Introduction

- The study of electronics is incomplete without an understand of how AC circuits work
- AC circuit theory is a basis for understanding how signals containing information can be processed in circuits
- We will begin by examining how circuits with a *sinusoidal* voltage or current source of single frequency, and in following sections of the course we will examine how circuits respond to a wide range of frequency
    - Note: If non-sinusoidal waveforms (such as square waves, or ramp waveform sources are used then different circuit analysis would be required)
- Our exploration of AC circuits leads to understanding how frequency selective circuits work, and these circuits are of importance in systems that process continuous (i.e. analog signals)
- For example, every audio system (such as your cell phone, or speakers on your computer) can produce sound that reproduced from a digital recording or stream of data
    - This involves transforming digital data to analog waveforms that are converted to sound waves by a speaker
- The analog portion of the system is designed using AC circuit theory
- Although it is possible to analyze what happens in a DC circuit due to step-changes in voltage and current, this requires transient analysis
    - relevant parts of transient analysis are explored in another part of this course
- To begin, we will explore steady state AC circuits that involve inductors, capacitors, and resistors
- All of the DC theory we have covered so far also applies to AC circuits

Electrical Impedance, Voltage, and Current in AC Circuits

- Although it is not usually called Ohm's Law, the relation for complex voltage is virtually the same with the exception that all quantities are complex values
- In AC circuits, the relationship between complex voltage and current is:

$$\boldsymbol{V} = \boldsymbol{I}\,\boldsymbol{Z}$$

Where:

$\boldsymbol{Z}$ is a <u>reactance</u> of a circuit network (or component) in ohms [Ω]

$\boldsymbol{I}$ is a sinusoidal current flowing through the component in amps [A]

- Recall from math curriculum that a complex number with a real and imaginary term according to Euler's Relation

$$r(\cos\theta + jsin\theta) = r(e^{j\theta})$$

Where:

$$j = \sqrt{-1}$$

- Any complex number can be expressed in polar or rectangular form

$$\boldsymbol{A} = |\boldsymbol{A}|(\cos\theta + jsin\theta) \qquad \text{rectangular form}$$

$$\boldsymbol{A} = |\boldsymbol{A}|\angle\theta \qquad \text{polar form}$$

Impedance of Inductors

- The voltage drop across and inductor with a sinusoidal current is:

$$\boldsymbol{V_L} = \boldsymbol{I_L}\,\boldsymbol{Z_L}$$

Where the complex impedance of an inductor ($\boldsymbol{Z_L}$) is imaginary, and has units of ohms [Ω]:

$$\boldsymbol{Z_L} = jX_L$$

And the value of (X_L) is referred to as the <u>inductor reactance</u> (a scalar quantity):

$$X_L = \omega L$$

Where:

ω is the radian velocity in units of [rad/s]

L is the inductance in henries [H]

Since the relation between radian velocity (ω) in [rad/s] and frequency (f) of a sinusoid in hertz [Hz] is:

$$\omega = 2\pi f$$

And substituting $2\pi f$ for ω into the expression for impedance, an expression for complex impedance of an inductor is obtained:

$$\boldsymbol{Z_L} = j2\pi f L$$

- In AC circuits, the frequency dependance of inductor impedance is an important trait of inductors because it allows the creation of circuits that are responsive to changes in frequency
- It is useful to remember that the impedance of an inductor is *directly* proportional to frequency
  - As frequency increases, the reactance of the inductor increases
  - As frequency decreases, the reactance of the inductor decreases

And, it is helpful to remember what happens at extremely low and extremely high frequencies:

- At extremely low frequency, or DC:

$$X_C \rightarrow 0$$

an inductor behaves like a short circuit (zero ohms)

- If we imagine a constantly increasing frequency:

$$X_L \rightarrow \infty$$

an inductor behaves like an open circuit (approaches infinite ohms)

Impedance of Capacitors

- The voltage drop across a capacitor with a sinusoidal current is:

$$\boldsymbol{V_C} = \boldsymbol{I_C}\,\boldsymbol{Z_C}$$

Where the complex impedance of an inductor ($\boldsymbol{Z_C}$) is imaginary, and has units of ohms [Ω]:

$$\boldsymbol{Z_C} = jX_C$$

And the value of (X_C) is referred to as the capacitor reactance (a scalar quantity):

$$X_C = \frac{1}{\omega C}$$

Where:

ω is the radian velocity in units of [rad/s]

C is the capacitance in farads [F]

Since the relation between radian velocity (ω) in [rad/s] and frequency (f) of a sinusoid in hertz [Hz] is:

$$\omega = 2\pi f$$

And substituting $2\pi f$ for ω into the expression for impedance, an expression for complex impedance of a capacitor is obtained:

$$\boldsymbol{Z_C} = \frac{1}{j2\pi fC}$$

or

$$\boldsymbol{Z_C} = -j\frac{1}{2\pi fC}$$

- In AC circuits, the frequency dependance of capacitor impedance is an important trait of capacitors because it allows the creation of circuits that are responsive to changes in frequency
- It is useful to remember that the impedance of a capacitor is *inversely* proportional to frequency:
  - As frequency increases, the reactance of a capacitor decreases
  - As frequency decreases, the reactance of a capacitor increases

And, it is helpful to remember what happens at extremely low and extremely high frequencies:

- At extremely low frequency, or DC:

$$X_C \rightarrow \infty$$

  a capacitor behaves like an open circuit (infinity ohms)

- If we imagine a constantly increasing frequency:

$$X_C \rightarrow 0$$

  a capacitor behaves like a short circuit (approaches zero ohms)

Circuit Phase Angle

- Circuit phase angle is the angle between the source voltage and the source current
  - We normally use the current phase angle as the reference
  - Assume that the source current has a phase angle of zero degrees ($\boldsymbol{I_S} = |I_s|\angle 0°$)
- To illustrate the meaning of circuit phase angle mathematically, we may start with the impedance relation for AC circuits where all variables are phasors:

$$\boldsymbol{V} = \boldsymbol{IZ}$$

Each phasor can be expressed in terms of its magnitude and angle:

$$|\boldsymbol{V}|\angle V = |\boldsymbol{I}|\angle I(|\boldsymbol{Z}|\angle Z)$$

By solving for impedance ($Z\angle Z$), we can find an expression for the circuit phase angle:

$$Z\angle Z = \frac{V\angle V}{I\angle I}$$

Since the impedance magnitude is found by dividing the magnitude of the two phasors, and the angle is found by subtracting the phase angle of the two phasors:

$$Z\angle Z = \frac{V}{I}\angle(\angle V - \angle I)$$

We see that the phase angle of impedance is obtained by subtracting the angle of the voltage phasor from the angle of the current phasor:

$$\angle Z = \angle V - \angle I$$

The circuit phase angle can also be understood as the impedance phase angle, with the caveat that **the source current is assumed to be the reference phasor such that ($\boldsymbol{I_S} = |\boldsymbol{I_s}|\angle \boldsymbol{0}°$)** such that:

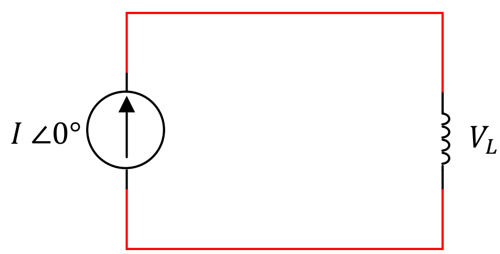
$$\angle Z_{Circuit} = \angle V_{Source} - \angle I_{Source}$$

- Phase angle is described as either leading or lagging in relation to phase angle of voltage compare to the phase angle of the current

