Bi-Directional Cooperative NOMA
Without Full CSIT

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Abstract—In this paper, we propose bi-directional cooperative non-orthogonal multiple access (NOMA). Compared to conventional NOMA, the main contributions of bi-directional cooperative NOMA can be explained in two directions: 1) the proposed NOMA system is still efficient when the channel gains of scheduled users are almost the same and 2) the proposed NOMA system operates well without accurate channel-state information at the base station. In a two-user scenario, the closed-form ergodic capacity of bi-directional cooperative NOMA is derived, and it is proven to be better than those of other techniques. Based on the ergodic capacity, the algorithms to find optimal power allocations maximizing the user fairness and sum rate are presented. Outage probability is also derived, and we show that bi-directional cooperative NOMA achieves a power gain over uni-directional cooperative NOMA and a diversity gain over non-cooperative NOMA and orthogonal multiple access (OMA). We finally extend the bi-directional cooperative NOMA to a multiuser model. The analysis of ergodic capacity and outage probability in a two-user scenario is numerically verified. Also, simulation results show that bi-directional cooperative NOMA provides better data rates than the existing NOMA schemes as well as OMA in a multiuser scenario.

Index Terms—Non-orthogonal multiple access, inaccurate CSI, cooperative NOMA, ergodic capacity, outage probability, power allocation, user fairness problem, max-sum-rate problem.

I. INTRODUCTION

NON-ORTHOGONAL multiple access (NOMA) based on power multiplexing has been introduced to utilize radio resources efficiently for a massive number of user terminals [1]. The 4G networks mainly operate based on orthogonal multiple access (OMA), allocating orthogonal resources to multiple users. However, as a massive number of various devices is deployed in the network, OMA is no longer able to maximize resource efficiency and to serve all devices simultaneously. For 5G communication systems, much higher data rates are expected compared to 4G, and efficient and flexible uses of energy and spectrum have become critical issues [2], [3]. To this end, NOMA has been actively researched as a promising technology in 5G networks [4], [5].

NOMA superposes the multiuser signals within the same frequency, time or spatial domain. The advanced receivers with successive interference cancelation (SIC) are typically considered to detect non-orthogonally multiplexed signals. In theory, NOMA provides a significant benefit in improving the cell throughput by using perfect SIC [6]. Performance analysis has also been conducted to examine the effectiveness of NOMA in practical environments [7], [8]. User scheduling for non-orthogonally multiplexed signaling has been studied in [9]. Optimal power allocation at the BS for NOMA users is an important issue [10] for several system goals, e.g., sum-rate maximization [11] and user fairness [12]. Recently, joint optimization of power allocation and user scheduling has been also studied for NOMA systems [13].

NOMA has been extensively researched in conjunction with various technologies. There have been some studies on the system applying NOMA to MIMO [14], [15] and on analyzing ergodic capacity of a MIMO-NOMA system [16]. NOMA has been also considered to increase the data rate of the cell-edge user in coordinated multipoint (CoMP) [17] and to maximize the user fairness and sum rate in distributed antenna systems [18]. Recently, the application of NOMA to simultaneous wireless information and power transfer (SWIPT) [19], [24] and physical security [20] have been studied.

This paper proposes bi-directional cooperative NOMA which targets two practical channel environments: 1) when the BS knows statistical CSI but channel gain differences among users are not large, and 2) when the BS does not know CSI at all. The proposed NOMA scheme is a kind of cooperative NOMA [21], which allows cooperation among users via short-range communications based on instantaneous CSI at transmitter (CSIT). The cooperative NOMA system of [21] improves the outage probability performance compared to conventional NOMA and it is applicable to relay communications [22] and SWIPT [24]. However, it is difficult to figure out which user has the better instantaneous channel gain and which user transmits the cooperation signal to others, when only statistical CSIT or no CSIT is available. In the bi-directional cooperative NOMA system, the direction of cooperation among users can be figured out by allowing users to exchange channel information via short-range communications.

The main contributions of this paper are as follows:
- The closed-form ergodic capacity of bi-directional cooperative NOMA is derived, especially for a two-user scenario. Also, the ergodic capacity of bi-directional cooperative NOMA is shown to be better than those of

other existing NOMA schemes and OMA, even when the channel variances between the users are small.

- On the basis of the ergodic capacity analysis, this paper presents the optimal power allocation algorithms to maximize the user fairness and sum rate for bi-directional cooperative NOMA.
- Outage probabilities of bi-directional cooperative NOMA are derived in a two-user scenario. For the non-SIC user, it is shown that the proposed system has a power gain over the existing NOMA schemes. For the SIC user, our scheme is shown to have a diversity gain over conventional NOMA and OMA.
- The extension of bi-directional cooperative NOMA to the multiuser model is presented by performing cooperation on signal-by-signal basis, not on user-by-user [21]. The cooperation on signal-by-signal basis does not require additional power allocations for the cooperation phases.
- Simulation results verify the analysis of ergodic capacity and outage probability. Moreover, bi-directional cooperative NOMA is shown to provide better data rates than other NOMA schemes and OMA even without instantaneous CSI, necessarily required for conventional NOMA.

We first propose the two-user scenario of bi-directional cooperative NOMA in Section II. Ergodic capacity analysis is performed in Section III and the optimal power allocation algorithms to maximize the user fairness and sum rate are presented in Section IV. In Section V, the outage probability of the proposed system is analyzed. Bi-directional cooperative NOMA is extended to the multiuser model in Section VI, and simulation results are shown in Section VII. Lastly, we conclude the paper in Section VIII.

II. SYSTEM MODEL

A. Channel Model

Consider cellular downlink communications in which a BS transmits signals to two users simultaneously. Extension to the multi-user model will be shown in Section VI. The Rayleigh fading channel from the BS to user i is defined as $h_i = \sqrt{L_i} g_i$ for $i = 1, 2$, where $L_i = 1/d_i^2$ controls the path loss, d_i is the BS–user-i distance, and g_i represents the fast fading component having a complex Gaussian distribution, $g_i \sim CN(0, 1)$.

In this paper, two cases are considered in terms of the CSI knowledge: only statistical CSIT and no CSIT. Here, the statistical CSIT means that the BS knows only users' channel variances. The BS usually allocates more power to the user having the smaller channel variance. The user having a larger channel variance performs SIC first to subtract inter-user interference, and then decodes its data. However, if the distances from the BS to two users are similar, then randomly generated channels cannot guarantee that the user of the larger variance experiences the stronger channel gain. In this case, the performance gain of NOMA over OMA mostly vanishes.

In practice, there exist harsh environments where CSI is hardly known at the BS. For example, in an IoT environment, a clumsy device acts as a transmitter but cannot handle substantial processing tasks, i.e., channel tracking and elaborate user scheduling, therefore CSI is not available at the transmitter side. In this no-CSIT case, the BS cannot judge which user has a larger or smaller channel variance, therefore the system should arbitrarily decide the given user whether to perform SIC or not. Also, the optimal power allocation cannot be found without any CSI, hence fixed power ratios for the users will be assumed. The problem for the no-CSIT case occurs when the user with the weaker channel gain is selected to perform SIC. In this case, there is no merit of employing conventional NOMA. This paper proposes a new cooperative NOMA system for reliable downlink transmissions for both statistical-CSIT only and no-CSIT cases.

B. Direct Transmission Phase

Denote γ_i as the power ratio allocated to user $i \in \{1, 2\}$, satisfying $\gamma_1 + \gamma_2 = 1$. The signal received by user i is given by

$$r_i = h_i(\sqrt{\gamma_1} s_1 + \sqrt{\gamma_2} s_2) + n_i, \qquad (1)$$

where s_i and n_i are the transmitted symbol and noise for user i, respectively, and $n_i \sim CN(0, \sigma_n^2)$. σ_n^2 is the normalized noise variance. Assume a normalized unit power at the BS, $\mathbb{E}[|s_i|^2] = 1$. Throughout the paper, users 1 and 2 are the non-SIC user and SIC user, respectively.

Let $V_{i,k}$ be the SINR of user i to decode s_k. Then, the received SINRs at both users become

$$V_{1,1} = \frac{|h_1|^2 \gamma_1}{|h_1|^2 \gamma_2 + \sigma_n^2} \qquad (2)$$

$$V_{2,2} = \frac{|h_2|^2 \gamma_2}{\sigma_n^2}, \qquad (3)$$

and the SINR for SIC at user 2 is given by

$$V_{2,1} = \frac{|h_2|^2 \gamma_1}{|h_2|^2 \gamma_2 + \sigma_n^2}. \qquad (4)$$

Here, user 2 performs SIC for s_1 first with $V_{2,1}$, and decodes s_2 with $V_{2,2}$.

C. Channel Information Exchange Phase

Since both users decode s_1 in the direct transmission phase, cooperation between users for improved decoding of s_1 is possible. Although both users' decoding processes at the direct transmission phase can be reliable, the risk is that all users receive and exploit the cooperation signal, when decoding of the user with the weaker channel gain fails. Therefore, the system allows only user i_0, satisfying $i_0 = \arg\max_{i \in \{1,2\}} |h_i|^2$, to transmit the cooperation signal. This indicates that transmission of the cooperation signal can be bi-directional, but actual cooperation at each time is performed at only the user of the weaker channel gain. To find user i_0, users exchange their CSI or just the received channel power in this phase. We assume that all users are located nearby and the exchange of CSI is performed via short-range communications; therefore, this phase would not take too much time. A highly crowded stadium is one example, where the distances between the BS and users do not differ greatly and the BS with

statistical CSIT only and no CSIT would hardly determine the direction of cooperation appropriately.

Remark: Obviously, the CSI exchange among users allows the BS not to collect all users' CSI at the expense of the additional delay and signaling overhead. Therefore, the proposed scheme can be more advantageous than existing cooperative NOMA with full CSIT, only when the CSI exchange step requires less time and overhead compared to transmissions of CSI feedbacks. In the case of statistical CSIT only or no CSIT, CSI feedbacks are not required, but much larger data rates can be obtained by allowing the exchanges of CSI among users at the expense of the additional delay and overhead, as shwon in Section VII.

One more thing to remark is that even if the CSI exchange step incurs longer delays than transmissions of CSI feedbacks, the proposed technique can be still beneficial over conventional schemes, especially when the channel coherence time is very short. In the proposed scheme, since all users already received NOMA signals from the BS and obtained the desired CSI at the direct transmission phase, even though the channel conditions change during the phase of the CSI exchange, users can find the appropriate direction of cooperation. On the other hand, in conventional NOMA where the BS should collect the users' CSI feedbacks, channel conditions at the time when users send CSI feedbacks to the BS could be changed when the BS transmits the NOMA signal to all users. Also, as responsibility for determining the direction of cooperation signals is shifted to user sides, the BS does not need to handle substantial processing tasks for estimating the exact CSI, e.g., channel tracking.

