Lab 1: Ad-hoc Earthquake Calibration

(Note, while this lab is not directly assessed, the final exam will make explicit reference to your experience with this lab and the tasks completed.)

Problem introduction

In addition to the ground shaking that occurs during an earthquake, the Earth's surface may become permanently deformed (Fig. 1). These surface displacements – direction and magnitude – are measured by GPS at particular locations (the stations). Following an earthquake, these GPS observations can be used to constrain a model of the ruptured fault, typically approximated as an elastic dislocation on a buried fault plane.

The Okada elastic dislocation model is the workhorse of so-called geodetic earthquake inversion. The model's inputs describe the geometry of the fault, the amount of slip on it during the earthquake, and the elastic properties of the crust. The output is a vector of displacement at any location on a flat surface somewhere above the fault (approximating the Earth's surface).

In this lab, you will be calibrating a model of the M 6.6 Lake Grassmere earthquake, which occurred near Blenheim in 2013. The event caused some damage to roads and buildings.

Figure 1: Ground displacement during the M 6.5 Edgecumbe earthquake (1987). The lefthand side of the fault has been uplifted about 0.5 m, while the righthand side has subsided a similar amount.

Tasks

As you complete the tasks below, you will encounter several *italicised* questions. While there is nothing to submit for this lab and no grade assessed, you may nevertheless wish to write down answers to these questions as an aid for your end of year exam revision.

Download *lab1_files.zip* from Canvas and extract the contents. We will only be working with the *main.m* file. The other files are related to execution and plotting of the Okada model. Open *main.m* in MatLab.

1.1 Familiarization with the Okada earthquake model

The main input parameters to the Okada elastic dislocation solution (our forward model) are:

- **Strike (free)**: the angle formed between North and the line that the fault makes when it intersects the surface.
- **Length (free)**: dimension of the fault along its **strike** direction.
- Height (fixed): dimension of the fault along its dip direction, assumed to be the same as *length*.
- Depth (fixed, 8 km): distance below ground surface of the fault centre.
- Dip (fixed, 68°): angle between the fault and the horizontal ground surface.
- Rake (fixed, 167°): relative direction of movement of the fault blocks during an earthquake.

- Slip (fixed, 1.7 m): the relative displacement between two sides of the fault after the earthquake.
- Shear modulus (fixed, 30 GPa): an elastic stiffness parameter, similar to Young's modulus, approximately 30 GPa for most crustal rocks.

For demonstration purposes, we shall assume that all parameters are known for this earthquake, except for **strike** and **length**, which are the focus of ad-hoc calibration (manual adjustment for good model fit). In practice, all eight parameters above could be unknown.

Run the code in CELL 1 of *main.m* by placing the cursor in the cell and hitting CTRL+ENTER.

The figure generated shows observations of GPS displacement at stations in the Wellington and Marlborough regions. These are compared against the predictions of an initial Okada model:

- GPS observations of horizontal displacement *direction* are shown as red arrows. In the righthand plot, their relative length corresponds to the relative *magnitude* of the displacement at the different locations.
- Black arrows are the surface displacement modelled by the Okada solution at various surface locations. Unlike the GPS stations, we can choose to model surface displacement anywhere we wish (including underneath Cook Strait).
- The yellow-orange contours indicate the *magnitude* of the modelled displacement.
- The blue line shows the location and orientation of the modelled fault. The small ticks indicate the downthrown direction, i.e., the direction that the fault dips towards.

Compare the direction and magnitude of surface displacement at the station locations with the model prediction:

- o *Do they agree with each other?*
- o *Would you consider this a well-calibrated model?*

1.2 The 2013 Lake Grassmere earthquake: ad-hoc model calibration

The aim is to produce a model that fits the GPS displacement data best.

Calibrate the Lake Grassmere earthquake model:

1. In CELL 2, choose new values for the parameters `strike` and `length`.
2. Execute the command `PlotH` in CELL 2. This produces two copies of the earlier figure – a previous (top, old parameters) and current calibration (bottom, new parameters).
3. Assess the fit between observed and modelled displacement vectors.
4. Repeat steps 1-3 until you are satisfied with your model.

Calibration is a process of diminishing returns. You should stop when you have a model that is "good enough" (a subjective assessment that can be difficult to justify, so err on the side of caution!)

- o *Is it easier to vary both of the parameters (strike and length) at once, or one at a time?*
- o *How did the size of the parameter changes that you made evolve as your calibration improved?*

The objective function (given in a box in the bottom left of each plot) is a least-squares measure of goodness-of-fit between the observations and the model. Our aim in calibrating the model is to minimize this quantity. The two *gradient* terms tell us *how* the objective function changes when we make a corresponding parameter change.

Use this information to make further improvements to your calibrated model.

- *How did you use information about the objective function and it's gradients to guide your calibration of the model?*

Observations are only available at particular locations (the GPS stations).

- *How does this impact the model calibration process?*
- *If you had a budget to install additional stations, where would you place them?*

1.3 Visualising the calibration process

Ad-hoc calibration amounts to a sort-of "walk" through parameter space (sometimes aimless, sometime directed) – we choose a direction, move a short distance, and then assess where we are and where we should head next.

Execute the commands in CELL 3 and inspect the figure that is generated.

This is a visualization of the path taken through parameter space as you calibrated the model. The coloured contours show the value of the objective function for different combinations of the parameters *strike* and *length*. The markers indicate the steps through parameter space that you took.

This plot shows clearly that there are two local minima, although one has a smaller objective function than the other.

- *Which local minimum did you end up choosing as the best-calibrated model? Which is better?*
- *Before viewing this plot, were you aware that there was more than one minimum? How could you approach the calibration process in a way that the existence of multiple minima might be revealed?*

Next week, we will look at automatic methods of model calibration, in particular *gradient descent*. As an introductory exercise, think of the plotted contours as if they represented topographic relief on a map (red = mountains, blue = valleys).

- *If you placed a ball at some point on the map, in which direction would it roll? Where would it end up?*

1.4 Computing earthquake magnitude

Earthquake magnitude is often computed from a quantity called the *moment*, which is a measure of energy release. The moment magnitude is given

$$M_W = \frac{2}{3}\log(GLHu) - 6,$$

where G is the shear modulus, L and H are the length and height of the rupture, u is the average slip on the fault.

Using your calibrated model value for L, and making a sensible assumption about H, compute a magnitude for the Lake Grassmere earthquake (you can assume G = 30 GPa and u = 1.7 m). Remember to use SI units.

- *How does your estimate compare with the seismologically determined magnitude of 6.6?*