# Question 1: Quasi-balanced Search Trees

**Quasi-balanced Search Trees**

In the following diagram, which we call a *quasi-balanced search tree (QUBSET),* every node represents a student. Each student has a name and each student has a student number, both of which are shown in each node. Every diagram shown is structured by the same set of rules. *You can assume for the whole question that all names/IDs are unique.*
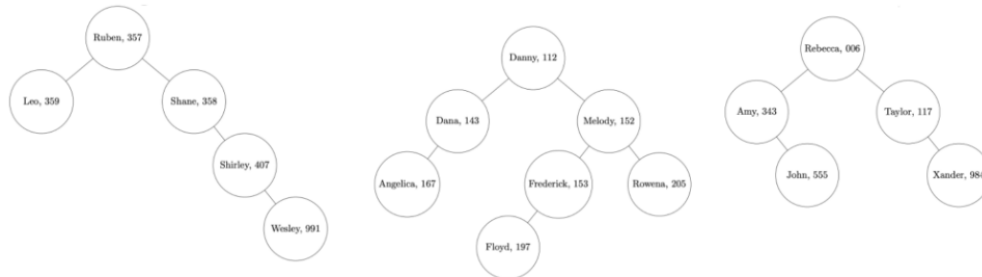


## Part A

i. If you remove the student IDs and only consider the names, what do you notice about the order/structure? Explain your answer. **0.5 marks**

ii. If you remove the names and only consider the student IDs, what do you notice about the order/structure? Explain your answer. **0.5 marks**

*At the bottom of this page, we provide two examples that show what happens when we add a new student (node) to our diagram.*

## Part B

If we add the student Fin who has a student ID of 2, the *QUBSET* changes from left to right.
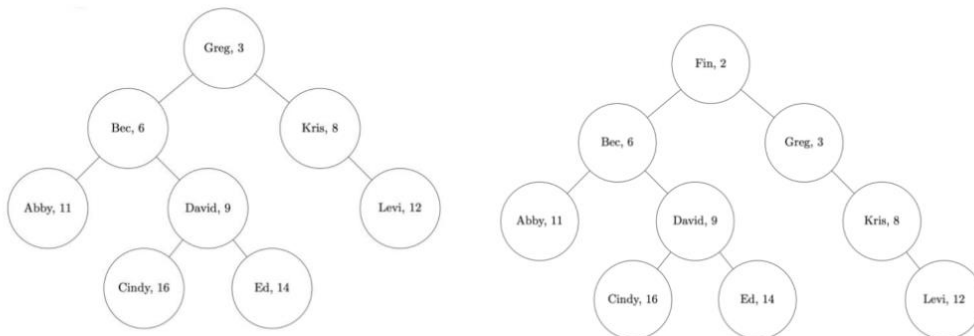
Write down all the missing steps in this process. You should provide just as much detail as the examples shown at the bottom. **2 marks**

## Part C

Given an arbitrary *QUBSET*, $T$, and a new student $S$, write a new function add_student($T$, $S$) that adds the new student to the diagram. Assume the operation is done in place (there should be no return value). You can assume $T_{\text{name}}$ and $T_{\text{id}}$ give the student name/ID respectively. $T$ and $S$ are of the same type and you can assume $S$ has no children. Your pseudocode should look like the pseudocode that is given in Lecture 9. **3 marks**

## Part D

We want the height of *QUBSET* to be as small as possible. Give an example of the worst case height when we add $5$ students to an empty *QUBSET*. **1 mark**

## Part E

Suppose we have a group of $n$ students that are listed in alphabetical order. If we add them to a binary search tree (sorted just by name and ignoring their student IDs), it can be shown that it degenerates to a tree of height $n$. Explain why the *QUBSET* we have used in this question is likely to have a height much smaller than $n$. **1 mark**
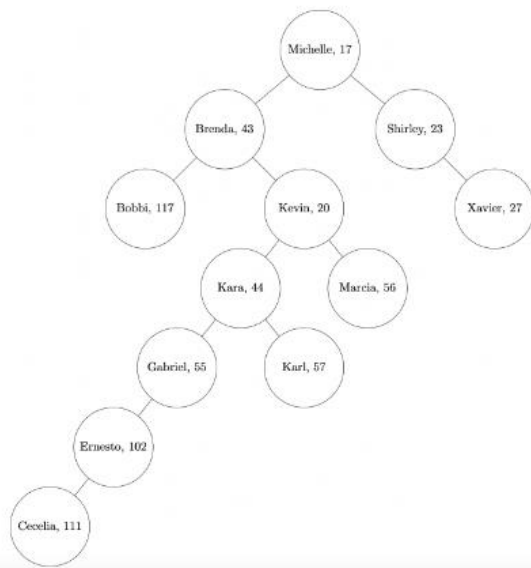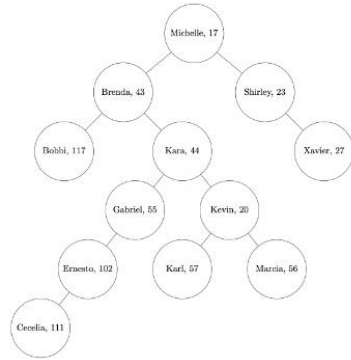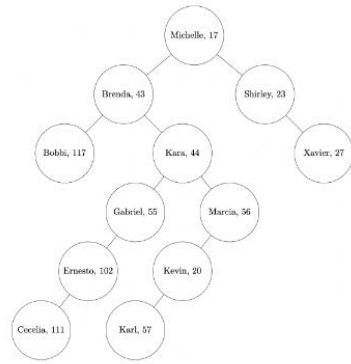
**Example 1**
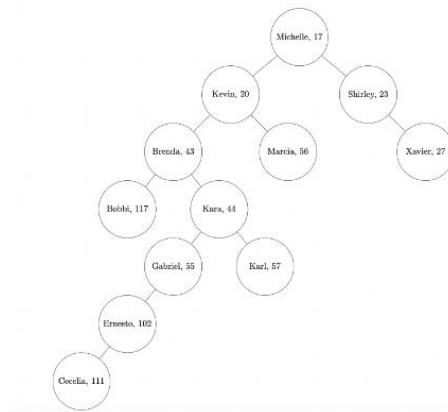
We start with the following *QUBSET*
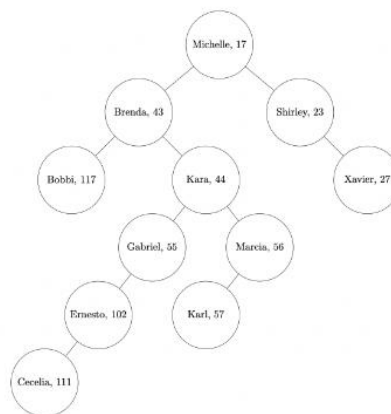