D. Bi-Directional Cooperative Phase

In this phase, the cooperation signal is transmitted from the user with a stronger channel gain to the user with a weaker gain. The cooperation signal can help user 1 to decode its data, or user 2 to perform SIC better. The cooperation signal received by user i is given by

$$c_i = g_{k,i} s_c + n_{c,i} \tag{5}$$

where $i \neq k$, $g_{k,i}$ is a Rayleigh fading channel coefficient from user k to user i, and $s_c = s_1$ here. As mentioned in the channel information exchange phase, when $|h_1|^2 > |h_2|^2$, only c_2 exists, and when $|h_1|^2 < |h_2|^2$, only c_1 is transmitted from user 2. The received SINR at user i is

$$W_i = \frac{|g_{k,i}|^2}{\sigma_n^2}, \tag{6}$$

and the SINR for decoding s_1 is given by

$$Z_{\text{cN},1}^{\text{bi}} = \begin{cases} \min\{\max\{V_{1,1}, W_1\}, V_{2,1}\} & \text{if } |h_1|^2 < |h_2|^2 \\ \min\{V_{1,1}, \max\{V_{2,1}, W_2\}\} & \text{otherwise.} \end{cases} \tag{7}$$

Here, if a certain user receives the cooperation signal, she chooses the better of the signals received in the direct transmission and bi-directional cooperative phases. When maximal-ratio combining is exploited [21], [22], we achieve

$$Z_{\text{cN},1}^{\text{bi}} = \begin{cases} \min\{V_{1,1} + W_1, V_{2,1}\} & \text{if } |h_1|^2 < |h_2|^2 \\ \min\{V_{1,1}, V_{2,1} + W_2\} & \text{otherwise.} \end{cases} \tag{8}$$

Since only s_1 is shared for cooperation, the SINR for decoding s_2 is $Z_{\text{cN},2}^{\text{bi}} = V_{2,2}$.

Assume that both users are located close to each other, i.e., $W_1, W_2 \gg V_{1,1}, V_{2,1}, V_{2,2}$. Then, $Z_{\text{cN},1}^{\text{bi}} \approx V_{2,1}$ when $|h_1|^2 < |h_2|^2$ or $Z_{\text{cN},1}^{\text{bi}} \approx V_{1,1}$ otherwise, and both (7) and (8) are simplified to

$$\tilde{Z}_{\text{cN},1}^{\text{bi}} \simeq \max\{V_{1,1}, V_{2,1}\}. \tag{9}$$

This assumption is used throughout the paper. The data rate of s_i in bi-directional cooperative NOMA becomes $R_{\text{cN},i}^{\text{bi}} = \log_2(1 + Z_{\text{cN},i}^{\text{bi}})$.

The difference of our work from [21] is that the direction of cooperation is determined on user sides by exchanging CSI among users, especially when the BS knows only statistical CSIT or no CSIT at all. Cooperative NOMA in [21] is based on instantaneous CSIT, and the BS can determine the user with a stronger channel gain as the SIC user and the other one with a weaker gain as the non-SIC user. This makes the direction of cooperation be always from the SIC user to the non-SIC user in the cooperative NOMA scheme of [21]. In the statistical-CSIT only or no CSIT cases, however, there is no guarantee that the SIC user's instantaneous channel is better than that of the non-SIC user. Therefore, exchanging the channel information among users is necessary for bi-directional cooperative NOMA to force the user with a stronger channel gain transmit the cooperation signal. We consider for comparison purposes uni-directional cooperative NOMA where the direction of cooperation is always from the SIC user to the non-SIC user.

Since uni-directional cooperative NOMA only allows the SIC user to transmit the cooperation signal to the non-SIC user, the SINR for decoding s_1 is $Z_{\text{cN},1}^{\text{uni}} = \min\{\max\{V_{1,1}, W_1\}, V_{2,1}\}$, and $\tilde{Z}_{\text{cN},1}^{\text{uni}} \simeq V_{2,1}$ is obtained with the assumption that both users are located close to each other. Also, the SINR for decoding s_1 in conventional NOMA is $Z_1^{\text{N}} = \min\{V_{1,1}, V_{2,1}\}$ [8]. Comparing $\tilde{Z}_{\text{cN},1}^{\text{bi}}$ with $\tilde{Z}_{\text{cN},1}^{\text{uni}}$ and Z_1^{N}, we can find that bi-directional cooperative NOMA exploits channel diversity. It is clear that $\tilde{R}_{\text{cN},1}^{\text{bi}}$ is larger than or equal to $\tilde{R}_{\text{cN},1}^{\text{uni}}$ and $R_{\text{N},1}$. On the other hand, since the cooperation is only helpful for s_1, the data rate of s_2 is the same for all considered schemes, i.e., $R_{\text{cN},2}^{\text{bi}} = R_{\text{cN},2}^{\text{uni}} = R_{\text{N},2}$. The sum rate is obtained by $R_{\text{cN}}^{\text{bi}} = R_{\text{cN},1}^{\text{bi}} + R_{\text{cN},2}^{\text{bi}}$.

When targeted data rates are already determined, the outage event of a certain user is the criterion for determining whether the other user should receive the cooperation signal or not. Let ϵ_1 and ϵ_2 be SINR thresholds for decoding s_1 and s_2, respectively. Then, in this two-user scenario, even though $|h_1|^2 < |h_2|^2$, user 1 cannot receive the cooperation signal from user 2 when $V_{2,1} < \epsilon_1$. Since $|h_1|^2 < |h_2|^2$, it is also clear that $V_{1,1} < \epsilon_1$; thus decoding of s_1 fails on both user sides. On the other hand, when $V_{2,1} > \epsilon_1$, user 1 can decode s_1 by using cooperation from user 2, even if $V_{1,1} < \epsilon_1$. The analysis of the outage probability is given in Section V.

III. ERGODIC CAPACITY ANALYSIS

When users' data rates are opportunistically determined by their Quality of Service (QoS) requirements, ergodic capacity analysis is important. Some key lemmas are established first

in deriving the closed-form ergodic capacity of bi-directional cooperative NOMA.

Lemma 1: For real constants $a, b > 0$ and a chi-square random variable X, an expected value of the function $\log_2(1 + \frac{aX}{b})$ becomes

$$\mathbb{E}\left[\log_2\left(1 + \frac{aX}{b}\right)\right] = C_1\left(\frac{2a}{b}\right), \tag{10}$$

where $C_1(x) = \frac{1}{\ln 2} e^{1/x} \int_1^\infty \frac{1}{t} e^{-t/x} dt$, for $x > 0$

Proof: Let the nonnegative random variable $Z = \log_2(1 + aX/b)$; then $\mathbb{E}[Z] = \int_0^\infty P[Z \geq z] dz$ is satisfied. Therefore,

$$\mathbb{E}[Z] = \int_0^\infty \left(1 - P[Z \leq z] dz\right) = \int_1^\infty \frac{1}{t \ln 2} e^{-\frac{b}{2a}(t-1)} dt$$
$$= C_1\left(\frac{2a}{b}\right), \tag{11}$$

where $t = 2^z$. $\quad\square$

Lemma 2: For real constants $a > 0$, b, and a chi-square random variable X,

$$\int_0^\infty e^{-bx} \log_2(1 + ax) dx = \frac{1}{b} C_1\left(\frac{a}{b}\right) \tag{12}$$

Proof:

$$\int_0^\infty e^{-bx} \log_2(1 + ax) dx$$
$$= \left[-\frac{1}{b} e^{-bx} \log_2(1 + ax)\right]_0^\infty + \int_0^\infty \frac{1}{\ln 2} \cdot \frac{a}{b(1 + ax)} e^{-bx} dx \tag{13}$$
$$= \int_1^\infty \frac{1}{b \ln 2} \cdot \frac{1}{t} e^{-\frac{b}{a}(t-1)} dt = \frac{1}{b} C_1\left(\frac{a}{b}\right), \tag{14}$$

where $t = 1 + ax$. $\quad\square$

Lemma 3: $C_1(x)$ is an increasing function of $x > 0$.

Proof:

$$\frac{d}{dx} C_1(x) = -\frac{1}{x^2 \ln 2} e^{1/x} \int_1^\infty \frac{e^{-t/x}}{t} dt + \frac{1}{x \ln 2} \tag{15}$$
$$> -\frac{1}{x^2 \ln 2} \ln(1 + x) + \frac{1}{x \ln 2}, \tag{16}$$

where the last inequality is satisfied according to $e^{-1/x} \ln(1 + x) > \int_1^\infty \frac{e^{-t/x}}{t} dt$ [25]. Since $x > 0$ and $x > \ln(1 + x)$, $\frac{d}{dx} C_1(x) > 0$, thus $C_1(x)$ is an increasing function of $x > 0$. $\quad\square$

With the assumption that both users are located nearby, Theorem 1 gives the closed-form ergodic capacity of the bi-directional cooperative NOMA system. Also, Theorem 2 shows that the ergodic capacity of bi-directional cooperative NOMA is larger than those of uni-directional cooperative NOMA and conventional NOMA, no matter which user's channel gain is larger.

Theorem 1: Assuming that both users are located close to each other, the closed-form ergodic capacity of two-user bi-directional cooperative NOMA is

$$\mathbb{E}[\tilde{R}_{cN}^{bi}] = C_1\left(\frac{L_1}{\sigma_n^2}\right) - C_1\left(\frac{\gamma_2 L_1}{\sigma_n^2}\right) + C_1\left(\frac{L_2}{\sigma_n^2}\right)$$
$$- C_1\left(\frac{L_1 L_2}{(L_1 + L_2)\sigma_n^2}\right) + C_1\left(\frac{\gamma_2 L_1 L_2}{(L_1 + L_2)\sigma_n^2}\right). \tag{17}$$

Proof: See Appendix A. $\quad\square$

Theorem 2: Assuming that both users are located close to each other,

$$\mathbb{E}[\tilde{R}_{cN}^{bi}] \geq \mathbb{E}[\tilde{R}_{cN}^{uni}] \geq \mathbb{E}[R_N]. \tag{18}$$

Proof: See Appendix B. $\quad\square$

To verify that bi-directional cooperative NOMA is applicable, comparison with OMA is also necessary. Theorem 3 shows that the ergodic capacity of bi-directional cooperative NOMA is better than that of OMA when the channel variances of two users are identical. Lemma 4 is introduced first before stating Theorem 3.

Lemma 4: $C_1(x) - C_1(\beta x)$ is an increasing function of $x > 0$, for any $0 < \beta < 1$.

