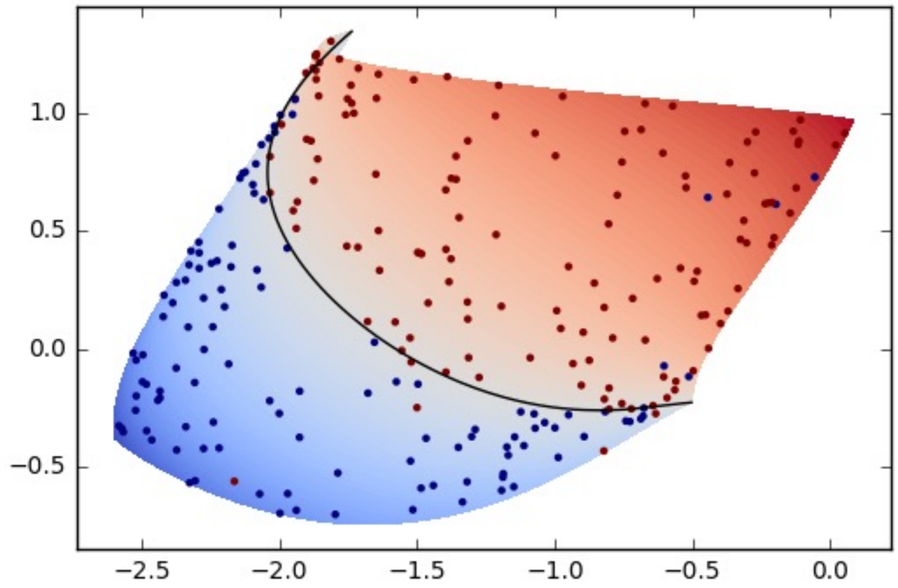
Lecture 24: Reinforcement Learning



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Spring 2023

Some slides adapted from Padhraic Smyth, Alex Ihler

Announcements

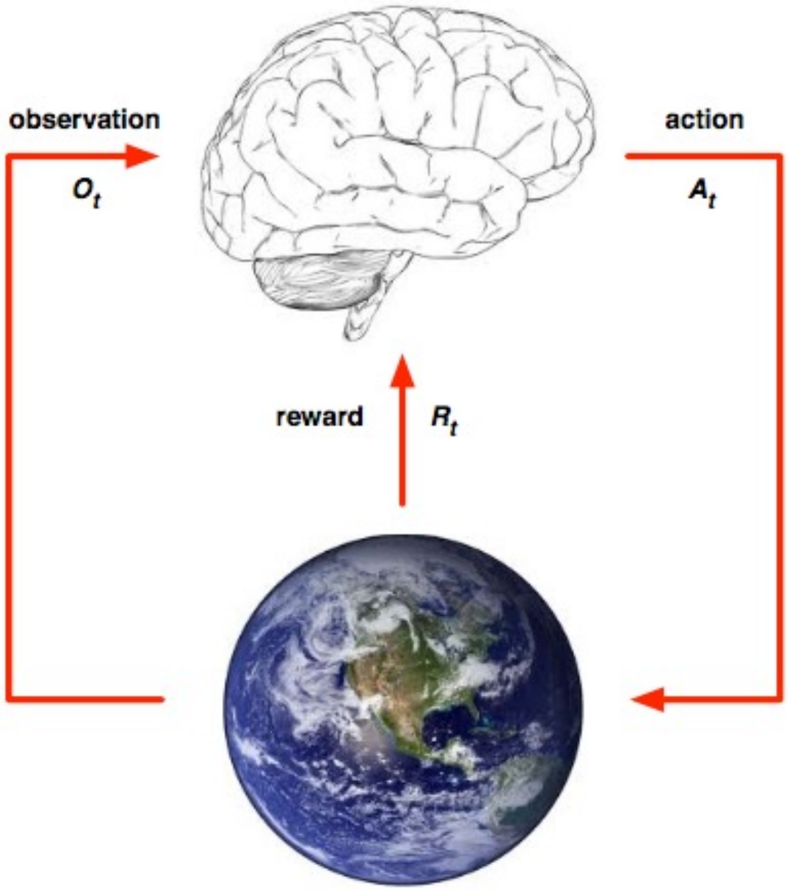
- HW4 due tonight
- HW5 released by Monday
  - Due in 2 weeks (Friday 6/9, last day of class)
  - Short assignment on clustering
- Final exam details discussed next week
  - Similar format to midterm
- No class on Monday (Memorial day)

Intro to Reinforcement Learning

Multi-Armed Bandits

Markov Decision Processes

Agent-Environment Interface



Agent

- Decides on an action
- Observes the state of the environment
- Receives a reward
- Goal: take actions that result in the highest total reward

Environment

- Executes the action
- Computes the next observation
- Computes the next reward

Reinforcement Learning is Hard

Agent takes in a history of states, actions, and rewards

Tries to predict the best action to take next

- No direct supervision: only rewards
  - Agent must figure out what the best actions are -- not provided as training data
  - "Exploration"
- Feedback is often delayed
  - Example: only getting a reward if you win a Chess game
- Data has a temporal/sequential aspect
- Environment is changing
  - Agent's actions have an effect

