Binary search tree

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In computer science, a **binary search tree** (**BST**), which may sometimes also be called an **ordered** or **sorted binary tree**, is a node-based binary tree data structure which has the following properties:[1]

- The left subtree of a node contains only nodes with keys less than the node's key.
- The right subtree of a node contains only nodes with keys greater than the node's key.
- Both the left and right subtrees must also be binary search trees.
- There must be no duplicate nodes.

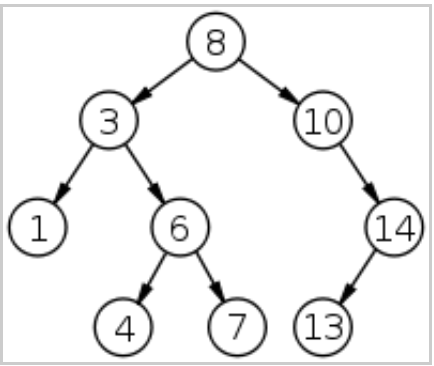
Binary search tree

Type	Tree	
	Time complexity in big O notation	
	Average	Worst case
Space	O(n)	O(n)
Search	O(log n)	O(n)
Insert	O(log n)	O(n)
Delete	O(log n)	O(n)

Generally, the information represented by each node is a record rather than a single data element. However, for sequencing purposes, nodes are compared according to their keys rather than any part of their associated records.

The major advantage of binary search trees over other data structures is that the related sorting algorithms and search algorithms such as in-order traversal can be very efficient.

Binary search trees are a fundamental data structure used to construct more abstract data structures such as sets, multisets, and associative arrays.



A binary search tree of size 9 and depth 3, with root 8 and leaves 1, 4, 7 and 13

Contents

Operations

Operations on a binary search tree require comparisons between nodes. These comparisons are made with calls to a comparator, which is a subroutine that computes the total order (linear order) on any two keys. This comparator can be explicitly or implicitly defined, depending on the language in which the BST is implemented.

Searching

Searching a binary search tree for a specific key can be a recursive or iterative process.

We begin by examining the root node. If the tree is null, the key we are searching for does not exist in the tree. Otherwise, if the key equals that of the root, the search is successful. If the key is less than the root, search the left subtree. Similarly, if it is greater than the root, search the right subtree. This process is repeated until the key is found or the remaining subtree is null. If the searched key is not found before a null subtree is reached, then the item must not be present in the tree.

Here is the search algorithm in pseudocode (iterative version, finds a BST node):

```
algorithm Find(key, root):
    current-node := root
    while current-node is not Nil do
        if current-node.key = key then
            return current-node
        else if key < current-node.key then
            current-node := current-node.left
        else
            current-node := current-node.right
```

The following recursive version is equivalent:

```
algorithm Find-recursive(key, node):  // call initially with node = root
    if node.key = key then
        node
    else if key < node.key then
        Find-recursive(key, node.left)
    else
        Find-recursive(key, node.right)
```

This operation requires O(log n) time in the average case, but needs O(n) time in the worst case, when the unbalanced tree resembles a linked list (degenerate tree).

Insertion

Insertion begins as a search would begin; if the key is not equal to that of the root, we search the left or right subtrees as before. Eventually, we will reach an external node and add the new key-value pair (here encoded as a record 'newNode') as its right or left child, depending on the node's key. In other words, we examine the root

and recursively insert the new node to the left subtree if its key is less than that of the root, or the right subtree if its key is greater than or equal to the root.

Here's how a typical binary search tree insertion might be performed in C++:

```
/* Inserts the node pointed to by "newNode" into the subtree rooted at "treeNode" */
void InsertNode(Node* &treeNode, Node *newNode)
{
    if (treeNode == NULL)
      treeNode = newNode;
    else if (newNode->key < treeNode->key)
      InsertNode(treeNode->left, newNode);
    else
      InsertNode(treeNode->right, newNode);
}
```

The above *destructive* procedural variant modifies the tree in place. It uses only constant heap space (and the iterative version uses constant stack space as well), but the prior version of the tree is lost. Alternatively, as in the following Python example, we can reconstruct all ancestors of the inserted node; any reference to the original tree root remains valid, making the tree a persistent data structure:

```
def binary_tree_insert(node, key, value):
    if node is None:
        return TreeNode(None, key, value, None)
    if key == node.key:
        return TreeNode(node.left, key, value, node.right)
    if key < node.key:
        return TreeNode(binary_tree_insert(node.left, key, value), node.key, node.value, node.right)
    else:
        return TreeNode(node.left, node.key, node.value, binary_tree_insert(node.right, key, value))
```

The part that is rebuilt uses $\Theta(\log n)$ space in the average case and $O(n)$ in the worst case (see big-O notation).