Proof: According to (74) and (78), the closed-form ergodic capacity of the uni-directional cooperative NOMA system becomes

$$\mathbb{E}[\tilde{R}_{cN}^{uni}] = C_1\left(\frac{L_1}{\sigma_n^2}\right) - C_1\left(\frac{\gamma_2 L_1}{\sigma_n^2}\right) + C_1\left(\frac{\gamma_2 L_2}{\sigma_n^2}\right). \tag{19}$$

According to (17) and (19), the following inequality holds by Theorem 2,

$$C_1\left(\frac{L_1}{\sigma_n^2}\right) - C_1\left(\frac{\gamma_2 L_1}{\sigma_n^2}\right) + C_1\left(\frac{L_2}{\sigma_n^2}\right)$$
$$- C_1\left(\frac{L_1 L_2}{(L_1 + L_2)\sigma_n^2}\right) + C_1\left(\frac{\gamma_2 L_1 L_2}{(L_1 + L_2)\sigma_n^2}\right)$$
$$\geq C_1\left(\frac{L_1}{\sigma_n^2}\right) - C_1\left(\frac{\gamma_2 L_1}{\sigma_n^2}\right) + C_1\left(\frac{\gamma_2 L_2}{\sigma_n^2}\right) \tag{20}$$
$$\Leftrightarrow C_1\left(\frac{L_2}{\sigma_n^2}\right) - C_1\left(\frac{\gamma_2 L_2}{\sigma_n^2}\right)$$
$$- \left\{C_1\left(\frac{L_1 L_2}{(L_1 + L_2)\sigma_n^2}\right) - C_1\left(\frac{\gamma_2 L_1 L_2}{(L_1 + L_2)\sigma_n^2}\right)\right\} \geq 0. \tag{21}$$

The equation (21) holds for any $L_1, L_2 > 0$ and $0 < \gamma_2 < 1$. Thus, $C_1(x) - C_1(\beta x)$ is an increasing function of $x > 0$. $\quad\square$

Theorem 3: When $L_1 = L_2 = L$, $\mathbb{E}[\tilde{R}_{cN}^{bi}] > \mathbb{E}[R_O]$, provided $\frac{L}{\sigma_n^2}, \frac{\gamma_1 L}{\alpha_1 \sigma_n^2}, \frac{\gamma_2 L}{\alpha_2 \sigma_n^2} > 1$.

Proof: See Appendix C. $\quad\square$

It is reasonable that $\mathbb{E}[\tilde{R}_{cN}^{bi}]$ becomes much larger than $\mathbb{E}[R_O]$ as L_2 increases above L_1. Also, $\mathbb{E}[\tilde{R}_{cN}^{bi}] - \mathbb{E}[R_O] > 0$ when $L_1 = L_2$ by Theorem 3, therefore it can also be expected that $\mathbb{E}[\tilde{R}_{cN}^{bi}]$ could be still larger than $\mathbb{E}[R_O]$ when $L_1 = L_2 + \delta$ for small $\delta > 0$. In Section VII, numerical results show that bi-directional cooperative NOMA still has a rate gain compared to OMA even when the SIC user (user 2) experiences the weaker channel condition than the non-SIC user (user 1).

IV. OPTIMAL POWER ALLOCATION RULE

On the basis of the ergodic capacity analysis, we present the optimal power allocation rule for bi-directional cooperative NOMA. Two optimization goals are considered: user fairness and sum rate. Note that the BS should know statistical CSI at least for the optimal power allocation, and we do not consider the no-CSIT case here. Assume $L_1 < L_2$ in this section.

Algorithm 1 Bisection Method for Power Allocation of User Fairness Problem

1: Initialize $\gamma_- = 0$, $\gamma_+ = 1$.
2: **while** $\gamma_+ - \gamma_- \geq \epsilon$ **do**
3:     $\gamma_2^* = (\gamma_+ + \gamma_-)/2$
4:     **if** $\mathbb{E}[\tilde{R}_{\text{cN},1}^{\text{bi}}] < \mathbb{E}[R_{\text{cN},2}^{\text{bi}}]$ **then** $\gamma_+ = \gamma_2^*$
5:     **else** $\gamma_- = \gamma_2^*$
6:     **end if**
7: **end while**

A. User Fairness Problem

As in [12] and [18], the max-min optimization problem is formulated for user fairness as

$$\gamma_2^* = \underset{0 < \gamma_2 < 1}{\arg\max} \ \min(\mathbb{E}[\tilde{R}_{\text{cN},1}^{\text{bi}}], \mathbb{E}[R_{\text{cN},2}^{\text{bi}}]), \qquad (22)$$

where γ_2^* is the optimal power ratio for user 2 and recall that $\gamma_1 + \gamma_2 = 1$. The following lemma helps to solve the above optimization problem.

Lemma 5: $\mathbb{E}[\tilde{R}_{\text{cN},1}^{\text{bi}}]$ is a decreasing function of γ_2 and $\mathbb{E}[R_{\text{cN},2}^{\text{bi}}]$ is an increasing function of γ_2.

Proof: $\tilde{R}_{\text{cN},1}^{\text{bi}} = \max\{Z_1, Z_2\}$, where $Z_1 = \log_2(1 + \frac{|h_1|^2 \gamma_1}{|h_1|^2 \gamma_2 + \sigma_n^2}) = \log_2(1 + \frac{|h_1|^2}{\sigma_n^2}) - \log_2(1 + \frac{|h_1|^2 \gamma_2}{\sigma_n^2})$, and $Z_2 = \log_2(1 + \frac{|h_2|^2 \gamma_1}{|h_2|^2 \gamma_2 + \sigma_n^2}) = \log_2(1 + \frac{|h_2|^2}{\sigma_n^2}) - \log_2(1 + \frac{|h_2|^2 \gamma_2}{\sigma_n^2})$. Since $\log_2(1 + \frac{|h_i|^2 \gamma_2}{\sigma_n^2})$ is an increasing function of γ_2, Z_1 and Z_2 are decreasing functions; therefore $\mathbb{E}[\tilde{R}_{\text{cN},1}^{\text{bi}}]$ is a decreasing function of γ_2. Also, $R_{\text{cN},2}^{\text{bi}} = \log_2(1 + \frac{|h_2|^2 \gamma_2}{\sigma_n^2})$, and $\mathbb{E}[R_{\text{cN},2}^{\text{bi}}]$ is an increasing function of γ_2. $\square$

By Lemma 5, the optimal solution of (22) is directly obtained when $\mathbb{E}[\tilde{R}_{\text{cN},1}^{\text{bi}}] = \mathbb{E}[R_{\text{cN},2}^{\text{bi}}]$. Since $\mathbb{E}[\tilde{R}_{\text{cN},1}^{\text{bi}}], \mathbb{E}[R_{\text{cN},2}^{\text{bi}}] \geq 0$ for any $\gamma_1, \gamma_2 \in [0,1]$, $\mathbb{E}[\tilde{R}_{\text{cN},1}^{\text{bi}}] = 0$ at $\gamma_1 = 0$, and $\mathbb{E}[R_{\text{cN},2}^{\text{bi}}] = 0$ at $\gamma_2 = 0$, the solution of $\mathbb{E}[\tilde{R}_{\text{cN},1}^{\text{bi}}] = \mathbb{E}[R_{\text{cN},2}^{\text{bi}}]$ would satisfy $0 \leq \gamma_2^* \leq 1$. However, the closed-form solution of γ_2^* is difficult to derive because of the expectation operations. Therefore, the bisection method is used to solve (22). Algorithm 1 shows the detail.

B. Max-Sum-Rate Problem

To maximize the sum-rate of NOMA system, allocating all power to the strong user is a simple solution. However, it destroys the user-fairness completely; the sum-rate performance is usually studied under a minimum rate constraint as in [18]. The problem can be formulated as

$$\gamma_2^* = \underset{0 < \gamma_2 < 1}{\arg\max} \ \mathbb{E}[\tilde{R}_{\text{cN},1}^{\text{bi}}] + \mathbb{E}[R_{\text{cN},2}^{\text{bi}}] \qquad (23)$$

$$\text{s.t.} \ \min(\mathbb{E}[\tilde{R}_{\text{cN},1}^{\text{bi}}], \mathbb{E}[R_{\text{cN},2}^{\text{bi}}]) \geq R_t, \qquad (24)$$

where R_t is the minimum data rate constraint for user fairness. Lemma 6 is introduced for solving the above optimization problem.

Lemma 6: $\mathbb{E}[\tilde{R}_{\text{cN},1}^{\text{bi}}] + \mathbb{E}[R_{\text{cN},2}^{\text{bi}}]$ is a decreasing function of γ_2, for any $L_1, L_2 > 0$.

Proof: According to (17),

$$\frac{\text{d}}{\text{d}\gamma_2}\left(\mathbb{E}[\tilde{R}_{\text{cN},1}^{\text{bi}}] + \mathbb{E}[R_{\text{cN},2}^{\text{bi}}]\right)$$
$$= \frac{\text{d}}{\text{d}\gamma_2}\left\{ -C_1\left(\frac{\gamma_2 L_1}{\sigma_n^2}\right) + C_1\left(\frac{\gamma_2 L_1 L_2}{(L_1 + L_2)\sigma_n^2}\right)\right\} < 0, \quad (25)$$

by Lemma 4 and $0 < \frac{L_1}{L_1 + L_2} < 1$. $\square$

By Lemmas 5 and 6, the optimal solution of (23) is obtained when $\mathbb{E}[R_{\text{cN},2}^{\text{bi}}] = R_t$. Similar to the user fairness problem, the bisection method can be used to find γ_2^* of (23). However, if the assumption that both users are located nearby is not satisfied, Lemma 6 does not hold anymore. In this case, we consider some special cases depending on the relative amounts of W_1 and W_2 compared to $V_{1,1}$ and $V_{2,1}$. Suppose $V_{1,1} < V_{2,1}$, then Lemma 6 holds when $V_{2,1} < W_1$. However, when $V_{1,1} < W_1 < V_{2,1}$, $R_{\text{cN},1}^{\text{bi}}$ becomes $\log_2(1 + \frac{|g_c|^2}{\sigma_n^2})$; thus $\mathbb{E}[R_{\text{cN},1}^{\text{bi}}]$ does not depend on the power allocation ratio. Then, $\mathbb{E}[R_{\text{cN},1}^{\text{bi}}] + \mathbb{E}[R_{\text{cN},2}^{\text{bi}}]$ becomes an increasing function of γ_2 because $\mathbb{E}[R_{\text{cN},2}^{\text{bi}}]$ does. On the other hand, when $W_1 < V_{1,1}$, $R_{\text{cN},1}^{\text{bi}}$ becomes $\log_2(1 + \frac{\gamma_1 |h_1|^2}{\gamma_2 |h_2|^2 + \sigma_n^2})$, and it can be proven that $\mathbb{E}[R_{\text{cN},1}^{\text{bi}}] + \mathbb{E}[R_{\text{cN},2}^{\text{bi}}]$ is a decreasing function of γ_2 in a method similar to the proof of Lemma 6.

