COMP 9334

Assignment 1

Question 1

(a) Number of completed jobs $= C = 676$

CPU busy time $= B(cpu) = 4729s$

Disk busy time $= B(disk) = 2565s$

$$X(0) = \frac{C}{T} = \frac{676}{90\times60} = \frac{169}{1350} \text{ jobs/s}$$

$$U(cpu) = \frac{B(cpu)}{T} = \frac{4729}{5400} \approx 0.876$$

$$U(disk) = \frac{B(disk)}{T} = \frac{2565}{90\times60} = 0.475$$

$$D(cpu) = \frac{U(cpu)}{X(0)} = 6.996s$$

$$D(disk) = \frac{U(disk)}{X(0)} = 3.794s$$

(b) Yes, from lecture 02A, we know that we needs only utilisation of various components to determine the bottleneck. In question 1, the utilisation of cpu is much higer than the disk, so cpu is the bottleneck of the whole system.

(c) From lecture notes, we know that:

$$X(0) \le \min\left[\frac{1}{\max D_i}, \frac{N}{\sum_{i=1}^{k} D_i + Z}\right]$$

$$\frac{1}{\max D_i} = \frac{1}{D(cpu)} = \frac{1}{6.996} = 0.143$$

$N = 30$, $Z = 31$

$$\frac{N}{\sum_{i=1}^{K} D_i + Z} = \frac{N}{D(cpu) + D(disk) + Z} = \frac{30}{6.996 + 3.794 + 31}$$

$$= 0.718$$

$X(0) \le \min[0.143, 0.718]$

The asymptoic bound is 0.143 jobs/s

(d) $M = X(0) * (Z + R)$

$$R = \frac{M}{X(0)} - Z$$

$$= \frac{30}{0.143} - 31 = 178.79s$$

Question 2

Since the arrival rate is Poisson distributed and both system 1 and system 2 have their own queue, the server farm consists of two M/M/1 queue

(a) $\rho_1 = \frac{p\lambda}{\mu_1}$, $\rho_2 = \frac{(1-p)\lambda}{\mu_2}$

$$\frac{20p}{10} = \frac{(1-p)\cdot 20}{15}$$

$$500p = 200$$

$$p = 0.4$$

When p is 0.4, System 1 and System 2 have the same utilisation.

(b) $\lambda_1 = 20 \times 0.4 = 8$, $\lambda_2 = 20 - 8 = 12$

System 1 and System 2 are both M/M/1 queue

$$T_1 = \frac{1}{\mu_1 - \lambda_1} = \frac{1}{10-8} = 0.5$$

$$T_2 = \frac{1}{\mu_2 - \lambda_2} = \frac{1}{15-12} = \frac{1}{3}$$

$$T = pT_1 + (1-p)T_2 = 0.4 \cdot 0.5 + 0.6 \cdot \frac{1}{3} = 0.4s$$

(c) $T_1 = \frac{1}{\mu_1 - \lambda_1}$, $T_2 = \frac{1}{\mu_2 - \lambda_2}$

$$T = pT_1 + (1-p)T_2$$

$$= \frac{p}{10-20p} + \frac{1-p}{15-(1-p)\cdot 20}$$

$$= \frac{4p^2 - 3.5p + 1}{-40p^2 + 30p - 5}$$

I used python to draw the image of this function, the file is called "draw_image.py".

From the image, we can see that the minimum value of the response time is around 0.395 (very close to 0.4) when p is around 0.3876.

Question 3

$M=4$, time-to next-failure rate $= \lambda = \frac{1}{600}$

repair rate of the team leader $= \mu_1 = \frac{1}{60}$

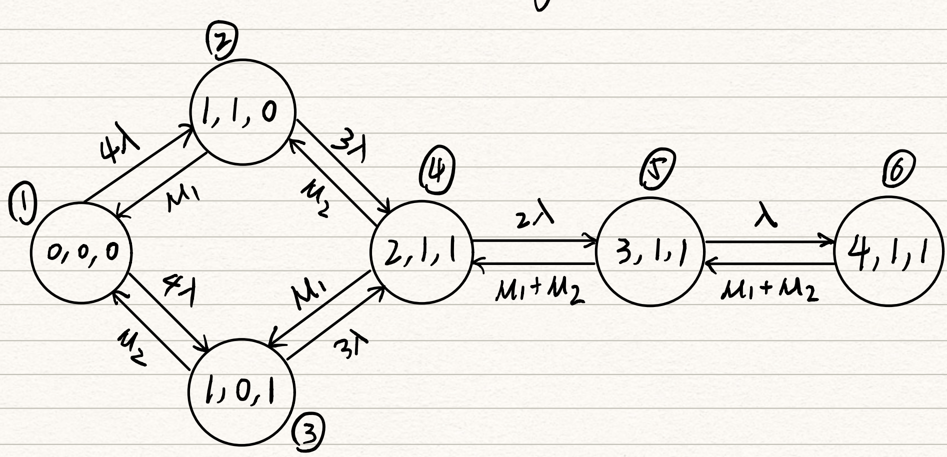
repair rate of the trainee $= \mu_2 = \frac{1}{90}$

