Competitive Programming Note

Moon Jam

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2 Segment Tree

```
#define lc id * 2 + 1
#define rc id * 2 + 2
#define M ((L + R) >> 1)

struct Nodes {
    Info info;
    Tag tag;
} seg[N * 4];

void push(int L, int R, int id) {
    if (R > L + 1) {
        seg[lc].tag... seg[id].tag;
        seg[rc].tag... seg[id].tag;
    }
    seg[id].info = ... seg[id].tag;  // Update the info from tag
    seg[id].tag = NULL;              // clear seg[id].tag
}

void pull(int L, int R, int id) {
    push(L, M, lc), push(M, R, rc);
    // Update the info of id from lc and rc
    seg[id].info = seg[lc].info... seg[rc].info;
}

void modify(int l, int r, Tag tag, int L = 0, int R = n, int id = 0) {
    push(id);
    if (l == L && r == R) {
        seg[id].tag = tag;
    } else {
        if (l >= M)
            modify(l, r, tag, M, R, rc);
        else if (r =< M)
            modify(l, r, tag, L, M, lc);
        else
            modify(l, M, tag, L, M, lc), modify(M, r, tag, M, R, rc);
        pull(id);
    }
}

Info query(int l, int r, int L = 0, int R = n, int id = 0) {
    push(id);
    if (l == L && r == R) return id.info;
    if (l >= M) return query(l, r, M, R, rc);
    if (r <= M) return query(l, r, lc);
    // calculate answer from lc and rc
    return query(l, M, L, M, lc)... query(M, r, M, R, rc);
}
```

3 Binary Indexed Tree

```
struct BIT {
    vector<int> bit;
    int n;

    BIT(int _n) {
        n = _n;
        bit.resize(n + 1);
    }
    void modify(int pos, int val) {
        for (; pos <= n; pos += pos & (-pos)) bit[pos] += val;
    }
    int query(int pos) {
        int ans = 0;
        for (; pos > 0; pos -= pos & (-pos)) ans += bit[pos];
        return ans;
    }
};
```

4 Sparse Table

5 莫隊

- 可以離線處理(沒有修改)
- 區間眾數、區間最大連續和、區間某個數字的數量

Range Pairing Query

```
int n, q, s;
struct qu {
    int l, r, id;
    bool operator<(const qu &B) const {
        if (l / s != B.l / s) return l < B.l;
        return (l / s & 1) ? r < B.r : r > B.r;
    }
};

signed main() {
    cin >> n;
    vector<int> arr(n + 1);
    for (int i = 1; i <= n; i++) cin >> arr[i];
    s = sqrt(n);

    cin >> q;
    vector<qu> qus(q);
    for (int i = 0; i < q; i++)
        cin >> qus[i].l >> qus[i].r, qus[i].id=i;
    sort(qus.begin(), qus.end());

    int l = 1, r = 1, p = 0;
    set<pair<int, int>> ans;
    vector<int> cur(n + 1);
    cur[arr[1]] = 1;
    for (qu i : qus) {
        while (i.r > r) {
            r++;
            if (cur[arr[r]] & 1) p++;
            cur[arr[r]]++;
        }
        while (i.l > l) {
            cur[arr[l]]--;
            if (cur[arr[l]] & 1) p--;
            l++;
        }
        while (i.r < r) {
            cur[arr[r]]--;
            if (cur[arr[r]] & 1) p--;
            r--;
        }
        while (i.l < l) {
            l--;
            if (cur[arr[l]] & 1) p++;
            cur[arr[l]]++;
        }
        ans.insert({i.id, p});
    }
    for (auto i : ans) cout << i.second << '\n';
    return 0;
}
```

6 DSU

7 PBDS & ROPE

```cpp
#include <bits/extc++.h>
using namespace __gnu_pbds;
using namespace std;

template <class T>
using Tree =
    tree<T, null_type, less<T>, rb_tree_tag, tree_order_statistics_node_update>;
template <class T>
using multiTree =
    tree<T, null_type, less_equal<T>, rb_tree_tag, tree_order_statistics_node_update>;

int main() {
    Tree<int> tr;
    tr.insert(1), tr.insert(8), tr.insert(3);
    // an iterator of the (k+1)th smallest element
    cout << *tr.find_by_order(1) << '\n';
    // the number of elements are less than ktyl,
    cout << tr.order_of_key(6) << '\n';
    tr.erase(8);
    return 0;
}
```

```cpp
#include <bits/extc++.h>
using namespace __gnu_cxx;
using namespace std;
int main() {
    rope<int> r, a, b;
    b += 10;          // {10}
    r += 2;           // {2}
    r += 3;           // {2, 3}
    a = r + r + b;    // {2, 3, 2, 3, 10}
    for (int i : a) cout << i << '-';
    cout << '\n';
    b += a.substr(2, 3);  // get 3 elements from a_2
    for (int i : b) cout << i << '-';
    cout << '\n';
    r.pop_back();         // {3}
    r.pop_front();        // empty
    r.push_back(1);       // {1}
    r.push_front(10);     // {10, 1}
    r.replace(1, 100);    // replace r_1 to 100
    r.erase(0, 1);        // erase 1 element from 0
    r.insert(0, 7);       // insert 7 at r_0
    for (int i : r) cout << i << '-';
}
```