The situation where $V_{1,1} > V_{2,1}$ can be also considered similar to $V_{1,1} < V_{2,1}$. However, this case is not applied to solve the max-sum-rate problem in (23). The reason is that $\mathbb{E}[V_{1,1}], \mathbb{E}[V_{2,1}], \mathbb{E}[W_1]$ and $\mathbb{E}[W_2]$ are used instead of $V_{1,1}, V_{2,1}, W_1$ and W_2 in the statistical CSIT case. This approximation does not consider the case of $V_{1,1} > V_{2,1}$, because we assume $L_2 > L_1$ first therefore $\mathbb{E}[V_{1,1}] < \mathbb{E}[V_{2,1}]$ always. Thus, this approximation makes γ_2^* of (23) a suboptimal solution.

Algorithm 2 Bisection Method of Power Allocation for Max-Sum-Rate Problem

1: Initialize $\gamma_- = 0$, $\gamma_+ = 1$.
2: **while** $\gamma_+ - \gamma_- \geq \epsilon$ **do**
3:     $\gamma_2^* = (\gamma_+ + \gamma_-)/2$
4:     **if** $R_0 > R_t$ **then**
5:         **if** $\min\{\mathbb{E}[R_{\text{cN},1}^{\text{bi}}], \mathbb{E}[R_{\text{cN},2}^{\text{bi}}]\} \leq R_t$ **then**
6:             **if** $\mathbb{E}[R_{\text{cN},1}^{\text{bi}}] > \mathbb{E}[R_{\text{cN},2}^{\text{bi}}]$ **then** $\gamma_- = \gamma_2^*$
7:             **else** $\gamma_+ = \gamma_2^*$
8:         **end if**
9:         **else**
10:             **if** $\mathbb{E}[V_{1,1}] < \mathbb{E}[W_1] < \mathbb{E}[V_{2,1}]$ **then** $\gamma_- = \gamma_2^*$
11:             **else** $\gamma_+ = \gamma_2^*$
12:         **end if**
13:         **end if**
14:     **else** System outage occurs.
15:     **end if**
16: **end while**

In summary, if $V_{1,1} < W_1 < V_{2,1}$, $\mathbb{E}[R_{\text{cN},1}^{\text{bi}}] + \mathbb{E}[R_{\text{cN},2}^{\text{bi}}]$ is an increasing function of γ_2; then the solution is obtained when $\mathbb{E}[R_{\text{cN},2}^{\text{bi}}] = R_t$. If not, $\mathbb{E}[R_{\text{cN},1}^{\text{bi}}] + \mathbb{E}[R_{\text{cN},2}^{\text{bi}}]$ is a decreasing function of γ_2, and γ_2^* is found when $\mathbb{E}[R_{\text{cN},2}^{\text{bi}}] = R_t$. Based on these behaviors, the suboptimal bisection method for maximizing the sum rate of bi-directional cooperative NOMA in the statistical CSIT case is presented in Algorithm 2.

Note that the outage event occurs when the minimum rate constraint in (24) is not satisfied. In addition, according to Lemma 5, we can recognize that $R_0 = \mathbb{E}[R_{\text{cN},1}^{\text{bi}}] = \mathbb{E}[R_{\text{cN},2}^{\text{bi}}]$ should be larger than R_t; otherwise, the system cannot avoid outage.

V. OUTAGE PROBABILITY

When the targeted data rates, $R_{t,1}$ and $R_{t,2}$, are determined by the users' QoS requirements, the outage probability is an important performance criterion. If the outage event occurs at the non-SIC user, the SIC user does not use the cooperation signal, and the outage of the SIC user does not allow the cooperation from the SIC user to the non-SIC user. The outage probability at the non-SIC user (user 1) in bi-directional cooperative NOMA is given by

$$P_{\text{cN},1}^{\text{bi}} = P\{V_{1,1} < \epsilon_1, \ V_{2,1} < \epsilon_1\} \\ + P\{\max\{V_{1,1}, \ W_1\} < \epsilon_1, \ V_{2,1} > \epsilon_1\}, \quad (26)$$

where $\epsilon_i = 2^{R_{t,i}} - 1$.

Again, $|h_i|^2 = L_i X_i / 2$ for $i \in \{1, 2\}$. The first term of (26) becomes

$$P\left\{\frac{\gamma_1 \ L_1 \ X_1}{\gamma_2 \ L_1 \ X_1 + 2\sigma_n^2} < \epsilon_1\right\} \cdot P\left\{\frac{\gamma_1 \ L_2 \ X_2}{\gamma_2 \ L_2 \ X_2 + 2\sigma_n^2} < \epsilon_1\right\}$$
$$= \begin{cases} 1 & \text{if } \frac{\gamma_1}{\gamma_2} < \epsilon_1 \\ (1 - e^{-\xi/L_1})(1 - e^{-\xi/L_2}) & \text{otherwise,} \end{cases} \quad (27)$$

where $\epsilon_i = 2^{R_{ti}} - 1$ and $\xi = \frac{\sigma_n^2 \epsilon_1}{\gamma_1 - \epsilon_1 \gamma_2}$, and the second term of (26) becomes

$$P\left\{\frac{\gamma_1 \ L_1 \ X_1}{\gamma_2 \ L_1 \ X_1 + 2\sigma_n^2} < \epsilon_1\right\} \cdot P\left\{\frac{L_c X_c}{2\sigma_n^2} < \epsilon_1\right\}$$
$$\cdot P\left\{\frac{\gamma_1 \ L_2 \ X_2}{\gamma_2 \ L_2 \ X_2 + 2\sigma_n^2} > \epsilon_1\right\} \quad (28)$$
$$= \begin{cases} 0 & \text{if } \frac{\gamma_1}{\gamma_2} < \epsilon_1 \\ (1 - e^{-\xi/L_1})(1 - e^{-\sigma_n^2 \epsilon_1/L_c})e^{-\xi/L_2} & \text{otherwise.} \end{cases} \quad (29)$$

By (26), (27) and (29), $P_{\text{cN},1}^{\text{bi}}$ is given by

$$P_{\text{cN},1}^{\text{bi}} = \begin{cases} 1 & \text{if } \frac{\gamma_1}{\gamma_2} < \epsilon_1 \\ (1 - e^{-\xi/L_1})(1 - e^{-\sigma_n^2 \epsilon_1/L_c - \xi/L_2}) & \text{otherwise.} \end{cases} \quad (30)$$

$P_{\text{cN},1}^{\text{bi}}$ conditioned on $\gamma_1/\gamma_2 > \epsilon_1$ is approximated in the high SNR region by

$$P_{\text{cN},1}^{\text{bi}} \approx \frac{\sigma_n^2 \epsilon_1}{L_1(\gamma_1 - \epsilon_1 \gamma_2)} \cdot \left(\frac{\sigma_n^2 \epsilon_1}{L_c} + \frac{\sigma_n^2 \epsilon_1}{L_2(\gamma_1 - \epsilon_1 \gamma_2)}\right), \quad (31)$$

and it indicates that user 1 achieves a diversity order of 2.

Here, the outage probability of user 1 in bi-directional cooperative NOMA is the same as that of uni-directional cooperative NOMA, i.e., $P_{\text{cN},1}^{\text{bi}} = P_{\text{cN},1}^{\text{uni}}$, because user 1 receives the cooperation signal from user 2 also in uni-directional cooperative NOMA. On the other hand, conventional NOMA and OMA are different. For conventional NOMA, the outage

probability of user 1 of conventional NOMA, $P_{\text{N},1}$, is given by

$$P_{\text{N},1} = P\left\{\frac{\gamma_1 L_1 X_1}{\gamma_2 L_1 X_1 + 2\sigma_n^2} < \epsilon_1\right\} = 1 - e^{-\xi/L_1} \quad (32)$$
$$\approx \frac{\sigma_n^2 \epsilon_1}{L_1(\gamma_1 - \epsilon_1 \gamma_2)}, \quad (33)$$

and it just has a diversity order of 1. The equation (33) is achieved by a high-SNR approximation.

Likewise, user 1 of OMA also achieves a diversity order of 1, as shown in (35) and (36).

$$P_{\text{O},1} = P\left\{\frac{\gamma_1 L_1 X_1}{2\alpha_1 \sigma_n^2} < \epsilon_{O,1}\right\} \quad (34)$$
$$= 1 - \exp\left\{-\frac{\alpha_1 \sigma_n^2 \epsilon_{O,1}}{\gamma_1 L_1}\right\} \quad (35)$$
$$\approx \frac{\alpha_1 \sigma_n^2 \epsilon_{O,1}}{\gamma_1 L_1}, \quad (36)$$

where $\epsilon_{O,i} = 2^{R_{t,i}/\alpha_i} - 1$ for $i \in \{1, 2\}$. We can easily note that bi- and uni-directional cooperative NOMA systems can achieve multiuser diversity.

Next, consider the outage probability of user 2 in bi-directional cooperative NOMA system. If user 1 avoids the outage event, user 2 can use the cooperation signal transmitted from user 1 for its SIC process. However, if the user 1's data rate is less than $R_{t,1}$, the cooperation from user 1 to user 2 cannot be performed. The outage probability of user 2 in bi-directional cooperative NOMA is given by

$$P_{\text{cN},2}^{\text{bi}} = P\{(V_{2,2} < \epsilon_2 \cup V_{2,1} < \epsilon_1), \ V_{1,1} < \epsilon_1\} \\ + P\{(V_{2,2} < \epsilon_2 \cup \max\{V_{2,1}, \ W_2\} < \epsilon_1), \ V_{1,1} > \epsilon_1\}. \quad (37)$$

The first term of (37) becomes

$$(1 - P\{Z_2 > \epsilon_2, \ V_{2,1} > \epsilon_1\})P\{V_{1,1} < \epsilon_1\} \quad (38)$$
$$= \left(1 - P\left\{\frac{\gamma_2 L_2 X_2}{2\sigma_n^2} > \epsilon_2, \ \frac{\gamma_1 L_2 X_2}{\gamma_2 L_2 X_2 + 2\sigma_n^2} > \epsilon_1\right\}\right)$$
$$\cdot P\left\{\frac{\gamma_1 L_1 X_1}{\gamma_2 L_1 X_1 + 2\sigma_n^2} < \epsilon_1\right\}. \quad (39)$$

