3 L_2 L_1 + X_2 L_1 + X_1 \\ &= X_4 \prod_{j=1}^{4-1} L_j + X_3 \prod_{j=1}^{3-1} L_j + X_2 \prod_{j=1}^{2-1} L_j + X_1 \prod_{j=1}^{1-1} L_j \\ &= \sum_{i=1}^{4} \left(X_i \prod_{j=1}^{i-1} L_j \right) \end{aligned}$$

Hence, for d-dimension(s):

$$\boxed{\text{index} = \sum_{i=1}^{d} \left(X_i \prod_{j=1}^{i-1} L_j \right)}$$

- index to coordinates:

* remarks: % is the modulo operator
// is the integer division operator

For 4-dimensions:

$$X_1 = \text{index} \,\%\, L_1$$

$$X_2 = (\text{index} \,//\, L_1) \,\%\, L_2$$

$$X_3 = (\text{index} \,//\, (L_1 \cdot L_2)) \,\%\, L_3$$

$$X_4 = (\text{index} \,//\, (L_1 \cdot L_2 \cdot L_3)) \,\%\, L_4$$

Hence, for d-dimension(s):

$$\boxed{X_d = \left(\text{index} \,//\, \prod_{i=1}^{d-1} L_i\right) \,\%\, L_d}$$