Sequential Decision Making

Actions have long-term consequences

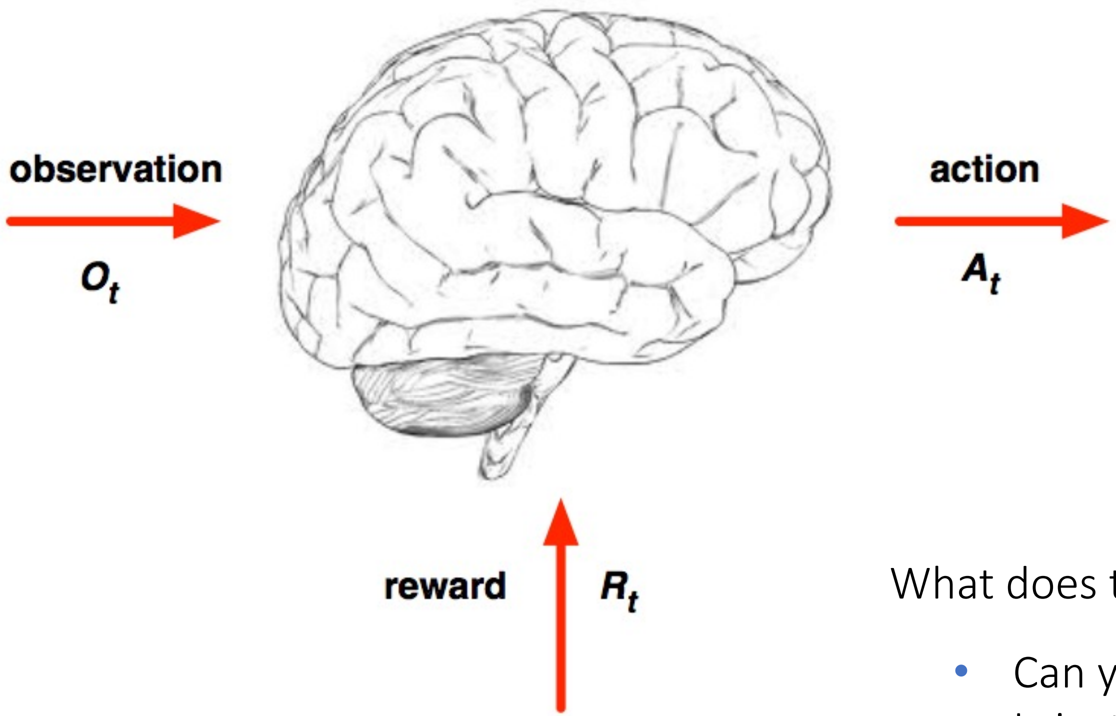
May be better to sacrifice short-term rewards for long-term benefit

Examples:

- Buying Apple stock 20 years ago
- Going into debt to go to college
- Starting on a course project early in the quarter

One aspect of *general intelligence* is the ability to plan far into the future

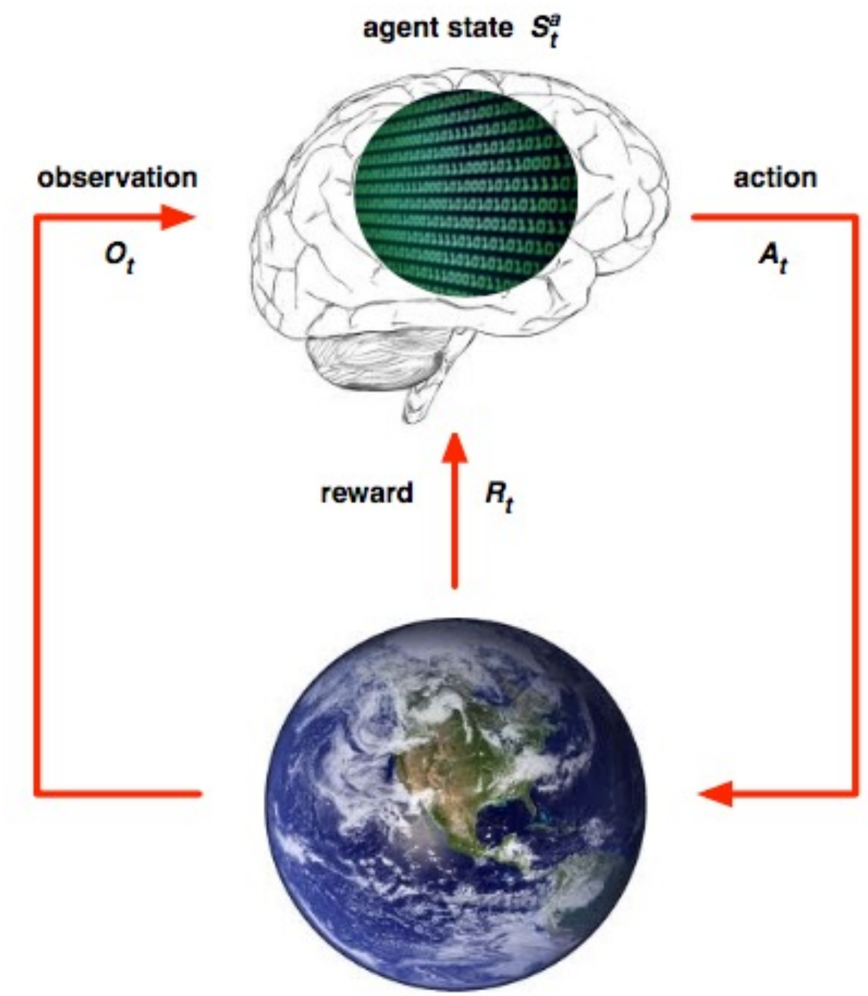
Agents



What does the choice of action depend on?

- Can you ignore O_t completely?
- Is just O_t enough? Or (O_t, A_t)?
- Is it last few observations?
- Is it all observations so far?