Similarly, the second term of (37) becomes

$$\left[P\{V_{2,2} < \epsilon_2\} + P\{\max\{V_{2,1}, \ W_2\} < \epsilon_1\} \\ - P\{V_{2,2} < \epsilon_2 \cap \max\{V_{2,1}, \ W_2\} < \epsilon_1\}\right] \cdot P\{V_{1,1} > \epsilon_1\}, \quad (40)$$

where

$$P\{\max\{V_{2,1}, \ W_2\} < \epsilon_1\} = P\{V_{2,1} < \epsilon_1\}P\{W_2 < \epsilon_1\}$$
$$= \begin{cases} 1 & \text{if } \frac{\gamma_1}{\gamma_2} < \epsilon_1 \\ (1 - e^{-\xi/L_2})(1 - e^{-\sigma_n^2 \epsilon_1/L_c}) & \text{otherwise} \end{cases} \quad (41)$$

and

$$P\{V_{2,2} < \epsilon_2 \cap \max\{V_{2,1},\ W_2\} < \epsilon_1\}$$
$$= \begin{cases} 1 & \text{if } \dfrac{\gamma_1}{\gamma_2} < \epsilon_1 \\[2mm] (1 - e^{-\xi/L_2})(1 - e^{-\sigma_n^2 \epsilon_1/L_c}) & \text{else if } \dfrac{\epsilon_2}{\gamma_2} > \dfrac{\xi}{\sigma_n^2} \\[2mm] (1 - e^{-\sigma_n^2 \epsilon_2/\gamma_2 L_2})(1 - e^{-\sigma_n^2 \epsilon_1/L_c}) & \text{else.} \end{cases}$$
$$\tag{42}$$

According to (37), (39), (41), and (42), $P_{\text{cN},2}^{\text{bi}}$ is given by (43), as shown in the bottom of this page. The approximations in (43) are obtained in the high-SNR region. Unlike $P_{\text{cN},1}^{\text{bi}}$, $P_{\text{cN},2}^{\text{bi}}$ has a diversity order of 1. The reason is that user 2 should perform SIC before its data decoding. Uni-directional cooperative NOMA does not allow user 2 to receive the cooperation signal, therefore its outage probability is obtained by

$$P_{\text{cN},2}^{\text{uni}} = P\{V_{2,2} < \epsilon_2 \cup V_{2,1} < \epsilon_1\} \tag{44}$$
$$= 1 - P\{V_{2,2} > \epsilon_2,\ V_{2,1} > \epsilon_1\} \tag{45}$$
$$= \begin{cases} 1 & \text{if } \dfrac{\gamma_1}{\gamma_2} < \epsilon_1 \\[2mm] 1 - \exp\{-\dfrac{\sigma_n^2 \epsilon_2}{\gamma_2 L_2}\} & \\[1mm] \approx \dfrac{\sigma_n^2 \epsilon_2}{\gamma_2 L_2} & \text{else if } \dfrac{\epsilon_2}{\gamma_2} > \dfrac{\xi}{\sigma_n^2} \\[3mm] 1 - \exp\{-\dfrac{\sigma_n^2 \epsilon_1}{L_2(\gamma_1 - \epsilon_1\gamma_2)}\} & \\[1mm] \approx \dfrac{\sigma_n^2 \epsilon_1}{L_2(\gamma_1 - \epsilon_1\gamma_2)} & \text{else.} \end{cases}$$
$$\tag{46}$$

The high-SNR approximation in (46) shows that user 2 in uni-directional cooperative NOMA realizes a diversity order of 1, the same as bi-directional cooperative NOMA. Conditioned on $\frac{\gamma_1}{\gamma_2} > \epsilon_1$, $P_{\text{cN},2}^{\text{bi}} = P_{\text{cN},2}^{\text{uni}}$ when $\frac{\epsilon_2}{\gamma_2} > \frac{\epsilon_1}{\gamma_1 - \epsilon_1\gamma_2}$, but $P_{\text{cN},2}^{\text{bi}}$ has a power gain compared to $P_{\text{cN},2}^{\text{uni}}$ when $\frac{\epsilon_2}{\gamma_2} < \frac{\epsilon_1}{\gamma_1 - \epsilon_1\gamma_2}$. Since there is no cooperation signal from user 1 to user 2 in uni-directional cooperative NOMA, $P_{\text{cN},2}^{\text{uni}}$ is the same as the outage probability of user 2 of conventional NOMA, i.e., $P_{\text{cN},2}^{\text{uni}} = P_{\text{N},2}$. Meanwhile, the outage probability of the user 2 of OMA is given by

$$P_{\text{O},2} = 1 - \exp\left\{-\dfrac{\alpha_2 \sigma_n^2 \epsilon_{O,2}}{\gamma_2 L_2}\right\} \approx \dfrac{\alpha_2 \sigma_n^2 \epsilon_{O,2}}{\gamma_2 L_2}, \tag{47}$$

and it also realizes a diversity order of 1.

In summary, bi-directional cooperative NOMA achieves a power gain for the SIC user (user 2) conditioned on $\frac{\epsilon_2}{\gamma_2} < \frac{\epsilon_1}{\gamma_1 - \epsilon_1\gamma_2}$ and $\frac{\gamma_1}{\gamma_2} > \epsilon_1$ compared to uni-directional cooperative

NOMA and conventional NOMA. On the other hand, the non-SIC user of bi- or uni-directional cooperative NOMA achieves better diversity order than that of conventional NOMA and OMA, as shown by (31).

VI. EXTENSION TO MULTIUSER SCENARIO

Thus far, we considered the two-user model for bi-directional cooperative NOMA. However, the proposed system can be extended to the multiuser scenario. Assume that the BS serves K users by NOMA. Bi-directional cooperative NOMA consists of $K + 2$ phases. The first and the second phases correspond to direct transmission and channel information exchange, respectively, and others are for cooperation. In the statistical-CSIT case, $L_1 < L_2 < \cdots < L_K$ is assumed, and suppose that $\gamma_1 > \gamma_2 > \cdots > \gamma_K$ for both cases of statistical CSIT and no CSIT. All phases of bi-directional cooperative NOMA in the multiuser scenario are explained below:

1) Direct Transmission Phase: The BS transmits the NOMA signal to all users. The signal received by user i is

$$r_i = h_i \sum_{k=1}^{K} \sqrt{\gamma_k} s_k + n_i. \tag{48}$$

2) Channel Information Exchange Phase: Users exchange their channel information to determine the direction of cooperation. As we will see in the j-th cooperative phase, the cooperation signal is transmitted at the user i_0, whose channel gain is the strongest among users $j, \cdots, K$. Therefore, users should know the order of channel gains of all users and this phase requires K time slots. For each slot, a user delivers its channel information to the others via short-range communications. Note that in the proposed scheme for a K-user scenario, each time slot is actually reduced for the exchange of channel information when the users are crowded.

Remark: As mentioned earlier in Section II-C, the CSI exchange step causes additional delays and overheads, and those penalties grow as K increases. However, even without the CSI exchange step, this problem also arises in the existing systems where the BS receives CSI feedbacks. A large K also causes a huge computational burden for SIC and requires a large power budget to enable multiple steps of SIC. Therefore, only two or four-user NOMA signaling has been considered in the practical system, and the proposed scheme can be applied well with the appropriate K for practical scenarios.

3) The j-th Cooperative Phase: The cooperation phase consists of K phases. Cooperation is performed on signal-by-signal basis, i.e., decoding of s_j is performed in the j-th cooperative phase. Users $j, \cdots, K$ decode s_j, and this

$$P_{\text{cN},2}^{\text{bi}} = \begin{cases} 1 & \text{if } \dfrac{\gamma_1}{\gamma_2} < \epsilon_1 \\[3mm] 1 - e^{-\sigma_n^2 \epsilon_2/\gamma_2 L_2} \approx \dfrac{\sigma_n^2 \epsilon_2}{\gamma_2 L_2} & \text{else if } \dfrac{\epsilon_2}{\gamma_2} > \dfrac{\xi}{\sigma_n^2} \\[3mm] (1 - e^{-\xi/L_2}) - e^{-\sigma_n^2 \epsilon_1/L_c - \xi/L_1}\left(e^{-\sigma_n^2 \epsilon_2/\gamma_2 L_2} - e^{-\xi/L_2}\right) & \\[2mm] \approx \dfrac{\sigma_n^2 \epsilon_1}{L_2(\gamma_1 - \epsilon_1\gamma_2)} - \left(\dfrac{\sigma_n^2 \epsilon_1}{L_2(\gamma_1 - \epsilon_1\gamma_2)} - \dfrac{\sigma_n^2 \epsilon_2}{\gamma_2 L_2}\right)\left(1 - \dfrac{\sigma_n^2 \epsilon_1}{L_c} - \dfrac{\sigma_n^2 \epsilon_1}{L_1(\gamma_1 - \epsilon_1\gamma_2)}\right) & \text{else.} \end{cases} \tag{43}$$

phase corresponds to one of the SIC steps, especially for users $j+1, \cdots, K$. Let $V_{i,j}$ be the SINR for decoding s_j at user i, where $i \geq j$, and

$$V_{i,j} = \frac{|h_i|^2 \gamma_j}{|h_i|^2 \sum_{k=j+1}^{K} \gamma_k + \sigma_n^2}. \tag{49}$$

Therefore, cooperation among users $j, \cdots, K$ for improved decoding of s_j is possible in the j-th cooperative phase. Among users $j, \cdots, K$, the system allows one whose channel condition is the strongest to transmit the cooperation signal. Let user i_0 be the strongest, i.e., $i_0 = \arg\max_{j \leq i \leq K} |h_i|^2 = \arg\max_{j \leq i \leq K} |V_{i,j}|^2$. Then, user i for $j \leq i \leq K$ and $i \neq i_0$ receives the cooperation signals c_i^j to help decoding of s_j from user i_0 given by

$$c_i^j = g_{i,i_0} s_j + n_i, \quad \forall i \in \{j, \cdots, K\}, \; i \neq i_0 \tag{50}$$

where g_{i,i_0} is channel fading from user i_0 to user i. User i_0 does not receive any cooperation signal. Let W_{i,i_0} be the SINR of the cooperation signal from user i_0 to user i, as written by

$$W_{i,i_0} = \frac{|g_{i,i_0}|^2}{\sigma_n^2}. \tag{51}$$

The j-th cooperation step can increase the data rate of user j, and/or help other users $j+1, \cdots, K$ to perform SIC better. The SINR for decoding s_j at user i is denoted by $Z_{\mathrm{cN},i,j}^{\mathrm{bi}}$ and obtained by

$$Z_{\mathrm{cN},i,j}^{\mathrm{bi}} = \begin{cases} V_{i,j} & i = i_0 \\ \max\{V_{i,j}, W_{i,i_0}\} & i \neq i_0, \end{cases} \quad \forall i \in \{j, \cdots, K\}. \tag{52}$$

Therefore, the total SINR for decoding s_j becomes as follows:

$$Z_{\mathrm{cN},j}^{\mathrm{bi}} = \min\{Z_{\mathrm{cN},j,j}^{\mathrm{bi}}, \cdots, Z_{\mathrm{cN},K,j}^{\mathrm{bi}}\}. \tag{53}$$