(a) There are total six states: $(0,0,0)$, $(1,1,0)$, $(1,0,1)$, $(2,1,1)$, $(3,1,1)$, $(4,1,1)$.

Since failure of a machine is independent of the others, the probability of first machine failure is $4\lambda = \frac{4}{600}$. Similiarly, when there is already one machine failed, the probability of second failure is $3\lambda = \frac{3}{600}$. The third failure rate is 2λ, the fourth failure rate is λ.

When both leader and trainee are busy, the machine waiting to be repaired rate is $(\mu_1+\mu_2)$ because once the leader or trainee become available, he will work on the newly failed machine immediately. This can be looked as a M/M/2 queue.

Here's the state transition diagram:



(b) $\lambda = \frac{1}{600}$, $\mu_1 = \frac{1}{60}$, $\mu_2 = \frac{1}{90}$

The state balance equations are:

(1) $8 \cdot \frac{1}{600} P(0,0,0) - \frac{1}{60} P(1,1,0) - \frac{1}{90} P(1,0,1) = 0$

(2) $-4 \cdot \frac{1}{600} P(0,0,0) + (\frac{1}{60} + 3 \cdot \frac{1}{600}) P(1,1,0) - \frac{1}{90} P(2,1,1) = 0$

(3) $-4 \cdot \frac{1}{600} P(0,0,0) + (\frac{1}{90} + 3 \cdot \frac{1}{600}) P(1,0,1) - \frac{1}{60} P(2,1,1) = 0$

(4) $-3 \cdot \frac{1}{600} P(1,1,0) - 3 \cdot \frac{1}{600} P(1,0,1)$

$+ (\frac{1}{90} + \frac{1}{60} + 2 \cdot \frac{1}{600}) P(2,1,1) - (\frac{1}{60} + \frac{1}{90}) P(3,1,1) = 0$

(5) $-2 \cdot \frac{1}{600} P(2,1,1) + (\frac{1}{60} + \frac{1}{90} + \frac{1}{600}) P(3,1,1) - (\frac{1}{60} + \frac{1}{90}) P(4,1,1) = 0$

(6) $-\frac{1}{600} P(3,1,1) + (\frac{1}{60} + \frac{1}{90}) P(4,1,1) = 0$

There are only 5 linearly independent equations above, we need one more equation:

$P(0,0,0) + P(1,1,0) + P(1,0,1) + P(2,1,1) + P(3,1,1) + P(4,1,1) = 1$

(c)

I used python to solve this equation set, the file is called "solve_equation.py".

The steady state probability of all the states are:

$P(0,0,0) = 0.4539$ $\quad P(2,1,1) = 0.0817$

$P(1,1,0) = 0.1816$ $\quad P(3,1,1) = 0.0098$

$P(1,0,1) = 0.2724$ $\quad P(4,1,1) = 0.0006$

(d) the probability at least three machines are available

$= P(0,0,0) + P(1,1,0) + P(1,0,1)$

$= 0.9097$

(e) mean number failed machines

$= 0 \times P(0,0,0) + 1 \times (P(1,1,0) + P(1,0,1)) + 2 \times P(2,1,1)$
$+ 3 \times P(3,1,1) + 4 \times P(4,1,1)$
$= 0.6492$

(f) MTTR = Queueing time for repair + actual repair time, we can use Little's law to compute it's value.

Throughput $= X = 4\lambda P(0,0,0) + 3\lambda(P(1,1,0) + P(1,0,1))$
$+ 2\lambda P(2,1,1) + \lambda P(3,1,1) + 0 \cdot P(4,1,1)$
$= 0.0056$

$$MMTR = \frac{N}{X} = \frac{0.6492}{0.0056} = 115.9286 \text{ min}$$

---- End of assignment ----