ENGSCI 255 Lab 2

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Question 1

```
set.seed(99)  # sets a seed for random number generator
spam.df = read.csv("spamdata.csv",header=TRUE) # read in the spam data
```

a) k-means clustering using all independent attributes:

```
# perform a k-means clustering with 20 iterations, using all 57 attributes
spamCluster = kmeans(spam.df, 2, nstart = 20, iter.max = 20)
spamCluster$size    # size of each cluster
```

```
## [1] 4357  244
```

Before we check the results of the clustering, we should find out how many emails are actually spam.

```
numOfSpam = 0
for (i in seq(1,length(spam.df$IsSpam))) {
     if (spam.df$IsSpam[i] == 1) {
         numOfSpam = numOfSpam + 1
     }
}
print(numOfSpam)
```

```
## [1] 1813
```

We can see there are two clusters of size 4357 and 244, but there are 1813 spam emails in the dataset. So there will be a lot of false predictions in our clustered data.

b) Generate a table for the clustering:

Next, let’s generate a table for the clustering results:

```
# columns are is/is not spam, rows are the two clusters created
table(spamCluster$cluster, spam.df$IsSpam, dnn = c("Cluster No", "Is/Is not Spam"))
```

```
##             Is/Is not Spam
## Cluster No    0    1
##           1 2735 1622
##           2   53  191
```

It seems like cluster 1 is predicting the group that isn’t spam, and cluster 2 is predicting the group that is spam.

We can make the attribute each cluster is predicting the name of the cluster, so it is easier to understand the calculations. We say that something being spam is the positive result (1 or yes).

```r
for (i in seq(1,length(spamCluster$cluster))) {
     # reduce the value by 1 so that each cluster represents what it is predicting
     spamCluster$cluster[i] = spamCluster$cluster[i] - 1
}

table(spamCluster$cluster, spam.df$IsSpam, dnn = c("Prediction", "Actual"))
```

```
##             Actual
## Prediction    0    1
##           0 2735 1622
##           1   53  191
```

If we used this clustering as a foundation for a spam filter, what is the accuracy, sensitivity, specificity and precision of the filter?

Number of true negatives (TN) = 2735. Number of true positives (TP) = 191. Number of false positives (FP) = 53. Number of false negatives (FN) = 1622.

$\text{Accuracy} = \text{proportion of datapoints classified correctly} = \frac{TN+TP}{\text{total \# of datapoints}} = \frac{2735+191}{2735+191+53+1622} = 0.6359$

$\text{Sensitivity} = \text{true positive rate} = \frac{TP}{TP+FN} = \frac{191}{191+1622} = 0.1054$. This is the proportion of datapoints which are predicted to be yes, and are actually yes.

$\text{Specificity} = \text{true negative rate} = \frac{TN}{TN+FP} = \frac{2735}{2735+53} = 0.9810$. This is the proportion of datapoints which are predicted to be no, and are actually no.

$\text{Precision} = \frac{TP}{TP+FP} = \frac{191}{191+53} = 0.7828$. This is the proportion of datapoints which are actually yes, when they are predicted to be yes.

Question 2

```r
set.seed(50)  # sets a seed for random number generator
```

Generate a training dataset of 2400 datapoints, where the remaining points will be the test set:

```r
# training subset of 2400 datapoints, half is random spam emails, half is random not spam emai
ls
# dataset is ordered so we know we are getting a 50/50 mix of
train = c(sample(1:numOfSpam,1200),sample(numOfSpam+1:length(spam.df$IsSpam),1200))

# treat spam classification as a factor instead of numeric
spam.df$IsSpam = as.factor(spam.df$IsSpam)
```

a) Loss Matrices:

i.

Set the termination criteria of max depth of 10. Disable any other termination criteria:

```
# cp to 0 as we want to branch any time possible, minimum complexity parameter, minsplit 1
controlParms = rpart.control(maxdepth = 10, cp = 0, minsplit = 1)
```

ii.

We want to modify the loss matrix to generate 6 different trees using all independent attributes, ranging from no FP's to no FN's in the training data. This means we should put a heavy cost on the false positives and low cost on false negatives, and then constantly make FP's lower cost and FN's higher cost. The different elements in the matrix correspond to the costs of the following:

```
list(loss = matrix(c("TN","FN","FP","TP"), nrow = 2))
```

```
## $loss
##      [,1] [,2]
## [1,] "TN" "FP"
## [2,] "FN" "TP"
```

Tree 1 (aiming for no false positives), cost of 95 for FP's, 5 for FN's:

```
# 95/5 cost for FP/FN
lossMatrix1 = list(loss = matrix(c(0,95,5,0), nrow = 2))

# create the tree using the preset control and the loss matrix
tree1 = rpart(IsSpam~., data = spam.df, subset = train, control = controlParms, parms = lossMa
trix1)

# in-sample prediction using the training data:
ISCM1 = table(predict(tree1,spam.df[train,],type="class"),spam.df[train,"IsSpam"], dnn = c("Pr
ediction", "Actual"))
ISCM1
```

```
##           Actual
## Prediction    0    1
##          0  524    0
##          1  212 1200
```

Tree 2 (still a high weight on false positives), cost of 70 for FP's, 30 for FN's:

```
# 70/30 cost for FP/FN
lossMatrix2 = list(loss = matrix(c(0,70,30,0), nrow = 2))

# create the tree using the preset control and the loss matrix
tree2 = rpart(IsSpam~., data = spam.df, subset = train, control = controlParms, parms = lossMa
trix2)

# in-sample prediction using the training data:
ISCM2 = table(predict(tree2,spam.df[train,],type="class"),spam.df[train,"IsSpam"], dnn = c("Pr
ediction", "Actual"))
ISCM2
```

```
##           Actual
## Prediction    0    1
##          0  688   14
##          1   48 1186
```

Tree 3 (slightly higher weight on false negatives vs false positives), cost of 45 for FP’s, 65 for FN’s:

```
# 45/65 cost for FP/FN
lossMatrix3 = list(loss = matrix(c(0,45,65,0), nrow = 2))

# create the tree using the preset control and the loss matrix
tree3 = rpart(IsSpam~., data = spam.df, subset = train, control = controlParms, parms = lossMa
trix3)

# in-sample prediction using the training data:
ISCM3 = table(predict(tree3,spam.df[train,],type="class"),spam.df[train,"IsSpam"], dnn = c("Pr
ediction", "Actual"))
ISCM3
```

```
##           Actual
## Prediction    0    1
##          0  726   53
##          1   10 1147
```