- Phase angle is **positive** if source voltage **leads** source current, the circuit has more **inductive** reactance than **capacitive** reactance so that the net imaginary component is **positive**
- Phase angle is **negative** if source voltage **lags** positive current, the circuit has more **capacitive** reactance than **inductance** reactance so that the net imaginary component is **negative**

Phase Angle for Inductors

- To determine the impedance phase angle for inductors, we may start with the expression for inductor voltage, while letting the phase angle of the source be zero ($\boldsymbol{I} = |I|\angle 0°$)



$$\boldsymbol{V_L} = \boldsymbol{IZ_L}$$
$$= (|I|\ \angle 0°)(2\pi f L \angle 90°)$$

Let:

$$|V_L| = |I| 2\pi f L$$

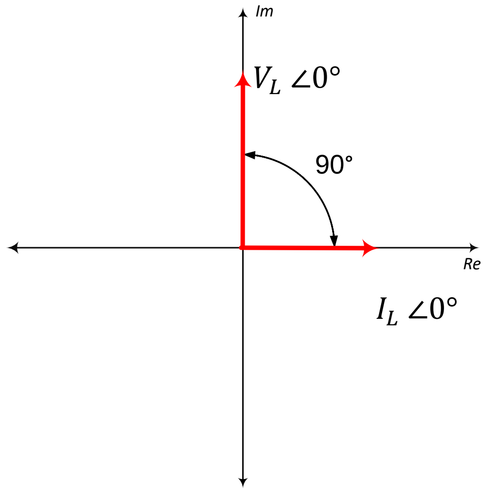
Then:

$$\boldsymbol{V_L} = |V_L|\angle 90°$$

And the impedance phase angle can be calculated:

$$\angle Z = \angle V - \angle I$$
$$= 90° - 0°$$
$$= 90°$$

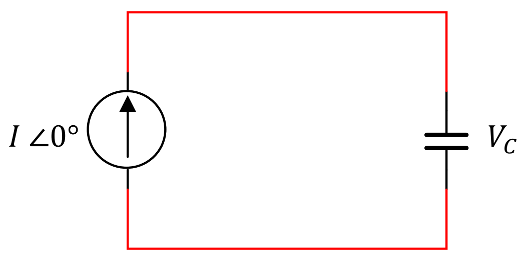
Thus, we see that **the voltage across an inductor *leads* the current phasor by an angle of 90 degrees**, and this relationship can also be depicted graphically on the complex plane, as shown below:



Phase Angle for Capacitors

- To determine the impedance phase angle for capacitors, we can use the same analysis as explained above for inductor but take careful note that the impedance of the circuit is Z_C

  we may start with the expression for inductor voltage, while letting the phase angle of the source be zero ($\boldsymbol{I} = |I|\angle 0°$)



$$\boldsymbol{V_C} = \boldsymbol{IZ_C}$$

$$\boldsymbol{V_C} = (|I| \angle 0°)\left(\frac{1}{2\pi f C}\angle -90°\right)$$

Let:

$$|V_C| = |I|\frac{1}{2\pi f C}$$

, then:

$$\boldsymbol{V_C} = |V_C|\angle -90°$$

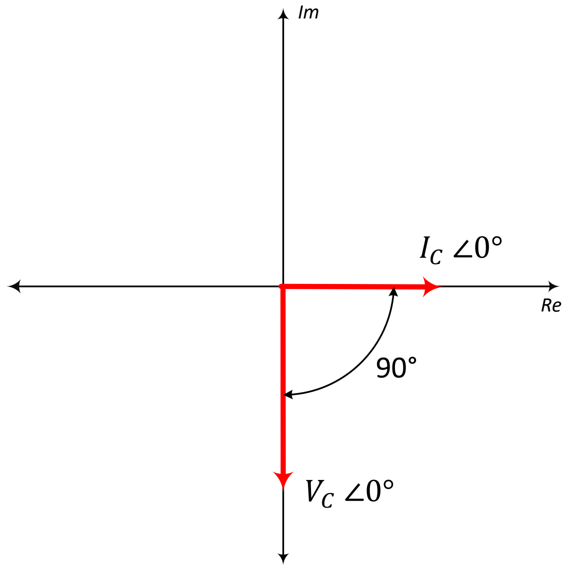
And the impedance phase angle can be calculated:

$$\angle Z = \angle V - \angle I$$

$$\angle Z = -90° - 0°$$
$$\angle Z = -90°$$

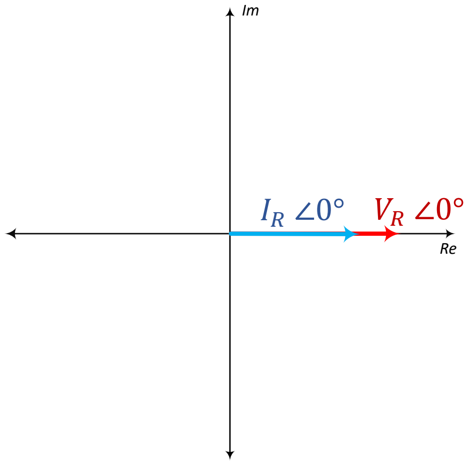
Thus, we see that the **voltage across a capacitor *lags* the current phasor by an angle of 90 degrees**

This relationship can also be depicted graphically on the complex plane, as shown below:



- Two mnemonics that are helpful tools for remembering phase relationships for inductors and capacitors are:
    - **ELI the ICE man**
        - ELI – stands for "*E Leads I*" for inductors *(note L stands for Inductor)*
        - ICE – stands for "*I Leads E*" for capacitors *(note C stands for capacitor)*
    - **CIVIL**
        - "CIV" for in capacitors - I leads V
        - "VIL" for inductors - V leads I
- The phase angle of passive components (resistors, capacitors, and inductors) are summarized on the next page:

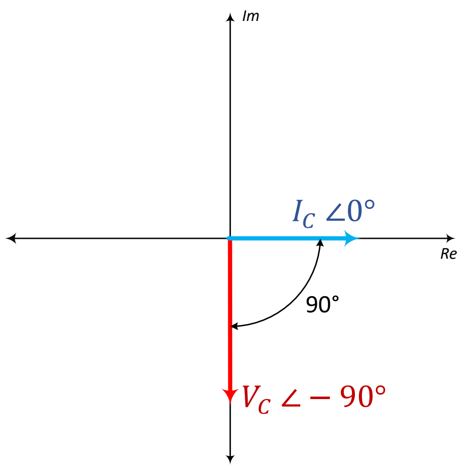
Resistor



- Resistor impedance has no imaginary component
- The voltage across a resistor is in phase with the current flowing through it
- Circuit phase angle for a resistor (or completely resistive circuit) is 0°

$$\boldsymbol{V} = \boldsymbol{IZ}$$
$$\boldsymbol{V} = (I\angle 0°)R$$
$$\boldsymbol{V} = IR\angle 0°$$

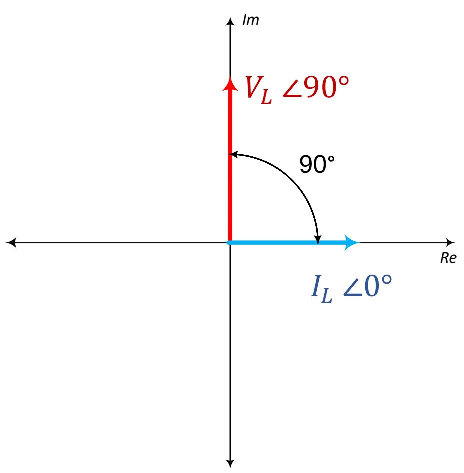
Capacitor



- Capacitor impedance has a negative imaginary component
- Voltage across a capacitor is 90 degrees ***lagging*** behind the phase of the current flowing through it
- Circuit phase angle for a capacitor (or completely capacitive circuit) is -90°

$$\boldsymbol{V_C} = \boldsymbol{I_C Z_C}$$
$$\boldsymbol{V_C} = (I\angle 0°)(X_C\angle -90°)$$
$$\boldsymbol{V_C} = IX_C\angle -90°$$