Agent State, S_t



History: everything that happened so far

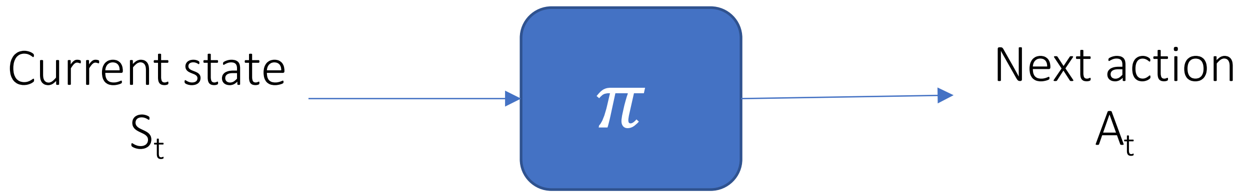
$H_t = O_1R_1A_1O_2R_2A_2O_3R_3,\ldots,A_{t-1}O_tR_t$

State, S_t could be… O_t
O_tR_t
$A_{t-1}O_tR_t$
$O_{t-3}O_{t-2}O_{t-1}O_t$

In general, $S_t = f(H_t)$

You, as AI designer, specify this function

Agent Policy, π



Deterministic Policy: $A_t = \pi(S_t)$

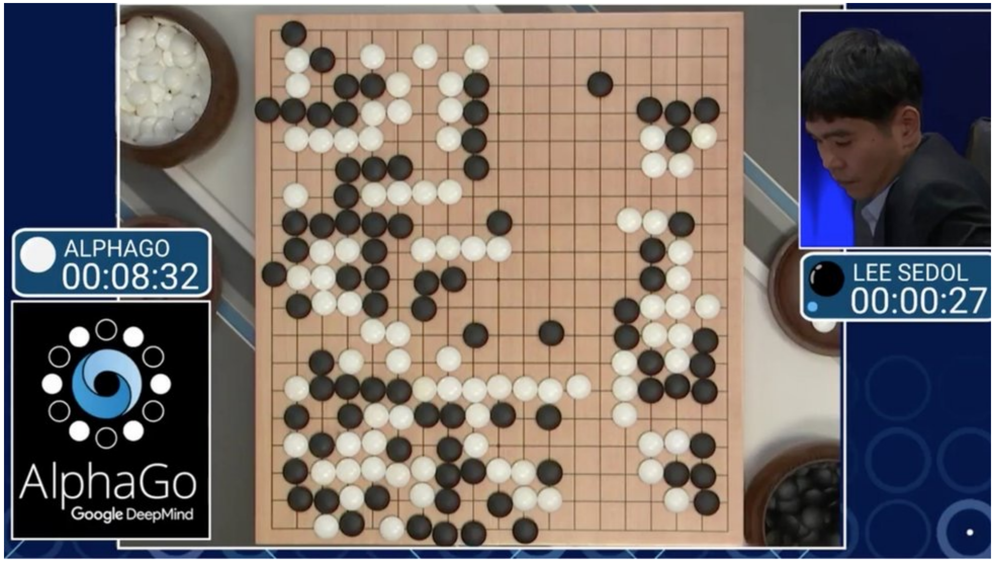
Stochastic Policy: $\pi(a|s) = P(A_t = a|S_t = s)$

Good policy: Leads to larger cumulative reward

Bad policy: Leads to worse cumulative reward

Examples of RL Problems

Learning to Play Go



https://www.bbc.com/news/technology-35785875

State:

- Location of all pieces on the board
- History of last few board positions

Actions:

- Where to place a piece

Rewards:

- Positive if you win; negative if you lose

Examples of RL Problems

Learning to Get Rich

https://www.cnet.com/personal-finance/investing/its-been-a-wild-ride-stock-market-predictions-for-the-next-year/

State:

- Prices of many stocks
- How much money you have in the bank
- Additional market data

Actions:

- What stocks to buy/sell

Rewards:

- Proportional to how much profit you make

Examples of RL Problems

Learning to Fly Autonomous Drones

https://www.ansys.com/blog/challenges-developing-fully-autonomous-drone-technology

State:

- Location and orientation of drone
- Velocity of drone
- Current weather

Actions:

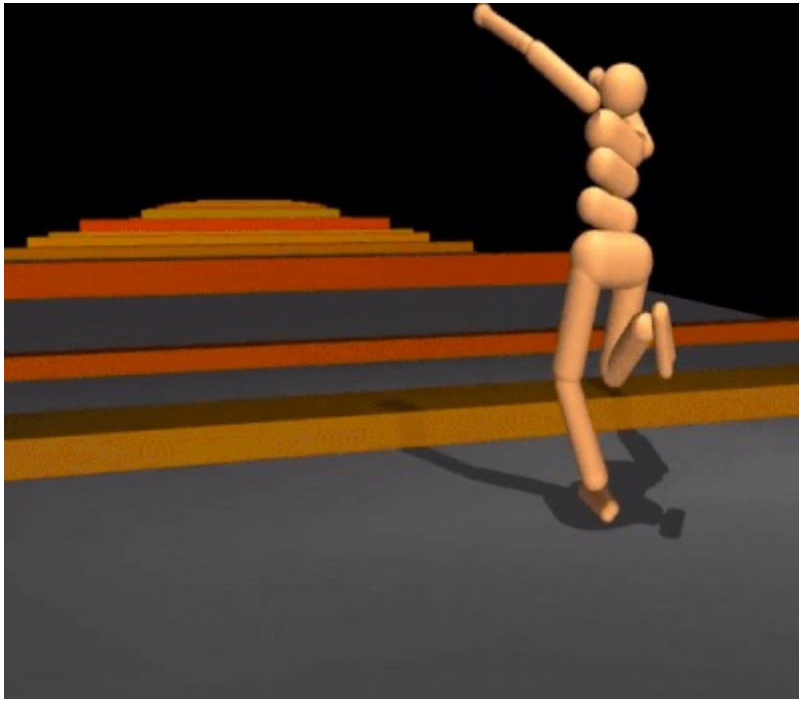
- Moving propellers in a particular fashion

Rewards:

- Positive reward if drone successfully flies to destination
- Negative reward if the drone crashes

