Feb 29

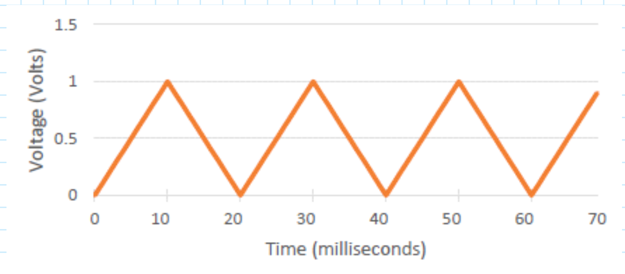
Wednesday, February 28, 2024 8:08 AM

Alternating Current (AC)

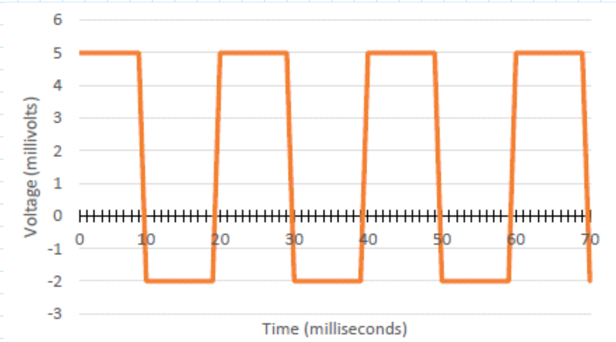
→ circuits involving time-varying voltages or currents are called Alternating Current (AC) circuits.

→ waveform - a graph of a quantity as a function of time. Periodic waveforms are the ones that repeat over a regular interval of time.

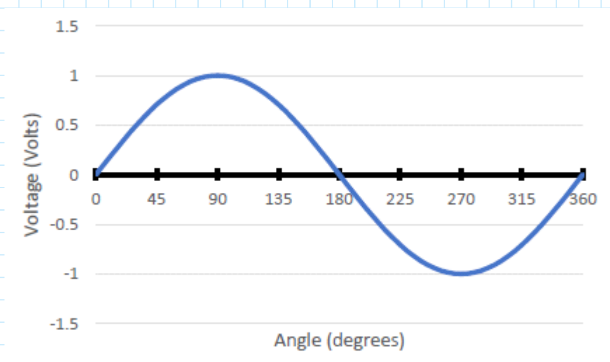
→ AC waveforms: ramp waveform, square waveform, sinusoidal waveform.



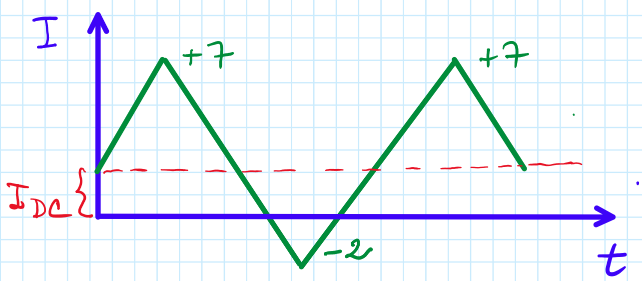
varies in time



varies in time and magnitude



varies in time and magnitude



AC with DC offset

→ sinusoidal waveforms occur naturally in AC power systems. Normally, when we discuss AC circuits, it is assumed that the sources involved are sinusoidal.

Characteristics

→ instantaneous value – the value of the quantity at a specific time.

→ period T – the time interval between successive repetitions (time necessary for a complete cycle), in s.

→ frequency f – the number of cycles that occur in one second, in Hz (Hertz).