Inductor

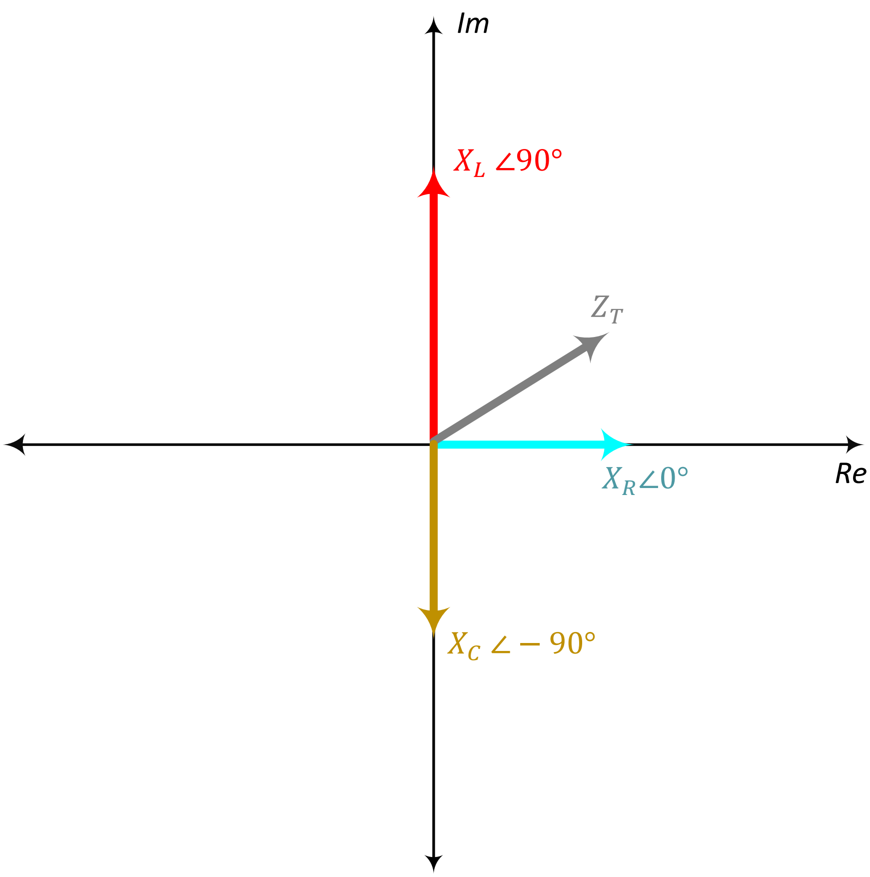


- Inductor impedance has a positive imaginary component
- Voltage across an inductor is 90 degrees ***leading*** the phase of the current flowing through it
- Circuit phase angle for an inductor (or completely inductive circuit) is 90°

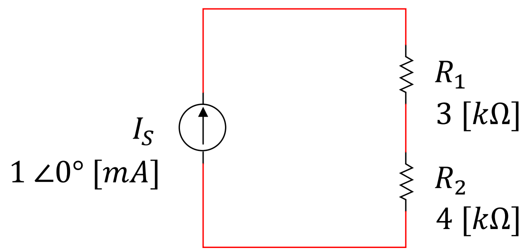
$$\boldsymbol{V_L} = \boldsymbol{I_L Z_L}$$
$$\boldsymbol{V_L} = (I\angle 0°)(X_L\angle 90°)$$
$$\boldsymbol{V_L} = IX_L\angle 90°$$

Impedance Diagrams

- A <u>phasor diagram</u> shows the relationships between currents and voltages
- An <u>impedance diagram</u> shows vectors that represent impedances in a circuit
- Impedances on the complex plane *do not rotate*
    - X_L is always at 90°
    - X_C is always at -90°
    - R is always at 0°
- Total impedance of the circuit that is *seen by the source* is a *vector sum* of the inductive, capacitive, and resistive capacitance



- **Example 1 – Resistive Circuit**
  - Draw the phasor diagram and the impedance diagram for the resistive circuit below



  - Determine all currents

$$\boldsymbol{I_S} = \boldsymbol{I_{R_1}} = \boldsymbol{I_{R_1}}$$

  - Resistor voltage drops may be determined using Ohm's Law

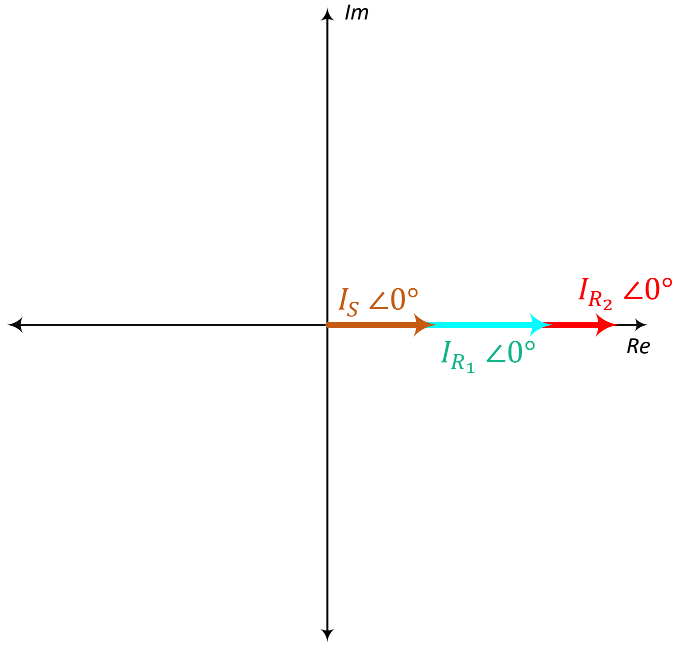
$$\begin{aligned}\boldsymbol{V_{R_1}} &= \boldsymbol{I_{R_1}R_1} \\ &= (1E-3)\angle 0°(3E3)\angle 0° \\ &= 3.00\ \angle 0°\ [mV_{RMS}]\end{aligned}$$

$$\begin{aligned}\boldsymbol{V_{R_2}} &= \boldsymbol{I_{R_2}R_2} \\ &= (1E-3)\angle 0°(4E3)\angle 0° \\ &= 4.00\ \angle 0°\ [mV_{RMS}]\end{aligned}$$