In either version, this operation requires time proportional to the height of the tree in the worst case, which is $O(\log n)$ time in the average case over all trees, but $O(n)$ time in the worst case.

Another way to explain insertion is that in order to insert a new node in the tree, its key is first compared with that of the root. If its key is less than the root's, it is then compared with the key of the root's left child. If its key is greater, it is compared with the root's right child. This process continues, until the new node is compared with a leaf node, and then it is added as this node's right or left child, depending on its key.

There are other ways of inserting nodes into a binary tree, but this is the only way of inserting nodes at the leaves and at the same time preserving the BST structure.

Here is an iterative approach to inserting into a binary search tree in Java:

```
private Node m_root;

public void insert(int data) {
    if (m_root == null) {
        m_root = new TreeNode(data, null, null);
        return;
    }
    Node root = m_root;
    while (root != null) {
        // Choose not add 'data' if already present (an implementation decision)
        if (data == root.getData()) {
            return;
        } else if (data < root.getData()) {
            // insert left
            if (root.getLeft() == null) {
                root.setLeft(new TreeNode(data, null, null));
                return;
            } else {
                root = root.getLeft();
            }
        } else {
            // insert right
            if (root.getRight() == null) {
                root.setRight(new TreeNode(data, null, null));
                return;
            } else {
                root = root.getRight();
            }
        }
    }
}
```

Below is a recursive approach to the insertion method.

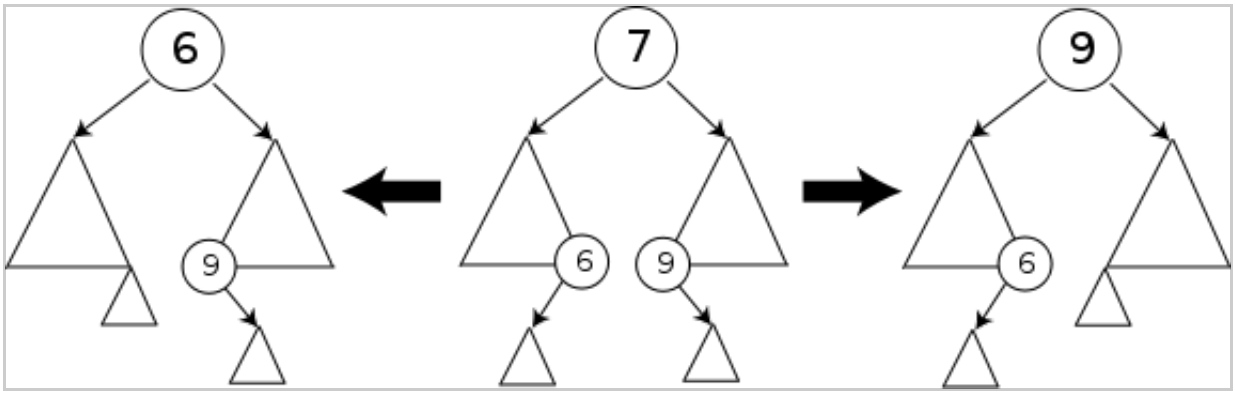
```
private Node m_root;

public void insert(int data){
    if (m_root == null) {
        m_root = new TreeNode(data, null, null);
    }else{
        internalInsert(m_root, data);
    }
}

private static void internalInsert(Node node, int data){
    // Choose not add 'data' if already present (an implementation decision)
    if (data == node.getKey()) {
        return;
    } else if (data < node.getKey()) {
        if (node.getLeft() == null) {
            node.setLeft(new TreeNode(data, null, null));
        }else{
            internalInsert(node.getLeft(), data);
        }
    }else{
        if (node.getRight() == null) {
            node.setRight(new TreeNode(data, null, null));
        }else{
            internalInsert(node.getRight(), data);
        }
    }
}
```

Deletion

There are three possible cases to consider:

- **Deleting a leaf (node with no children):** Deleting a leaf is easy, as we can simply remove it from the tree.
- **Deleting a node with one child:** Remove the node and replace it with its child.
- **Deleting a node with two children:** Call the node to be deleted *N*. Do not delete *N*. Instead, choose either its in-order successor node or its in-order predecessor node, *R*. Replace the value of *N* with the value of *R*, then delete *R*.

As with all binary trees, a node's in-order successor is the left-most child of its right subtree, and a node's in-order predecessor is the right-most child of its left subtree. In either case, this node will have zero or one children. Delete it according to one of the two simpler cases above.



Deleting a node with two children from a binary search tree. The triangles represent subtrees of arbitrary size, each with its leftmost and rightmost child nodes at the bottom two vertices.

Consistently using the in-order successor or the in-order predecessor for every instance of the two-child case can lead to an unbalanced tree, so good implementations add inconsistency to this selection.