Examples of RL Problems

Learning to Walk



https://www.deepmind.com/blog/producing-flexible-behaviours-in-simulated-environments

State:

- Velocity, orientation
- Position of all joints

Actions:

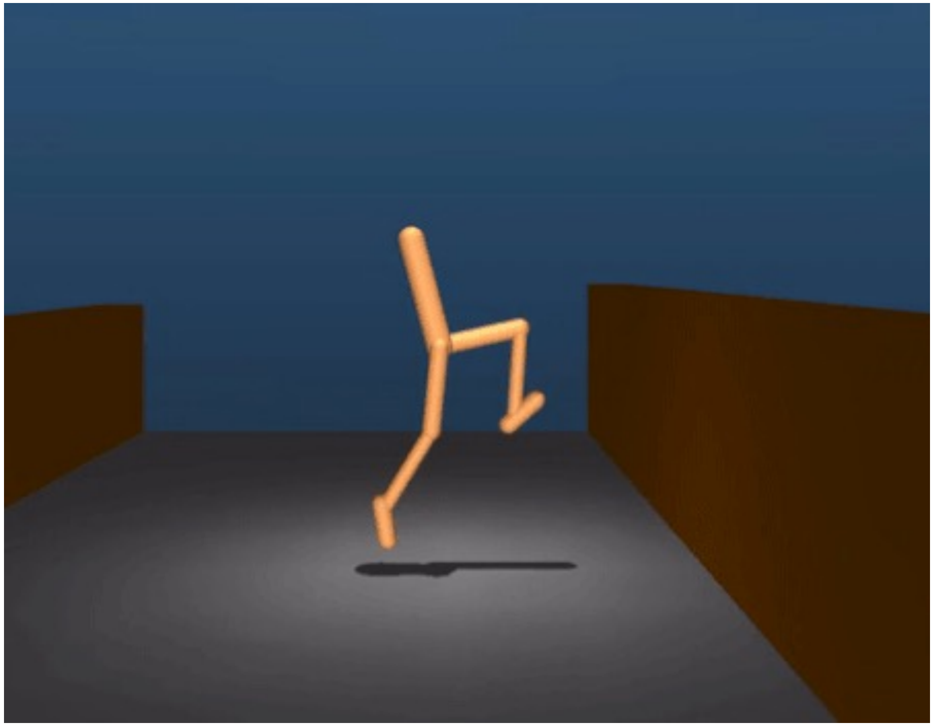
- How to move each individual joint

Rewards:

- Positive reward if you stay upright, make it to goal
- Negative reward if you fall

Examples of RL Problems

Learning to Walk



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State:

- Velocity, orientation
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Actions:

- How to move each individual joint

Rewards:

- Positive reward if you stay upright, make it to goal
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Examples of RL Problems

Playing Atari Games

State:

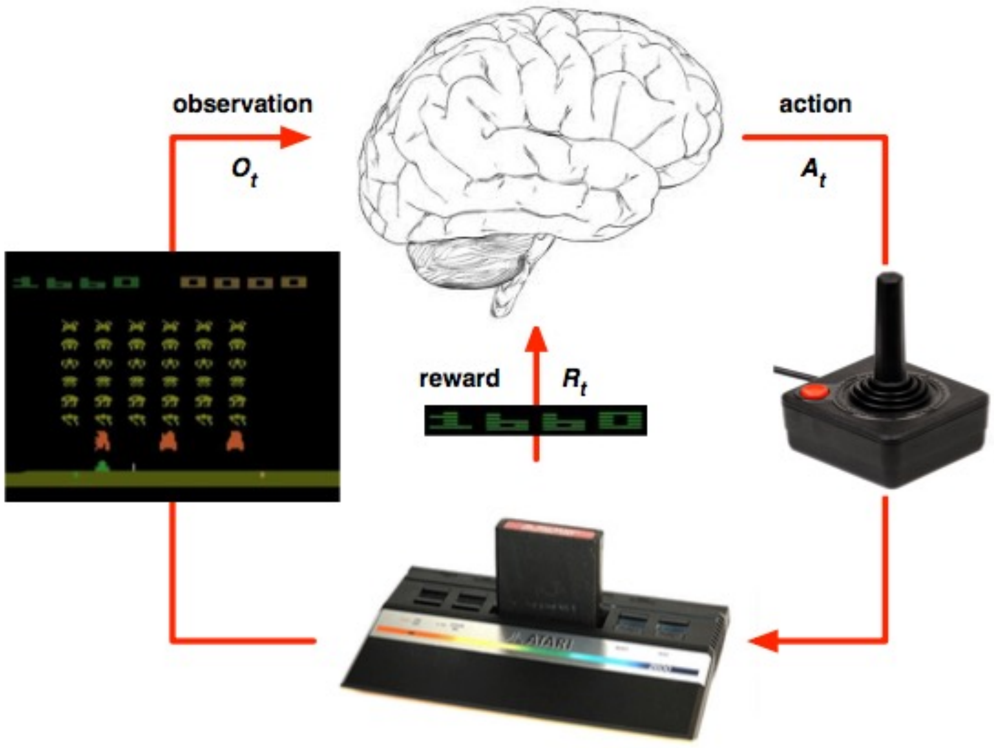
- Pixels on screen

Actions:

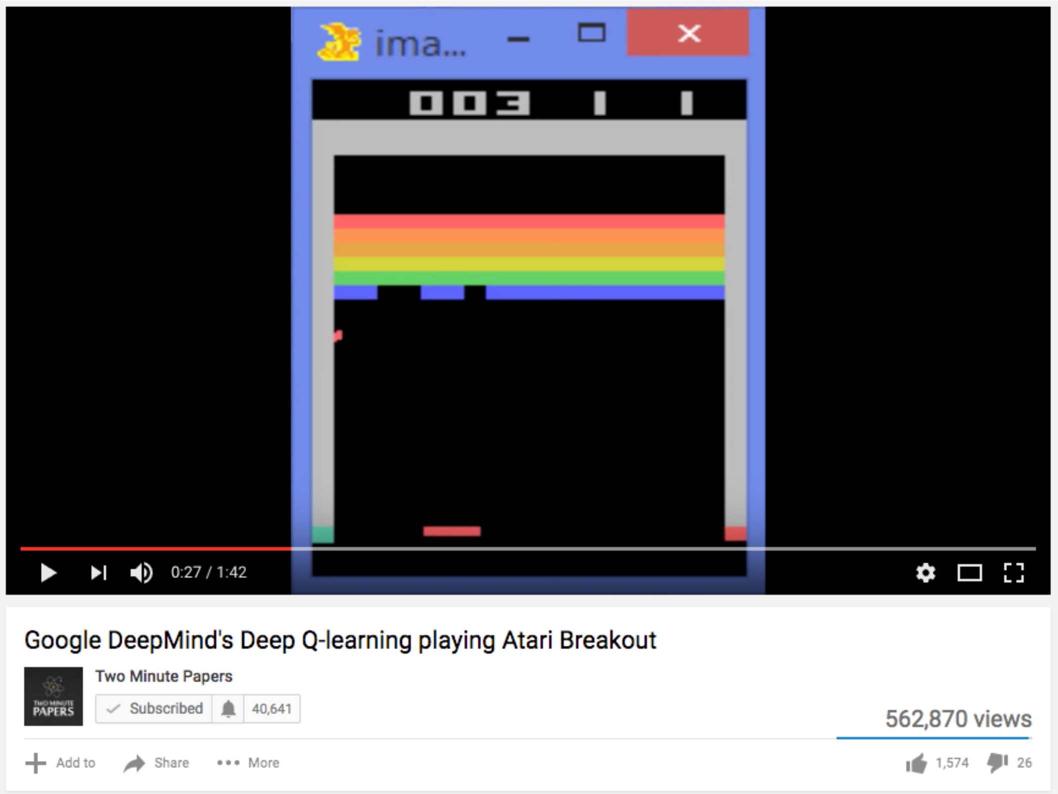
- Moving joystick
- Pressing buttons

Rewards:

- Computed based on current score, number of lives left, etc.



Examples of RL Problems



https://www.youtube.com/watch?v=V1eYniJ0Rnk

Questions?

Intro to Reinforcement Learning

Multi-Armed Bandits

Markov Decision Processes

Multi-Armed Bandits

A simple RL problem we will explore in-depth

Basic problem:

- Have K different slot machines ("Bandits")
- At each time step, agent can choose one machine and receives a random reward
- Each has some unknown average reward θ_i (e.g. a number between 0 and 1)

Multi-Armed Bandits

A simple RL problem we will explore in-depth

Agent must balance between…

- Playing machines where little is known about the reward (“Exploration”)
- Playing machines where the reward is believed to be high (“Exploitation”)

Examples of MAB Problems

Online advertising: which ad will result in the most clicks?

Show Ad 1

Show Ad 2

. . . . . .

Show Ad K

Reward:

- +1 if user clicks on the ad
- 0 if user does not click on ad

Average reward for each ad is what proportion of users click on the ad

Examples of MAB Problems

Clinical trials: which medicine will result in the best health outcomes?

Medication 1

Medication 2

. . . . .

Medication K

Reward:

- +1 if patient is cured
- 0 if patient's health does not change
- -1 if patient's health gets worse

Average reward over many trials summarizes how often drug is helpful/harmful

MAB as an RL problem

Actions:

- Which arm to pull at each trial

State:

- Agent's current estimate of average reward per arm
- $S_t = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_K]$ is a K-dimensional vector
- $\hat{\theta}_j =$ (total reward received from arm j) / (total number of times arm j was pulled)

Goal of our agent is to maximize the total reward across T trials

Strategies for the MAB Problem

Note: in each strategy, agent updates S_t after every trial

Strategy 1: Random Guessing

- At each trial, pull a random arm
- Not very smart: we don't adapt to choose arms which give high rewards
- All explore, no exploit

Strategy 2: Explore-then-Exploit

- Pull random arms for the first $S < T$ trials
- Pull the best arm for the remaining $T – S$ trials
- What is wrong with this?
    - Usually want S to be small
    - Can get unlucky and choose a suboptimal arm

Strategies for the MAB Problem

Note: in each strategy, agent updates S_t after every trial

Strategy 3: ϵ-Greedy

- Fix a number $0 < \epsilon < 1$
- At each trial:
    - pull the best arm (highest $\hat{\theta}_j$) with probability 1 - ϵ
    - pull a random arm with probability ϵ
- ϵ explicitly trades-off between exploration and exploitation
    - Needs to be chosen by us, usually small (0.05-0.1)

Strategies for the MAB Problem

Note: in each strategy, agent updates S_t after every trial

Strategy 4: ϵ-Decreasing

- Fix numbers $0 < \epsilon < 1$ and $0 < \alpha < 1$
  - α is a "decay" factor
- At trial t:
  - pull the best arm (highest $\hat{\theta}_j$) with probability 1 - $\epsilon \cdot \alpha^t$
  - pull a random arm with probability $\epsilon \cdot \alpha^t$
- Idea:
  - When t is small, want to explore more (high ϵ)
  - When t is large, want to exploit more (low ϵ)