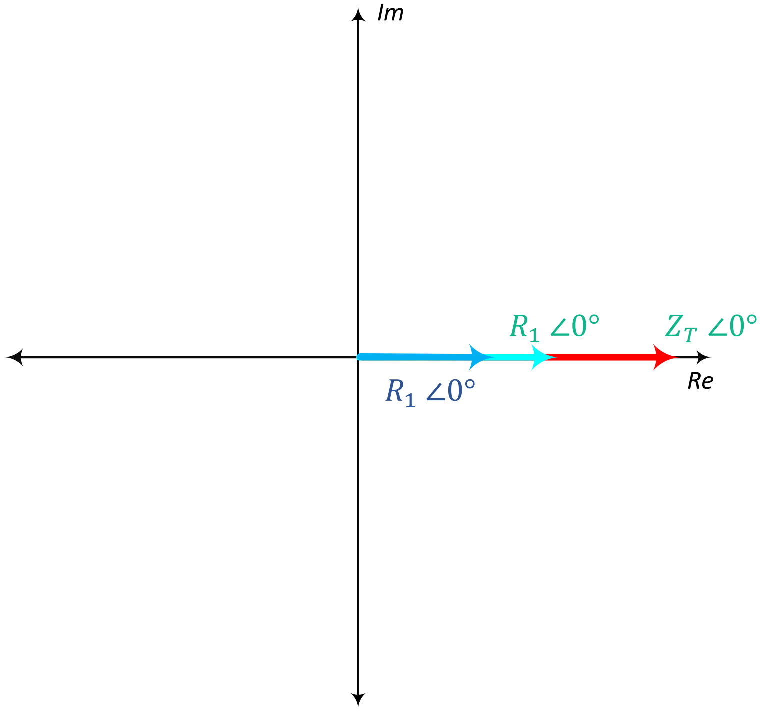
  - Phase relationships
    - $\boldsymbol{I_{R_1}}$ is in phase with $\boldsymbol{V_{R_1}}$ (or 0°)
    - $\boldsymbol{I_{R_2}}$ is in phase with $\boldsymbol{V_{R_2}}$ (or 0°)
  - Circuit phase angle

$$\begin{aligned}\phi &= \angle E_S - \angle I_S \\ &= 0° - 0° \\ &= 0°\end{aligned}$$

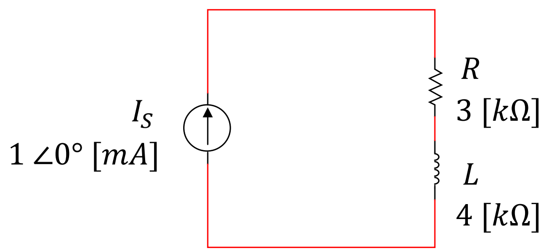
- Phasor diagram



- Impedance diagram



- **Example 2 – RL Circuit**
  - Draw the phasor diagram and the impedance diagram for the RL circuit below



  - Determine all currents

$$I_S = I_R = I_L = 1\angle 0° \, [mA]$$

  - Component voltage drops may be determined using Ohm's Law

$$\begin{aligned} V_R &= I\,R_1 \\ &= (1E-3\angle 0°)(3E3\angle 0°) \\ &= 3.00\,\angle 0° \,[V] \end{aligned}$$

$$\begin{aligned} V_L &= I_S X_{L_1} \\ &= (1E-3\angle 0°)(4E3\angle 90°) \\ &= 4.00\,\angle 90° \,[V] \end{aligned}$$

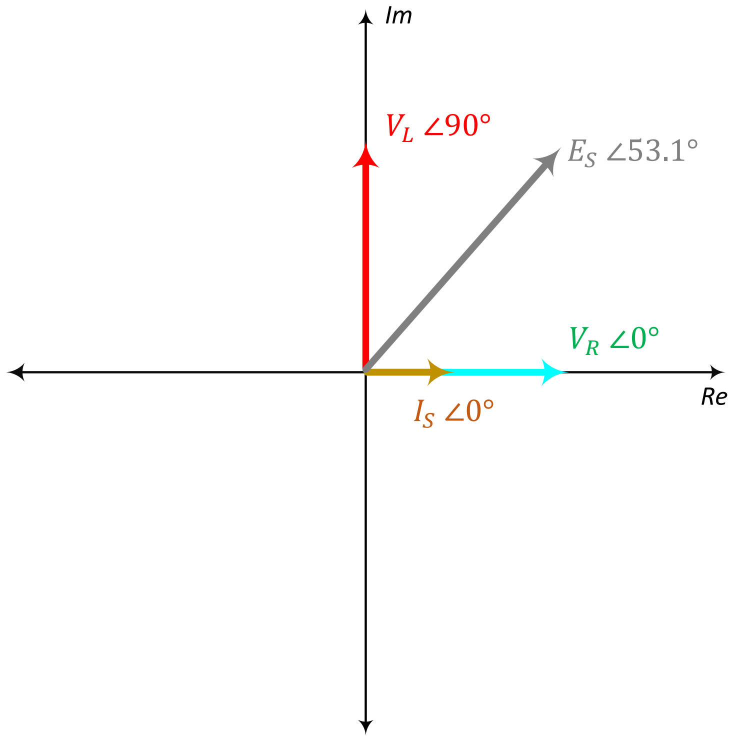
  - Voltage and Current Phase Relationships
    - Current flowing in the circuit has an angle of zero degrees due to the source, which is forcing it to be that way
    - I_R and V_R are in phase with the same angle of zero degrees
    - V_L leads I_L by 90° (*recall ELI or CIVIL*)
    - $\boldsymbol{E_s}$ is determined by KVK, and is a vector sum

$$\begin{aligned} \boldsymbol{E_S} &= \boldsymbol{V_R} + \boldsymbol{V_{X_L}} \\ &= \boldsymbol{I_S R} + \boldsymbol{I_S Z_L} \\ &= I_S R + |I_S|\angle 0°(X_{L_1}\angle 90°) \\ &= (1E-3\angle 0°)(3E3\angle 0°) + (1E-3\,\angle 0°)(4E3\angle 90°) \\ &= 3.00\angle 0° + 4.00\angle 90° \\ &= 3.00 + j4.00 \\ &= 5.00\,\angle 53.1° \,[V] \end{aligned}$$

  - Circuit phase angle

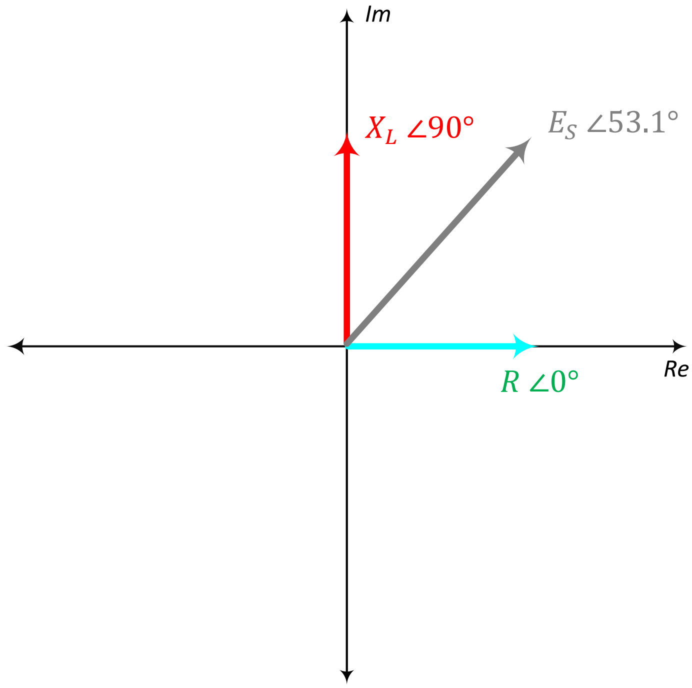
$$\phi = \angle E_S - \angle I_S$$
$$= 53.1° - 0°$$
$$= 53.1°$$

- Phasor diagram

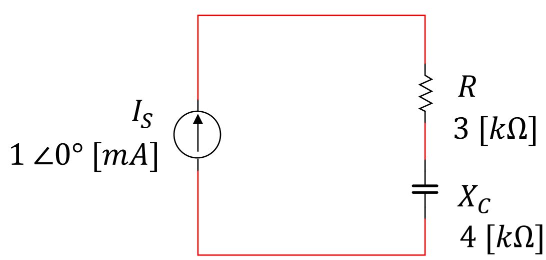


- Impedance Diagram
  - By dividing the voltage of each impedance in the phasor diagram by the current in each component, we can determine their impedances
    - $\frac{V_L}{I_L} = Z_L$
    - $\frac{V_R}{I_R} = R$
  - The vector sum of the impedances is the total circuit impedance, which could also be determined using Ohm's Law
    - $Z_T = R + jX_L$