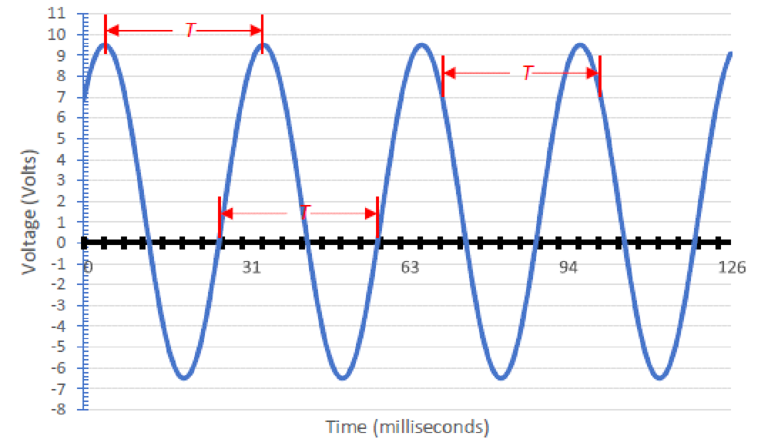
$$f = \frac{1}{T} \qquad T = \frac{1}{f}$$

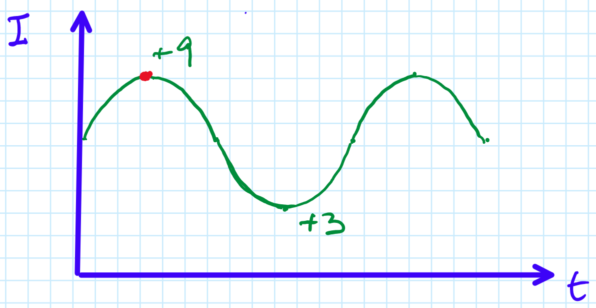
Examples:

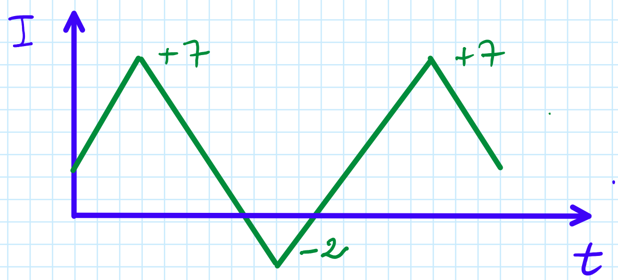
▪ Human Hearing	20 [Hz] to 20 [kHz]
▪ AM Radio Broadcast	535 [kHz] to 1300 [kHz]
▪ Television Broadcast Channel 2	54MHz to 60 MHz
▪ FM Radio Broadcast	88 [MHz] to 106 [MHz]
▪ Cellular Phones	880 [MHz]
▪ Satellite TV	4 [GHz]
▪ Police Radar	10 to 12 [GHz]

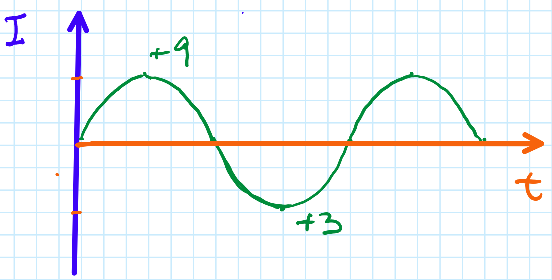
→ peak value – the maximum value on a waveform. V_p , I_p

→ peak-to-peak value – the range of values from maximum to minimum that a waveform traverses. For sinusoidal signals, it is twice the peak value. (V_{p-p} , I_{p-p}) – uppercase letters

