
ECE 1100H

Introduction to Engineering

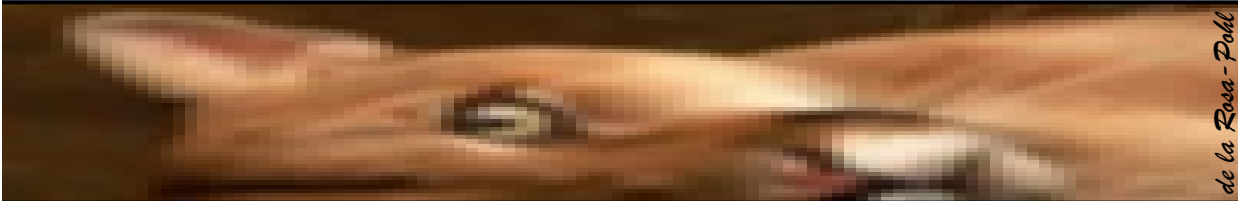
CIRCUIT ANALYSIS MADE EASY!

PART III: KVL, KCL, AND OHM'S LAW

University of Houston
Len Trombetta



WHERE WE'RE GOING



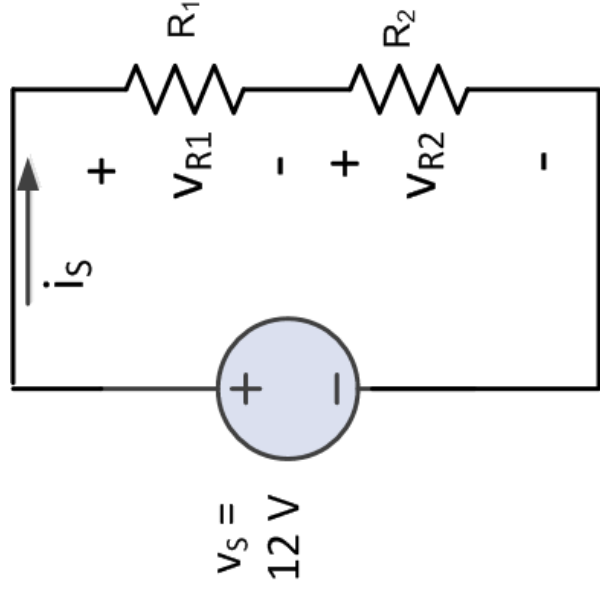
de la Rosa - Pohl



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Learning. Leading.

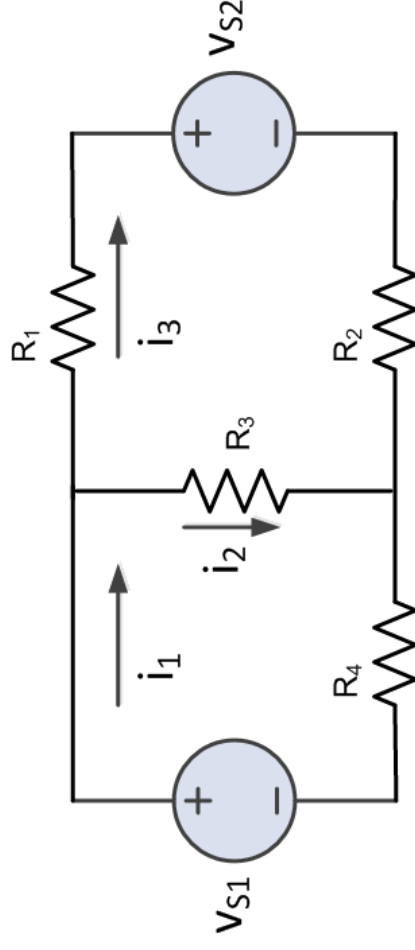
Where Are We Going?

Where we've been:

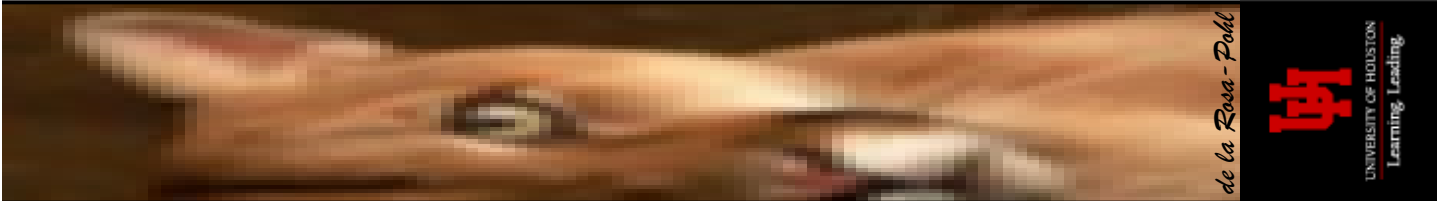


This is a simple circuit with one loop, and one current. We analyzed it using KVL and Ohm's Law.

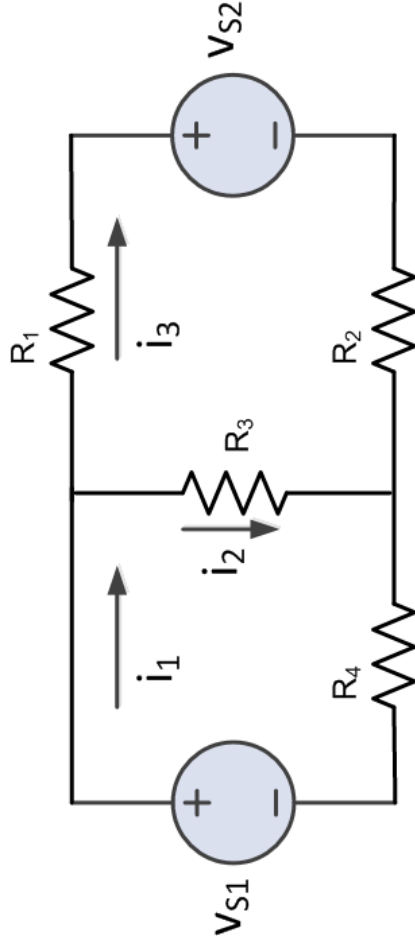
Where we're going:



We will analyze circuits like this one, which has three currents. This will require Kirchhoff's Current Law (KCL).



Where Are We Going?



We will look at circuits with voltage sources and resistors only. We will not worry ourselves with

current sources
capacitors or inductors
integrated circuits (LM741, LM386, digital logic devices)
solid state devices (BJT, MOSFET, diode)

These things can be handled easily later if we understand the basics.



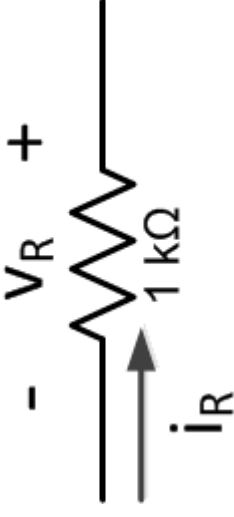
OHM'S LAW REVIEW

Ohm's Law Review

Ohm's Law depends on how we label current and voltage:

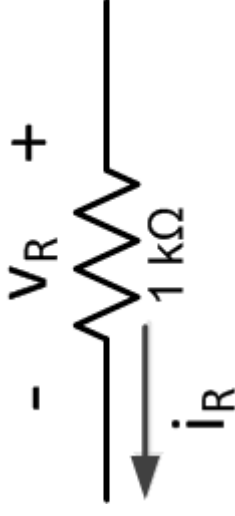


$$v_R = i_R R$$

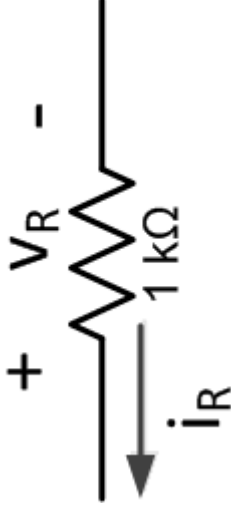


$$v_R = -i_R R$$

These are of course the same things



$$v_R = i_R R$$

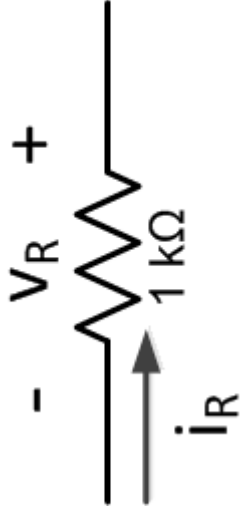


$$v_R = -i_R R$$

Let's Try Some



$$V_R = 7\text{ V} \quad i_R = ??$$



$$V_R = 120\text{ V} \quad i_R = ??$$

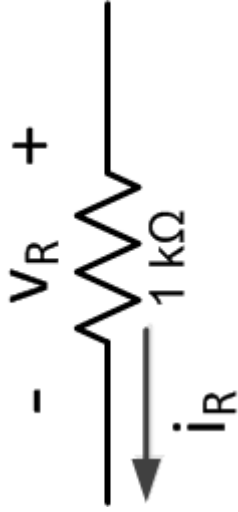


$$V_R = -120\text{ V} \quad i_R = ??$$

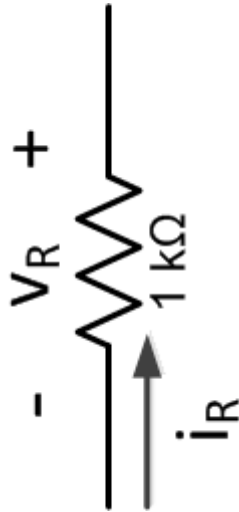
Let's Try Some



$$i_R = 7\text{ mA} \quad V_R = ??$$



$$i_R = -12\text{ mA} \quad V_R = ??$$



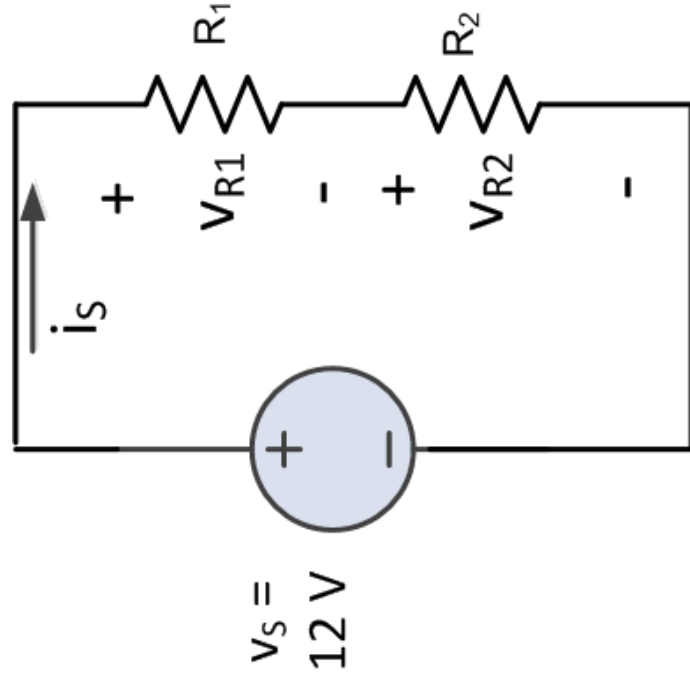
$$i_R = 50\text{ mA} \quad V_R = ??$$



VOLTAGE DIVIDER RULE REVIEW

Voltage Divider Review

- Voltage Divider Rule



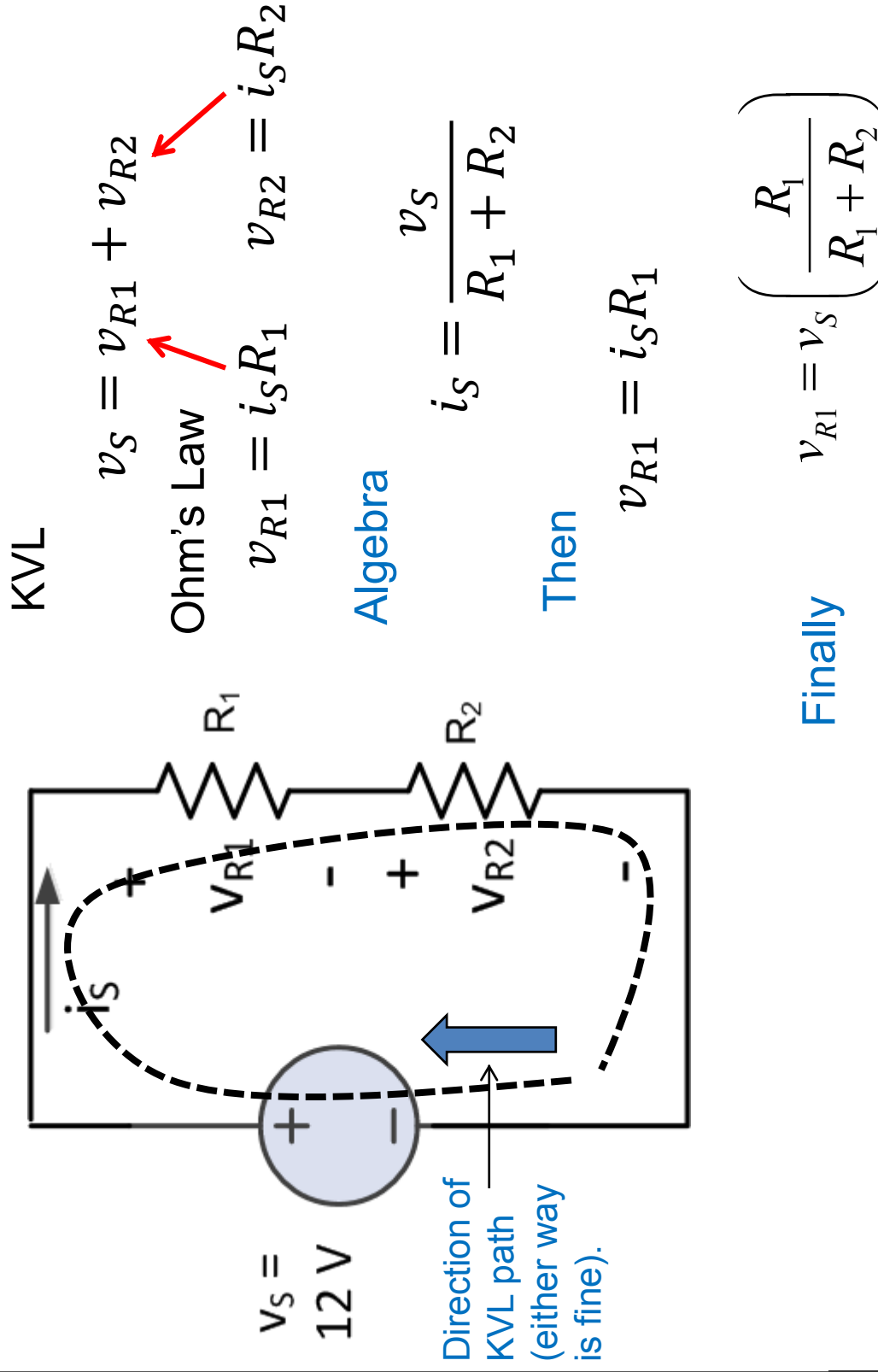
- Ohm's Law and KVL give us the following equations

$$V_{R1} = V_S \left(\frac{R1}{R1 + R2} \right)$$

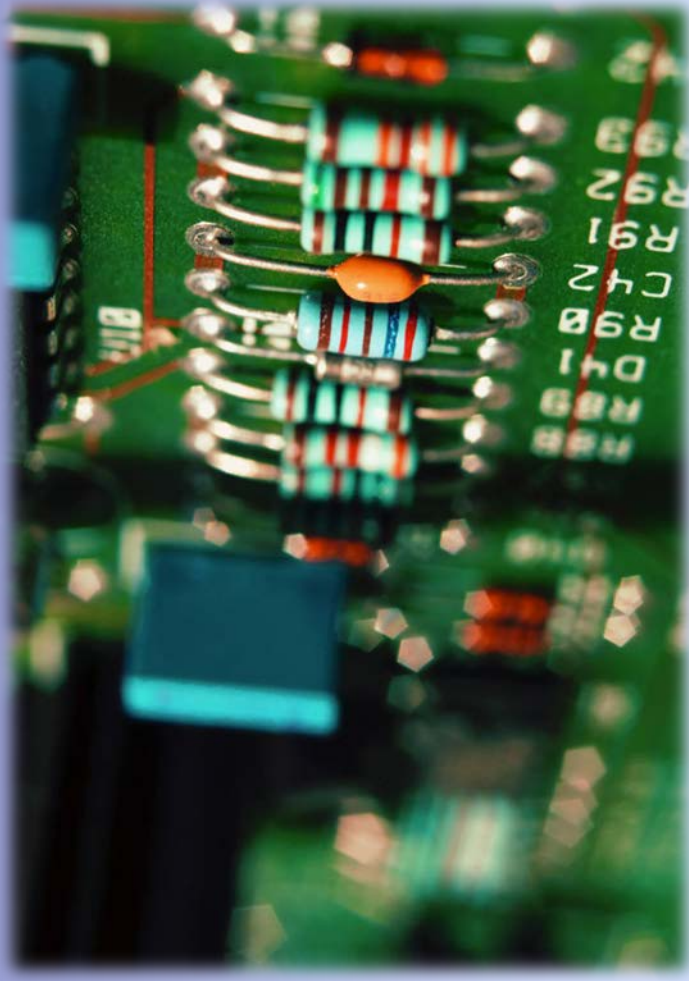
$$V_{R2} = V_S \left(\frac{R2}{R1 + R2} \right)$$

How did we get those results?