    Or…
    - $\frac{E_S}{I_S} = Z_T$
  - The diagram is shown on the next page…



- **Example 3 – RC Circuit**
  - Draw the phasor diagram and the impedance diagram for the RC circuit below



  - Find all currents

$$\boldsymbol{I_S} = \boldsymbol{I_R} = \boldsymbol{I_C} = 1 \angle 0° \ [mA]$$

  - Find all voltage drops

$$\begin{aligned} \boldsymbol{V_R} &= \boldsymbol{I_S R} \\ &= (1E-3\angle 0°)(3E3\angle 0°) \\ &= 3.00 \angle 0° \ [V] \end{aligned}$$

$$\begin{aligned}\boldsymbol{V_C} &= \boldsymbol{I_S Z_C}\\ &= \boldsymbol{I_S}(X_C\angle -90°)\\ &= (1E-3\ \angle 0°)(4E3\angle -90°)\\ &= 4.00\angle -90°\ [V]\end{aligned}$$

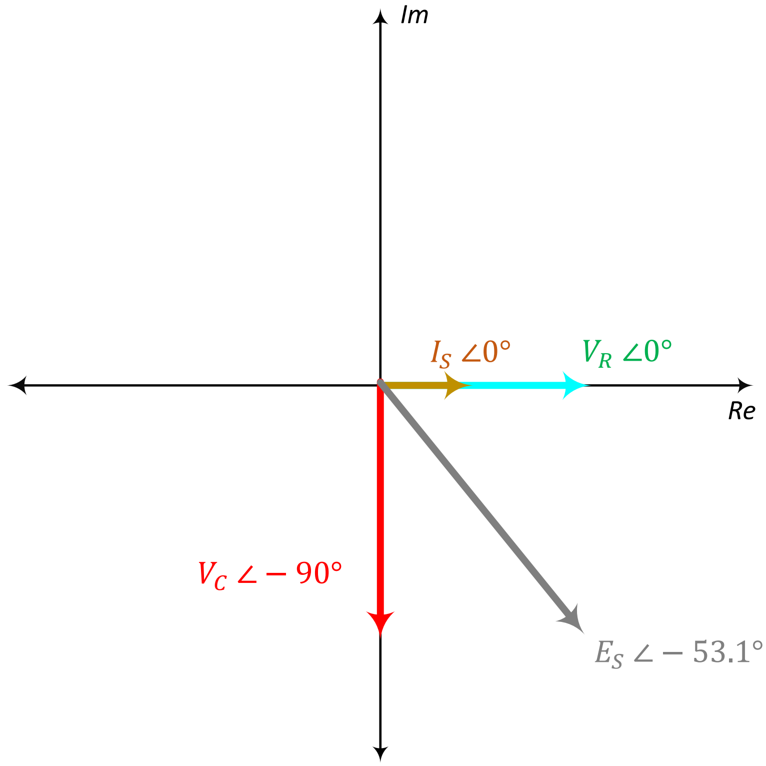
- Voltage and Current Phase Relationships
  - V_R is in phase with I_S at zero degrees
  - $\boldsymbol{V_C}$ lags $\boldsymbol{I_S}$ by 90°
  - $\boldsymbol{E_s}$ may be found using KVL, and is a vector sum

$$\begin{aligned}\boldsymbol{E_S} &= \boldsymbol{V_c} + \boldsymbol{V_R}\\ &= \boldsymbol{I_s Z_C} + \boldsymbol{I_S R}\\ &= \boldsymbol{I_S}(X_C\angle -90°) + \boldsymbol{I_S}(R\angle 0°)\\ &= (1E-3\angle 0°)(4E3\angle -90°) + (1E-3\angle 0°)(3E3\angle 0°)\\ &= 4\angle -90° + 3\angle 0°\\ &= -j4 + 3\\ &= 5.00\ \angle -53.1°\ [V]\end{aligned}$$

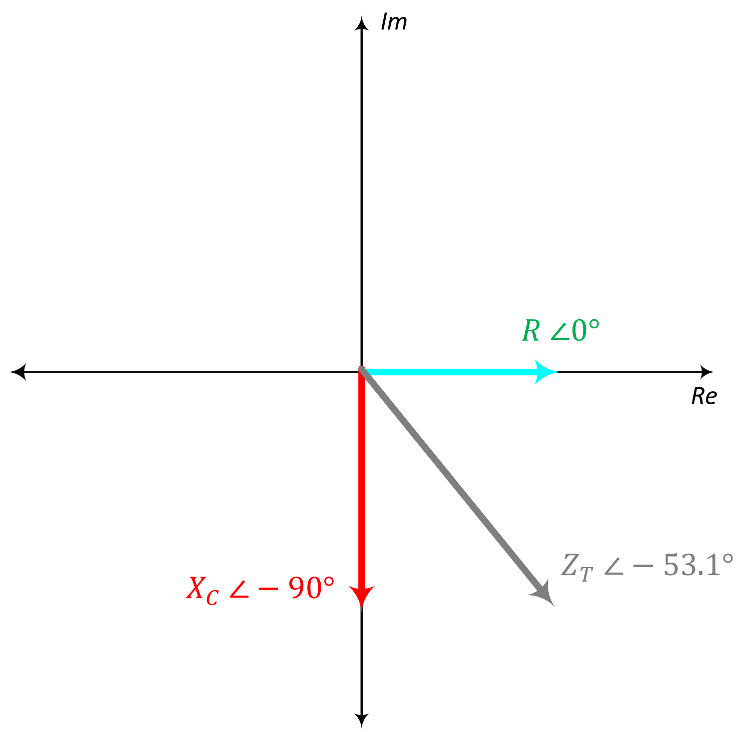
  - Circuit phase angle

$$\begin{aligned}\phi &= \angle E_S - \angle I_S\\ &= -53.1° - 0°\\ &= -53.1°\end{aligned}$$

- Phasor diagram



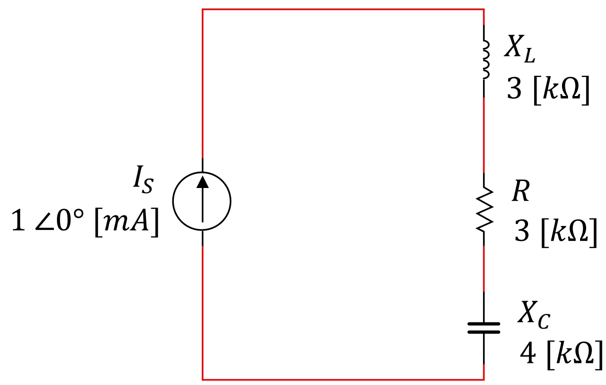
- Impedance diagram



- **Example 4 – RLC Circuit**

- Draw the phasor diagram and the impedance diagram for the RLC circuit below

- Note: Although this circuit has both inductive and capacitive components, its overall impedance is ***capacitive***



- Find all currents

$$\boldsymbol{I_S} = \boldsymbol{I_L} = \boldsymbol{I_R} = \boldsymbol{I_C} = 1\angle 0°\ [mA]$$

