

1

演習課題 9

- RBMの対数尤度関数 $\ln L(\theta)$ を可視ユニット v_i のバイアス a_i , 隠れユニット h_j のバイアス b_j で偏微分したときの値 $\frac{\partial \ln L(\theta)}{\partial a_i}$, $\frac{\partial \ln L(\theta)}{\partial b_j}$ が次の式になることを示せ.

$$\frac{\partial \ln L(\theta)}{\partial a_i} = \sum_{n=1}^N v_i^n - NE_{\theta}[v_i]$$

$$\frac{\partial \ln L(\theta)}{\partial b_j} = \sum_{n=1}^N \frac{1}{1+e^{-\lambda_j^n}} - NE_{\theta}[h_j]$$

10/19/2023

2

演習課題 9 解答

$$\begin{aligned} & \frac{\partial \ln L(\theta)}{\partial a_i} \\ &= \sum_{n=1}^N \left(\frac{\partial \sum_i a_i v_i^n}{\partial a_i} + \frac{\partial \sum_j \ln(1+e^{b_j+\sum_i w_{j,i} v_i^n})}{\partial a_i} - \frac{\partial \ln z(\theta)}{\partial a_i} \right) \\ &= \sum_{n=1}^N (v_i^n - E_{\theta}[v_i]) \\ &= \sum_{n=1}^N v_i^n - NE_{\theta}[v_i] \end{aligned}$$

10/19/2023

3

演習課題 9 解答

$$\begin{aligned}
& \frac{\partial \ln L(\theta)}{\partial b_j} \\
&= \sum_{n=1}^N \left(\frac{\partial \sum_i a_i v_i^n}{\partial b_j} + \frac{\partial \sum_j \ln(1 + e^{b_j + \sum_i w_{j,i} v_i^n})}{\partial b_j} - \frac{\partial \ln z(\theta)}{\partial b_j} \right) \\
& \frac{\partial \sum_j \ln(1 + e^{b_j + \sum_i w_{j,i} v_i^n})}{\partial b_j} = \frac{1}{1 + e^{b_j + \sum_i w_{j,i} v_i^n}} e^{b_j + \sum_i w_{j,i} v_i^n} \frac{\partial (b_j + \sum_i w_{j,i} v_i^n)}{\partial b_j} \\
&= \frac{1}{1 + e^{b_j + \sum_i w_{j,i} v_i^n}} e^{b_j + \sum_i w_{j,i} v_i^n} \\
&= \frac{1}{1 + e^{-\lambda_j^n}} \\
& \frac{\partial \ln L(\theta)}{\partial b_j} = \sum_{n=1}^N \left(\frac{1}{1 + e^{-\lambda_j^n}} - E_{\theta}[h_j] \right) = \sum_{n=1}^N \frac{1}{1 + e^{-\lambda_j^n}} - N E_{\theta}[h_j]
\end{aligned}$$

10/19/2023