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## 演習課題 8

- 対数尤度関数  $\ln L(\theta)$  をバイアス  $b_i$  で偏微分したときの  $\frac{\partial \ln L(\theta)}{\partial b_i}$  を求めよ.

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## 演習課題 8 解答

$$\begin{aligned} \frac{\partial \ln L(\theta)}{\partial b_i} &= \frac{\partial \sum_{n=1}^N (-\Phi(x_n, \theta) - \ln z(\theta))}{\partial b_i} \\ &= \sum_{n=1}^N \left( -\frac{\partial \Phi(x_n, \theta)}{\partial b_i} - \frac{\partial \ln z(\theta)}{\partial b_i} \right) \end{aligned}$$

$$-\frac{\partial \Phi(x_n, \theta)}{\partial b_i} = \frac{\partial (\sum_{(i,j)} w_{j,i} x_i^n x_j^n + \sum_i b_i x_i^n)}{\partial b_i} = x_i^n$$

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## 演習課題 8 解答

$$- \frac{\partial \Phi(x_n, \theta)}{\partial b_i} = \frac{\partial (\sum_{(i,j)} w_{j,i} x_i^n x_j^n + \sum_i b_i x_i^n)}{\partial b_i} = x_i^n$$

$$\begin{aligned}
 - \frac{\partial \ln z(\theta)}{\partial b_i} &= - \frac{1}{z(\theta)} \frac{\partial z(\theta)}{\partial b_i} = - \frac{1}{z(\theta)} \frac{\partial \sum_x e^{-\Phi(x, \theta)}}{\partial b_i} \\
 &= - \frac{1}{z(\theta)} \sum_x \frac{\partial e^{-\Phi(x, \theta)}}{\partial b_i} = - \frac{1}{z(\theta)} \sum_x e^{-\Phi(x, \theta)} \frac{\partial (-\Phi(x, \theta))}{\partial b_i} \\
 &= - \sum_x \frac{1}{z(\theta)} e^{-\Phi(x, \theta)} x_i = - \sum_x P(x|\theta) x_i
 \end{aligned}$$

全ての  $x$  を考えたときの  $x_i$  の期待値  
( $E_\theta[x_i]$ と表す)

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## 演習課題 8 解答

$$\begin{aligned}
 \frac{\partial \ln L(\theta)}{\partial b_i} &= \frac{\partial \sum_{n=1}^N (-\Phi(x_n, \theta) - \ln z(\theta))}{\partial b_i} \\
 &= \sum_{n=1}^N \left( - \frac{\partial \Phi(x_n, \theta)}{\partial b_i} - \frac{\partial \ln z(\theta)}{\partial b_i} \right) \\
 &= \sum_{n=1}^N (x_i^n - E_\theta[x_i]) \\
 &= \sum_{n=1}^N (x_i^n) - N E_\theta[x_i]
 \end{aligned}$$

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