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今日の内容

■ n元論理代数方程式を解く

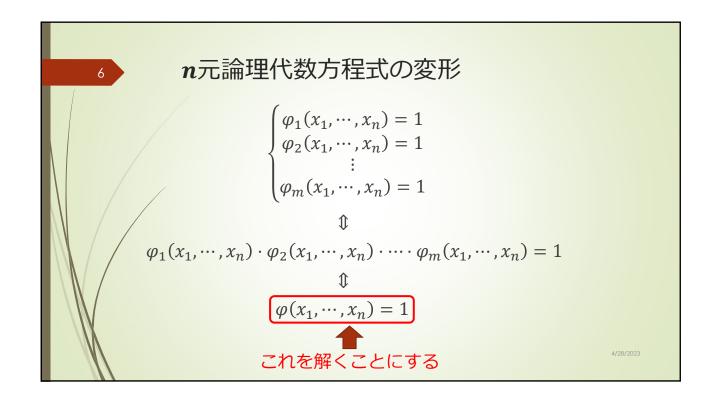
$$\begin{cases} f_1(x_1,\cdots,x_n) = g_1(x_1,\cdots,x_n) \\ f_2(x_1,\cdots,x_n) = g_2(x_1,\cdots,x_n) \end{cases}$$
を満たす x_1,\cdots,x_n を求めよ:
$$f_m(x_1,\cdots,x_n) = g_m(x_1,\cdots,x_n)$$

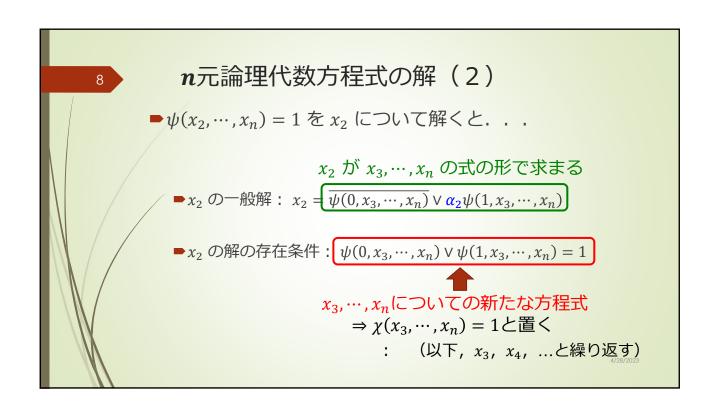
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n元論理代数方程式の変形

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n元論理代数方程式の解(3)

- x_n について解いたら, x_n から逆順に一般解に x_n, \dots, x_i を代入していき一般解に含まれる変数を消去する

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例題

■ 二元論理代数方程式 $\begin{cases} ax \lor by = 1 \\ xy = c \end{cases}$ を x, y について解け

$$xy = c \Leftrightarrow xyc \lor \overline{xy}\overline{c} = 1 \ \text{Lb}$$

$$\varphi(x,y) = (ax \lor by)(xyc \lor \overline{xy}\overline{c}) = 1$$
 を代わりに解く.

$$\varphi(0,y) = (a \cdot 0 \lor by) \cdot (0 \cdot yc \lor \overline{0 \cdot y}\overline{c}) = by\overline{c}$$

$$\varphi(1,y) = (a \cdot 1 \vee by) \cdot (1 \cdot yc \vee \overline{1 \cdot y}\overline{c}) = (a \vee by) \cdot (yc \vee \overline{y}\overline{c})$$

$$x$$
 の一般解は $x = \overline{\varphi(0,y)} \vee \alpha_1 \varphi(1,y)$

$$= \overline{by\bar{c}} \vee \alpha_1(a \vee by) \cdot (yc \vee \bar{y}\bar{c})$$

 $= \bar{b} \vee \bar{y} \vee c \vee \alpha_1 ayc \vee \alpha_1 a\bar{y}\bar{c} \vee \alpha_1 byyc \vee \alpha_1 by\bar{y}\bar{c}$

 $= \bar{b} \vee \bar{y} \vee c$

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\varphi(0,y) = by\bar{c} \qquad \varphi(1,y) = (a \lor by) \cdot (yc \lor \bar{y}\bar{c})
x \, \mathcal{O}解の存在条件は \, \varphi(0,y) \lor \varphi(1,y) = 1
したがって \psi(y) = \varphi(0,y) \lor \varphi(1,y) と置くと
\psi(y) = by\bar{c} \lor (a \lor by) \cdot (yc \lor \bar{y}\bar{c}) = 1
\psi(0) = b \cdot 0 \cdot \bar{c} \lor (a \lor b \cdot 0) \cdot (0 \cdot c \lor \bar{0} \cdot \bar{c}) = a\bar{c}
\psi(1) = b \cdot 1 \cdot \bar{c} \lor (a \lor b \cdot 1) \cdot (1 \cdot c \lor \bar{1} \cdot \bar{c}) = b\bar{c} \lor (a \lor b)c
= b \lor ac
y \, \mathcal{O} - \mathcal{W}
= \bar{a} \, \forall c \lor \alpha_2 b \lor \alpha_2 ac
= \bar{a} \lor c \lor \alpha_2 b \lor \alpha_2 ac
= \bar{a} \lor c \lor \alpha_2 b
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 $\psi(0) = a\bar{c}$ $\psi(1) = b \vee ac$ $y \, \mathcal{O}$ 解の存在条件は $\psi(0) \vee \psi(1) = 1$ したがって解の存在条件は $a\bar{c} \vee b \vee ac = a \vee b = 1$ $y = \bar{a} \vee c \vee \alpha_2 b \, \text{より}$ $x \, \mathcal{O}$ 一般解は $x = \bar{b} \vee \bar{y} \vee c$ $= \bar{b} \vee \bar{a} \overline{\vee} c \vee \alpha_2 b \vee c$ $= \bar{b} \vee a\bar{c} \overline{\alpha_2} b \vee c$ $= \bar{b} \vee a\bar{c} \overline{\alpha_2} \vee b \vee c$ $= \bar{b} \vee a\bar{c} \overline{\alpha_2} \vee a\bar{c} \bar{b} \vee c$ $= \bar{b} \vee a\bar{\alpha_2} \vee c$ $= \bar{b} \vee a\bar{\alpha_2} \vee c$