Tree 4 (even higher weighting on false negatives), cost of 20 for FP’s, 80 for FN’s:

```
# 20,80 cost for FP/FN
lossMatrix4 = list(loss = matrix(c(0,20,80,0), nrow = 2))

# create the tree using the preset control and the loss matrix
tree4 = rpart(IsSpam~., data = spam.df, subset = train, control = controlParms, parms = lossMa
trix4)

# in-sample prediction using the training data:
ISCM4 = table(predict(tree4,spam.df[train,],type="class"),spam.df[train,"IsSpam"], dnn = c("Pr
ediction", "Actual"))
ISCM4
```

```
##           Actual
## Prediction    0    1
##          0  735   83
##          1    1 1117
```

Tree 5 (aiming for no false negatives), cost of 5 for FP’s, 95 for FN’s:

```
# 5/95 cost for FP/FN
lossMatrix5 = list(loss = matrix(c(0,5,95,0), nrow = 2))

# create the tree using the preset control and the loss matrix
tree5 = rpart(IsSpam~., data = spam.df, subset = train, control = controlParms, parms = lossMa
trix5)

# in-sample prediction using the training data:
ISCM5 = table(predict(tree5,spam.df[train,],type="class"),spam.df[train,"IsSpam"], dnn = c("Pr
ediction", "Actual"))
ISCM5
```

```
##            Actual
## Prediction    0    1
##           0  736  151
##           1    0 1049
```

Tree 6 (let's see what happens if we put an even higher weight now that we have achieved no false negatives already), cost of 1 for FP's, 99 for FN's:

```
# 1/99 cost for FP/FN
lossMatrix6 = list(loss = matrix(c(0,1,99,0), nrow = 2))

# create the tree using the preset control and the loss matrix
tree6 = rpart(IsSpam~., data = spam.df, subset = train, control = controlParms, parms = lossMa
trix6)

# in-sample prediction using the training data:
ISCM6 = table(predict(tree6,spam.df[train,],type="class"),spam.df[train,"IsSpam"], dnn = c("Pr
ediction", "Actual"))
ISCM6
```

```
##            Actual
## Prediction   0   1
##           0 736 251
##           1   0 949
```

iii.

The in-sample confusion matrices have already been shown in part ii above, so I will just show the confusion matrices for the out-of-sample predictions, using the test set (original dataset minus the training set).

Tree 1:

```
# out-of-sample prediction using the test data:
OOSCM1 = table(predict(tree1,spam.df[-train,],type="class"),spam.df[-train,"IsSpam"], dnn = c(
"Prediction", "Actual"))
OOSCM1
```

```
##            Actual
## Prediction    0    1
##           0 1405   12
##           1  647  601
```

Tree 2:

```
# out-of-sample prediction using the test data:
OOSCM2 = table(predict(tree2,spam.df[-train,],type="class"),spam.df[-train,"IsSpam"], dnn = c(
"Prediction", "Actual"))
OOSCM2
```

```
##           Actual
## Prediction    0    1
##          0 1784   33
##          1  268  580
```

Tree 3:

```
# out-of-sample prediction using the test data:
OOSCM3 = table(predict(tree3,spam.df[-train,],type="class"),spam.df[-train,"IsSpam"], dnn = c(
"Prediction", "Actual"))
OOSCM3
```

```
##           Actual
## Prediction    0    1
##          0 1864   69
##          1  188  544
```

Tree 4:

```
# out-of-sample prediction using the test data:
OOSCM4 = table(predict(tree4,spam.df[-train,],type="class"),spam.df[-train,"IsSpam"], dnn = c(
"Prediction", "Actual"))
OOSCM4
```

```
##           Actual
## Prediction    0    1
##          0 1904   76
##          1  148  537
```

Tree 5:

```
# out-of-sample prediction using the test data:
OOSCM5 = table(predict(tree5,spam.df[-train,],type="class"),spam.df[-train,"IsSpam"], dnn = c(
"Prediction", "Actual"))
OOSCM5
```

```
##           Actual
## Prediction    0    1
##          0 1954  122
##          1   98  491
```

Tree 6:

```
# out-of-sample prediction using the test data:
OOSCM6 = table(predict(tree6,spam.df[-train,],type="class"),spam.df[-train,"IsSpam"], dnn = c(
"Prediction", "Actual"))
OOSCM6
```