Voltage Divider Revisited

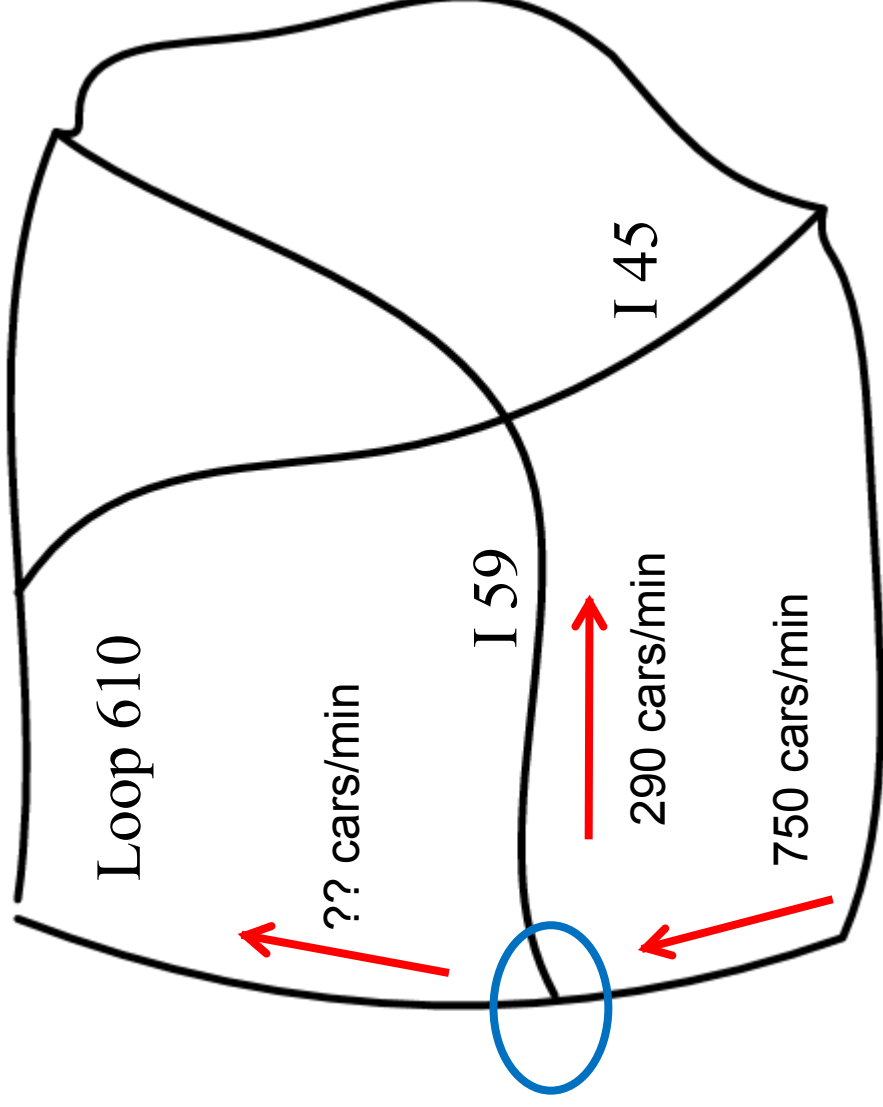


So we substituted Ohm's Law into KVL



KIRCHHOFF'S CURRENT LAW

Houston Traffic

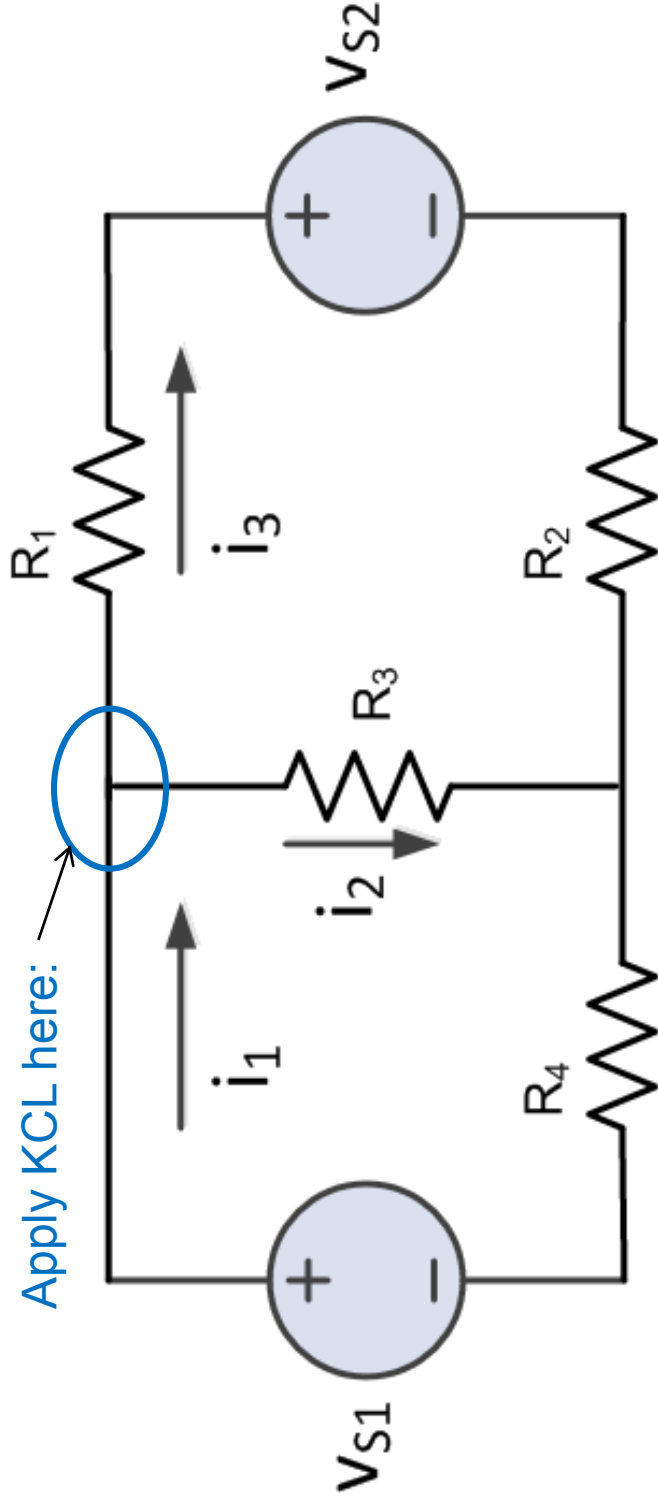


Kirchhoff's Current Law:

in = out

All other roads are closed for repairs!

Kirchhoff's Current Law



What current is flowing in R_4 ? R_2 ?... V_{S2} V_{S1} ?

What is the relationship between i_1 , i_2 , and i_3 ?

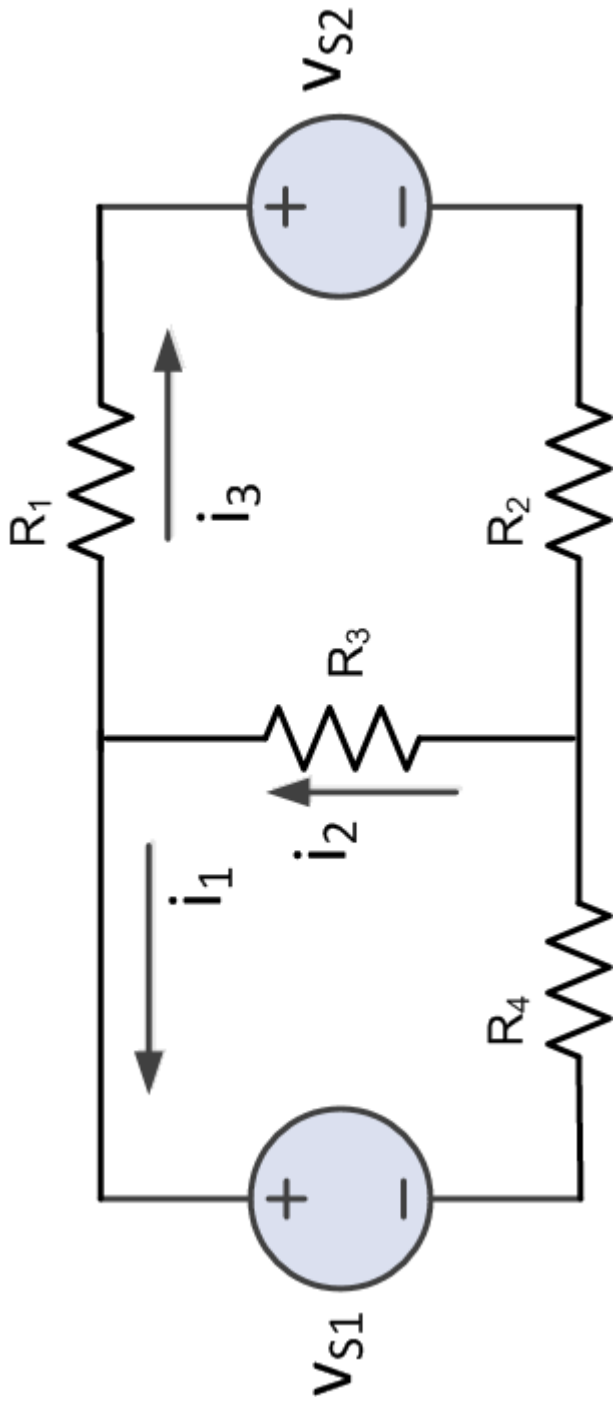
$I_n = \text{out}$

KCL:

$$i_1 = i_2 + i_3$$

Kirchhoff's Current Law

What if we had drawn it this way? Now what is KCL?



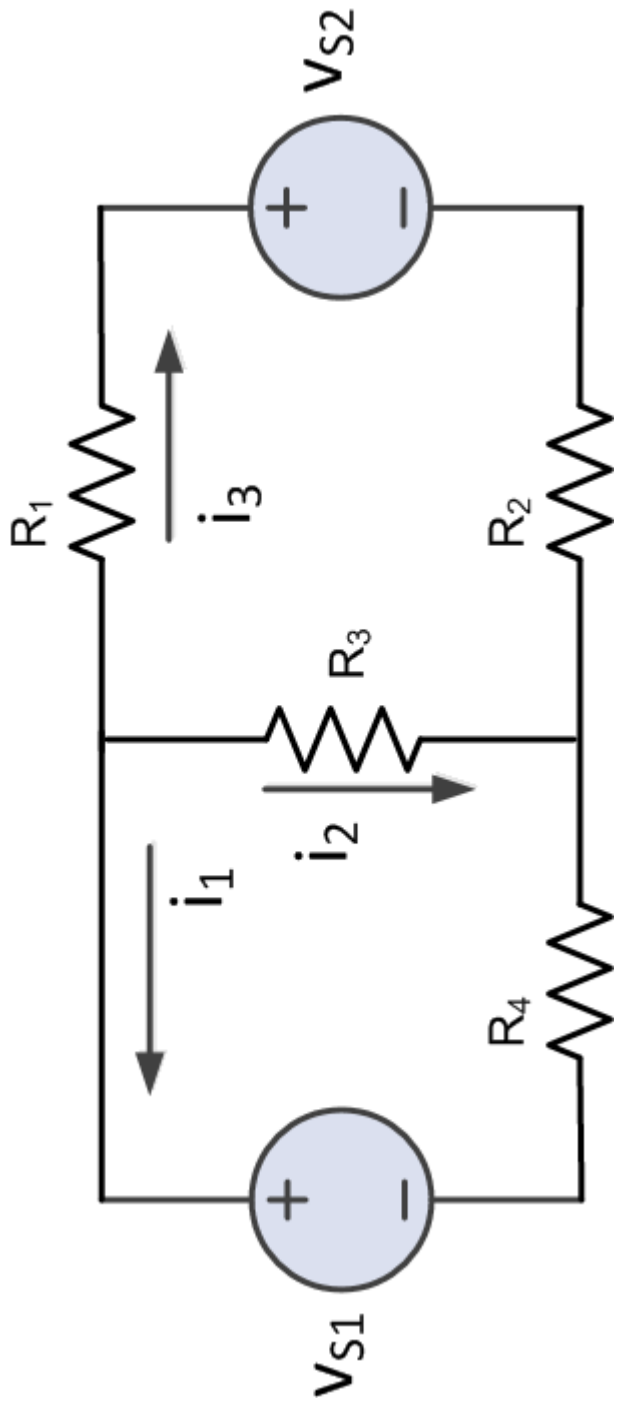
KCL:

$$i_2 = i_1 + i_3$$

in = out

Kirchhoff's Current Law

Can I do THIS???