Running time analysis: Although this operation does not always traverse the tree down to a leaf, this is always a possibility; thus in the worst case it requires time proportional to the height of the tree. It does not require more even when the node has two children, since it still follows a single path and does not visit any node twice.

Here is the code in Python:

```
def findMin(self):
    '''
    Finds the smallest element that is a child of *self*
    '''
    current_node = self
    while current_node.left_child:
        current_node = current_node.left_child
    return current_node

def replace_node_in_parent(self, new_value=None):
    '''
    Removes the reference to *self* from *self.parent* and replaces it with *new_value*.
    '''
    if self.parent:
        if self == self.parent.left_child:
            self.parent.left_child = new_value
        else:
            self.parent.right_child = new_value
    if new_value:
        new_value.parent = self.parent

def binary_tree_delete(self, key):
    if key < self.key:
        self.left_child.binary_tree_delete(key)
    elif key > self.key:
        self.right_child.binary_tree_delete(key)
    else: # delete the key here
        if self.left_child and self.right_child: # if both children are present
            # get the smallest node that's bigger than *self*
            successor = self.right_child.findMin()
            self.key = successor.key
            # if *successor* has a child, replace it with that
            # at this point, it can only have a *right_child*
            # if it has no children, *right_child* will be "None"
            successor.replace_node_in_parent(successor.right_child)
        elif self.left_child or self.right_child:   # if the node has only one child
            if self.left_child:
                self.replace_node_in_parent(self.left_child)
            else:
                self.replace_node_in_parent(self.right_child)
        else: # this node has no children
            self.replace_node_in_parent(None)
```

Traversal

Main article: Tree traversal

Once the binary search tree has been created, its elements can be retrieved in-order by recursively traversing the left subtree of the root node, accessing the node itself, then recursively traversing the right subtree of the node, continuing this pattern with each node in the tree as it's recursively accessed. As with all binary trees, one may conduct a pre-order traversal or a post-order traversal, but neither are likely to be useful for binary search trees.

The code for in-order traversal in Python is given below. It will call **callback** for every node in the tree.

```
def traverse_binary_tree(node, callback):
    if node is None:
        return
    traverse_binary_tree(node.leftChild, callback)
    callback(node.value)
    traverse_binary_tree(node.rightChild, callback)
```

Traversal requires O(n) time, since it must visit every node. This algorithm is also O(n), so it is asymptotically optimal.

An in-order traversal algorithm for C is given below.

```
void in_order_traversal(struct Node *n, void (*cb)(void*))
{
        struct Node *cur, *pre;

        if(!n)
                return;

        cur = n;

        while(cur) {
                if(!cur->left) {
                        cb(cur->val);
                        cur= cur->right;
                } else {
                        pre = cur->left;

                        while(pre->right && pre->right != cur)
                                pre = pre->right;

                        if (!pre->right) {
                                pre->right = cur;
                                cur = cur->left;
                        } else {
                                pre->right = NULL;
                                cb(cur->val);
                                cur = cur->right;
                        }
                }
        }
}
```

An alternate recursion-free algorithm for in-order traversal using a stack and `goto` statements is provided below. The stack contains nodes whose right subtrees have yet to be explored. If a node has an unexplored left subtree (a condition tested at the `try_left` label, then the node is pushed (marking its right subtree for future exploration) and the algorithm descends to the left subtree. The purpose of the `loop_top` label is to avoid moving to the left subtree when popping to a node (as popping to a node indicates that its left subtree has already been explored.)

```c
void in_order_traversal(struct Node *n, void (*cb)(void*))
{
        struct Node *cur;
        struct Stack *stack;

        if (!n)
                return;

        stack = stack_create();
        cur = n;

try_left:
        /* check for the left subtree */
        if (cur->left) {
                stack_push(stack, cur);
                cur = cur->left;
                goto try_left;
        }

loop_top:
        /* call callback */
        cb(cur->val);

        /* check for the right subtree */
        if (cur->right) {
                cur = cur->right;
                goto try_left;
        }

        cur = stack_pop(stack);
        if (cur)
                goto loop_top;

        stack_destroy(stack);
}
```

Sort

A binary search tree can be used to implement a simple but efficient sorting algorithm. Similar to heapsort, we insert all the values we wish to sort into a new ordered data structure—in this case a binary search tree—and then traverse it in order, building our result:

```python
def build_binary_tree(values):
    tree = None
    for v in values:
        tree = binary_tree_insert(tree, v)
    return tree

def get_inorder_traversal(root):
    '''
    Returns a list containing all the values in the tree, starting at *root*.
    Traverses the tree in-order(leftChild, root, rightChild).
    '''
    result = []
    traverse_binary_tree(root, lambda element: result.append(element))
    return result
```

The worst-case time of `build_binary_tree` is $O(n^2)$—if you feed it a sorted list of values, it chains them into a linked list with no left subtrees. For example, `build_binary_tree([1, 2, 3, 4, 5])` yields the tree `(1 (2 (3 (4 (5)))))`.