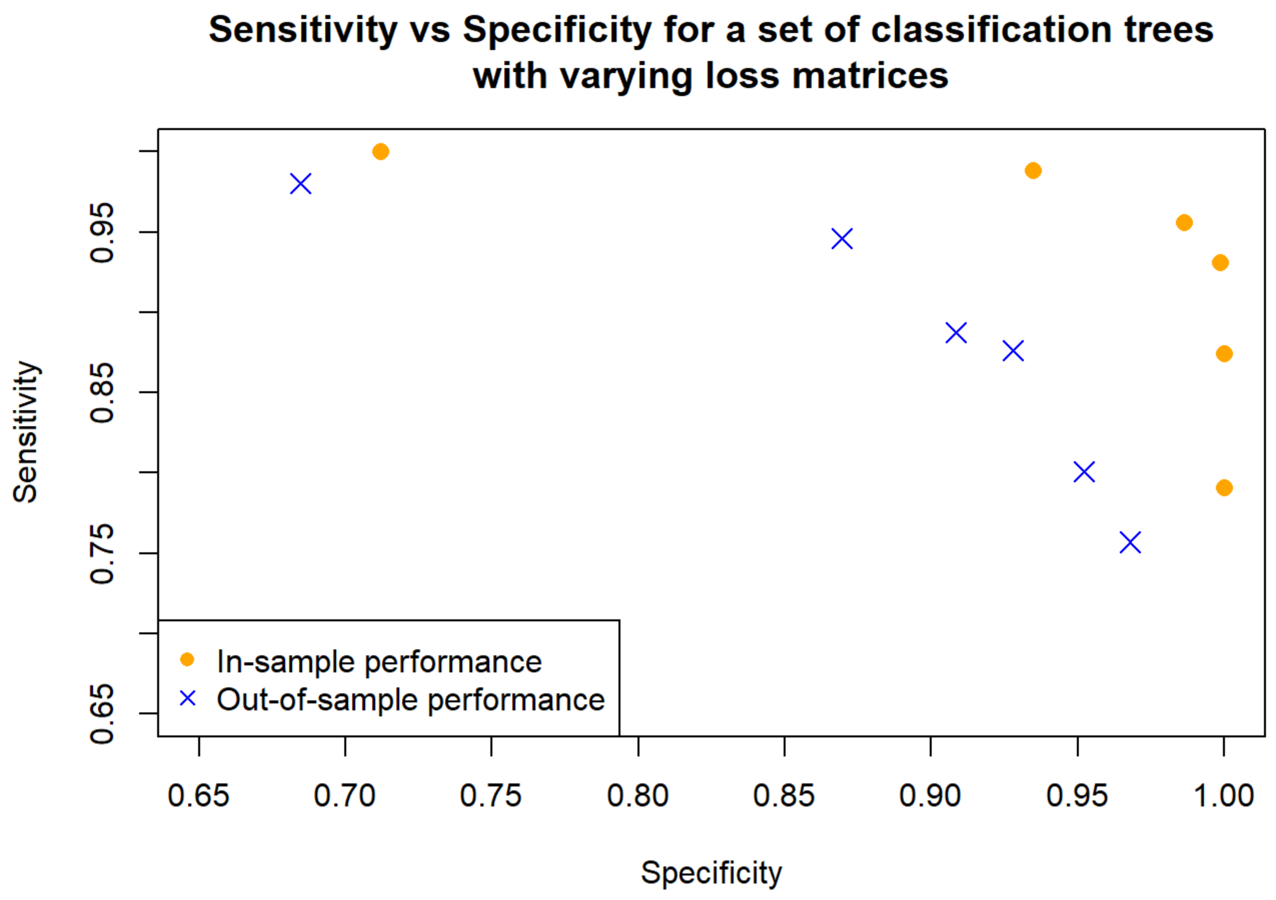
```
##           Actual
## Prediction    0    1
##          0 1986  149
##          1   66  464
```

iv.

Put the values of sensitivity and specificity for each classification model into a vector to then show on a scatterplot (including both in and out-of-sample performance): [Code for sensitivity/specificity calculations hidden because messy]

Show this information on a 2D scatterplot of sensitivity vs specificity, again, the in-sample performance was using the training dataset and out-of-sample performance used the original dataset minus the training set. Note the axes don't start from 0:

```
plot(inSampleSensitivity ~ inSampleSpecificity, data = performance1.df, col = "orange", pch =
16, cex = 1.2, ylim = c(0.65,1), xlim = c(0.65,1), ann = FALSE)
points(outOfSampleSensitivity ~ outOfSampleSpecificity, data = performance2.df, col = "blue",
pch = 4, cex = 1.4)
legend("bottomleft", legend = c("In-sample performance", "Out-of-sample performance"), pch = c
(16,4), col = c("orange","blue"))
title(main = "Sensitivity vs Specificity for a set of classification trees\nwith varying loss
matrices", xlab = "Specificity", ylab = "Sensitivity")
```



b) Tree Depth:

i.

We want to generate six classification trees that vary by their max depth (from 5 to 30). For this I have disabled any other stopping criteria as otherwise the higher depth trees will all look the same and not go until their max depth has been reached.

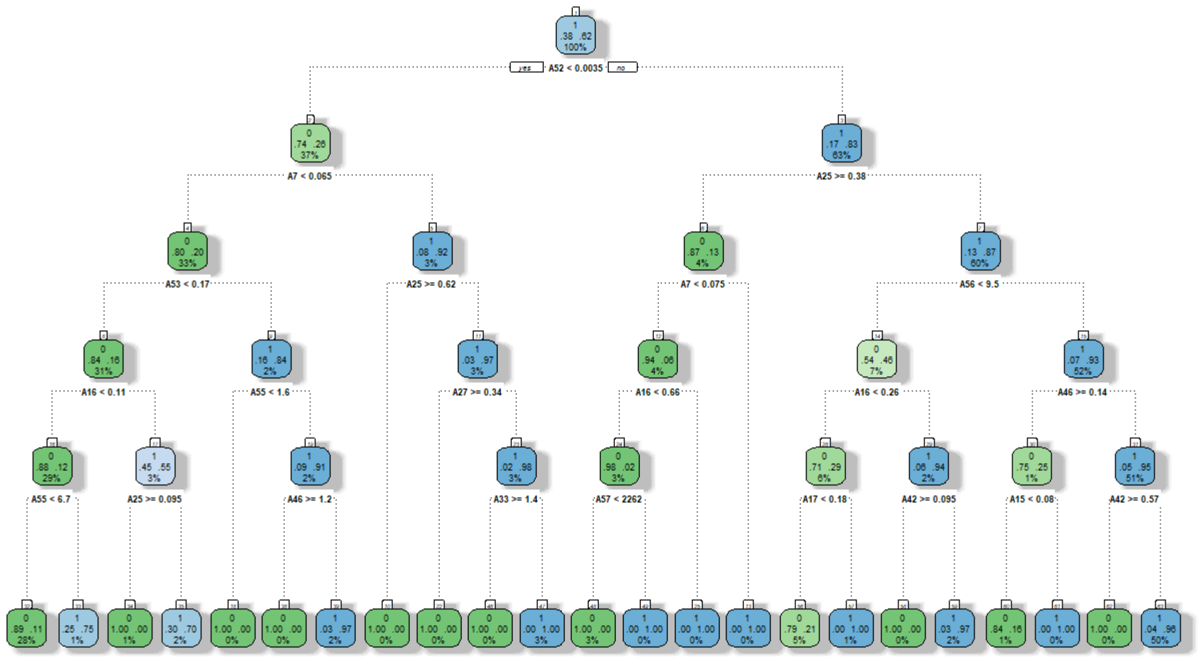
To save space, I used a list to store all 6 trees generated, indexed 1 through 6 corresponding to max depth of 5 through 30 in steps of 5 for each tree:

```r
# create a list to store the trees
trees = list()

for (i in seq(1,6)) {
  # max depth of 5 to 30, disable other stopping criteria
  controlParms = rpart.control(maxdepth = 5*i, cp = 0, minsplit = 1)

  # create a tree for each max depth
  trees[[i]] = rpart(IsSpam~.,data = spam.df, subset = train, control = controlParms)
}
```

ii.