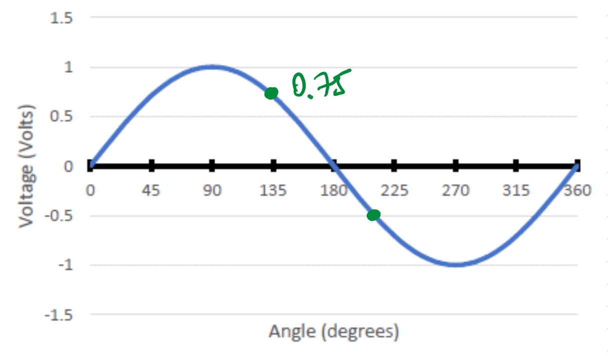


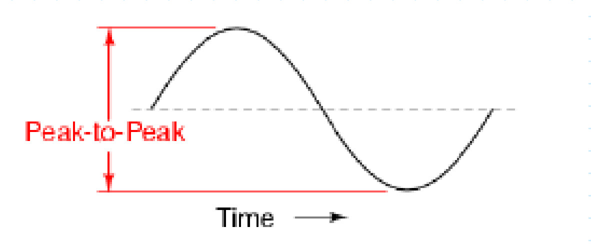






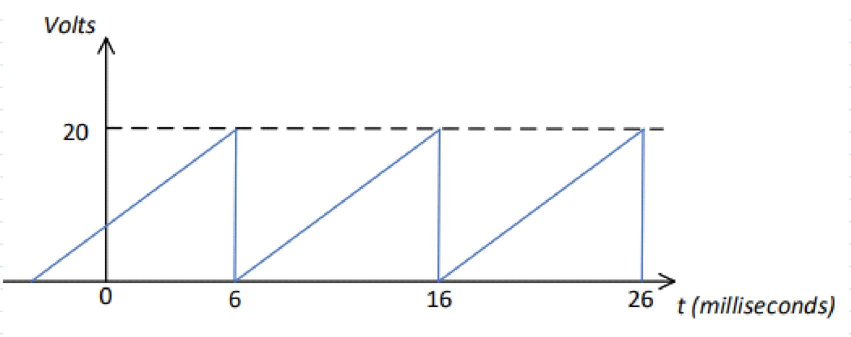
$\Rightarrow$ for sinewave $\boxed{V_{p-p} = 2V_p}$ and for DC offset sinewave $V_{p-p} < V_p$





$v(t_1) = +0.75\ V$
$v(t_2) = -0.5\ V$
} instantaneous values

Ex:



Sine Wave

→ all electrical utility systems in the world generate and deliver power using sinusoidal waveforms at specific voltages and currents.

Examples of single-phase plug load voltages used around the world:

• Hong Kong	220V at 50Hz
• Korea, South	220V at 60Hz
• India	230V at 50Hz
• Iraq	230V at 50 Hz
• England	230 V at 50Hz
• Japan	100V at 50/60Hz
• Canada	120V at 60Hz

→ sinusoidal waveforms have a characteristic shape that can be produced by a sine or cosine trigonometric function. An oscilloscope is used to display the waveform.

→ the horizontal axis can be time (s) or angle (degrees or radians).

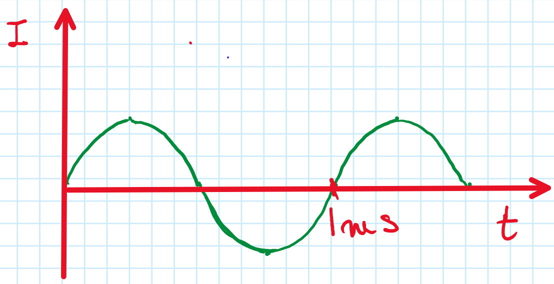
$$1 \text{ rad} = 360/(2\pi)$$

→ <u>angular velocity ω</u> - the change in angle over time

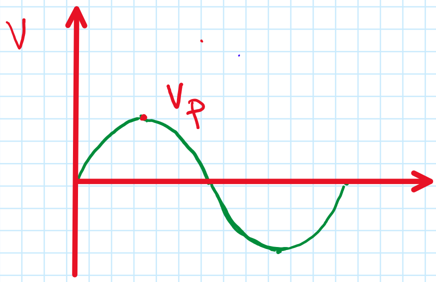
$$\omega = 2\pi f = \frac{2\pi}{T}$$

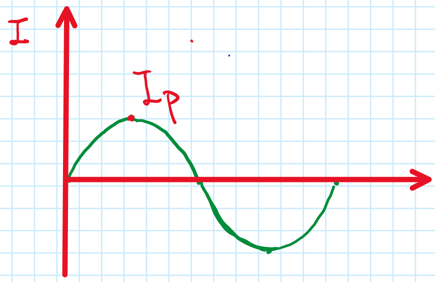
Ex: Calculate the angular velocity of the wave if the frequency is 50 Hz.

Ex: Determine angular velocity.



→ the equation of the sinewave will be written with lower case letters.