There are several schemes for overcoming this flaw with simple binary trees; the most common is the self-balancing binary search tree. If this same procedure is done using such a tree, the overall worst-case time is O(*n*log *n*), which is asymptotically optimal for a comparison sort. In practice, the poor cache performance and added overhead in time and space for a tree-based sort (particularly for node allocation) make it inferior to other asymptotically optimal sorts such as heapsort for static list sorting. On the other hand, it is one of the most efficient methods of *incremental sorting*, adding items to a list over time while keeping the list sorted at all times.

Types

There are many types of binary search trees. AVL trees and red-black trees are both forms of self-balancing binary search trees. A splay tree is a binary search tree that automatically moves frequently accessed elements nearer to the root. In a treap (*tree heap*), each node also holds a (randomly chosen) priority and the parent node has higher priority than its children. Tango trees are trees optimized for fast searches.

Two other titles describing binary search trees are that of a *complete* and *degenerate* tree.

A complete tree is a tree with n levels, where for each level d <= n - 1, the number of existing nodes at level d is equal to 2^d. This means all possible nodes exist at these levels. An additional requirement for a complete binary tree is that for the nth level, while every node does not have to exist, the nodes that do exist must fill from left to right.

A degenerate tree is a tree where for each parent node, there is only one associated child node. What this means is that in a performance measurement, the tree will essentially behave like a linked list data structure.

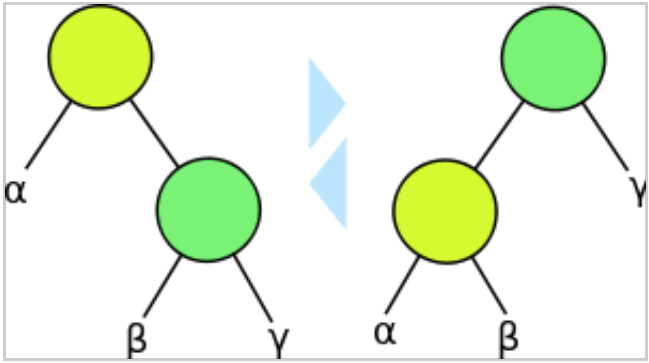
Performance comparisons

D. A. Heger (2004)[2] presented a performance comparison of binary search trees. Treap was found to have the best average performance, while red-black tree was found to have the smallest amount of performance variations.

Optimal binary search trees

If we do not plan on modifying a search tree, and we know exactly how often each item will be accessed, we can construct an *optimal binary search tree*, which is a search tree where the average cost of looking up an item (the *expected search cost*) is minimized.

Even if we only have estimates of the search costs, such a system can considerably speed up lookups on average. For example, if you have a BST of English words used in a spell checker, you might balance the tree based on word frequency in text corpora, placing words like *the* near the



Tree rotations are very common internal operations in binary trees to keep perfect, or near-to-perfect,

internal balance in the tree.

root and words like *agerasia* near the leaves. Such a tree might be compared with Huffman trees, which similarly seek to place frequently used items near the root in order to produce a dense information encoding; however, Huffman trees only store data elements in leaves and these elements need not be ordered.

If we do not know the sequence in which the elements in the tree will be accessed in advance, we can use splay trees which are asymptotically as good as any static search tree we can construct for any particular sequence of lookup operations.

Alphabetic trees are Huffman trees with the additional constraint on order, or, equivalently, search trees with the modification that all elements are stored in the leaves. Faster algorithms exist for *optimal alphabetic binary trees* (OABTs).

Example:

```
procedure Optimum Search Tree(f, f´, c):
 for j = 0 to n do
  c[j, j] = 0, F[j, j] = f´j
  for d = 1 to n do
   for i = 0 to (n − d) do
    j = i + d
    F[i, j] = F[i, j − 1] + f´ + f´j
    c[i, j] = MIN(i<k<=j){c[i, k − 1] + c[k, j]} + F[i, j]
```

See also

- Search tree
- Binary search algorithm
- Randomized binary search tree
- Tango trees

References

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2. ^ Heger, Dominique A. (2004), "A Disquisition on The Performance Behavior of Binary Search Tree Data Structures" (http://www.cepis.org/upgrade/files/full-2004-V.pdf) , *European Journal for the Informatics Professional* **5** (5): 67–75, http://www.cepis.org/upgrade/files/full-2004-V.pdf

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External links

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