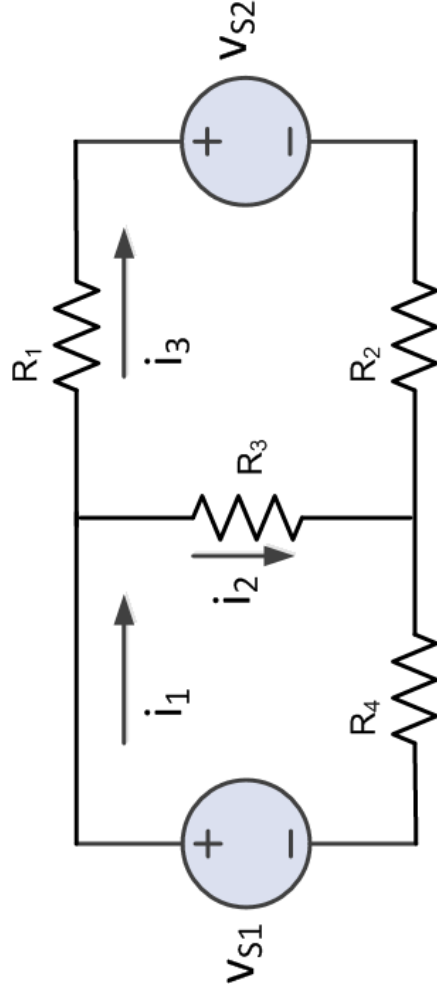
Sure!!

in = out

KCL:

$$0 = i_2 + i_1 + i_3$$

Kirchhoff's Current Law



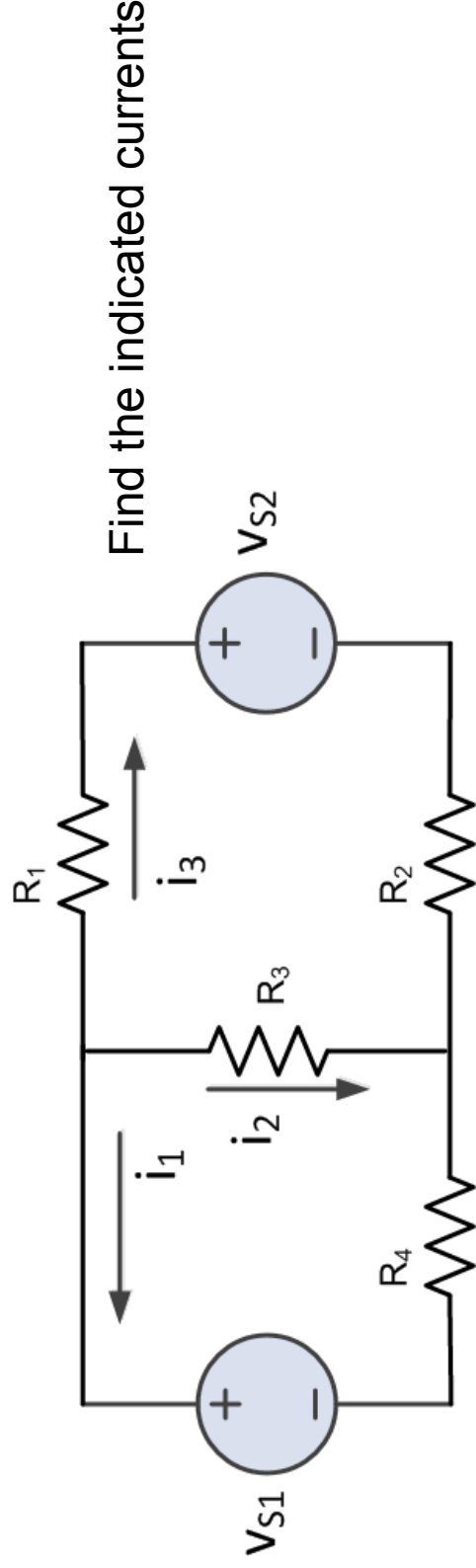
Find the indicated currents

$$\begin{aligned} i_1 &= 70 \text{ mA} \\ i_2 &= 15 \text{ mA} \\ i_3 &= ?? \end{aligned}$$

$$\begin{aligned} i_1 &= -24 \text{ mA} \\ i_2 &= ?? \\ i_3 &= 10 \text{ mA} \end{aligned}$$

$$\begin{aligned} i_1 &= ?? \\ i_2 &= 0.25 \text{ mA} \\ i_3 &= 0.20 \text{ mA} \end{aligned}$$

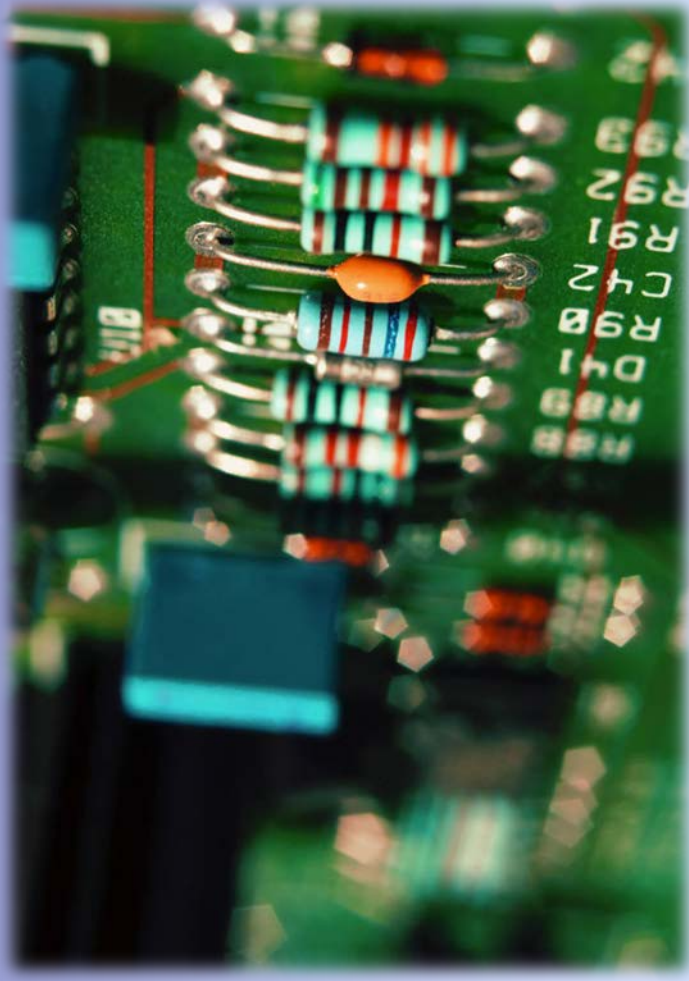
Kirchhoff's Current Law



$$\begin{aligned} i_1 &= 240 \text{ mA} \\ i_2 &= -150 \text{ mA} \\ i_3 &= ?? \end{aligned}$$

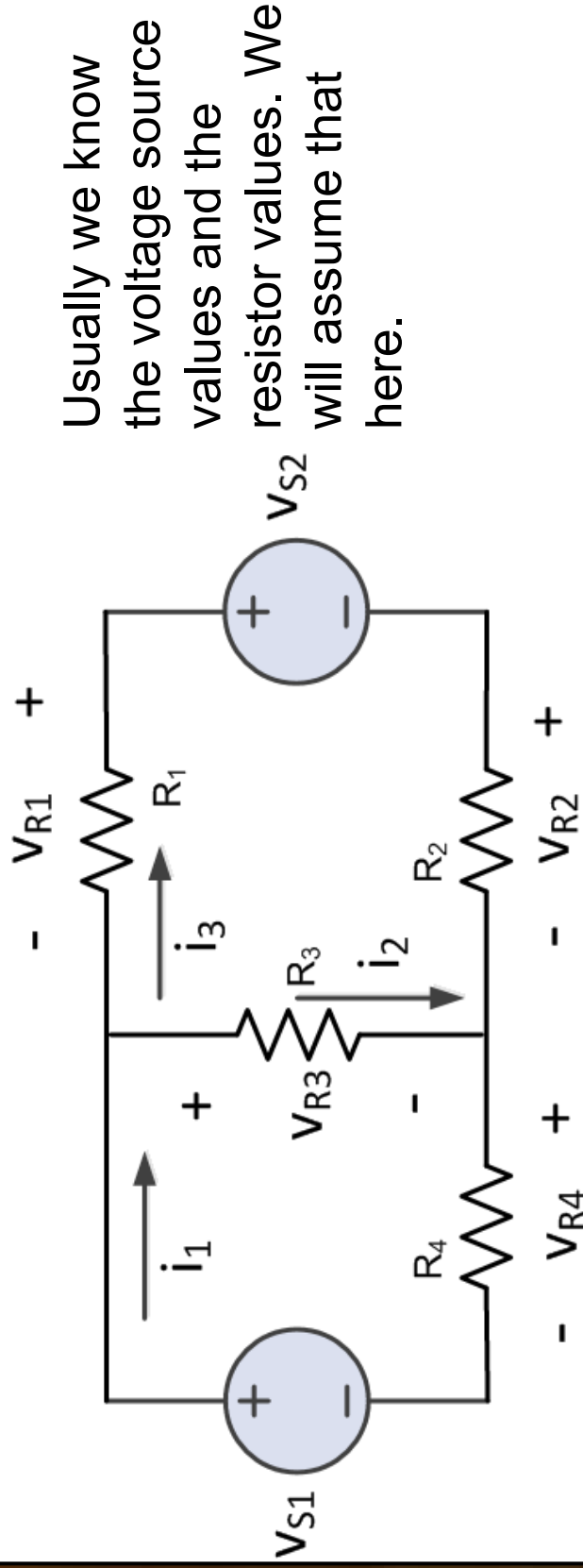
$$\begin{aligned} i_1 &= -240 \text{ mA} \\ i_2 &= ?? \\ i_3 &= 150 \text{ mA} \end{aligned}$$

$$\begin{aligned} i_1 &= ?? \\ i_2 &= 0.75 \text{ mA} \\ i_3 &= 0.20 \text{ mA} \end{aligned}$$