Many more strategies exist: we won't cover them all here

MAB Example

$\theta_1 = 0.1$

$\theta_2 = 0.3$

$\theta_3 = 0.7$

$\theta_4 = 0.2$

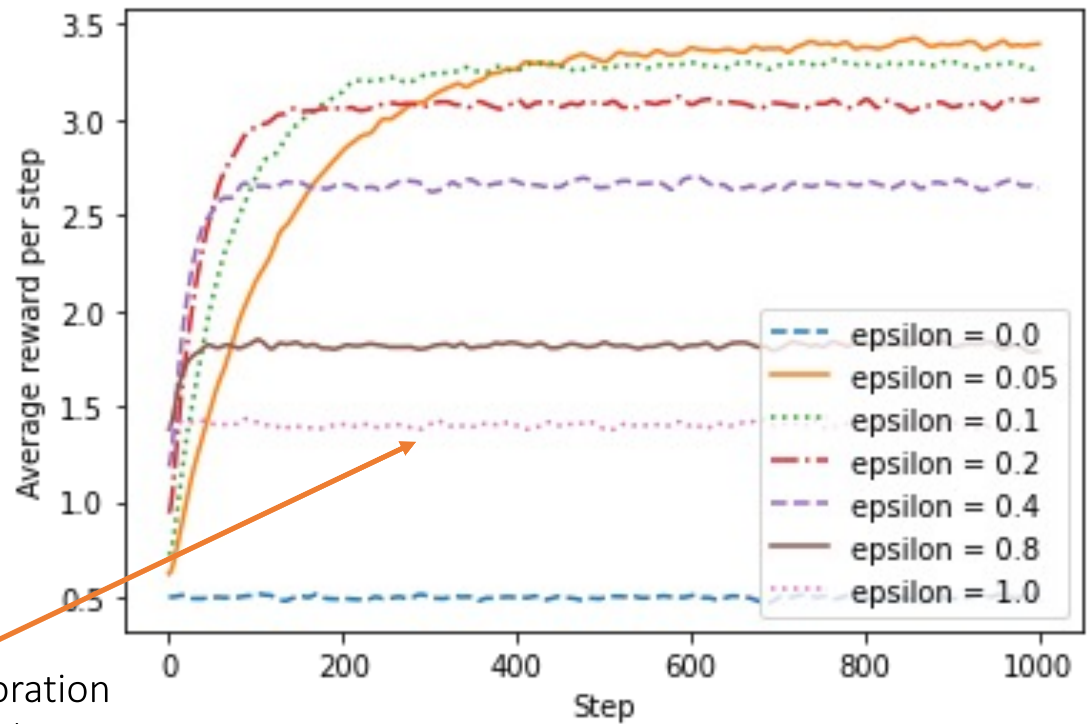
$\theta_5 = 0.1$

Arm k gives a random reward:

- +1 with probability θ_k
- 0 with probability 1 - θ_k

MAB Example

ϵ-Greedy Strategy



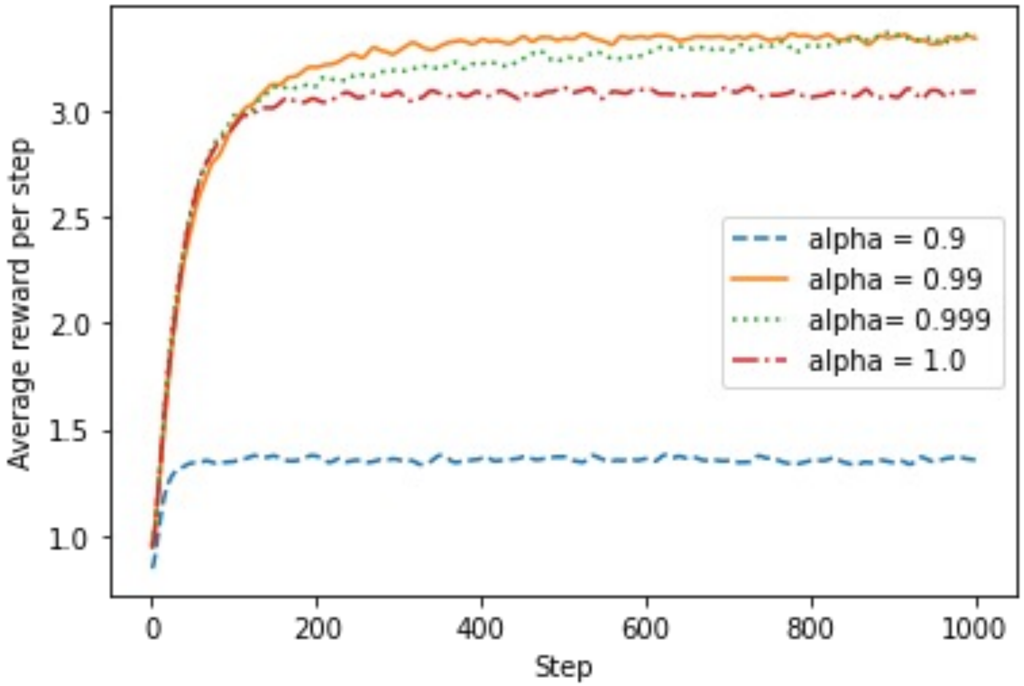
Pure exploration
(Strategy 1)

https://gibberblot.github.io/rl-notes/single-agent/multi-armed-bandits.html

MAB Example

Decreasing ϵ Strategy

Initial ϵ is 0.2



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Summary of MABs

- Simple RL problem that can model many real-world problems
  - Which ads to show, where to place them
  - Medical trials
  - General alternative to A/B testing

- Need to balance exploration against exploitation
  - Many strategies for explicitly doing so

- Fairly deep topic
  - Can prove bounds on the “regret”
  - comparisons between your strategy and the optimal strategy
  - See e.g. “Regret Analysis…” (Bubeck and Cesa-Bianchi, 2012)

Questions?