Now suppose we add Kevin who has a student ID of 20 to the *QUBSET* following the rule in part a) i.
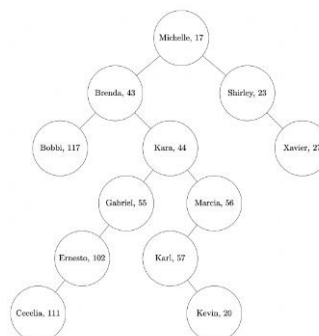


After this, the tree does not follow the rule in part a) ii. We continue to update the diagram until it follows the *QUBSET* rule.
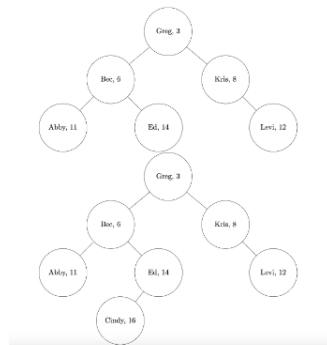
Tree 1:
Michelle, 17
- Brenda, 43
  - Bobbi, 117
  - Kara, 44
    - Gabriel, 55
      - Ernesto, 102
        - Cecelia, 111
      - Kevin, 20
        - Karl, 57
    - Marcia, 56
- Shirley, 23
  - Xavier, 27



Tree 2:
Michelle, 17
- Brenda, 43
  - Bobbi, 117
  - Kara, 44
    - Gabriel, 55
      - Ernesto, 102
        - Cecelia, 111
    - Kevin, 20
      - Karl, 57
      - Marcia, 56
- Shirley, 23
  - Xavier, 27



Tree 3:
Michelle, 17
- Brenda, 43
  - Bobbi, 117
  - Kevin, 20
    - Kara, 44
      - Gabriel, 55
        - Ernesto, 102
          - Cecelia, 111
      - Karl, 57
    - Marcia, 56
- Shirley, 23
  - Xavier, 27

Michelle, 17

Kevin, 20    Shirley, 23

Brenda, 43    Marcia, 56    Xavier, 27

Bobbi, 117    Kara, 44

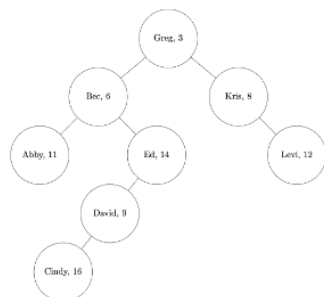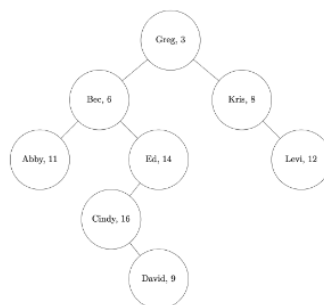Gabriel, 55    Karl, 57

Ernesto, 102

Cecelia, 111
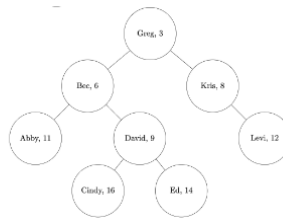
Now we satisfy the *QUBSET* rule in a) ii. and stop.

### Example 2

If we start with the following *QUBSET* and then add Cindy who has a student ID of 16, we do not need to change anything afterwards.

Greg, 3

Bec, 6    Kris, 8

Abby, 11    Ed, 14    Levi, 12

Greg, 3

Bec, 6    Kris, 8

Abby, 11    Ed, 14    Levi, 12

Cindy, 16

Now suppose we add David who has a student ID of 9 into our *QUBSET*. The following steps occur

Greg, 3

Bec, 6    Kris, 8

Abby, 11    Ed, 14    Levi, 12

Cindy, 16

David, 9

Greg, 3

Bec, 6    Kris, 8

Abby, 11    Ed, 14    Levi, 12

David, 9

Cindy, 16

And after this we are done.