Plot the tree for a depth of 5:

```r
fancyRpartPlot(trees[[1]], sub = "Tree using all independent vars, max depth 5")
```



Tree using all independent vars, max depth 5

iii.

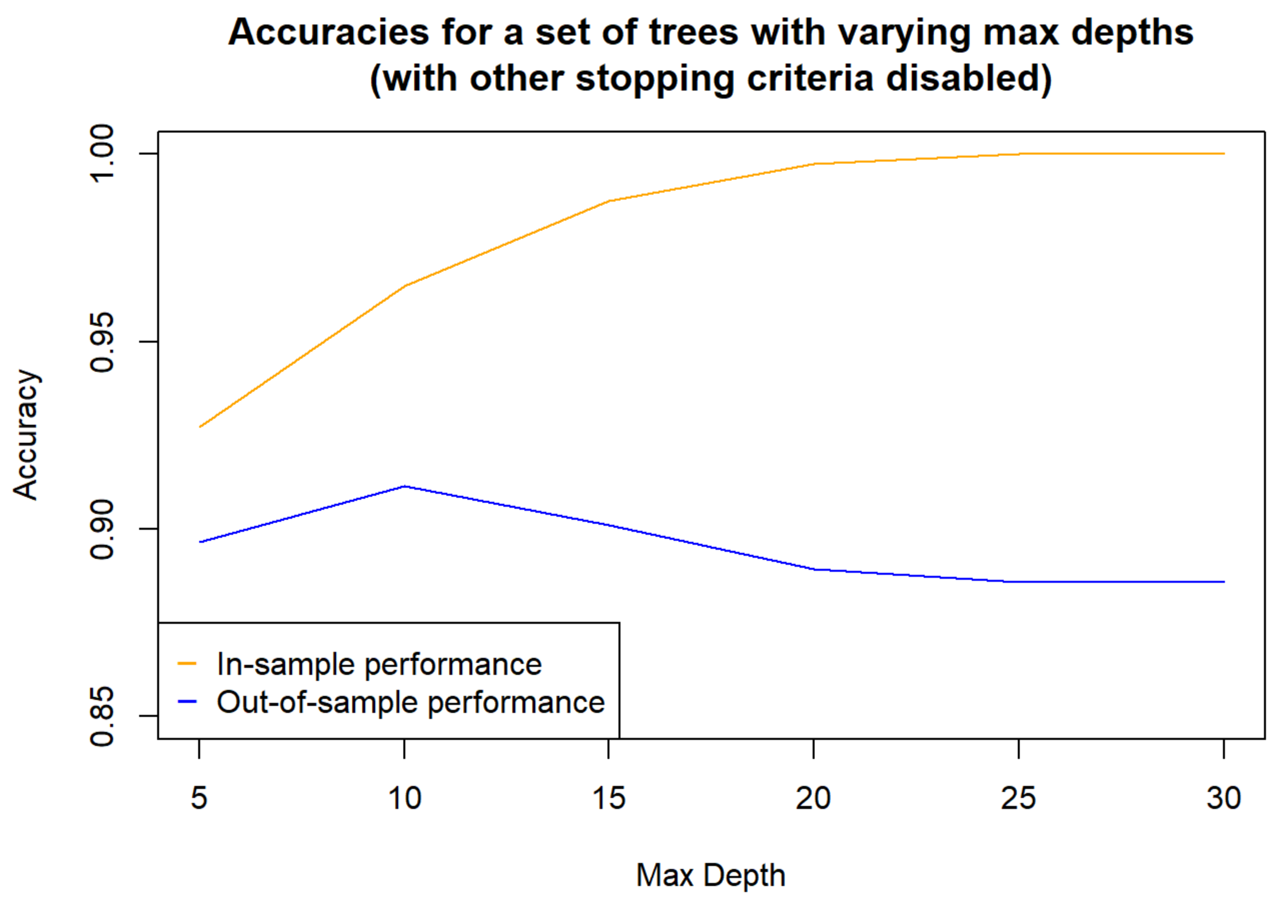
We now want to plot the *accuracies* of the classification models vs the max depths, one line for in-sample performance and one line for out-of-sample performance.

First of all, let’s get some vectors of the accuracies for each tree depth and their predictions for both in and out-of-sample data:

```
# create vectors to hold accuracies
ISAcc = c()
OOSAcc = c()

# generate confusion matrices, then put accuracies in lists for their respective predictions
for (i in seq(1,6)) {
  ISCM = table(predict(trees[[i]],spam.df[train,],type="class"),spam.df[train,"IsSpam"])
  OOSCM = table(predict(trees[[i]],spam.df[-train,],type="class"),spam.df[-train,"IsSpam"])

  # I couldn't simply use length(train) and length(spam.df$IsSpam - train) because the number
of
  # predictions generated did not match the number of datapoints in the training set/ test set
...
  # idk why, but the number of datapoints in the train+test results does sum to total # of dat
apoints
  ISAcc[i] = (ISCM[1]+ISCM[4])/(ISCM[1]+ISCM[2]+ISCM[3]+ISCM[4])
  OOSAcc[i] = (OOSCM[1]+OOSCM[4])/(OOSCM[1]+OOSCM[2]+OOSCM[3]+OOSCM[4])
}
```

Now let’s try plot the accuracies against the max depths for each tree. Note again the axes don’t start from 0:

```
# create a vector holding the max depths
maxDepths = seq(5,30,5)

# plot the data and add information about it
plot(ISAcc ~ maxDepths, type = "n", ylim = c(0.85,1), ann = FALSE)
lines(ISAcc ~ maxDepths, col = "orange")
lines(OOSAcc ~ maxDepths, col = "blue")
legend("bottomleft", legend = c("In-sample performance", "Out-of-sample performance"), pch = "
_", col = c("orange","blue"))
title(main = "Accuracies for a set of trees with varying max depths\n(with other stopping crit
eria disabled)", xlab = "Max Depth", ylab = "Accuracy")
```



c) Discuss the results above:

From the scatterplot (where max depth was constant and we changed the loss matrices for each tree), we can infer that in general, as you try to increase the sensitivity of a classification tree, the specificity will drop as a result, and vice versa for increasing specificity. For all trees, we have a decent ratio of specificity to sensitivity, meaning we have hit reasonably optimal solutions, assuming we are weighting specificity and sensitivity the same in a bi-objective optimasation problem.