CIRCUIT ANALYSIS USING KVL, KCL, AND OHM'S LAW

Let's Analyze That Thing



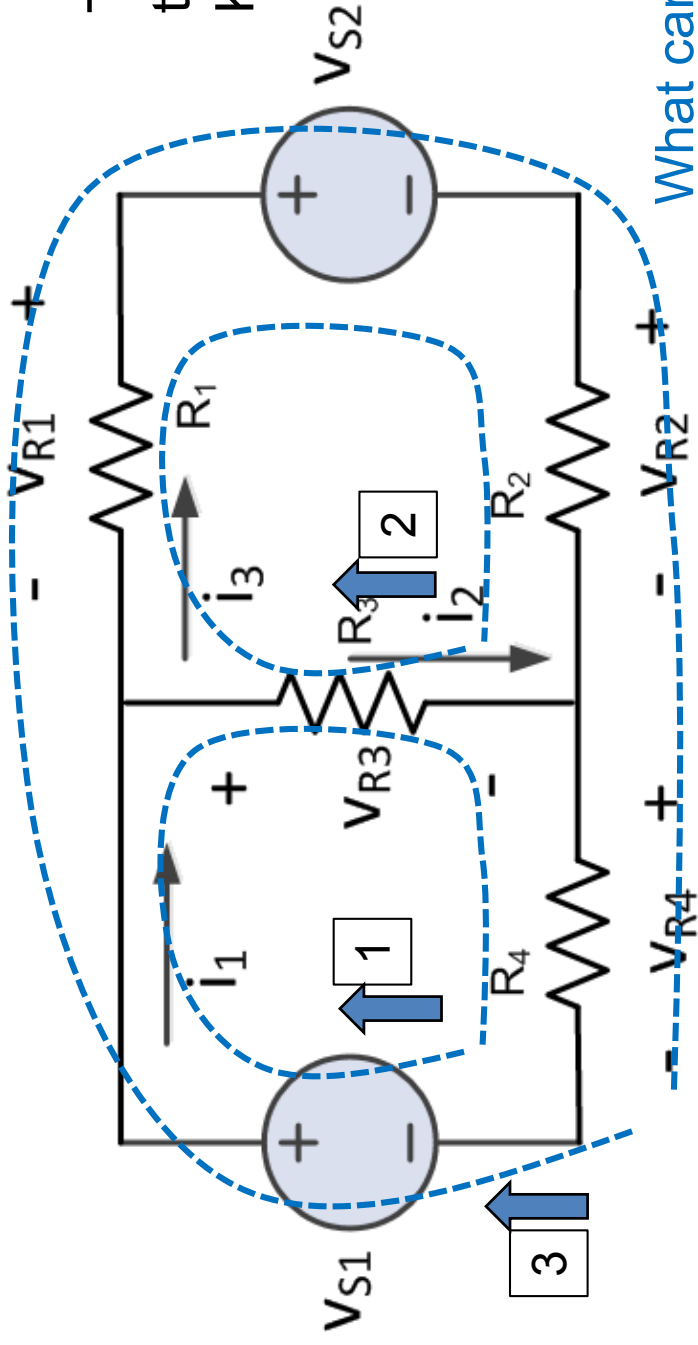
How many KVL's can we write for this circuit?

of KVL's: 3

How many KCL's can we write for this circuit?

of KCL's: 2

Let's Do It: KVL



There are three possible KVL paths.

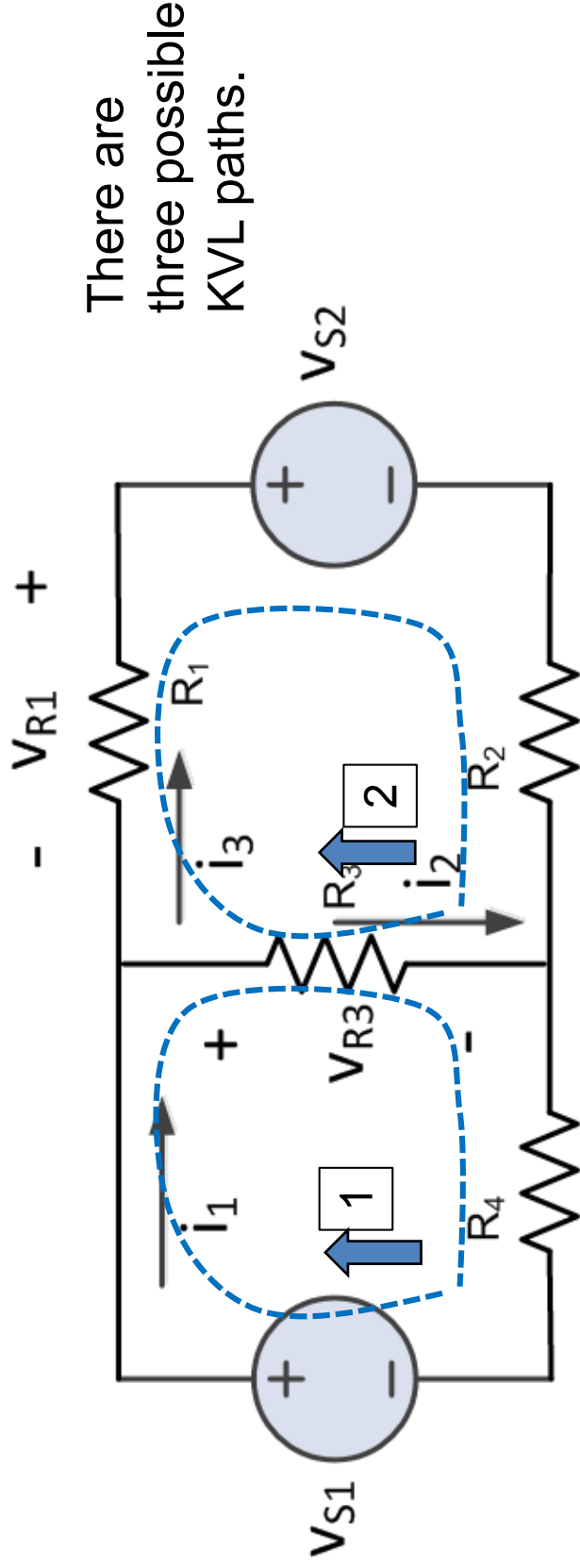
What can we say about the relationship among these three equations?

1 $-v_{S1} + v_{R3} + v_{R4} = 0$

2 $-v_{R3} - v_{R1} + v_{S2} + v_{R2} = 0$

3 $-v_{S1} - v_{R1} + v_{S2} + v_{R2} + v_{R4} = 0$

Let's Do It: KVL



There are three possible KVL paths.

They are not algebraically independent – we can only use two of them (any two will do).