With the assumption that all users are located nearby, the cooperation signal is much stronger than the signal from the BS, i.e., $W_{i,i_0} \gg V_{i,j}$, $Z_{\mathrm{cN},j}^{\mathrm{bi}}$ can be approximated by

$$Z_{\mathrm{cN},j}^{\mathrm{bi}} \approx \tilde{Z}_{\mathrm{cN},j}^{\mathrm{bi}} = \max\{V_{j,j}, \cdots, V_{K,j}\}. \tag{54}$$

On the other hand, in uni-directional cooperative NOMA, the direction of cooperation is already determined; therefore, all users except for user K receive the cooperation signals from user K, whose channel variance is the largest in the statistical-CSIT case, or who are arbitrarily determined in the no-CSIT case. Therefore, the SINRs are given by

$$Z_{\mathrm{cN},i,j}^{\mathrm{uni}} = \begin{cases} V_{i,j} & i = K \\ \max\{V_{i,j}, W_{j,K}\} & i \neq i_0, \end{cases} \quad \text{for } j \leq i \leq K, \tag{55}$$

$$Z_{\mathrm{cN},j}^{\mathrm{uni}} = \min\{Z_{\mathrm{cN},j,j}^{\mathrm{uni}}, \cdots, Z_{\mathrm{cN},K,j}^{\mathrm{uni}}\} \tag{56}$$

$$\approx \tilde{Z}_{\mathrm{cN},j}^{\mathrm{uni}} = V_{K,j}. \tag{57}$$

Conventional NOMA does not allow any cooperation, so its SINR for decoding s_j is given by

$$Z_{\mathrm{N},j} = \min\{V_{j,j}, V_{j+1,i}, \cdots, V_{K,i}\}. \tag{58}$$
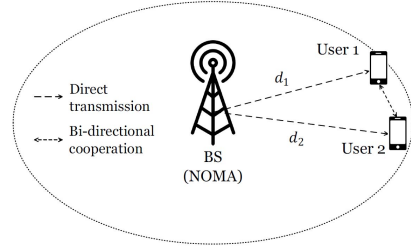

Fig. 1. Bi-directional NOMA model.

Mathematical analysis of ergodic capacity and outage probability for the multiuser model is omitted here, but we verify the advantages of bi-directional cooperative NOMA in the multiuser model by simulation. Section VII-D shows that bi-directional cooperative NOMA gives still better data rates than uni-directional cooperative NOMA, conventional NOMA and OMA for randomly generated multiple users.

Remark: Complexity for performing SIC is an important issue in the NOMA system. For conventional NOMA, $k-1$ times of SIC processes are required for the k-th strongest user and SIC processes of all users are performed independently. The number of required SIC processes for bi-directional cooperative NOMA is the same as that of conventional NOMA. Also, the cooperation phases are performed on signal-by-signal basis; therefore, the SIC step for decoding s_j is performed at every user $i \in \{j, \cdots, K\}$ in parallel. Therefore, bi-directional cooperative NOMA does not require the additional time slots for SIC processes of different users, as long as the j-th cooperation phase is successfully completed after SIC of s_j. On the other hand, the k-th strongest user of cooperative NOMA in [21] should wait for the others with the better channel conditions to finish the SIC processes and to transmit the cooperation signals.

VII. SIMULATION RESULTS

The simulation model is based on Fig. 1, and we assume $R = 50$. Without loss of generality, users 1 and 2 are assume to be the non-SIC user and SIC user, respectively. Short-hand notations "Bi-cN", "Uni-cN", and "NOMA" denote bi-directional cooperative NOMA, uni-directional cooperative NOMA, and conventional NOMA, respectively, in the figures.

A. Ergodic Capacity Under Statistical-CSIT Only

First, consider the statistical-CSIT case. For bi-directional cooperative NOMA, Algorithms 1 and 2 are used to find the optimal power allocations for maximization of the user fairness and sum-rate, respectively. For other schemes, uni-directional cooperative NOMA, conventional NOMA and OMA, the optimal power ratios are numerically found. The ergodic capacity plots versus d_2 are obtained with the fixed position of user 1, $d_1 = 40$. Since user 2 is the SIC user, $L_1 \leq L_2$, i.e., $d_2 \leq d_1$, in the statistical-CSIT case.

Figs. 2 and 3 show the performance of the user-fairness and sum rate in the two-user scenario with the transmit SNR of 10 dB, respectively. $R_{t,1} = R_{t,2} = 0.8$ is assumed for the sum-rate results. Each figure includes both ergodic capacity
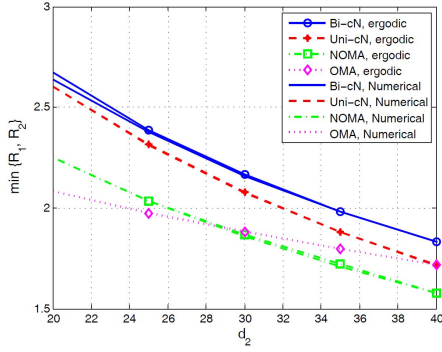
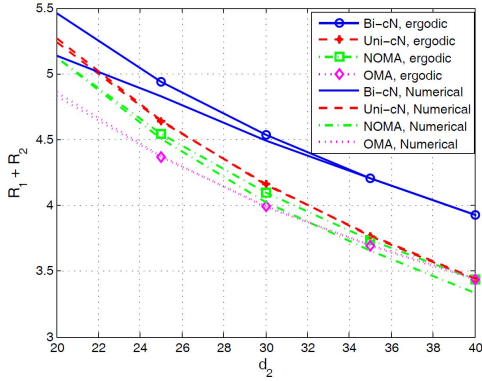Fig. 2.   User fairness in statistical-CSIT case, SNR=10 dB.

Fig. 3.   Sum rate in statistical-CSIT case, SNR=10 dB.

Fig. 4.   Sum-rate in no-CSIT case, SNR=10dB.

Fig. 5.   Outage probabilities when $d_1 = 40, d_2 = 20, R_{t1} = 0.7$, $R_{t2} = 1.5$.

and numerically obtained data rates, and we can easily see that both are almost the same as d_2 approaches to $d_1 = 40$. This indicates that the assumption that the users are located nearby is reliable. When d_2 is small, the ergodic capacity and the numerically obtained sum-rate of bi-directional cooperative NOMA are somewhat different because Algorithm 2 is sub-optimal, as mentioned earlier. In Figs. 2 and 3, bi-directional cooperative NOMA gives better performances of both user fairness and sum-rate than other schemes. As d_2 approaches to d_1, the channel gain difference between the users decreases, therefore the capacity gain of NOMA compared to OMA also decreases. Therefore, uni-directional cooperative NOMA has the same performance as OMA and conventional NOMA becomes even worse than OMA, when $d_1 \approx d_2$. Whereas, bi-directional cooperative NOMA is still better than OMA; thus it can be said that bi-directional cooperative NOMA is useful even when the channel gain difference of users is not large.

B. Ergodic Capacity Under No-CSIT

In this subsection, we consider the situation where the BS does not know users' CSI at all. Again, the two-user scenario is considered with $d_1 = 40$. Since no CSI is available at the BS, the BS arbitrarily determines users 1 and 2 as the non-SIC user and SIC user, respectively, therefore $d_2 > d_1$ is possible. It is impossible to find the optimal power allocation, therefore the fixed power allocation is used. $\gamma_1 = 0.8$ is assumed for NOMA schemes. $\gamma_1 = 0.5$ and $\alpha_1 = 0.5$ are used for OMA because the fair power allocation is preferable for OMA without CSI. Also, the only sum-rate performance is
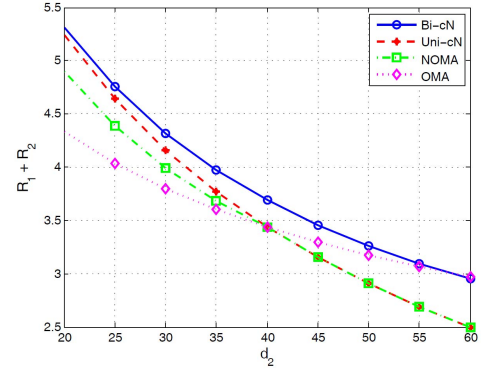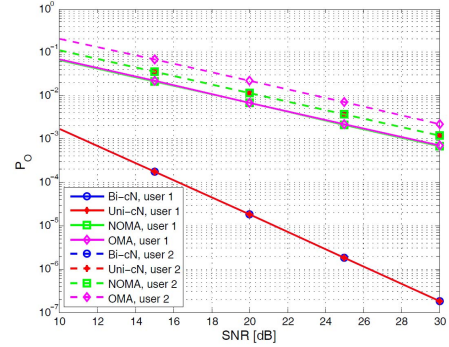
investigated in the no-CSIT case, because the trend of the user-fairness performance depends largely on the power allocation ratios.

Fig. 4 shows the sum-rate graphs with no CSIT and a transmit SNR of 10 dB. In Fig. 4, it is easily noticeable that bi-directional cooperative NOMA gives better capacity than other NOMA schemes and OMA. When $d_2 < d_1$, capacity gains of bi-directional cooperative NOMA over other NOMA schemes are not large, but its gain over OMA is large owing to SIC. When $d_2 \geq d_1$, since the channel gain of the SIC user usually becomes smaller than that of the non-SIC user, the advantage of NOMA and SIC vanishes. Therefore, uni-directional cooperative NOMA and conventional NOMA become worse than OMA. However, bi-directional cooperative NOMA still has a capacity gain compared to OMA even when $d_2 \geq d_1$. These results are consistent with Theorems 2 and 3. Especially when $d_1 = d_2 = 40$, bi-directional cooperative NOMA shows a capacity increase of 15% compared to OMA. As d_2 increases much, bi-directional cooperative NOMA would give a smaller sum-rate than OMA, but its gap is relatively small, compared to the region of $d_2 \leq d_1$.

C. Outage Probability

As shown in Section V, the outage probability is computed by channel variances and power allocation ratios. The power allocation and $R_{t,1}$ are appropriately chosen to satisfy $\frac{\gamma_1}{\gamma_2} > \epsilon_1$, and $\gamma_1 = 0.75$ is assumed here. Figs. 5 and 6 give the outage probability performance when $d_1 = 40$ and $d_2 = 20$. Fig. 5 assumes $R_{t,1} = 0.7$ and $R_{t,2} = 1.5$, and Fig. 6 is obtained with $R_{t,1} = 1.5$ and $R_{t,2} = 0.7$. In other words,
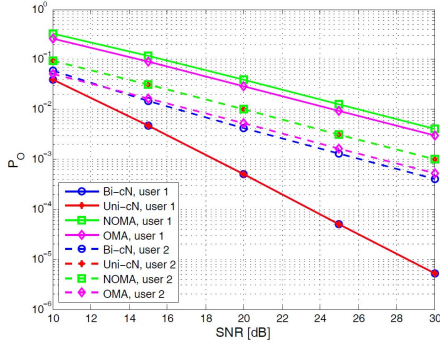
Fig. 6. Outage probabilities when $d_1 = 40, d_2 = 20, R_{t1} = 1.5, R_{t2} = 0.7$.