From the line graph (where the loss matrices were kept constant and we changed the max depth for each tree), we can see that there are two different trends. The in-sample performance kept improving as we increased the max depth of the tree, but the out-of-sample performance started dropping after the max depth of the tree was higher than 10. This is probably due to overfitting the classification model to the training data, so when applied to unseen data, it performs poorly as it has been conditioned too far with the training data. So we can see that the peak optimality (if we are aiming for accuracy) in this case is at a max tree depth of 10, which is what we were using for when we were varying the loss matrices in the first part of the question.

In both experiments, we can see that the out-of-sample performance was poorer than the in-sample performance, which is expected. We should also consider the fact that we may have overfit the trees for both experiments, and to see if there is a more optimal max depth to use, we should test further by checking varying max depths between 5 and 15 in smaller increments, and possibly change the stopping criteria so that it does have a minimum split of higher than 1 and requires some threshold change in complexity before branching out.

Question 3

```
set.seed(42)  # sets a seed for random number generator
```

a) Generate different random forests with 10, 100 and 100 trees:

```
# with 10 trees
spamForest1 = randomForest(IsSpam~., data = spam.df, ntree = 10)

# with 100 trees
spamForest2 = randomForest(IsSpam~., data = spam.df, ntree = 100)

# with 1000 trees
spamForest3 = randomForest(IsSpam~., data = spam.df, ntree = 1000)
```

b) Confusion matrices and calculations:

For forest with **10 trees**:

```
# generate confusion matrix
table(predict(spamForest1, spam.df, type = "response"), spam.df$IsSpam, dnn = c("Prediction",
"Actual"))
```

```
##             Actual
## Prediction    0    1
##          0 2782   31
##          1    6 1782
```

$$\text{Accuracy} = \frac{TN + TP}{\text{total \# of datapoints}} = \frac{2782 + 1782}{2782 + 31 + 6 + 1782} = 0.9920$$

$$\text{Sensitivity} = \frac{TP}{TP + FN} = \frac{1782}{1782 + 31} = 0.9829$$

$$\text{Specificity} = \frac{TN}{TN + FP} = \frac{2782}{2782 + 6} = 0.9978$$

$$\text{Precision} = \frac{TP}{TP + FP} = \frac{1782}{1782 + 6} = 0.9966$$

For forest with **100 trees**:

```
# generate confusion matrix
table(predict(spamForest2, spam.df, type = "response"), spam.df$IsSpam, dnn = c("Prediction",
"Actual"))
```

```
##             Actual
## Prediction    0    1
##          0 2788   19
##          1    0 1794
```

$$\text{Accuracy} = \frac{TN + TP}{\text{total \# of datapoints}} = \frac{2788 + 1794}{2788 + 19 + 0 + 1794} = 0.9959$$

$$\text{Sensitivity} = \frac{TP}{TP + FN} = \frac{1794}{1794 + 19} = 0.9895$$

$$\text{Specificity} = \frac{TN}{TN+FP} = \frac{2788}{2788+0} = 1.000$$

$$\text{Precision} = \frac{TP}{TP+FP} = \frac{1794}{1794+0} = 1.000$$

For forest with **1000 trees**:

```
# generate confusion matrix
table(predict(spamForest3, spam.df, type = "response"), spam.df$IsSpam, dnn = c("Prediction",
"Actual"))
```

```
##           Actual
## Prediction    0    1
##          0 2788   20
##          1    0 1793
```

$$\text{Accuracy} = \frac{TN+TP}{\text{total \# of datapoints}} = \frac{2788+1793}{2788+20+0+1793} = 0.9957$$

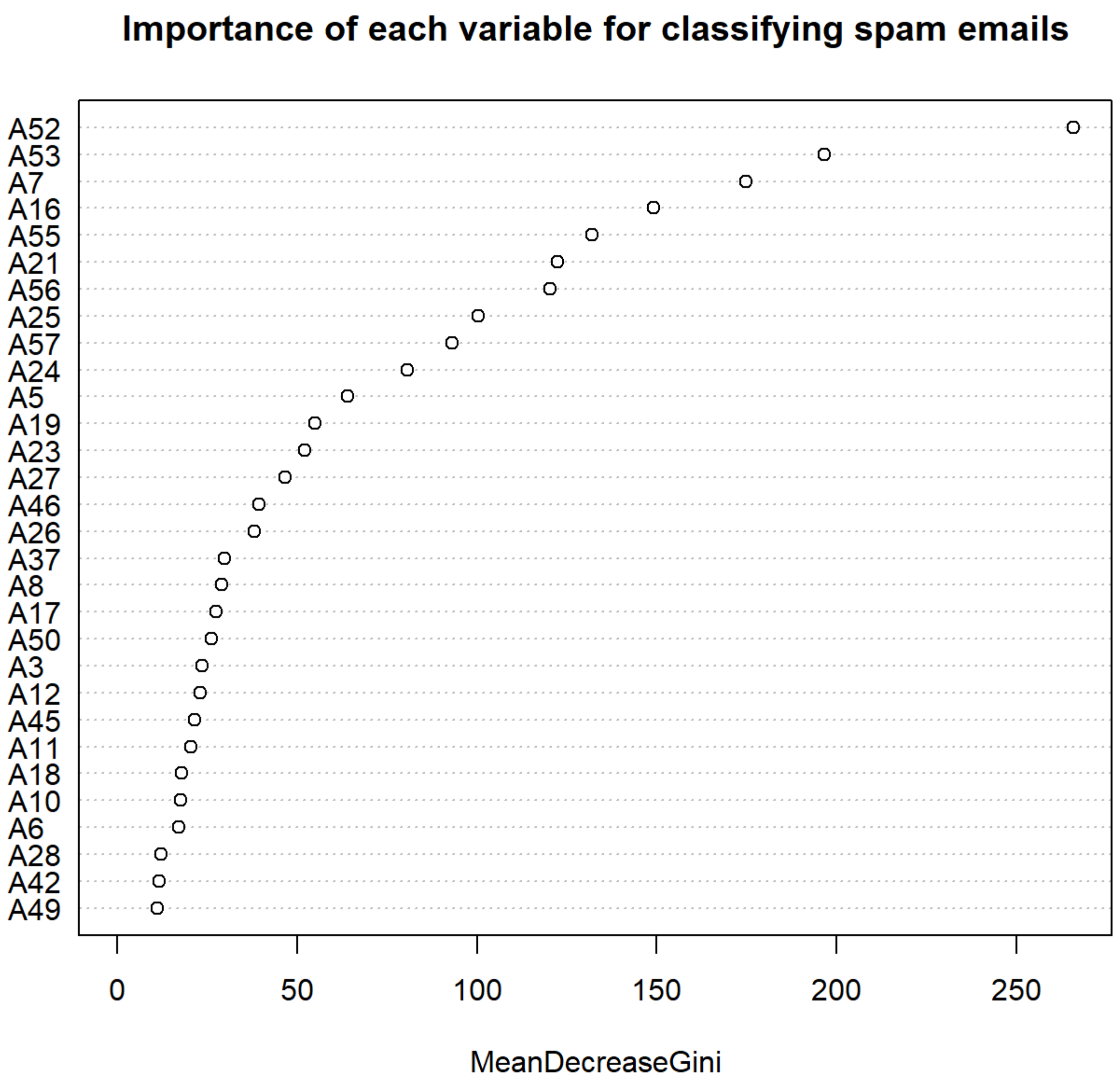
$$\text{Sensitivity} = \frac{TP}{TP+FN} = \frac{1793}{1793+20} = 0.9890$$

