# **Quadcopter Flight Dynamics**

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# What do you mean by flight dynamics?

Flight dynamics is the study of the performance, stability, and control of vehicles flying through the air or in outer space. It is concerned with how forces acting on the vehicle influence its speed and attitude with respect to time.

-Wikipedia



# Some terms related to flight...

Pitch – Rotation around the 'Y' axis.
 Refers to forward and backward tilting of an aircraft. Results in lift in the case of an airplane but forward/backward movement in the case of rotorcrafts.



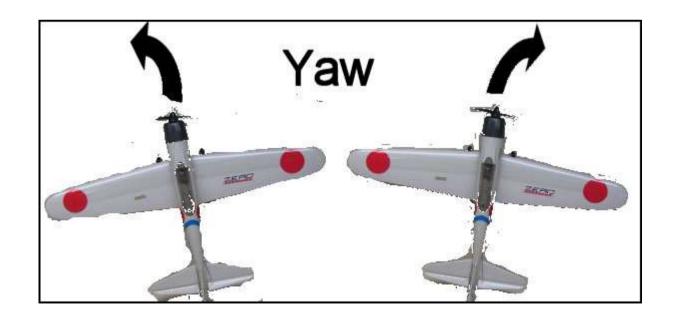


Roll – Rotation around the 'X' axis.
 It's the sideways banking of an aircraft. Results in sideways movements both in aircrafts and rotorcrafts.



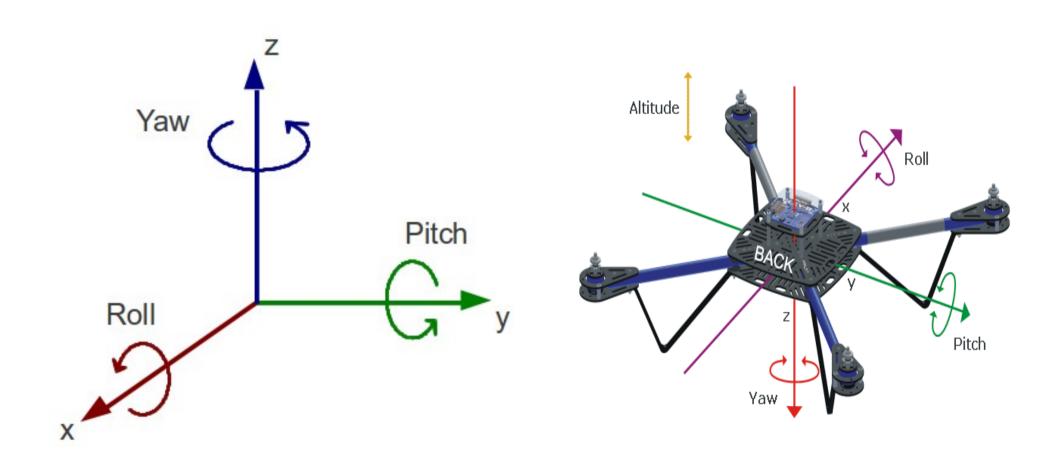


Yaw – Rotation around 'Z' axis.
 Refers to the turning movement of an aircraft. Allows an aircraft/rotorcraft to turn or rotate.





# Summary of the three rotations...





#### Attitude

It refers to the orientation of an aircraft at a particular time. It is described by pitch, roll, yaw angles and altitude.





# Forces on a quadcopter:-

 Thrust force – four rotating propellors, therefore four thrust vectors which are always perpendicular to the plane of the quadcopter.

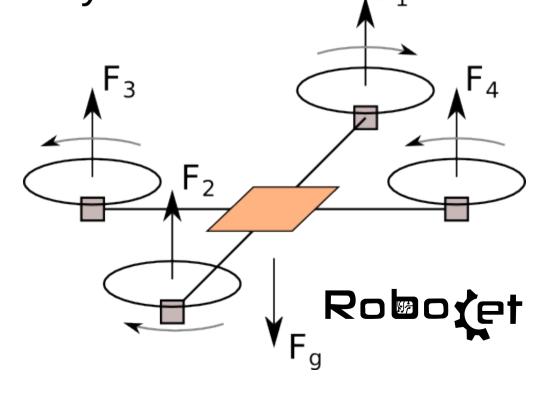
Magnitude of each thrust  $F_i$  is related to angular velocity  $\omega_i$  by :  $F_i = k^* \omega_i ^2$ 

The constant 'k' depends up on air density, propellor dimensions and the motor used.