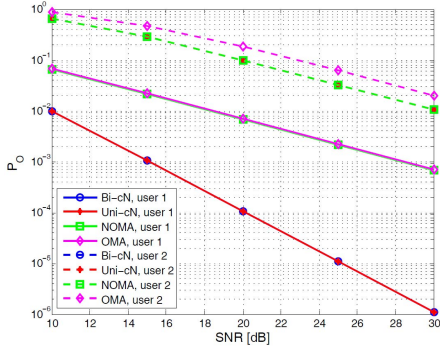
Fig. 7. Outage probabilities when $d_1 = 40, d_2 = 60, R_{t1} = 0.7, R_{t2} = 1.5$.
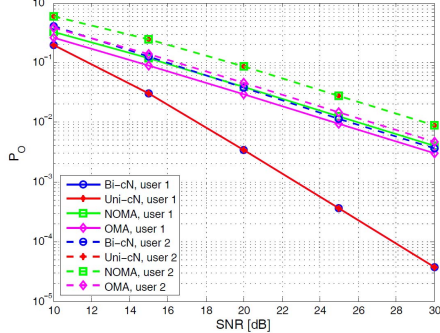

Fig. 8. Outage probabilities when $d_1 = 40, d_2 = 60, R_{t1} = 1.5, R_{t2} = 0.7$.

Fig. 5 satisfies the condition of $\frac{\epsilon_2}{\gamma_2} > \frac{\epsilon_1}{\gamma_1 - \epsilon_1 \gamma_2}$; therefore, bi- and uni-directional cooperative NOMA schemes show exactly the same outage probabilities for both users. Also, it is easily noted by the slopes of graphs that bi- and uni-directional cooperative NOMA schemes provide a better diversity order than conventional NOMA and OMA for user 1, but not for user 2. The difference in diversity order is also observed in Fig. 6, and two cooperative NOMA schemes still have the same outage probability of user 1. However, since $\frac{\epsilon_2}{\gamma_2} < \frac{\epsilon_1}{\gamma_1 - \epsilon_1 \gamma_2}$ is satisfied in Fig. 6, the outage probability of user 2 of bi-directional cooperative NOMA has a power gain compared to uni-directional cooperative NOMA.

Figs. 7 and 8 give the outage probability performance obtained with $d_1 = 40$ and $d_2 = 60$, respectively. Since $L_1 > L_2$ in Figs. 7 and 8, those results only correspond to the no-CSIT case. The plots in Figs. 7 and 8 show almost the
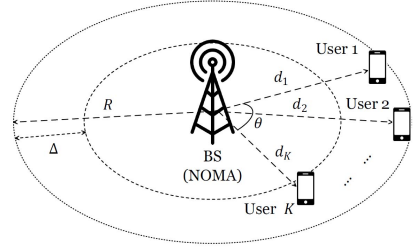

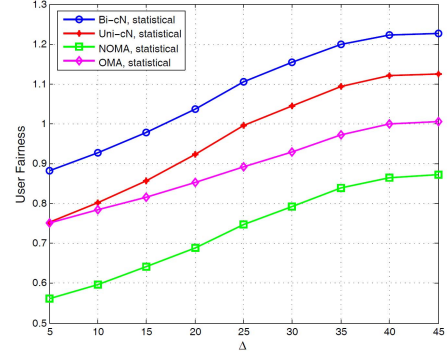Fig. 9. Cellular model of randomly generated multiple users.

Fig. 10. Capacity for user fairness in cellular model of randomly generated users.

same trends as those in Figs. 5 and 6, except for a little bit of power gain differences. The power gain of bi-directional cooperative NOMA over uni-directional cooperative NOMA and the diversity order gains over conventional NOMA and OMA are still guaranteed even when $d_1 = 40$ and $d_2 = 60$.

D. Capacity of Randomly Generated Users

This section considers the cellular model of randomly positioned multiple users, as shown in Fig. 9. In Fig. 9, K users are uniformly placed in the outer ring of the cell of radius $R = 50$, i.e., the region between inner and outer circles whose radii are $R - \Delta$ and R, respectively, therefore $d_k \in [R - \Delta, R]$, $\forall k = \{1, \cdots, K\}$. When Δ is very small, the only cell-edge users are chosen, and as Δ increases, almost all of the cell region is covered. Also, K users are separated by an angle smaller than θ to guarantee that all users are close to one another. It makes the exchange of CSI among users easier and the cooperation more helpful. We only presents an optimal power allocation rule for the two-user model, therefore the optimal power allocations for K users are numerically obtained in the statistical-CSIT case. On the other hand, in the case of no-CSIT, the fixed power allocation is applied. Assume that $\gamma_k = 2\gamma_{k+1}$ for NOMA schemes, and $\gamma_1 = \cdots = \gamma_K = 0.5$ and $\alpha_1 = \cdots = \alpha_K = 0.5$ are used for OMA. Similar to Section VII-B, the sum-rate performance is only considered in the no-CSIT case.

Figs. 10 and 11 show plots of data rates versus Δ obtained from the user-fairness and sum-rate problems, respectively, with $K = 4$ randomly located users. Solid and dashed graphs correspond to the cases of statistical CSIT and no CSIT, respectively. All graphs show increasing trends over Δ, because users are likely to have stronger channel gains with a larger Δ. We can easily see that bi-directional cooperative
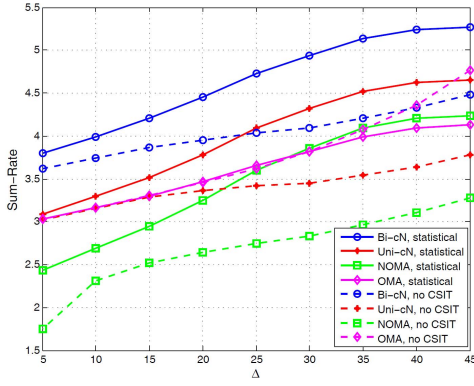
Fig. 11. Capacity for sum-rate in cellular model of randomly generated users.

NOMA gives the best data rates among comparison schemes both in the statistical-CSIT and no-CSIT cases. Especially in the statistical-CSIT case, when Δ is small, channel gain differences among users are not sufficiently large to show the advantage of NOMA schemes compared to OMA; thus, the performance of uni-directional cooperative NOMA is similar to that of OMA, and conventional NOMA is worse than OMA. On the other hand, bi-directional cooperative NOMA is still better than OMA even with small Δ in terms of both user fairness and sum rate.

The sum-rate performance in the no-CSIT case is much worse than that in the statistical-CSIT case as Δ increases. The reason is that when Δ is large, the situation in which users with the weaker channel conditions perform SIC for decoding the signal of users with the stronger channel gains so the advantage of NOMA vanishes happens frequently. In this situation, a smaller power is allocated to the weaker user than the stronger one, therefore uni-directional cooperative NOMA and conventional NOMA give worse sum rates than OMA in the no-CSIT case. However, the sum rates of bi-directional cooperative NOMA are still better than other NOMA schemes as well as OMA in most values of Δ, even when the BS does not have accurate knowledge of users' CSI and thus arbitrarily schedules the users for signal transmissions. This means that bi-directional cooperative NOMA is useful when there is little need for channel tracking or elaborate user scheduling.

VIII. CONCLUSION

This paper proposes bi-directional cooperative NOMA, in which NOMA users cooperate with each other by the channel information exchange with statistical CSIT only or no CSIT. The performance analysis has been conducted in terms of ergodic capacity and outage probability. The closed-form ergodic capacity of bi-directional cooperative NOMA in the two-user model is derived, and it is shown to be better than those of uni-directional cooperative NOMA, conventional NOMA, and OMA even when the scheduled users have similar channel gains. On the basis of the ergodic capacity analysis, algorithms to find the optimal power allocations are presented for the user-fairness and max-sum-rate problems. In addition, we show that the outage probability of the SIC user of bi-directional cooperative NOMA has a power

gain over that of uni-directional cooperative NOMA, and bi- and uni-directional cooperative NOMA schemes have a diversity gain over conventional NOMA and OMA. Also, the multi-user model of bi-directional cooperative NOMA is presented by using the cooperation among users on signal-by-signal basis. Simulation results verify the above performance analyses, and that bi-directional cooperative NOMA works well with multiple users in statistical- and no-CSIT cases. The proposed system is beneficial in some important practical scenarios: a highly crowded stadium in which many users experience similar channel gains, and an IoT environment in which an inexpensive transmitter should serve a massive number of machine-type devices but cannot handle substantial processing tasks; thus, not enough CSI is available at the BS.

APPENDIX A
PROOF OF THEOREM 1

$|h_1|^2$ and $|h_2|^2$ are chi-square distributed with variances of $L_1/2$ and $L_2/2$, respectively, so $|h_i|^2 = L_i X_i/2$, for $i = \{1, 2\}$, where X_i is a chi-square random variable of unit variance. According to (9),

$$\mathbb{E}[\tilde{R}_{cN,1}^{bi}] = \iint p(x_1, x_2)\tilde{R}_{cN,1}^{bi}\mathrm{d}x_1\mathrm{d}x_2 \tag{59}$$

$$= \iint_{|h_1|^2 > |h_2|^2} p(x_1, x_2)Z_1\mathrm{d}x_1\mathrm{d}x_2$$

$$+ \iint_{|h_1|^2 < |h_2|^2} p(x_1, x_2)Z_{2,SIC}\mathrm{d}x_1\mathrm{d}x_2. \tag{60}$$

Since X_1 and X_2 are independent, $p(x_1, x_2) = p(x_1)p(x_2)$, the first term of (60) becomes

$$\int_0^\infty \int_0^{\frac{L_1}{L_2}x_1} \frac{1}{4}e^{-\frac{x_1}{2}}e^{-\frac{x_2}{2}}$$

$$\times \left\{ \log_2\left(1 + \frac{\gamma_1\,L_1\,x_1}{\gamma_2\,L_1\,x_1 + 2\sigma_n^2}\right)\right\}\mathrm{d}x_1\mathrm{d}x_2 \tag{61}$$

$$= \int_0^\infty \frac{1}{2}(e^{-\frac{x_1}{2}} - e^{-\frac{L_1+L_2}{2L_2}x_1})$$

$$\times \left\{ \log_2\left(1 + \frac{L_1\,x_1}{2\sigma_n^2}\right) - \log_2\left(1 + \frac{\gamma_2\,L_1\,x_1}{2\sigma_n^2}\right)\right\}\mathrm{d}x_1 \tag{62}$$

$$= C_1\left(\frac{L_1}{\sigma_n^2}\right) - C_1\left(\frac{\gamma_2 L_1}{\sigma_n^2}\right) - \frac{L_2}{L_1 + L_2}$$

$$\times \left\{ C_1\left(\frac{L_1 L_2}{(L_1 + L_2)\sigma_n^2}\right) - C_1\left(\frac{\gamma_2 L_1 L_2}{(L_1 + L_2)\sigma_n^2}\right)\right\}. \tag{63}$$

The last equation (63) holds by Lemma 12.