- Find all voltage drops

$$\begin{aligned}\boldsymbol{V_L} &= I_S Z_L \\ &= \boldsymbol{I_S}(X_L\angle 90°) \\ &= (1E-3\angle 0°)(3E3\angle 90°) \\ &= 3.00\ \angle 90°\end{aligned}$$

$$\begin{aligned}\boldsymbol{V_R} &= I_S R \\ &= (1E-3\angle 0°)(3E3\angle 0°) \\ &= 3.00\ \angle 0°\ [V]\end{aligned}$$

$$\begin{aligned}\boldsymbol{V_C} &= \boldsymbol{I_S Z_C} \\ &= (I_S)(X_C\angle -90°) \\ &= (1E-3\angle 0°)(4E3\angle -90°) \\ &= 4.00\ \angle -90°\ [V]\end{aligned}$$

- Voltage and Current Phase Relationships

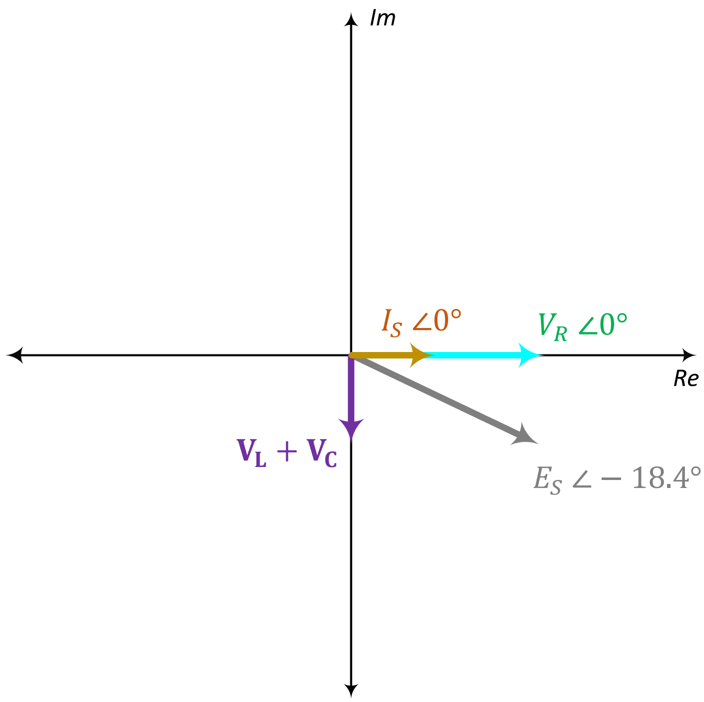
- V_{X_L} leads I_S by 90°
- V_R is in phase with I_S
- V_{X_C} lags I_S by 90°
- E_S may be found by KVL, and is a vector sum

$$\begin{aligned}
\boldsymbol{E_S} &= \boldsymbol{V_R} + \boldsymbol{V_L} + \boldsymbol{V_C} \\
&= 3.00\angle 0° + 3.00\angle 90° + 4.00\angle -90° \\
&= 3.00 + j3.00 - j4.00 \\
&= 3.00 - j1.00 \\
&= 3.16\angle -18.4° \ [V]
\end{aligned}$$

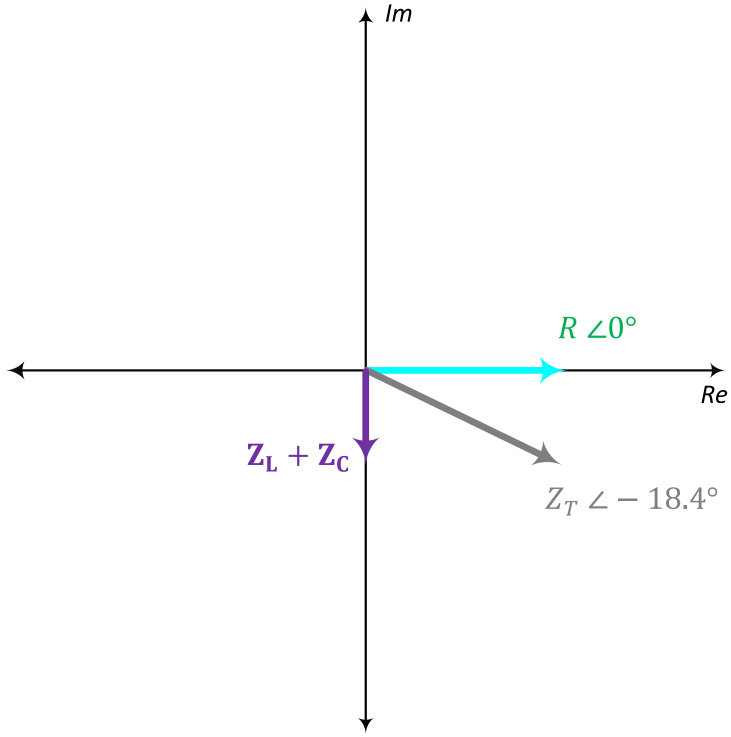
- Circuit phase angle

$$\begin{aligned}
\phi &= \angle E_S - \angle I_S \\
&= -18.4° - 0° \\
&= -18.4°
\end{aligned}$$

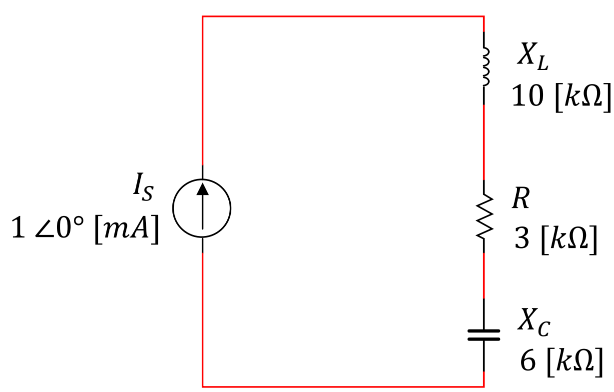
- Phasor diagram



- Impedance diagram



- **Example 5 – RLC Circuit**
  - Although this circuit has both inductive and capacitive components, its overall impedance is ***inductive***



  - Find all currents

$$\boldsymbol{I_S} = \boldsymbol{I_L} = \boldsymbol{I_R} = \boldsymbol{I_C} = 1\angle 0°\ [mA]$$

  - Find all voltage drops

$$\boldsymbol{V_L} = I_S Z_L$$

$$= \boldsymbol{I_S}(X_L\angle 90°)$$
$$= (1E-3\angle 0°)(10E3\angle 90°)$$
$$= 10.00\ \angle 90°$$

$$\boldsymbol{V_R} = I_S R$$
$$= (1E-3\angle 0°)(3E3\angle 0°)$$
$$= 3.00\ \angle 0°\ [V]$$

$$\boldsymbol{V_C} = \boldsymbol{I_S Z_C}$$
$$= (I_S)(X_C\angle -90°)$$
$$= (1E-3\angle 0°)(6E3\angle -90°)$$
$$= 6.00\ \angle -90°\ [V]$$