 Force due to gravity – The weight of the quadcopter can be assumed to act on its center of mass:

$$F_g = m*g$$

'm' is the mass of the quadcopter and 'g' is acceleration due to gravity.  $F_1$ 



 <u>Frictional force</u> – force due to air resistance. It is proportional to the velocity of the quadcopter. It has more effect when agile manoeuvres are performed. It can be modelled as:

$$F_x = -k_d*V_x$$
;  $F_y = -k_d*V_y$ ;  $F_z = -k_d*V_z$ 

Where  $k_d$  is friction constant and suffixes x, y and z denote the x, y and z direction.



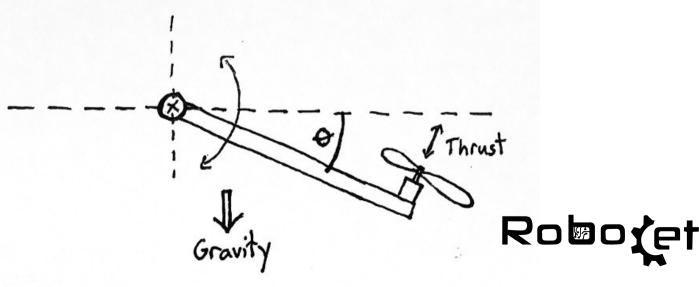
# Torques on a quadcopter:-

 Torques due to thrust – These torques are responsible for the pitch and roll motions of the quadcopter i.e., these torques are along the 'x' or 'y' axis. The equation for these torques is:

$$T_i = F_i * d_i = (k*\omega_i^2)*d_i$$

where  $d_i$  is the distance of each arm from the

center.



 <u>Drag torque</u> – This is caused due to the high speed rotation of the propellor against air. The torque due to this is along the 'z' axis. Therefore this torque will result in yaw movement. The equation for this torque is:

$$T_i = F_i * d_i + I * \alpha = (b*\omega_i ^2)*d_i + I*\alpha$$

Where T is the MOI of the motor propellor system, b is the drag coefficient and a is the angular acceleration of the propellor which will be zero most of the times.



## Now we're ready for some action..!!

First of all...

Quadcopter control is a fundamentally difficult and interesting problem. With six degrees of freedom (three translational and three rotational) and only four independent input (rotor speeds), quadcopters are severely underactuated. In order to achieve six degrees of freedom, rotational and translational motion are coupled. The resulting dynamics are highly nonlinear, especially after accounting for the complicated aerodynamic effects.



#### **Question**:

Which direction each propellor should rotate? Should all rotate in the same direction (i.e., CW/CCW) or should some rotate in CW and others in CCW?

Two of the propellors should rotate in CW and the other two should rotate in CCW direction. If this is not the case, there will be a yaw movement due to a net drag torque.

Therfore we need propellors with opposite angle of attacks:

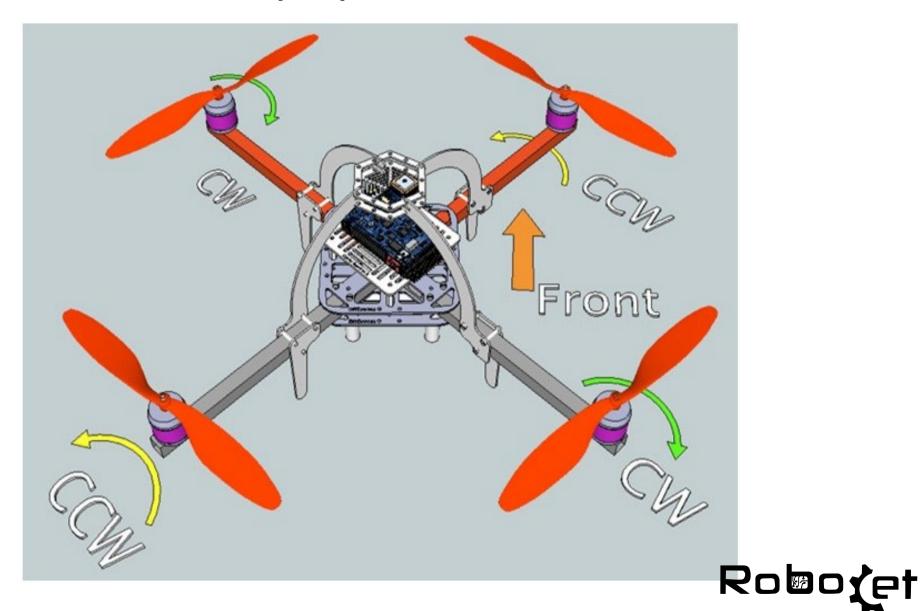


## CW and CCW propellors:





## Rotation of each propellor:



## **Condition for hovering?**

 Sum of all vertical thrusts should equal the weight. That is:

$$\sum F_{i(vertical)} = m*g$$

Also sum of all torques should equal zero:

$$\Sigma \tau_i = 0$$

One simple way to realise this to rotate all the propellors with the same speed.