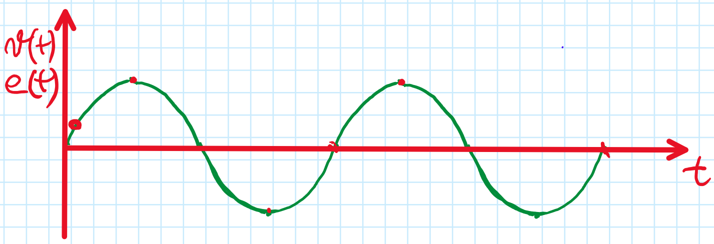
$v(t) = V_P \sin\theta$

$v(t) = V_P \sin\omega t$

$i(t) = I_P \sin\theta$

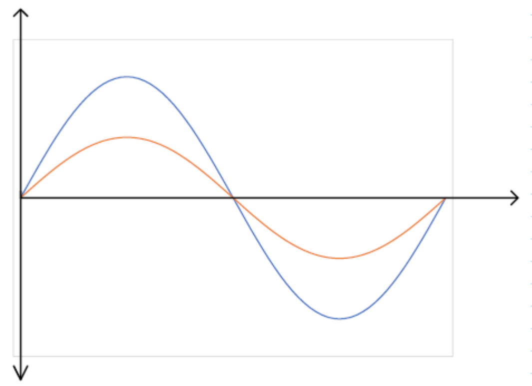
$i(t) = I_P \sin\omega t$

Ex: An AC voltage with a frequency of 1.5 kHz has a peak value of 3.3 V. Determine the value of the voltage at 0.65 μs and 1.2 ms.

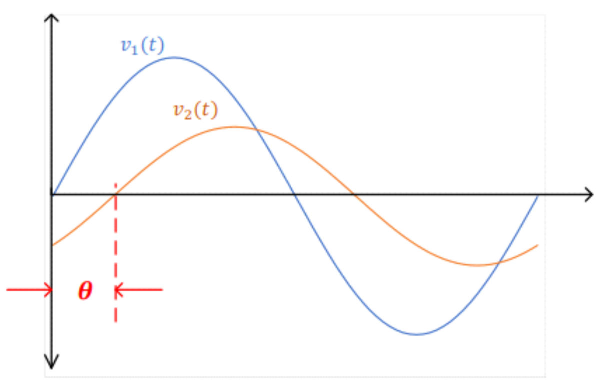


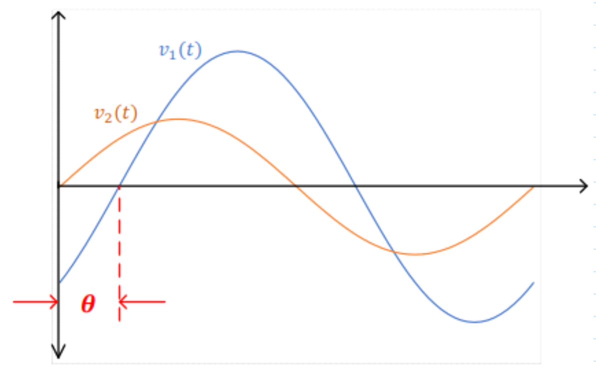
Phase

→ the two waveforms are in phase.



→ compare two waveforms of the same frequency. They are not in phase.