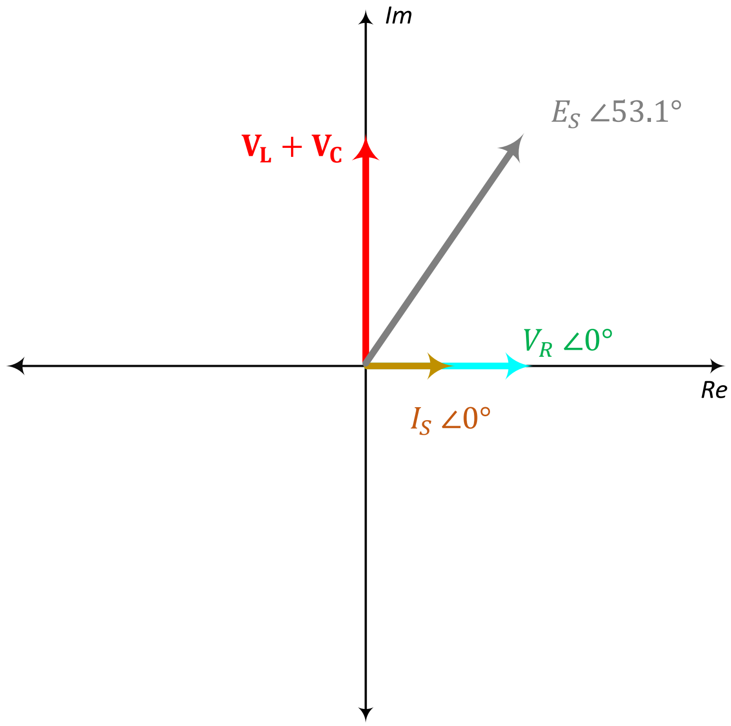
- Voltage and Current Phase Relationships
  - V_L leads I_S by 90°
  - V_R is in phase with I_S
  - V_C lags I_S by 90°
  - E_S may be found by KVL, and is a vector sum

$$\boldsymbol{E_S} = \boldsymbol{V_R} + \boldsymbol{V_L} + \boldsymbol{V_C}$$
$$= 3.00\angle 0° + 10.00\angle 90° + 6.00\angle -90°$$
$$= 3.00 + j10.00 - j6.00$$
$$= 3.00 + j4.00$$
$$= 5.00\ \angle\ 53.1°\ [V]$$

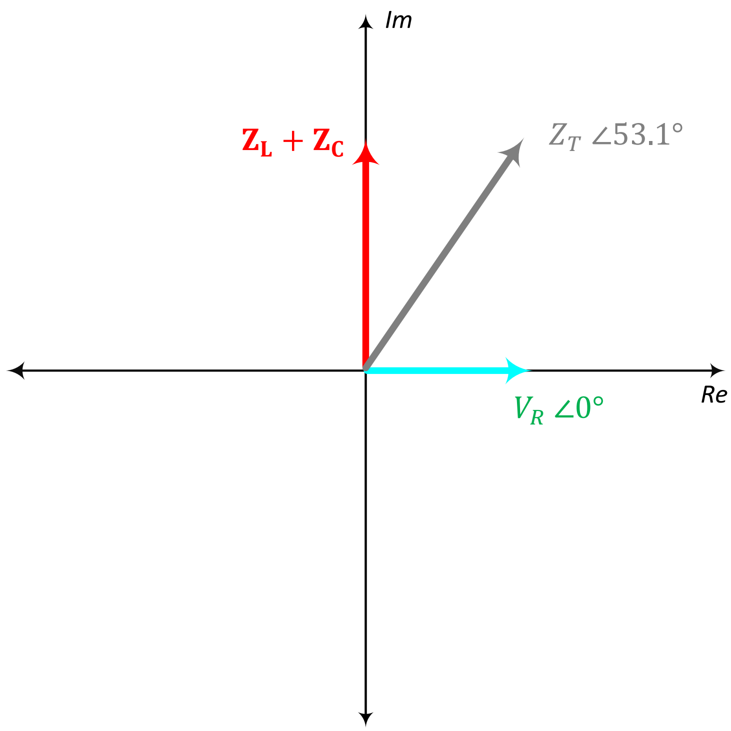
- Circuit phase angle

$$\phi = \angle E_S - \angle I_S$$
$$= 53.1° - 0°$$
$$= 53.1°$$

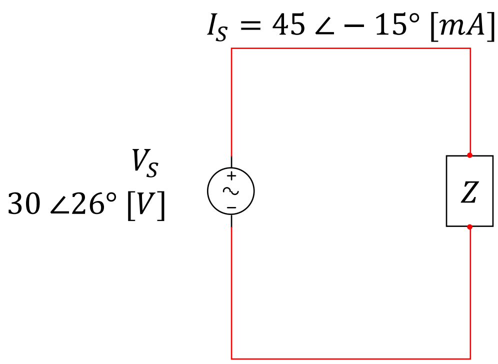
- Phasor diagram



- Impedance diagram



- **Example 6 – Phase Angle**
    - Find the circuit phase angle



- Recall that the circuit phase angle (ϕ) is the phase angle of the supply current subtracted from phase angle of the supply voltage:

$$\phi = \angle V_S - \angle I_S$$
$$\phi = \angle Z_T$$

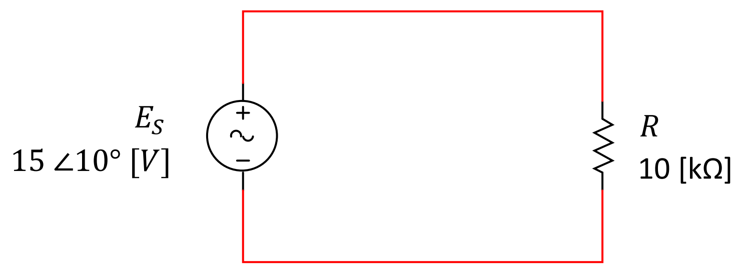
- Since circuit phase angle is the angle of the total impedance phasor, we may find the circuit phase angle as the angle of the impedance vector:

$$\begin{aligned} Z_T &= \frac{V_S}{I_S} \\ &= \frac{30\ \angle 26°}{45E-3\angle -15°} \\ &= 667\ \angle 41.0°\ [\Omega] \end{aligned}$$

- The phase angle is the angle of the circuit impedance:

$$\phi = 41.0°$$

- **Example 7 – Peak to peak curent**
  - Find the peak-to-peak resistor current ($I_{R_{PP}}$)
  - Draw the phasor diagram showing E_S, $I_{R_{PP}}$, and the circuit phase angle (ϕ)

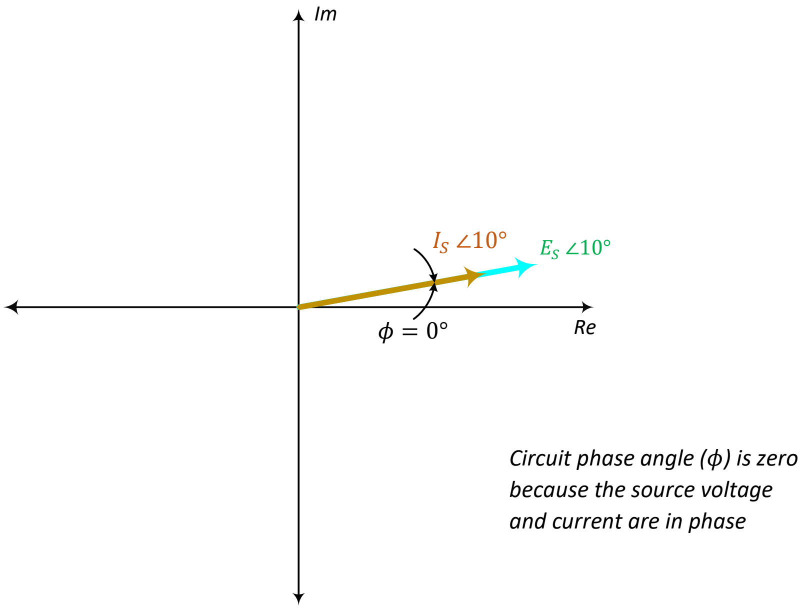


  - The peak-to-peak resistor current is found using Ohm's Law

$$\begin{aligned} \boldsymbol{I_R} &= \frac{E_S}{R} \\ &= \frac{15\angle 10°}{10E3\angle 0°} \\ &= 1.5000E-3\ \angle 10°\ [A] \end{aligned}$$

$$\begin{aligned} \boldsymbol{I_{R_{PP}}} &= 2I_{R_{PP}} \\ &= 2(I_R\sqrt{2}) \\ &= 2\sqrt{2}(1.5000E-3\angle 10°) \\ &= 4.24\ \angle 10°\ [mA] \end{aligned}$$