1

$$-v_{S1} + v_{R3} + v_{R4} = 0$$

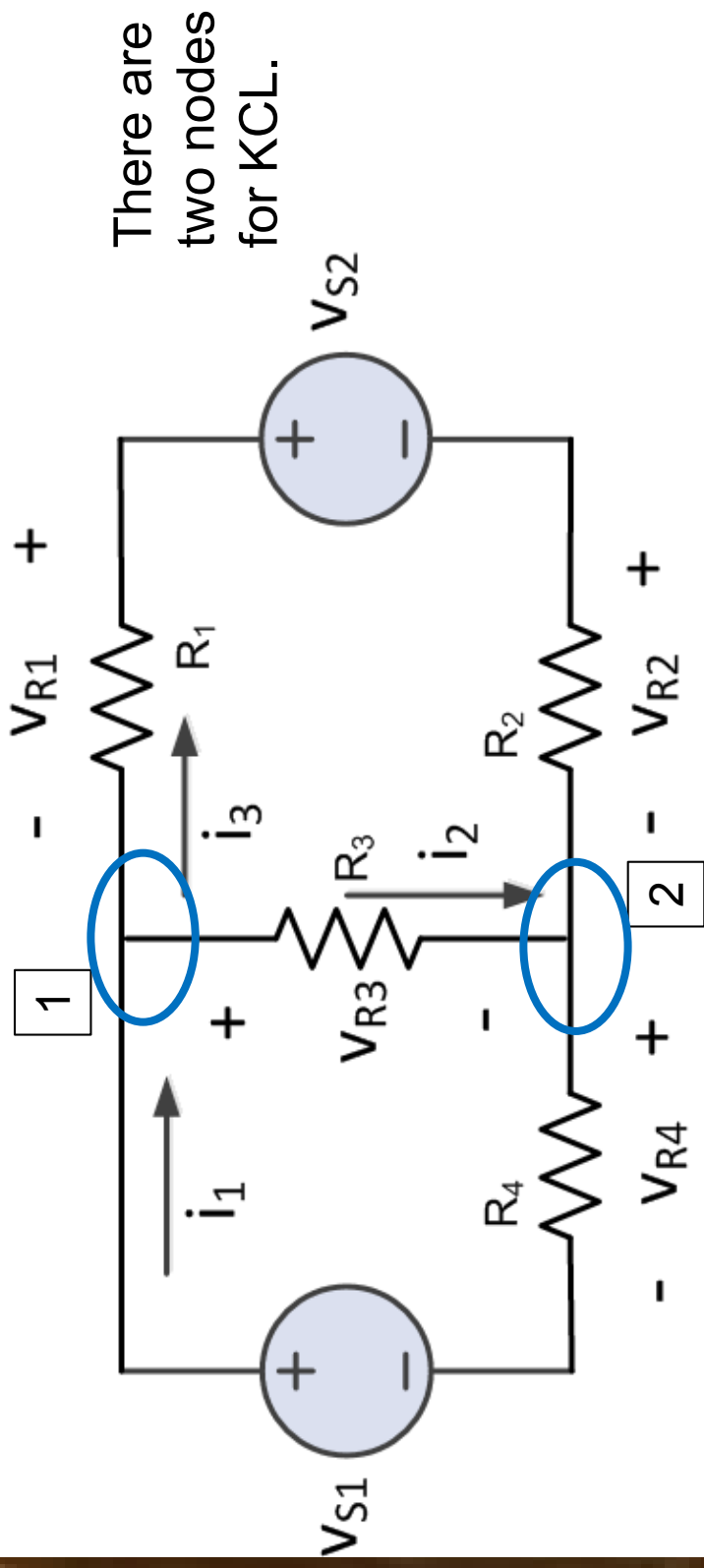
2

$$-v_{R3} - v_{R1} + v_{S2} + v_{R2} = 0$$

3

~~$$-v_{S1} - v_{R1} + v_{S2} + v_{R2} + v_{R2} = 0$$~~

Let's Do It: KCL

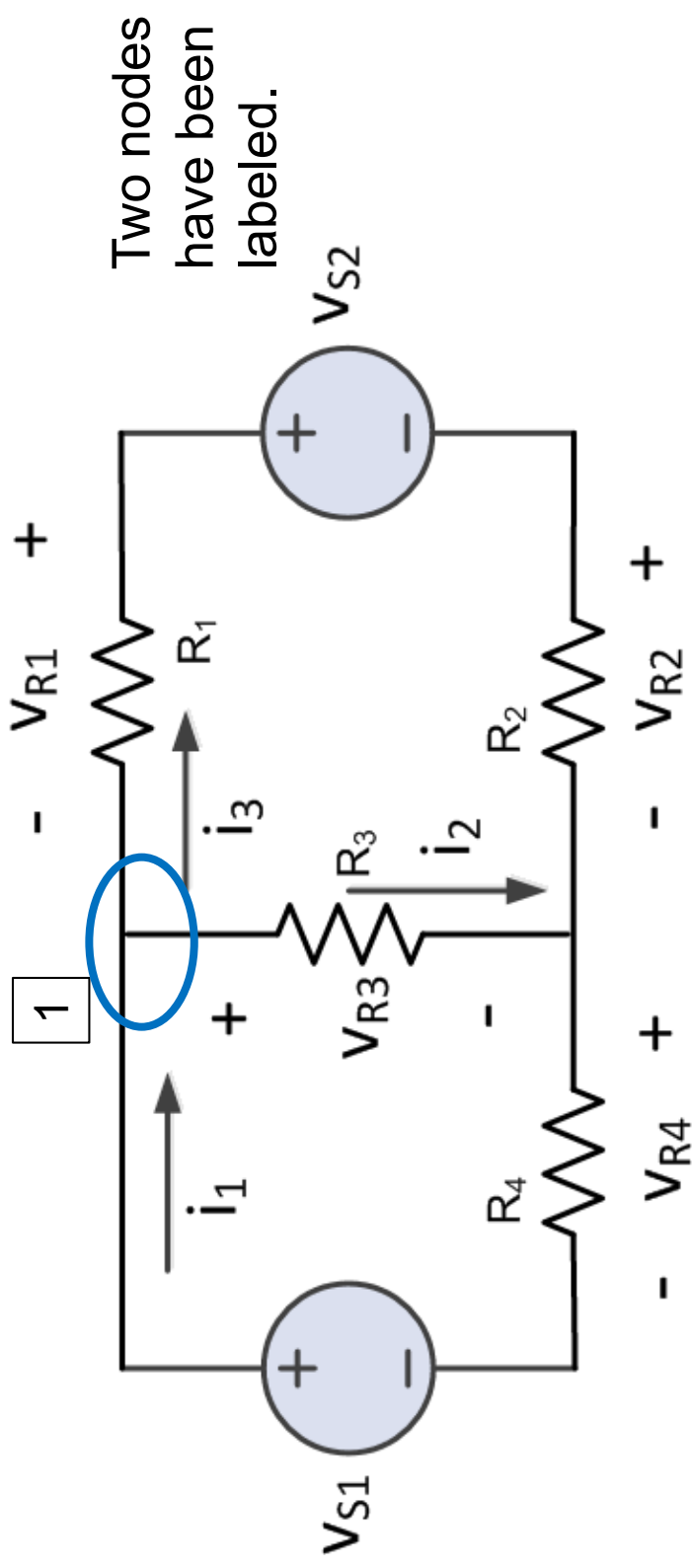


$$\boxed{1} \quad i_1 = i_2 + i_3$$

$$\boxed{2} \quad i_2 + i_3 = i_1$$

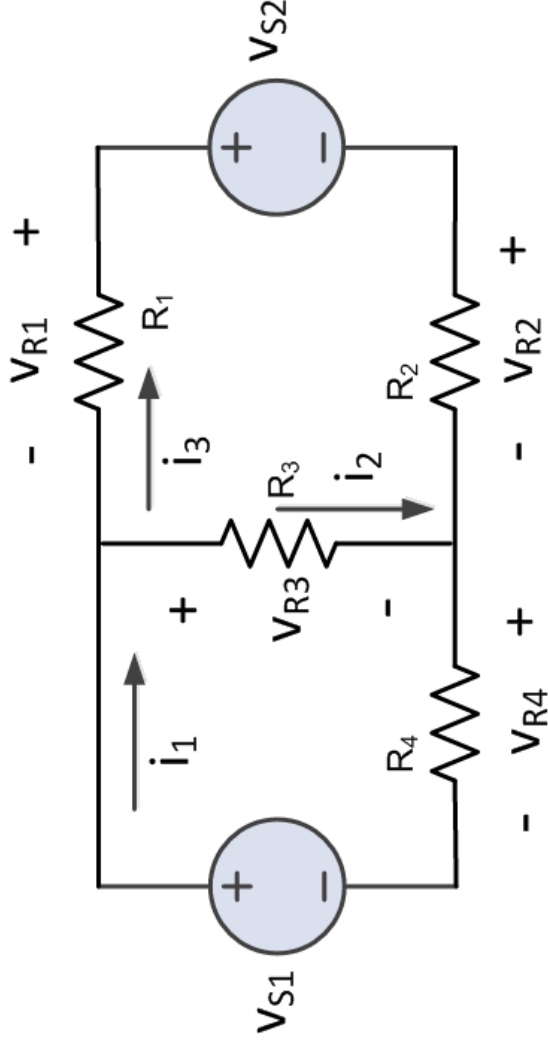
What can we say about these two KCL equations?

Let's Do It: KCL



- 1 $i_1 = i_2 + i_3$
- 2 ~~$i_2 + i_3 = i_1$~~
- They are algebraically identical. We can only use one of them.

Let's Do It: KCL and KVL



We have three equations: 2 KVL's and 1 KCL.

Remember: we assume we know the voltage sources and resistor values.

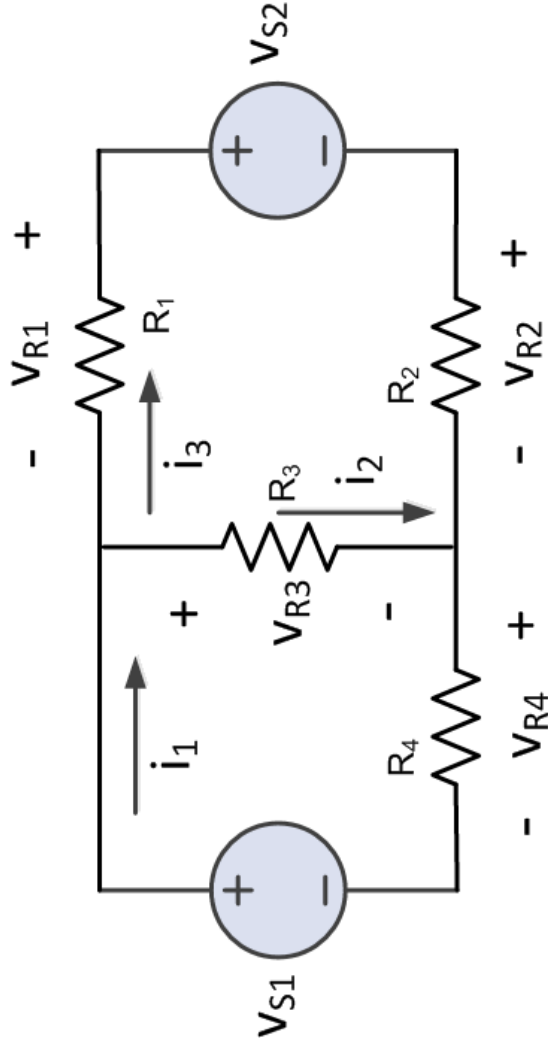
But now what? We have too many **unknowns** to go further – the resistor voltages and the three currents are all unknown.

$$1 \quad i_1 = i_2 + i_3$$

$$2 \quad -v_{S1} + v_{R3} + v_{R4} = 0$$

$$3 \quad -v_{R3} - v_{R1} + v_{S2} + v_{R2} = 0$$

Ohm's Law!!



We can use Ohm's Law to eliminate the resistor voltages watch the signs!