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Markov Decision Processes

Where We're Headed

Markov Property

"The future is independent of the past given the present"

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"The future is independent of the past given the present"

Definition

A state S_t is *Markov* if and only if

$$\mathbb{P}[S_{t+1} \mid S_t] = \mathbb{P}[S_{t+1} \mid S_1, ..., S_t]$$

Markov Property

"The future is independent of the past given the present"

Definition

A state S_t is *Markov* if and only if

$$\mathbb{P}[S_{t+1} \mid S_t] = \mathbb{P}[S_{t+1} \mid S_1, ..., S_t]$$

- The state captures all relevant information from the history
- Once the state is known, the history may be thrown away
- i.e. The state is a sufficient statistic of the future

State Transition Matrix

For a Markov state s and successor state s', the *state transition probability* is defined by

$$\mathcal{P}_{ss'} = \mathbb{P}\left[S_{t+1} = s' \mid S_t = s\right]$$

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$$\mathcal{P}_{ss'} = \mathbb{P}\left[S_{t+1} = s' \mid S_t = s\right]$$

State transition matrix $\mathcal{P}$ defines transition probabilities from all states s to all successor states s',

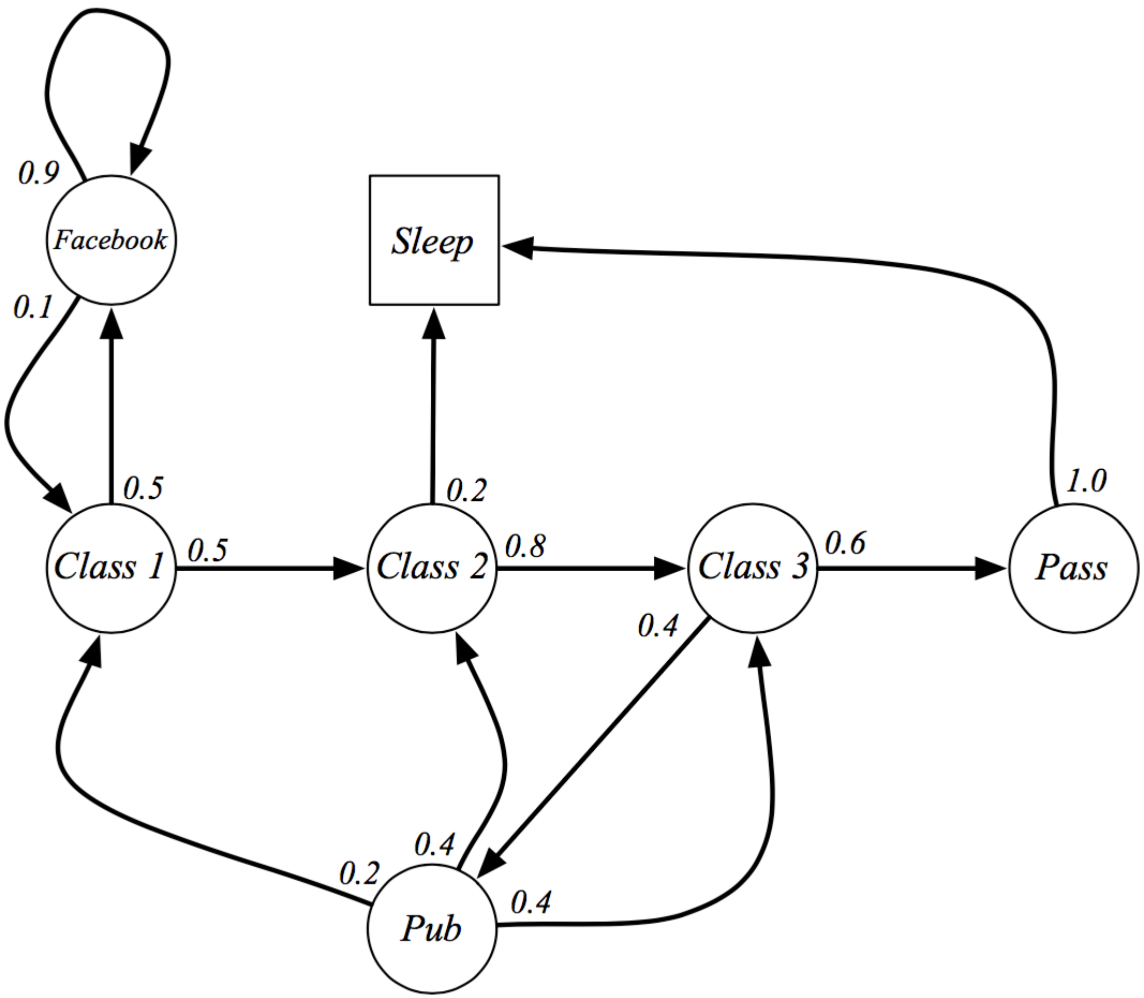
$$\mathcal{P} \;=\; \begin{array}{cc} & \textit{to} \\ \textit{from} & \begin{bmatrix} \mathcal{P}_{11} & \dots & \mathcal{P}_{1n} \\ \vdots & & \\ \mathcal{P}_{n1} & \dots & \mathcal{P}_{nn} \end{bmatrix} \end{array}$$

where each row of the matrix sums to 1.