$$\text{Specificity} = \frac{TN}{TN+FP} = \frac{2788}{2788+0} = 1.000$$

$$\text{Precision} = \frac{TP}{TP+FP} = \frac{1793}{1793+0} = 1.000$$

c) Plot importance of different attributes:

```
# using forest with 1000 trees
varImpPlot(spamForest3, main = "Importance of each variable for classifying spam emails")
```



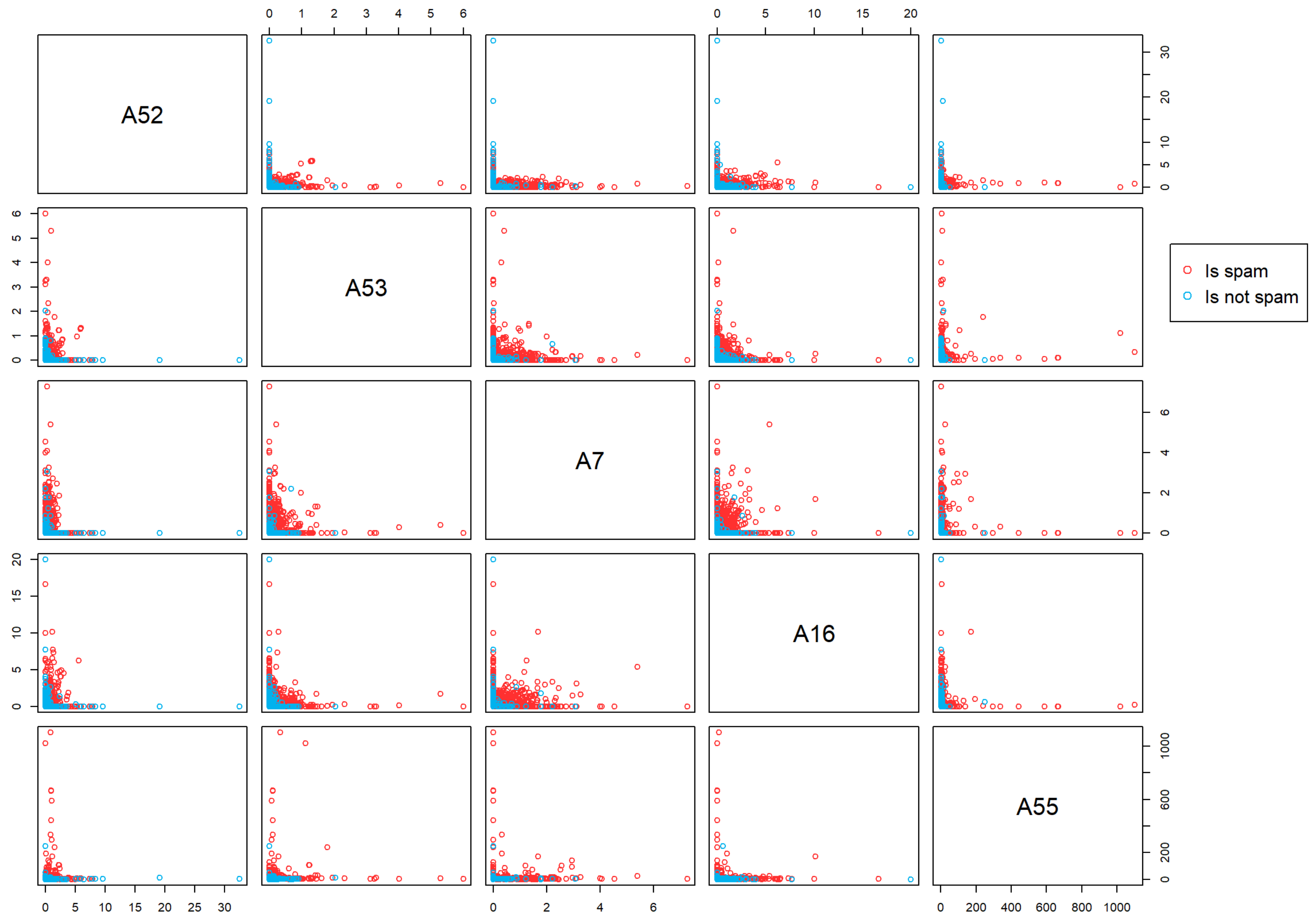
A higher mean decrease in gini index means the variable has a higher importance, so variables with points further to the right of the plot are of higher importance for distinguishing spam emails.

