March 21

Friday, March 15, 2024 9:53 AM

# Step Response of 1st Order (RC and RL circuits)

- **Capacitors and inductors are energy storage elements, and they do not dissipate energy**
  - Capacitors store energy as electric fields (accumulated charge)
  - Inductors store energy as magnetic fields (moving charge, or current)
  - Stored energy must be released later, but not converted to heat as resistors
    - Power relations ($P = VI$, $P = V^2/R$, and $P = I^2R$) do not apply ~~to~~ inductors and capacitors it does, ~~with~~ Z!
- Capacitors and inductors have nonlinear voltage-current relationships, and these quantities change with time due to storage and release of energy
  - They do ~~not obey~~ Ohm's Law they do!
  - Current and voltage change over time time, until fully charged or discharged

**First Order Circuits**

- The term first order is used to describe circuits that contain only one storage element
- Recalling that inductors and capacitors do not dissipate energy but rather temporarily store and then release their stored energy, any circuit that contains only one inductor or capacitor is considered a first order circuit
- It is assumed that the storage element is ideal (has no parasitic traits) and therefore does not have a stray resistance that would otherwise cause it to dissipate energy

→ in terms of behavior, inductors oppose to any change in current by producing a voltage, while capacitors oppose a change in voltage by producing a current.

→ for an inductor, the voltage is proportional to the rate of change in current, and the inductance.

$$v_L(t) = L\frac{\Delta i_L}{\Delta t}$$

→ for a capacitor, the current is proportional to the rate of change in voltage, and the capacitance.

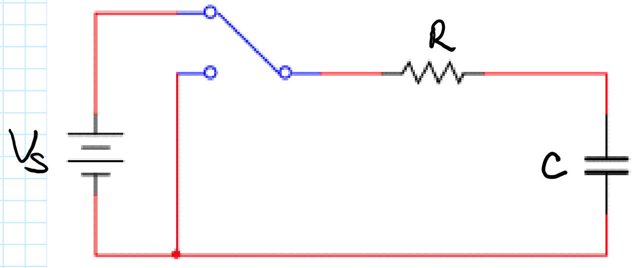
$$i_C(t) = C\frac{\Delta v_C}{\Delta t}$$

→ inductors and capacitors are in <u>steady state</u> when their current and voltage are not changing (they are either fully charged or discharged).

$$\frac{\Delta v_C}{\Delta t} = 0 \qquad \frac{\Delta i_L}{\Delta t} = 0$$

→ when not in steady state, they either store or release energy. This corresponds to the <u>transient period</u>.

## <u>Series RC Circuit Step Response</u>




→ consider initially the capacitor not charged. When the switch is in upper position, current goes through the resistor and the capacitor gets fully charged – this is <u>transient state</u>. At the end of the process, $v_C = V_S$ and $i = 0$. This is <u>steady state condition</u>.

→ when the switch is in the lower position, the charge stored in the capacitor will flow through the resistor that will dissipate the energy as heat – this is <u>transient state</u> again until the capacitor is fully discharged. At the end of the process it will be steady state condition again, with $v_C = 0$ and $i = 0$.

$$v_C(t) = V_{ss} - (V_{SS} - V_i)e^{-\frac{t}{\tau}}$$

$$i_C(t) = I_{ss} - (I_{SS} - I_i)e^{-\frac{t}{\tau}}$$

$\tau = RC$

$\tau$ – time constant (s) – the larger R or C is, the longer it takes for the capacitor to charge up.

$V_{SS}$ – steady state or final voltage across the capacitor after $5\tau$.

$V_i$ – initial voltage across the capacitor (V)

$I_{SS}$ – steady state or final current (A)

$I_i$ – initial current (A)

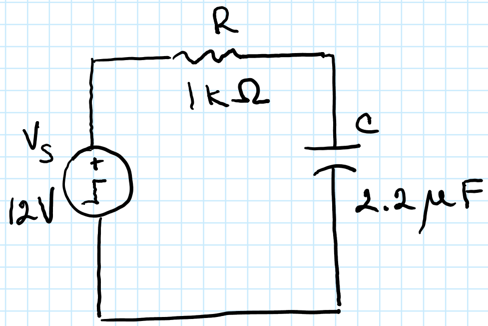
$t$ – time (s)