Substitute these equations into KVL and get

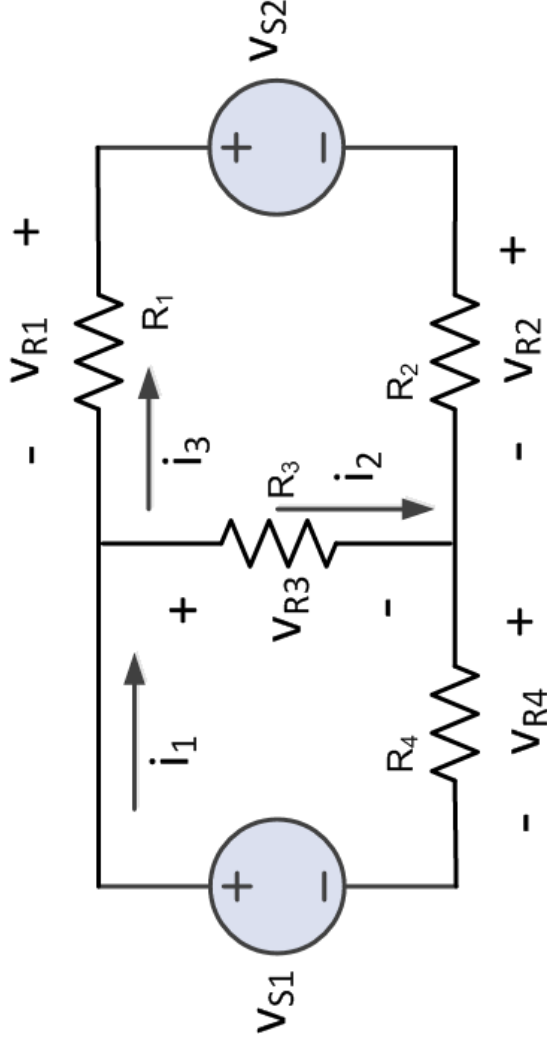
$$v_{R1} = -i_3 R_1$$

$$v_{R2} = i_3 R_2$$

$$v_{R3} = i_2 R_3$$

$$v_{R4} = i_1 R_4$$

Let's Do It: KCL and KVL

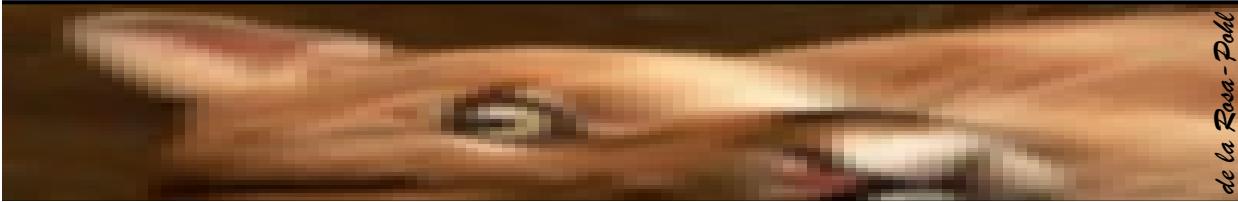


Now we have three equations in three **unknown** currents! We can solve for the currents and then use Ohm's Law to get the resistor voltages.

$$1 \quad i_1 = i_2 + i_3$$

$$2 \quad -v_{S1} + i_2 R_3 + i_1 R_4 = 0$$

$$3 \quad -i_2 R_3 + i_3 R_1 + v_{S2} + i_3 R_2 = 0$$



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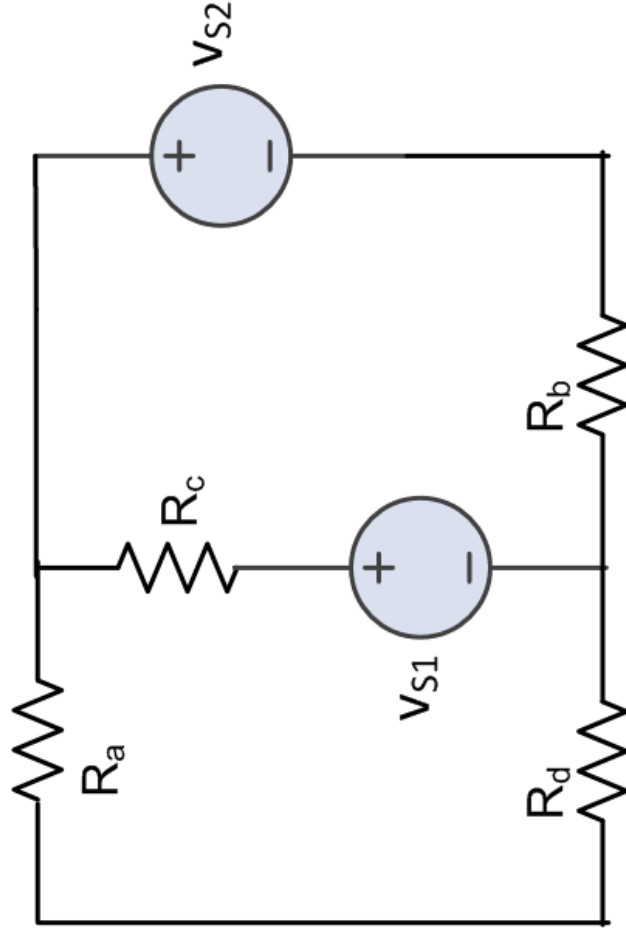


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YOU TRY!

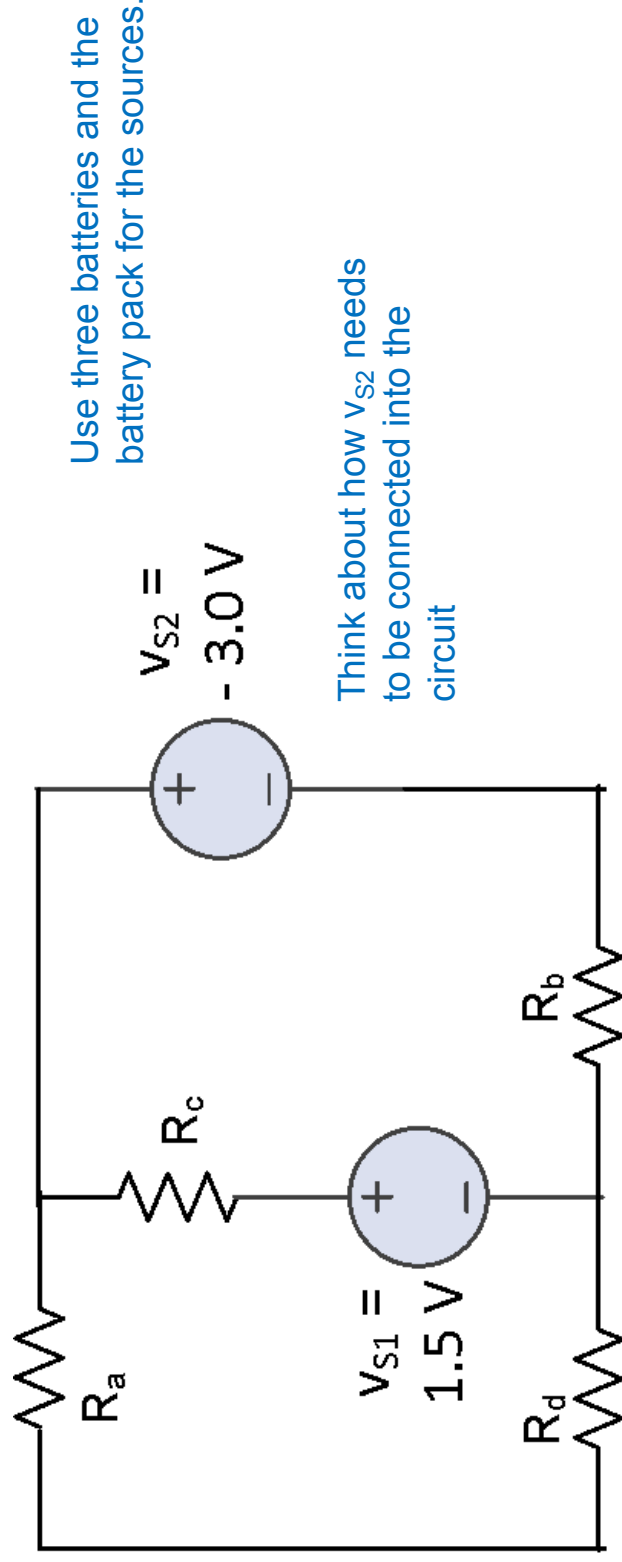


You Try!



1. Label 3 currents and all resistor voltages. Watch the label rules!
2. Write 2 KVL equations and 1 KCL equation.
3. Write Ohm's Law for each of the resistor voltage variables.
4. Re-write your equations in terms of three unknown currents.
5. Solve, given: $v_{S1} = 3.5 \text{ [V]}$, $v_{S2} = -5.7 \text{ [V]}$, $R_a = 1 \text{ [k}\Omega\text{]}$, $R_b = 2.2 \text{ [k}\Omega\text{]}$, $R_c = 470 \text{ [}\Omega\text{]}$, $R_d = 4.7 \text{ k[}\Omega\text{]}$.

Let's Build It: Practice Circuit!



1. Choose four resistors with red or orange in the 3rd band.
2. Use the battery pack to generate the voltages.
3. Solve for the three currents in your circuit (which will of course depend on your R's.)
4. Find the resistor voltages using Ohm's Law.
5. Measure the currents and voltages.

Solving the Practice Circuit

- You need to solve three simultaneous algebraic equations for three unknown currents.
- Once you have the currents you can find the resistor voltages.
 - Wolfram Alpha will solve simultaneous equations. This is available for free on line.
 - MultiSim will solve the circuit.