  - Phasor diagram



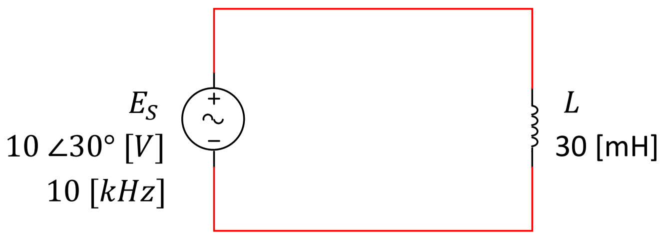
- **Example 8 – Inductive reactance and impedance**
  - Find the inductive ***reactance***, and the inductive ***impedance*** of a 30 [mH] inductor with a sinusoidal current of 60 [kHz] flowing through it
  - Solution
    - Inductive reactance is the magnitude of the inductor's impedance (it is a scalar quantity)

$$\begin{aligned} X_L &= \omega L \\ &= 2\pi f L \\ &= 2\pi(60E3)(30E-3) \\ &= 11.3\ [k\Omega] \end{aligned}$$

    - Inductive impedance is the complex quantity, which is purely imaginary on the complex plane

$$\begin{aligned} Z_L &= jX_L \\ &= j11.3\ [k\Omega] \\ &= 11.3\ \angle 90°\ [k\Omega] \end{aligned}$$

- **Example 9 – Current and Voltage phasors**
  - Find the source current (I_S), voltage across the inductor (V_L), and circuit phase angle (ϕ)



  - Solution
    - The source current is found using Ohm's Law, but inductor impedance must be first determined

$$\begin{aligned} \boldsymbol{Z_L} &= jX_L \\ &= j2\pi f L \\ &= j2\pi(10E3)(30E-3) \\ &= 1.8850\ \angle 90°\ [k\Omega] \end{aligned}$$

$$\begin{aligned} I_S &= \frac{E_S}{Z_L} \\ &= \frac{10\angle 30°}{1.8850E3\ \angle 90°} \\ &= 5.31\ \angle -60°\ [mA] \end{aligned}$$

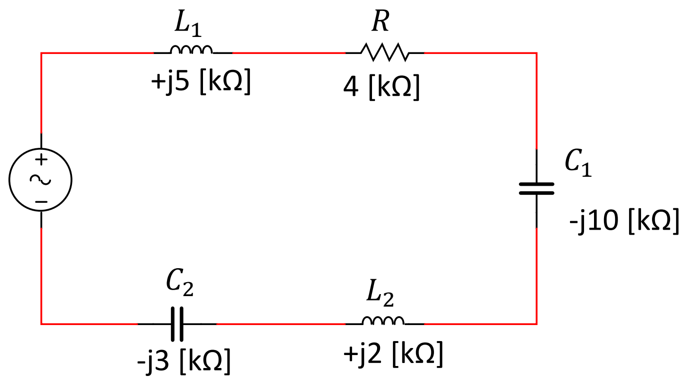
    - The voltage across the inductor is the same as the source voltage

$$V_L = E_S = 10\ \angle 30°\ [V]$$

    - Circuit phase angle may be found as the angle of the total circuit impedance, or as the difference between the source voltage and the source current angles

$$\begin{aligned} \phi &= \angle E_S - \angle I_S \\ &= 30° - (-60°) \\ &= 90° \end{aligned}$$

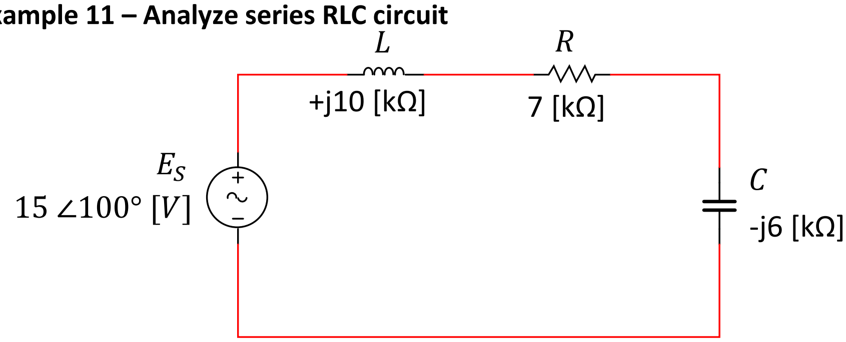
- **Example 10 – Total Impedance**
  - Determine the total impedance of the circuit, which the source current flows through



  - Solution
    - Like DC circuits, the total impedance is the sum of the individual impedances because they are connected in series

$$\boldsymbol{Z_T} = Z_{L_1} + R + Z_{C_1} + Z_{L_2} + Z_{C_2}$$
$$= j5E3 + 4E3 - j10E3 + j2E3 - j3E3$$
$$= 4E3 - j6E3$$
$$= 7.21 \angle -56.3° \ [k\Omega]$$

- **Example 11 – Analyze series RLC circuit**



  - Find:
    - Total impedance (Z_T)
    - Source current (I_S)
    - Voltage across each component (V_L), (V_R), and (V_C)
  - Solution
    - The total impedance is the sum of the component impedances

$$\boldsymbol{Z_T} = \boldsymbol{Z_L} + \boldsymbol{Z_R} + \boldsymbol{Z_C}$$

$$= +j10E3 + 7E3 - j6E3$$
$$= 7E3 + j4E3$$
$$= 8.0623E3\ \angle 29.745°\ [\Omega]$$

- The source current is found using Ohm's Law

$$\boldsymbol{I_S} = \frac{\boldsymbol{E_S}}{\boldsymbol{Z_T}}$$
$$= \frac{15\angle 100°}{8.0623E3\ \angle 29.745°}$$
$$= 1.8605E-3\ \angle 70.255°\ [A]$$
$$= 1.86\ \angle 70.3°\ [mA]$$

- The resistor, inductor, and capacitor voltage drops (V_R), (V_L), and (V_C) are found by Ohm's Law:

$$\boldsymbol{V_R} = \boldsymbol{IZ_R}$$
$$= (\ 1.8605E-3\ \angle 70.255°\ )7E3\angle 0°$$
$$= 13.023\ \angle 70.255°\ [V]$$
$$= 13.0\ \angle 70.3°[V]$$

$$\boldsymbol{V_L} = \boldsymbol{IZ_L}$$
$$= (\ 1.8605E-3\ \angle 70.255°\ )10E3\angle 90°$$
$$= 18.605\ \angle 160.255°\ [V]$$
$$= 18.6\ \angle 160°\ [V]$$

$$\boldsymbol{V_C} = \boldsymbol{IZ_C}$$
$$= (\ 1.8605E-3\ \angle 70.255°\ )6E3\angle -90°$$
$$= 11.163\ \angle -19.745°\ [V]$$
$$= 11.2\ \angle -19.7°\ [V]$$

To confirm that the voltage drops are correct, KVL may be applied to confirm that the sum of the individual voltage drops added together are equal to the supply voltage:

$$\boldsymbol{E_S} = \boldsymbol{V_R} + \boldsymbol{V_L} + \boldsymbol{V_C}$$
$$= 13.023\ \angle 70.255° + 18.605\ \angle 160.255° + 11.163\ \angle -19.745°$$

$$= 14.999 \angle 100° \ [V]$$

$$= 15.0 \angle 100° \ [V]$$