Likewise, the second term of (60) is

$$\int_0^\infty \int_0^{\frac{L_2}{L_1}x_2} \frac{1}{4}e^{-\frac{x_1}{2}}e^{-\frac{x_2}{2}}$$

$$\log_2\left(1 + \frac{\gamma_1\,L_2\,x_2}{\gamma_2\,L_2\,x_2 + 2\sigma_n^2}\right)\mathrm{d}x_1\mathrm{d}x_2 \tag{64}$$

$$= C_1\left(\frac{L_2}{\sigma_n^2}\right) - C_1\left(\frac{\gamma_2 L_2}{\sigma_n^2}\right) - \frac{L_1}{L_1 + L_2}$$

$$\times \left\{ C_1\left(\frac{L_1 L_2}{(L_1 + L_2)\sigma_n^2}\right) - C_1\left(\frac{\gamma_2 L_1 L_2}{(L_1 + L_2)\sigma_n^2}\right)\right\}. \tag{65}$$

$$\mathbb{E}[\tilde{R}_{cN,1}^{bi}] = C_1\Big(\frac{L_1}{\sigma_n^2}\Big) - C_1\Big(\frac{\gamma_2 L_1}{\sigma_n^2}\Big) + C_1\Big(\frac{L_2}{\sigma_n^2}\Big) - C_1\Big(\frac{\gamma_2 L_2}{\sigma_n^2}\Big) - C_1\Big(\frac{L_1 L_2}{(L_1+L_2)\sigma_n^2}\Big) + C_1\Big(\frac{\gamma_2 L_1 L_2}{(L_1+L_2)\sigma_n^2}\Big) \quad (66)$$

Therefore, $\mathbb{E}[\tilde{R}_{cN,1}^{bi}]$ is given by (66), as shown at the top of this page.

Next,

$$\mathbb{E}[R_{cN,2}^{bi}]$$
$$= \int_0^\infty \int_0^\infty \frac{1}{4} e^{-\frac{x_1}{2}} e^{-\frac{x_2}{2}} \log_2\Big(1 + \frac{\gamma_2\, L_2\, x_2}{2\sigma_n^2}\Big) \mathrm{d}x_1 \mathrm{d}x_2 \tag{67}$$
$$= \int_0^\infty \frac{1}{2} e^{-\frac{x_2}{2}} \log_2\Big(1 + \frac{\gamma_2\, L_2\, x_2}{2\sigma_n^2}\Big) \mathrm{d}x_2 \tag{68}$$
$$= C_1\Big(\frac{\gamma_2 L_2}{\sigma_n^2}\Big). \tag{69}$$

Equation (69) holds by Lemma 12. Thus, closed-form ergodic capacity (17) can be obtained.

APPENDIX B
PROOF OF THEOREM 2

$$\mathbb{E}[\tilde{R}_{cN,1}^{bi}]$$
$$= \mathbb{E}\Big[\max\Big\{ \log_2\Big(1 + \frac{|h_1|^2\gamma_1}{|h_1|^2\gamma_2 + \sigma_n^2}\Big),$$
$$\log_2\Big(1 + \frac{|h_2|^2\gamma_1}{|h_2|^2\gamma_2 + \sigma_n^2}\Big)\Big\}\Big] \tag{70}$$
$$\geq \max\Big\{ \mathbb{E}\Big[\log_2\Big(1 + \frac{|h_1|^2\gamma_1}{|h_1|^2\gamma_2 + \sigma_n^2}\Big)\Big],$$
$$\mathbb{E}\Big[\log_2\Big(1 + \frac{|h_2|^2\gamma_1}{|h_2|^2\gamma_2 + \sigma_n^2}\Big)\Big]\Big\} \tag{71}$$
$$= \max\Big\{ C_1\Big(\frac{L_1}{\sigma_n^2}\Big) - C_1\Big(\frac{\gamma_2 L_1}{\sigma_n^2}\Big),\ C_1\Big(\frac{L_2}{\sigma_n^2}\Big) - C_1\Big(\frac{\gamma_2 L_2}{\sigma_n^2}\Big)\Big\} \tag{72}$$

The last equation (72) holds by Lemma 1.

For uni-directional cooperative NOMA system, according to 1,

$$\mathbb{E}[\tilde{R}_{cN,1}^{uni}] = \mathbb{E}\Big[\log_2\Big(1 + \frac{|h_2|^2\gamma_1}{|h_2|^2\gamma_2 + \sigma_n^2}\Big)\Big] \tag{73}$$
$$= C_1\Big(\frac{L_2}{\sigma_n^2}\Big) - C_1\Big(\frac{\gamma_2 L_2}{\sigma_n^2}\Big) \tag{74}$$

Likewise, the upper bound on ergodic capacity of conventional NOMA system becomes

$$\mathbb{E}[R_{N,1}]$$
$$= \mathbb{E}\Big[\min\Big\{ \log_2\Big(1 + \frac{|h_1|^2\gamma_1}{|h_1|^2\gamma_2 + \sigma_n^2}\Big),$$
$$\log_2\Big(1 + \frac{|h_2|^2\gamma_1}{|h_2|^2\gamma_2 + \sigma_n^2}\Big)\Big\}\Big] \tag{75}$$
$$\leq \min\Big\{ \mathbb{E}\Big[\log_2\Big(1 + \frac{|h_1|^2\gamma_1}{|h_1|^2\gamma_2 + \sigma_n^2}\Big)\Big],$$
$$\mathbb{E}\Big[\log_2\Big(1 + \frac{|h_2|^2\gamma_1}{|h_2|^2\gamma_2 + \sigma_n^2}\Big)\Big]\Big\} \tag{76}$$
$$= \min\Big\{ C_1\Big(\frac{L_1}{\sigma_n^2}\Big) - C_1\Big(\frac{\gamma_2 L_1}{\sigma_n^2}\Big),\ C_1\Big(\frac{L_2}{\sigma_n^2}\Big) - C_1\Big(\frac{\gamma_2 L_2}{\sigma_n^2}\Big)\Big\} \tag{77}$$

Note that the data rates of s_2 of three schemes are all the same,

$$\mathbb{E}[R_{cN,2}^{bi}] = \mathbb{E}[R_{cN,2}^{uni}] = \mathbb{E}[R_{N,2}]$$
$$= \mathbb{E}\Big[\log_2\Big(1 + \frac{|h_2|^2\gamma_2}{\sigma_n^2}\Big)\Big] = C_1\Big(\frac{\gamma_2 L_2}{\sigma_n^2}\Big) \tag{78}$$

Since $\max\{x,y\} \geq x \geq \min\{x,y\}$, $\forall x,y \in \mathbb{R}$, $\mathbb{E}[\tilde{R}_{cN}^{bi}] \geq \mathbb{E}[\tilde{R}_{cN}^{uni}] \geq \mathbb{E}[R_N]$ is satisfied according to (72), (74), (77), and (78).

APPENDIX C
PROOF OF THEOREM 3

First, concavity of $C_1(x)$ for $x > 1$ is proved. Differentiating $C_1(x)$ twice,

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2} C_1(x) = \Big(\frac{2}{x^3} + \frac{1}{x^4}\Big)C_1(x) - \frac{1}{\ln 2}\Big(\frac{1}{x^3} + \frac{1}{x^2}\Big) \tag{79}$$

Let $f(x) = x^4 \frac{\mathrm{d}^2}{\mathrm{d}x^2} C_1(x) = (2x+1)C_1(x) - \frac{1}{\ln 2}(x^2 + x)$. Then,

$$x^2 \frac{\mathrm{d}}{\mathrm{d}x} f(x) = (2x^2 - x - 1)\Big(C_1(x) - \frac{x}{\ln 2}\Big) - xC_1(x) \tag{80}$$
$$< \frac{1}{\ln 2}(2x^2 - x - 1)(\ln(1+x) - x) - xC_1(x) \tag{81}$$

Since $2x^2 - x - 1 > 0$ when $x > 1$ and $x > \ln(1 + x)$, equation (81) is smaller than 0 when $x > 1$. $f(1) < 0$ and $\frac{\mathrm{d}}{\mathrm{d}x} f(x) < 0$, so $f(x)$ is a strictly decreasing function of x for $x > 1$. Therefore, $C_1(x)$ is a strictly concave function of x for $x > 1$.

Then,

$$\mathbb{E}[\tilde{R}_{cN}^{bi}] - \mathbb{E}[R_O] \tag{82}$$
$$= C_1\Big(\frac{L}{\sigma_n^2}\Big) - C_1\Big(\frac{\gamma_2 L}{\sigma_n^2}\Big) - \Big\{ C_1\Big(\frac{L}{2\sigma_n^2}\Big) - C_1\Big(\frac{\gamma_2 L}{2\sigma_n^2}\Big)\Big\}$$
$$+ C_1\Big(\frac{L}{\sigma_n^2}\Big) - \alpha_1 C_1\Big(\frac{\gamma_1 L}{\alpha_1 \sigma_n^2}\Big) - \alpha_2 C_1\Big(\frac{\gamma_2 L}{\alpha_2 \sigma_n^2}\Big) \tag{83}$$

By Lemma 4,

$$C_1\Big(\frac{L}{\sigma_n^2}\Big) - C_1\Big(\frac{\gamma_2 L}{\sigma_n^2}\Big) \geq C_1\Big(\frac{L}{2\sigma_n^2}\Big) - C_1\Big(\frac{\gamma_2 L}{2\sigma_n^2}\Big), \tag{84}$$

and since $C_1(x)$ is strictly concave for $x > 1$,

$$C_1\Big(\frac{L}{\sigma_n^2}\Big) > \alpha_1 C_1\Big(\frac{\gamma_1 L}{\alpha_1 \sigma_n^2}\Big) + \alpha_2 C_1\Big(\frac{\gamma_2 L}{\alpha_2 \sigma_n^2}\Big), \tag{85}$$

according to Jensen's inequality, if $\frac{L}{\sigma_n^2}, \frac{\gamma_1\, L}{\alpha_1 \sigma_n^2}, \frac{\gamma_2\, L}{\alpha_2 \sigma_n^2} > 1$. Theorem 3 is proved.

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