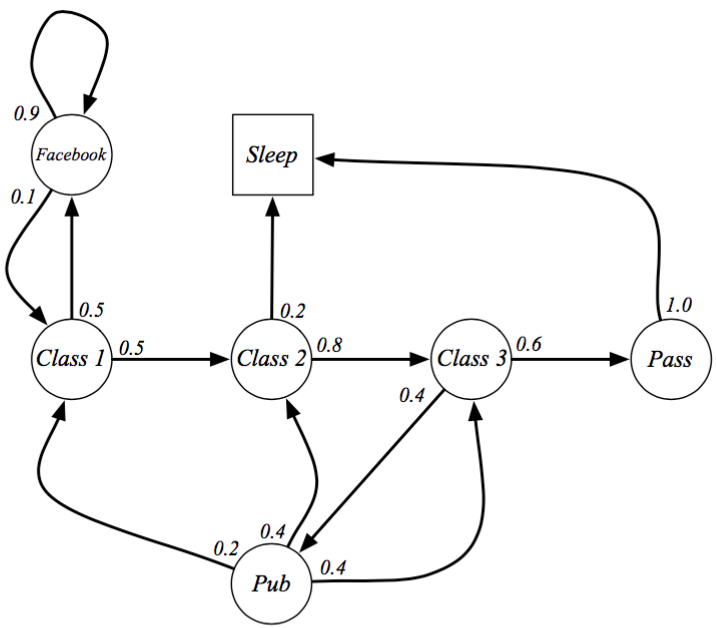
State Transition Matrix

A Markov process is a memoryless random process, i.e. a sequence of random states $S_1, S_2, ...$ with the Markov property.

Student Markov Chain



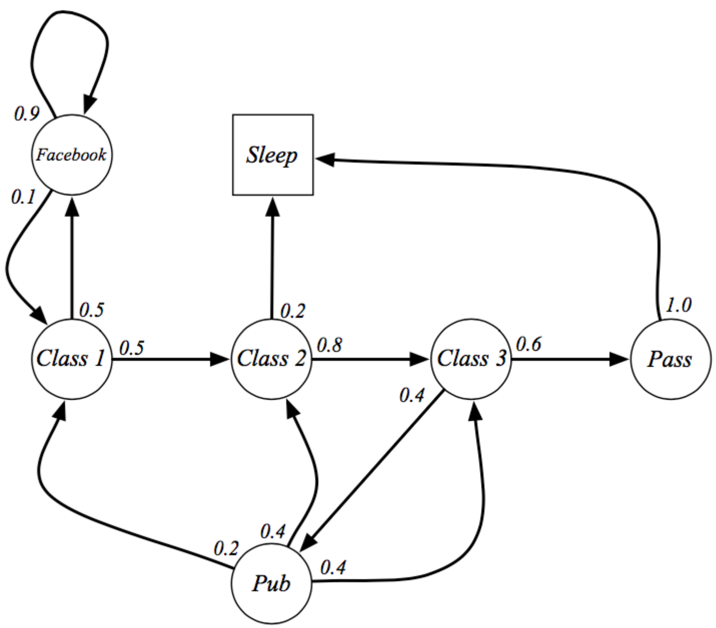
Student Markov Chain



Sample episodes for Student Markov Chain starting from $S_1 = \text{C1}$

$$S_1, S_2, \ldots, S_T$$

Student Markov Chain

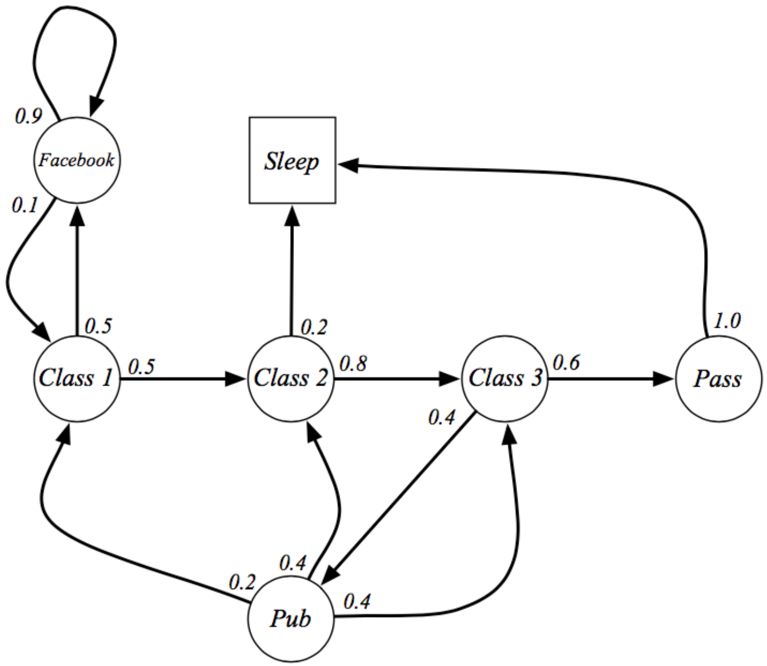


Sample episodes for Student Markov Chain starting from $S_1 = \text{C1}$

$$S_1, S_2, \ldots, S_T$$

- C1 C2 C3 Pass Sleep
- C1 FB FB C1 C2 Sleep
- C1 C2 C3 Pub C2 C3 Pass Sleep
- C1 FB FB C1 C2 C3 Pub C1 FB FB FB C1 C2 C3 Pub C2 Sleep

Student Markov Chain



$$\mathcal{P} = \begin{array}{c|ccccccc|} & C1 & C2 & C3 & Pass & Pub & FB & Sleep \\ \hline C1 & & 0.5 & & & & 0.5 & \\ C2 & & & 0.8 & & & & 0.2 \\ C3 & & & & 0.6 & 0.4 & & \\ Pass & & & & & & & 1.0 \\ Pub & 0.2 & 0.4 & 0.4 & & & & \\ FB & 0.1 & & & & & 0.9 & \\ Sleep & & & & & & & 1 \\ \hline \end{array}$$

Markov Chain Demo

http://setosa.io/ev/markov-chains/

Where We're Headed

Wrapup

- Reinforcement Learning
  - An agent interacts with an environment over time
  - Must explore this environment in order to maximize reward over time
- Multi-Armed Bandits
  - Simple RL problem with many real-world applications
- Markov Processes
  - Next state depends only on current state
  - Characterized by transition matrix