Wolfram Alpha

The screenshot shows the Wolfram Alpha website interface. The browser's address bar displays the URL: <http://www.wolframalpha.com/input/?i=-1.5-1000y+2000x=0,-1.5-1000y-3+1000z=0,x+y+z=0>. The search bar contains the input: $-1.5-1000y+2000x=0, -1.5-1000y-3+1000z=0, x+y+z=0$. The results are displayed in a structured format:

Input:
 $\{-1.5 - 1000 y + 2000 x = 0, -1.5 - 1000 y - 3 + 1000 z = 0, x + y + z = 0\}$

Result:
 $\{2000 x - 1000 y - 1.5 = 0, -1000 y + 1000 z - 4.5 = 0, x + y + z = 0\}$

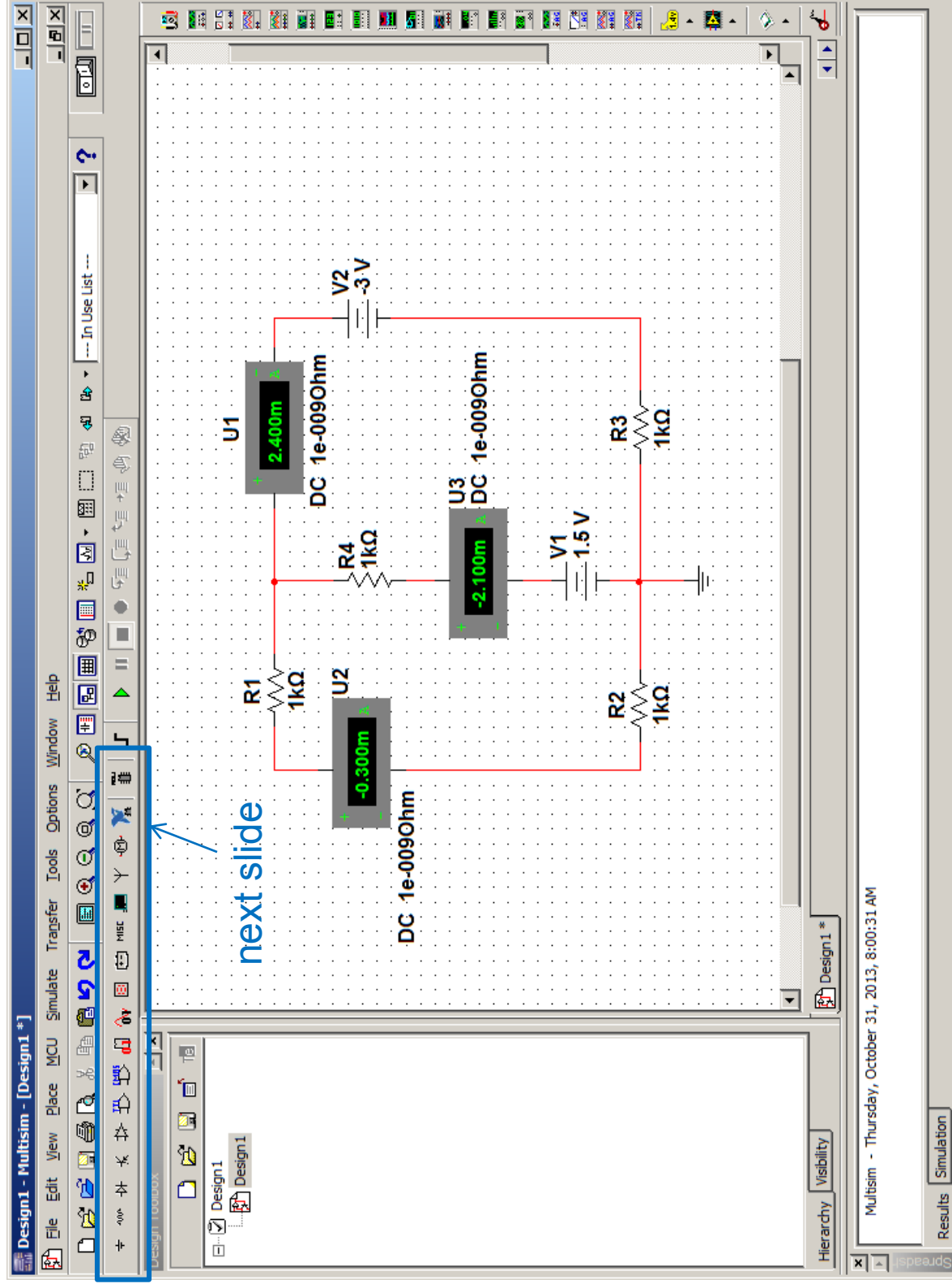
Solution:
 $x \approx -0.0003, y \approx -0.0021, z \approx 0.0024$

The interface also includes a sidebar with links to Favorites, History, Preferences, Downloads, Uploads, and Account. The top navigation bar features links to Examples, Random, and a Sign In button.

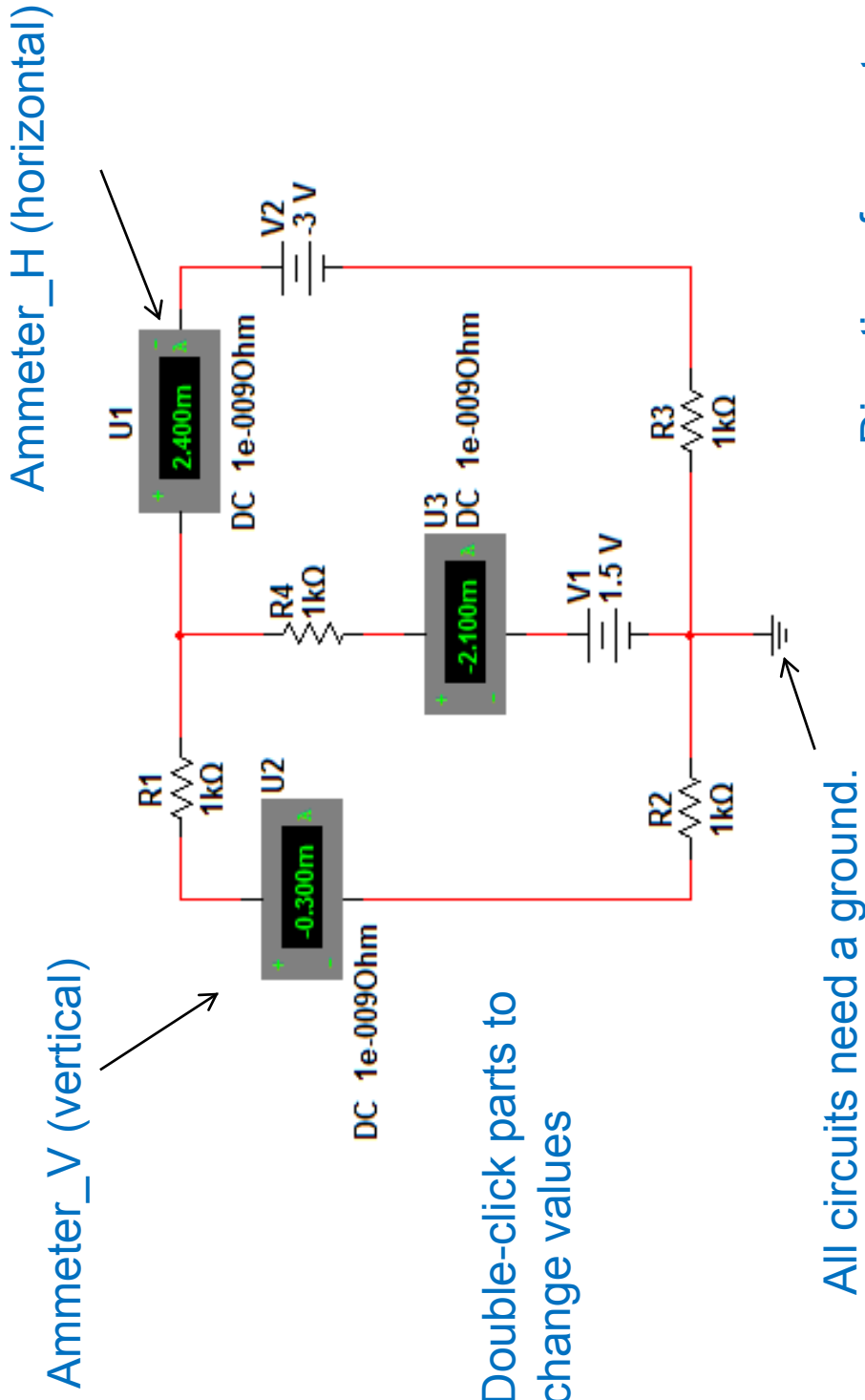
www.wolframalpha.com

!! WolframAlpha interprets "i2" as i^2 !!

Multisim

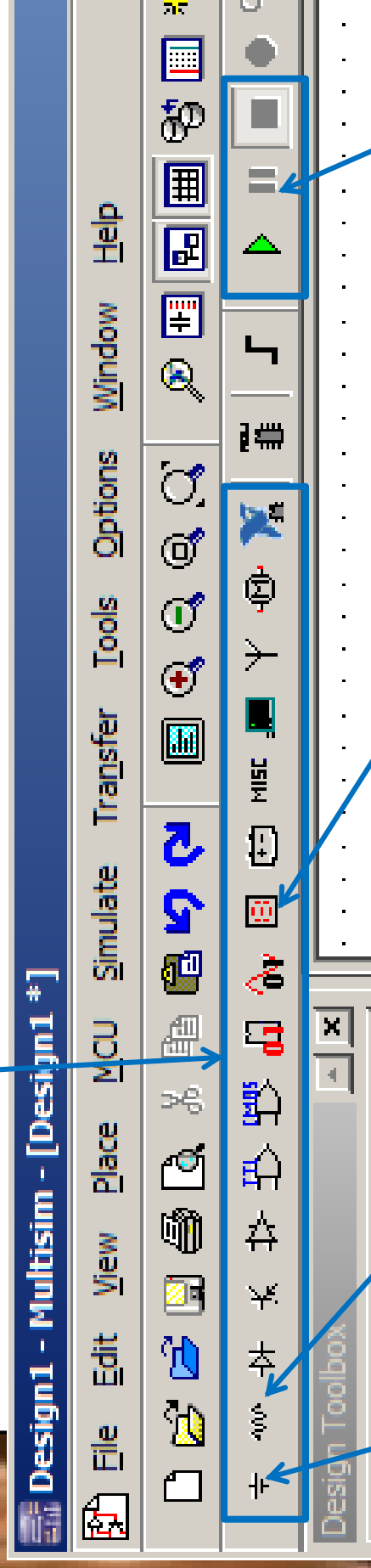


Multisim



Parts Palette

basic parts lists



Indicators: ammeter, voltmeter

Start, stop, pause

Resistors, caps,

Power supplies and ground

ON/OFF



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