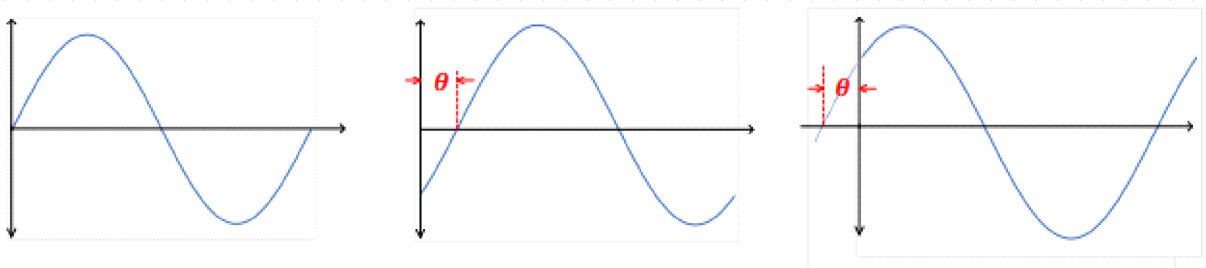


$v_1(t) = V_P \sin(\omega t + \theta)$
$v_2(t) = V_P \sin(\omega t - \theta)$

v_1 leads v_2 by θ
v_2 lags v_1 by θ

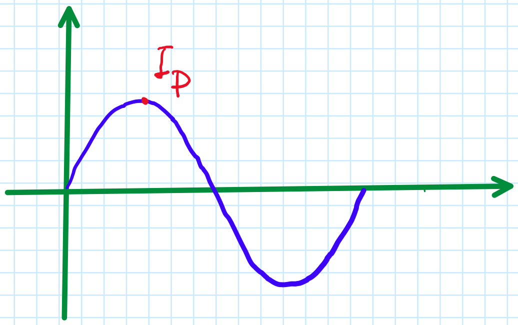
$v_1(t) = V_P \sin(\omega t - \theta)$ ϕ
$v_2(t) = V_P \sin(\omega t + \theta)$

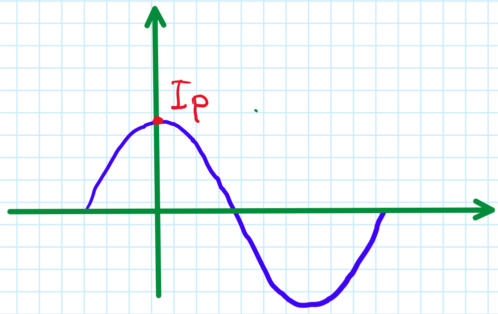
v_1 lags v_2 by θ
v_2 leads v_1 by θ



$V_P * \sin(\omega t)$ | $V_P * \sin(\omega t - \theta)$ | $V_P * \sin(\omega t + \theta)$

Zero phase shift | Lagging by θ degrees | Leading by θ degrees





$i = I_p \sin \omega t$

$i = I_p \cos \omega t = I_p \sin(\omega t + 90^\circ)$

$$v = V_p \cos \omega t = V_p \sin(\omega t + 90^\circ)$$

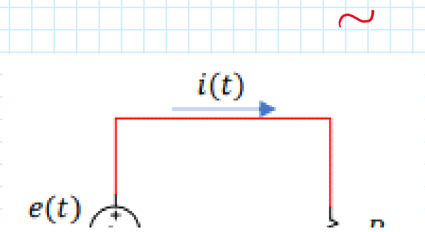
→ <u>wavelength λ</u> – the distance between two troughs, or two crests, in m (the length of the waveform for 1 T).

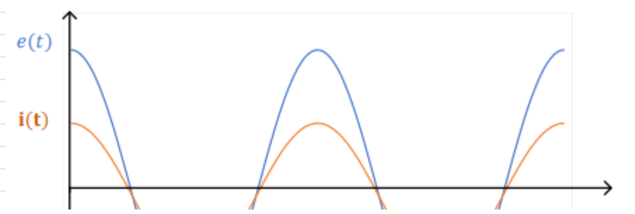
$\boxed{\lambda = cT}$ or $\boxed{\lambda = \frac{c}{f}}$

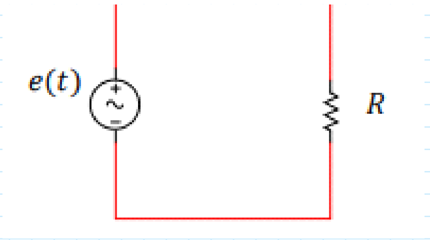
$c = 3 \times 10^8$ m/s is the speed of light in vacuum

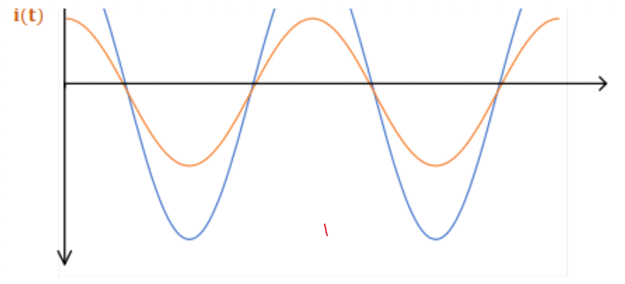
Ex: Calculate the wavelength of a signal with a frequency of 740 kHz (CBC AM radio broadcast signal).