→ if the capacitor is fully discharged before the source voltage is applied, then $V_i = 0$:

$$v_C(t) = V_{ss}\left(1 - e^{-\frac{t}{\tau}}\right)$$

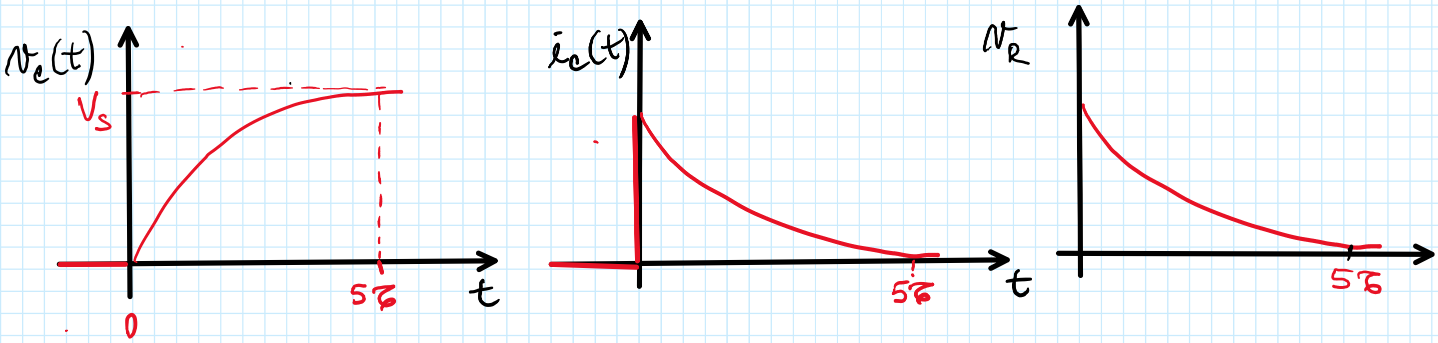
→ when $I_{SS} = 0$: $i_C(t) = I_i e^{-\frac{t}{\tau}}$ $I_i = \frac{V_S}{R}$

Ex: Find the expression for capacitor voltage and current step response (capacitor fully discharged initially).




→ in a steady state, the capacitor acts like an open circuit in DC, and like a short circuit in AC.

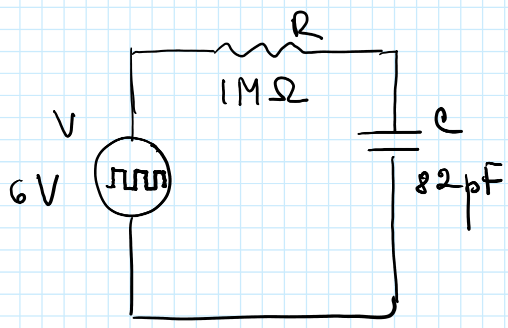
DC, and like a short circuit in AC.

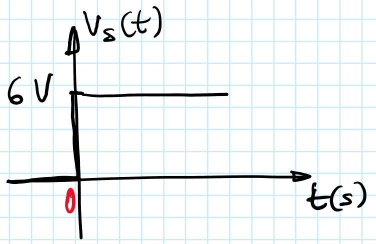



Ex: For the circuit below, calculate:

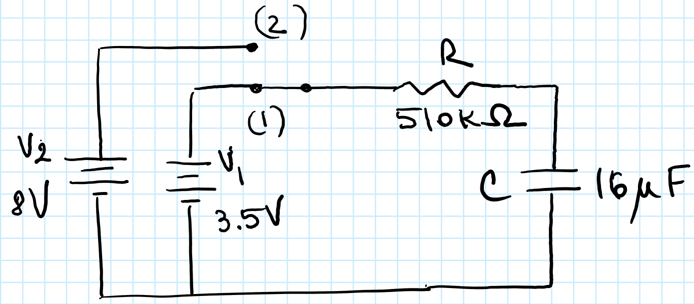
a. The current flowing in the circuit immediately after the source voltage switches from 0 to 6 V.

b. The voltage across the capacitor when $t = \tau$.



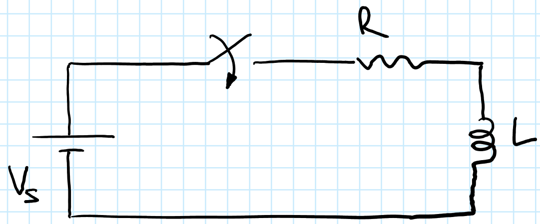
Ex: For the circuit below, assume that the switch moves from terminal 1 to terminal 2 at t=0. Calculate the voltage across the capacitor at t = 180 μs. Redo the calculations if the values of the voltages are switched.




## Series RL Circuit Step Response

$\tau = L/R$

$$\begin{cases} v_L(t) = (I_{SS}R)e^{-\frac{t}{\tau}} = V_S e^{-\frac{t}{\tau}} \\ i_L(t) = I_{ss} - (I_{SS} - I_i)e^{-\frac{t}{\tau}} \end{cases}$$



$I_{SS} = \frac{V_S}{R}$ $\qquad$ $I_i = 0$ (inductor fully discharged)

Ex: For the circuit below, calculate the values of the inductor voltage and current after 3 μs. Assume the inductor is fully discharged initially.