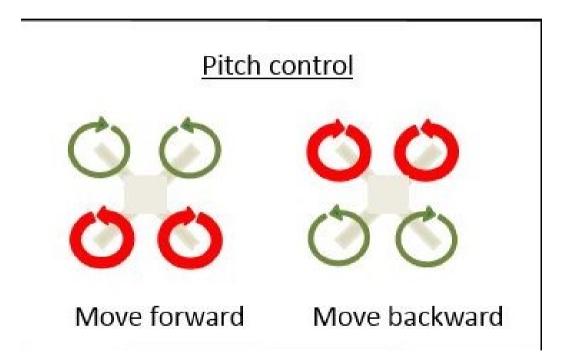


## How to realise pitch?

 Total force in the vertical direction should equal the weight and there should be a total horizontal component in the forward/backward direction.

This can be realised as follows:





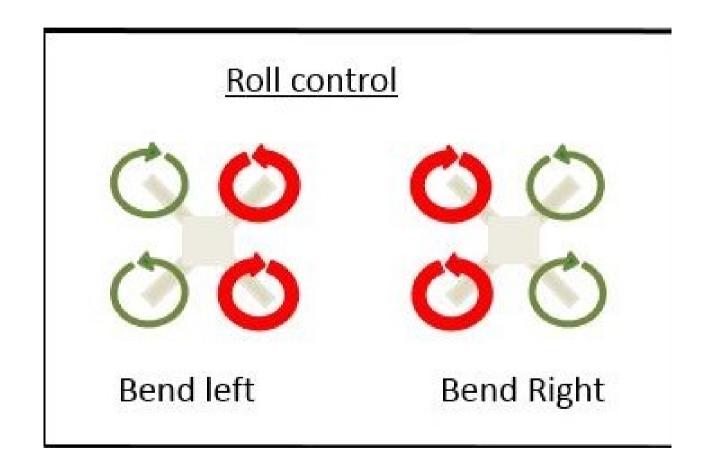


#### How to realise roll?

 We can use the same technique as pitch but for the sideways direction. That is, the total vertical force should equal weight but there should be a net horizontal force in the sideways direction.

This is realised as follows:





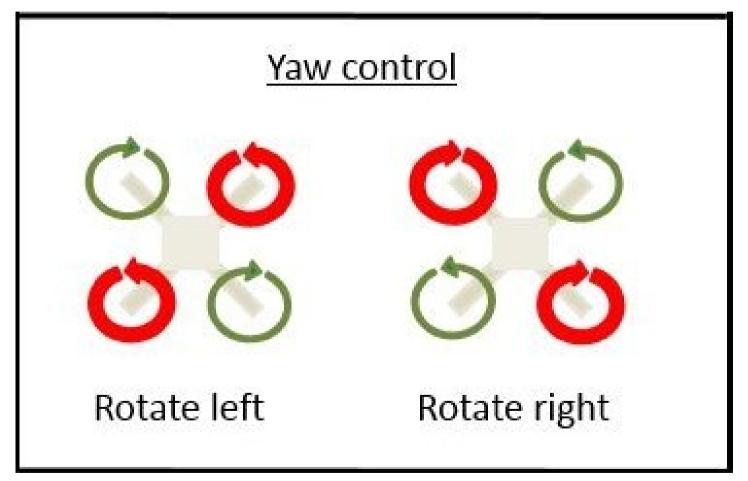


## How to realise yaw?

 Yaw in the CW direction can be realised by increasing the speed of the CCW propellors and decreasing the speed of the CW propellors by the same amount.

 Yaw in the CCW direction can be realised by increasing the speed of the CW propellors and decreasing the speed of the CCW propellors by the same amount.







How much do we need to increase/decrease the speed of each propellor to move at a particular speed in a particular direction?

It's decided by the control algorithm. See the slides on PID control of Quadcopters.