8 拓樸排序

9 Floyd

10 Dijkstra

11 Bellman-Ford

12 Prime

13 Kruskal

14 Tarjan

15 Kosaraju

```
int id = 0;
int n, m;
vi g[100005], rg[100005], ord;
int idx[100005];
bool vis[100005], rvis[100005];

void dfs(int rt){
    if(vis[rt]) return;
    vis[rt] = 1;
    for(int i : g[rt]) dfs(i);
    ord.eb(rt);
}

void rdfs(int rt){
    if(rvis[rt]) return;
    rvis[rt] = 1;
    for(int i : rg[rt]) rdfs(i);
    idx[rt] = id;
}

signed main() {
    ios;
    cin >> n >> m;
    rep(i, 1, m) {
        int a, b;
        cin >> a >> b;
        g[a].eb(b);
        rg[b].eb(a);
    }
    rep(i, 1, n) dfs(i);
    reverse(all(ord));
    for(int i : ord) if(!rvis[i]) id++,rdfs(i);
    cout << id << '\n';
    rep(i, 1, n) cout << idx[i] << ' ';
    return 0;
}
```

15.1 2-SAT

Each pair $(a_i \vee b_i)$ generates two edges: $\neg a_i \to b_i$ and $\neg b_i \to a_i$

1. find SCC of G (use kosaraju can find topological ordering at the same time)
2. for all x :
   if(x and $\neg x$ in the same SCC) return IMPOSSIBLE
3. for all x :
   if(topological ordering $\neg x > x$) set x to true
   else set x to false

16 Eulerian Path

16.1 雙向

```
int n, m, st=1, ed=1, cnt_odd = 0;
si g[100005];
vi ans;

void dfs(int rt){
    while(!g[rt].empty()){
        int cur = *g[rt].begin();
        g[rt].erase(g[rt].begin());
        g[cur].erase(rt);
        dfs(cur);
    }
    ans.eb(rt);
}

signed main() {
    ios;
    cin >> n >> m;
    si cg;
    rep(i, 1, m){
        int a, b;
        cin >> a >> b;
        g[a].insert(b), g[b].insert(a);
    }
    rep(i, 1, n) if(g[i].size()&1) ed = st, st = i, cnt_odd++;
    if(cnt_odd!=0) {
        cout << "IMPOSSIBLE\n";
        return 0;
    }
    dfs(st);
    rep(i, 1, n) if(!g[i].empty()) {
        cout << "IMPOSSIBLE\n";
        return 0;
    }
    reverse(all(ans));
    for(int i : ans) cout << i << ' ';
    return 0;
}
```

16.2 單向

```
// from 1 to n
int n, m, deg[100005];
vi g[100005], ans;

void dfs(int rt){
    while(!g[rt].empty()){
        int cur = g[rt].back();
        g[rt].pob();
        dfs(cur);
    }
    ans.eb(rt);
}

signed main() {
```

```
    ios;
    cin >> n >> m;
    rep(i, 1, m){
        int a, b;
        cin >> a >> b;
        g[a].eb(b);
        deg[a]++, deg[b]++;
    }
    rep(i, 1+1, n-1) if(deg[i]&1){
        cout << "IMPOSSIBLE\n";
        return 0;
    }
    if(!((deg[1]&1) && (deg[n]&1))){
        cout << "IMPOSSIBLE\n";
        return 0;
    }
    dfs(1);
    rep(i, 1, n) if(!g[i].empty()){
        cout << "IMPOSSIBLE\n";
        return 0;
    }
    reverse(all(ans));
    for(int i : ans) cout << i << ' ';
    return 0;
}
```

16.3 De Bruijn Sequence

```
int n;
vi g[100005], ans;

void dfs(int rt){
    while(!g[rt].empty()){
        int cur = g[rt].back();
        g[rt].pob();
        dfs(cur);
        ans.eb(rt&1);
    }
}

signed main() {
    ios;
    cin >> n;
    rev(i, (1<<(n-1))-1, 0){
        int nxt = (i<<1) & ((1<<(n-1))-1);
        g[i].eb(nxt);
        g[i].eb(nxt+1);
    }
    dfs(0);
    reverse(all(ans));
    rep(i,1,n-1) cout << 0;
    if(n==1) cout << "01";
    else for(int i : ans) cout << i;
    return 0;
}
```

17 凸包