The top 5 attributes are: “A52”, “A53”, “A7”, “A16” and “A55”, in descending order of importance.

d) Scatterplot matrix of the top five attributes and comments:

Let’s make a scatterplot matrix using the top 5 attributes for distinguishing whether or not an email is spam or not:

```
pairs(spam.df[,c("A52", "A53", "A7", "A16", "A55")], col = ifelse(spam.df$IsSpam == "1", "fire
brick1", "deepskyblue2"), oma=c(3,2,2,12))

# plot legend outside of the figure region
legend(0.91, 0.76, legend = c("Is spam", "Is not spam"), col = c("firebrick1", "deepskyblue2")
, pch = 1, xpd = TRUE)
```



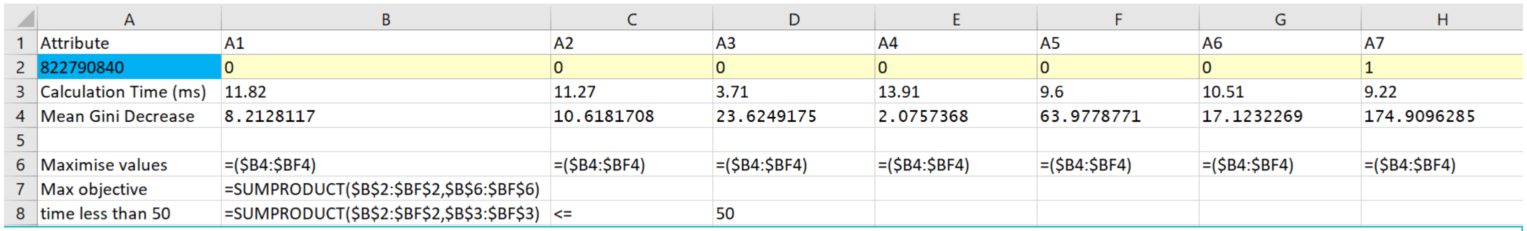
We can see most blue (not spam) datapoints are clustered on the left or bottom edge or both on each plot. This means that for every attribute shown in the pairs plot, there is a (seemingly) strong trend that if it's attributes (A52, A53, A7, A16 and A55) are near zero, it is likely to not be spam. We can see a much higher concentration of red datapoints as we go away from the bottom left corner in *any direction*, meaning that if any attribute increases, the email is more likely to be spam. As any attribute takes higher and higher positive values, the emails associated with those attributes are much more likely to be spam. This statement is less evident in the "A52" attribute, where even as A52 increases, emails are unlikely to be spam if the other attributes are near zero. Keep in mind that all the claims I made for the dataset here are only applicable to the five attributes mentioned before, and further testing will need to be done if we want to make more general claims about the entire dataset. However it still is ok for prediction purposes since these five attributes are the most important for distinguishing if an email is spam or not. This result is interesting because it may allow us to cut the time spent analysing the data for all attributes and make confident decisions using only a few attributes.

Question 4

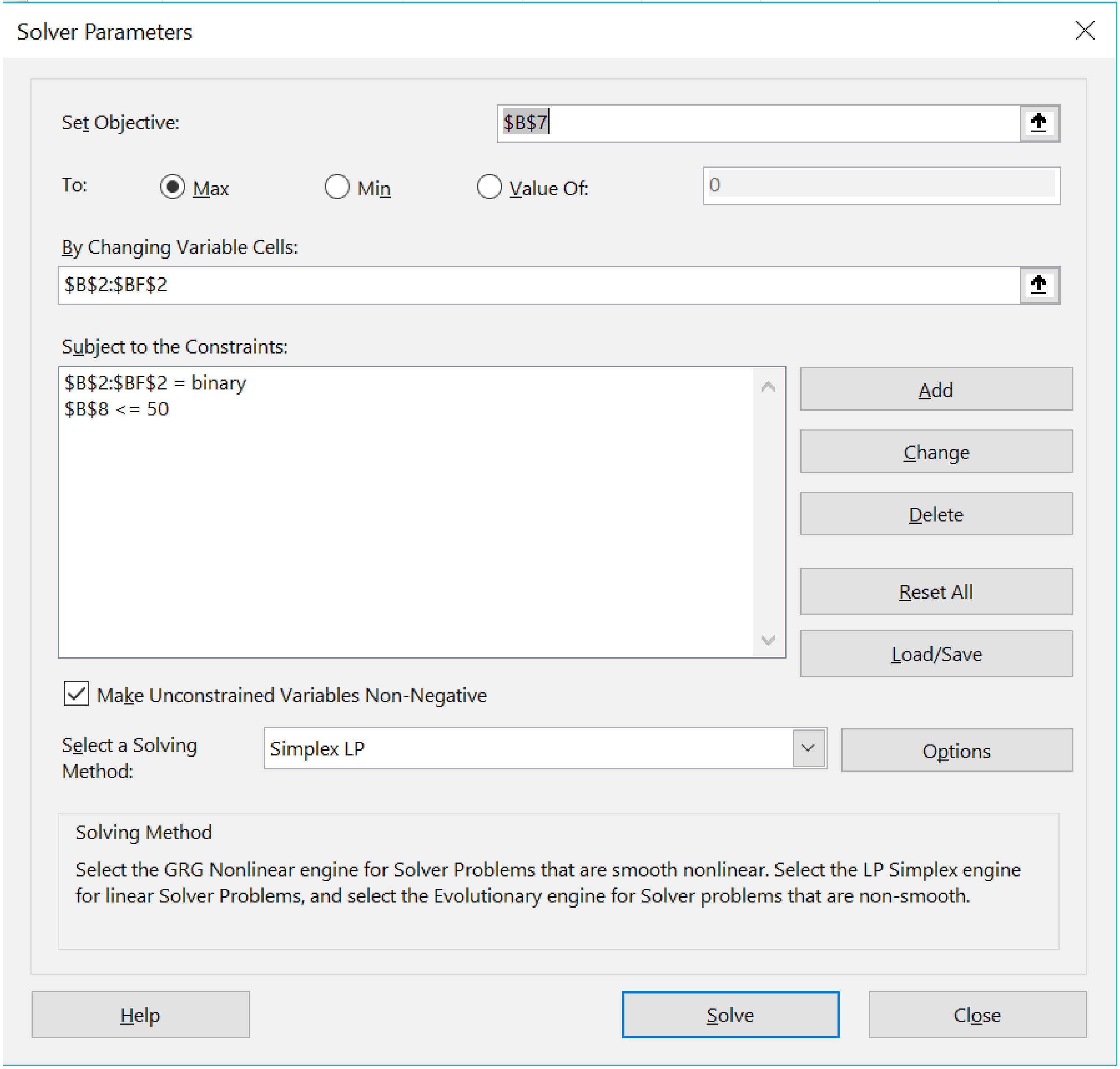
a) Formulate a binary integer program in Excel to determine what attributes to collect and use:

We want the most optimal selection of attributes to collect and use in the spam filtering proces, maximising mean decrease in gini index while also considering the time taken (we don't want to exceed 50ms for a single email)

After getting all the importance data from the "importance(spamForest3)" command, I input them into an Excel linear program. I divided importance score for each attribute by the time taken to scan that particular attribute to give the coefficients for the maximise objective function, and set contraints that total time could not exceed 50ms and the decision variables are all binary. A snippet containing all the important formula used is shown below:



	A	B	C	D	E	F	G	H
1	Attribute	A1	A2	A3	A4	A5	A6	A7
2	822790840	0	0	0	0	0	0	1
3	Calculation Time (ms)	11.82	11.27	3.71	13.91	9.6	10.51	9.22
4	Mean Gini Decrease	8.2128117	10.6181708	23.6249175	2.0757368	63.9778771	17.1232269	174.9096285
5								
6	Maximise values	=($B4:$BF4)	=($B4:$BF4)	=($B4:$BF4)	=($B4:$BF4)	=($B4:$BF4)	=($B4:$BF4)	=($B4:$BF4)
7	Max objective	=SUMPRODUCT($B$2:$BF$2,$B$6:$BF$6)						
8	time less than 50	=SUMPRODUCT($B$2:$BF$2,$B$3:$BF$3)	<=	50				



Solver Parameters

Set Objective: $B$7

To: [✓ Max] Min Value Of: 0

By Changing Variable Cells:

$B$2:$BF$2

Subject to the Constraints:

$B$2:$BF$2 = binary
$B$8 <= 50

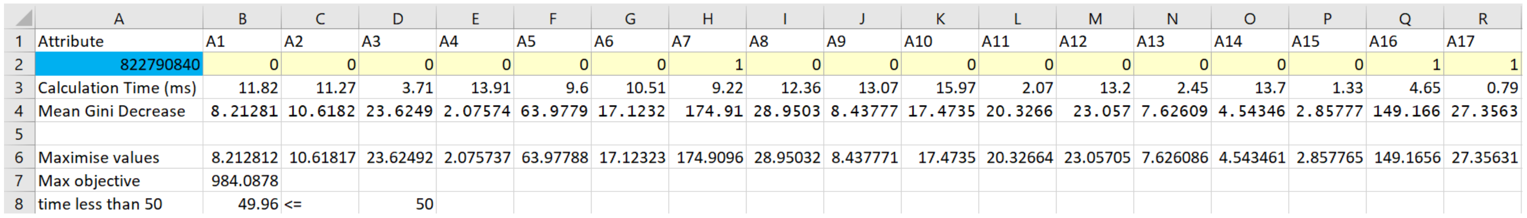
[✓] Make Unconstrained Variables Non-Negative

Select a Solving Method: Simplex LP

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

And the results:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	Attribute	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17
2	822790840	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1
3	Calculation Time (ms)	11.82	11.27	3.71	13.91	9.6	10.51	9.22	12.36	13.07	15.97	2.07	13.2	2.45	13.7	1.33	4.65	0.79
4	Mean Gini Decrease	8.21281	10.6182	23.6249	2.07574	63.9779	17.1232	174.91	28.9503	8.43777	17.4735	20.3266	23.057	7.62609	4.54346	2.85777	149.166	27.3563
5																		
6	Maximise values	8.212812	10.61817	23.62492	2.075737	63.97788	17.12323	174.9096	28.95032	8.437771	17.4735	20.32664	23.05705	7.626086	4.543461	2.857765	149.1656	27.35631
7	Max objective	984.0878																
8	time less than 50	49.96	<=	50														

From the results of solving the linear program, we only want to scan these attributes: A7, A16, A17, A21, A25, A28, A52, A55. Interestingly, we ditch some of the very high importance variables like A53 in favour of lower importance ones, as the scan time for those variables is too high to justify it being used.

b) Generate random forest using optimal attributes:

```r
# generate random forest using only the attributes that satisfy the 50ms requirement from abov
e
# column 58 for "IsSpam" data
spamForestOptimised = randomForest(IsSpam~., data = spam.df[,c(7,16,17,21,25,28,52,55,58)], nt
ree = 1000)
```

c) Compute the confusion matrix and related calculations:

```r
# generate confusion matrix for new random forest
table(predict(spamForestOptimised, spam.df, type = "response"), spam.df$IsSpam, dnn = c("Predi
ction", "Actual"))
```

```
##           Actual
## Prediction    0    1
##          0 2740  139
##          1   48 1674
```

$$\text{Accuracy} = \frac{TN + TP}{\text{total \# of datapoints}} = \frac{2740 + 1674}{2740 + 139 + 48 + 1674} = 0.9594$$

$$\text{Sensitivity} = \frac{TP}{TP + FN} = \frac{1674}{1674 + 139} = 0.9233$$

$$\text{Specificity} = \frac{TN}{TN + FP} = \frac{2740}{2740 + 48} = 0.9828$$

$$\text{Precision} = \frac{TP}{TP + FP} = \frac{1674}{1674 + 48} = 0.9721$$

d) Compare the results to that from Question 3:

```r
scantimes.df = read.csv("scantimes.csv",header=TRUE) # read in the scan time data

# sum the times from A1 to A57
sum(scantimes.df[2:58])
```

```
## [1] 465.85
```

So the time taken to scan all attributes from an email (A1 through A57), it would take around 465.85ms, while the optimised search only takes about 49.96ms to scan a single email. This is an 89.28% decrease in scan time for one email.

So how does the performance of the search compare?

Comparing the performance of the random forest of 1000 trees with all attributes used vs the optimised random forest of 1000 trees:

Accuracy: 0.9957 vs 0.9594 (3.65% decrease)

Sensitivity: 0.9890 vs 0.9233 (6.64% decrease)

Specificity: 1.000 vs 0.9828 (1.72% decrease)

Precision: 1.000 vs 0.9721 (2.79% decrease)

The largest decrease in performance is in the sensitivity of the filter dropping by nearly 7%, while the accuracy, specificity and precision only decrease from around 1% to 4%.

This is not a terrible drop in performance of the filter considering that we have decreased the time by more than 89%, which I believe is a valuable tradeoff for such small drop in performance. So I think this optimised filter is good as it is now for general purpose use, while if we wanted a *really* accurate filter for some specialised purpose, we would have to consider more things and spend more time scanning the emails as a result.