<u>AC Circuits</u>









→ in a purely resistive circuit, the voltage and current are in phase.

$e = E_P \sin \omega t$ $\qquad$ $i = I_P \sin \omega t$ $\qquad$ $\boxed{I_p = \frac{E_p}{R}}$

→ <u>effective value or Root Mean Square value (RMS)</u>

$V_{RMS} = \frac{V_p}{\sqrt{2}}$ $\qquad$ $I_{RMS} = \frac{I_p}{\sqrt{2}}$ $\qquad$ $V_p = V_{RMS}\sqrt{2}$

→ multimeters read RMS values; scope reads peak and peak-to-peak values.

→ <u>average power P_{AVE}</u>: always given as Watts RMS.

$$P_{AVG} = (I_{RMS})^2 R = \frac{(V_{RMS})^2}{R} = I_{RMS}V_{RMS}$$

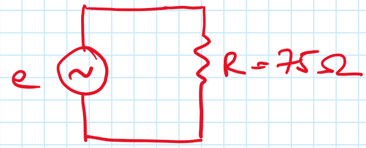
Ex: $E = 10\,V_{RMS}$

$I = 2\,mA_{RMS}$

P

Ex: $e = 8 \sin \omega t$ [V]

P_R



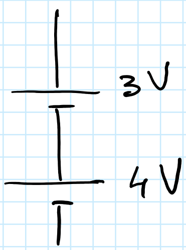
Phasors

→ phasors are rotating vectors that we use to analyze an AC circuit.

→ phasor diagrams - drawing a vector from the origin, representing the magnitude, at an angle measured counter clockwise.

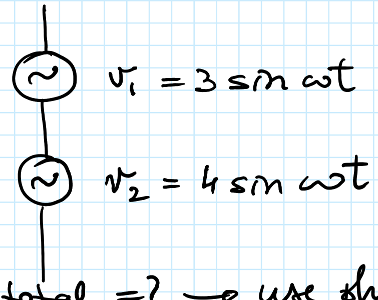
Ex: two sources

DC

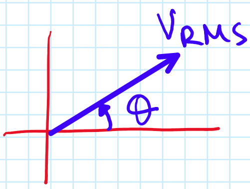


total 7V

AC



total =? → use phasors!



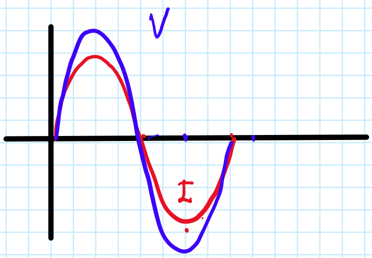
$v = V_p \sin(\omega t + \theta)$ rotate ccw for $+\theta$
cw for $-\theta$

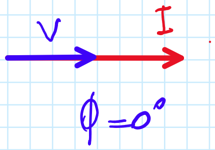
Ex. a. Represent the sinusoid $v(t)=8\sin(\omega t+45°)$. [V]

b. Represent the sinusoid $i(t)=4\sin(\omega t-30°)$. [mA]

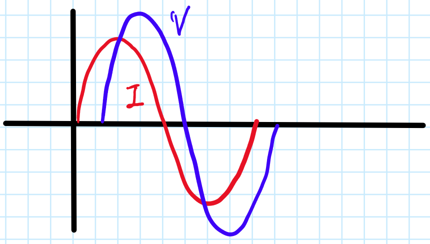
c. Does V lead I, and by how much?

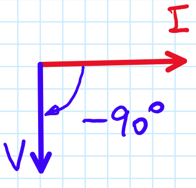
Resistors: I and V are in phase





Capacitors: I leads V by 90°.





Inductors: V leads I by 90°.

Inductors: V leads I by 90°.