



# *Business Statistics: A First Course*

## 6<sup>th</sup> Edition

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### **Chapter 9**

### 仮説検定

### Fundamentals of Hypothesis Testing: One-Sample Tests



# Learning Objectives

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## In this chapter, you learn:

- The basic principles of hypothesis testing 仮説検定の基本
- How to use hypothesis testing to test a mean or proportion  
平均を仮説検定する方法
- The assumptions of each hypothesis-testing procedure, how to evaluate them, and the consequences if they are seriously violated 仮説検定の前提とその前提に合わない場合の結果
- How to avoid the pitfalls involved in hypothesis testing  
仮説検定を行う時、落とし穴を回避する方法
- The ethical issues involved in hypothesis testing  
仮説検証に関わる倫理的問題

# What is a Hypothesis?

DCOVA

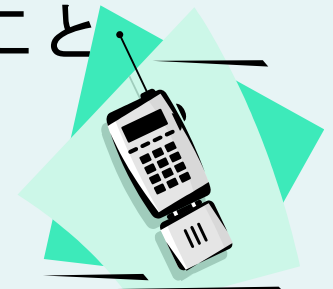
- A hypothesis is a claim (assertion) about a population parameter: 仮説とは、真偽はともかくとして、何らかの現象や法則性を説明するのに役立つ命題のこと

- population mean 母集団 平均

**Example: The mean monthly cell phone bill in this city is  $\mu = \$42$**

- population proportion 母集団 割合

**Example: The proportion of adults in this city with cell phones is  $\pi = 0.68$**



# The Null Hypothesis, $H_0$

## 帰無仮説

DCOVA

”帰無仮説とは、ある仮説”が正しいかどうかの判断のために立てられる仮説です。

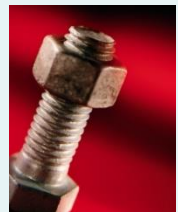
- States the claim or assertion to be tested

**Example:** The average diameter of a manufactured bolt is 30mm ( $H_0 : \mu = 30$  )

- Is always about a population parameter, not about a sample statistic

$$H_0 : \mu = 30$$

$$H_0 : \bar{X} = 30$$



# The Null Hypothesis, $H_0$

帰無仮説

DCOVA  
(continued)

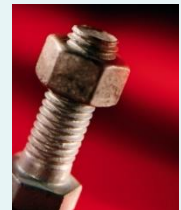
- Begin with the assumption that the null hypothesis is true まず、肯定されることを期待して立てられる
  - Similar to the notion of innocent until proven guilty
- Refers to the status quo or historical value
- Null always contains “=” sign
- May or may not be rejected



# The Alternative Hypothesis 対立仮説, $H_1$

DCOVA

- Is the opposite of the null hypothesis  
帰無仮説と対立する仮説である。
  - e.g., The average diameter of a manufactured bolt is not equal to 30mm (  $H_1: \mu \neq 30$  )
- Challenges the status quo
- Alternative never contains the “=” sign
- May or may not be proven
- Is generally the hypothesis that the researcher is trying to prove 帰無仮説は一般的に研究者が証明しようとしているという仮説である。

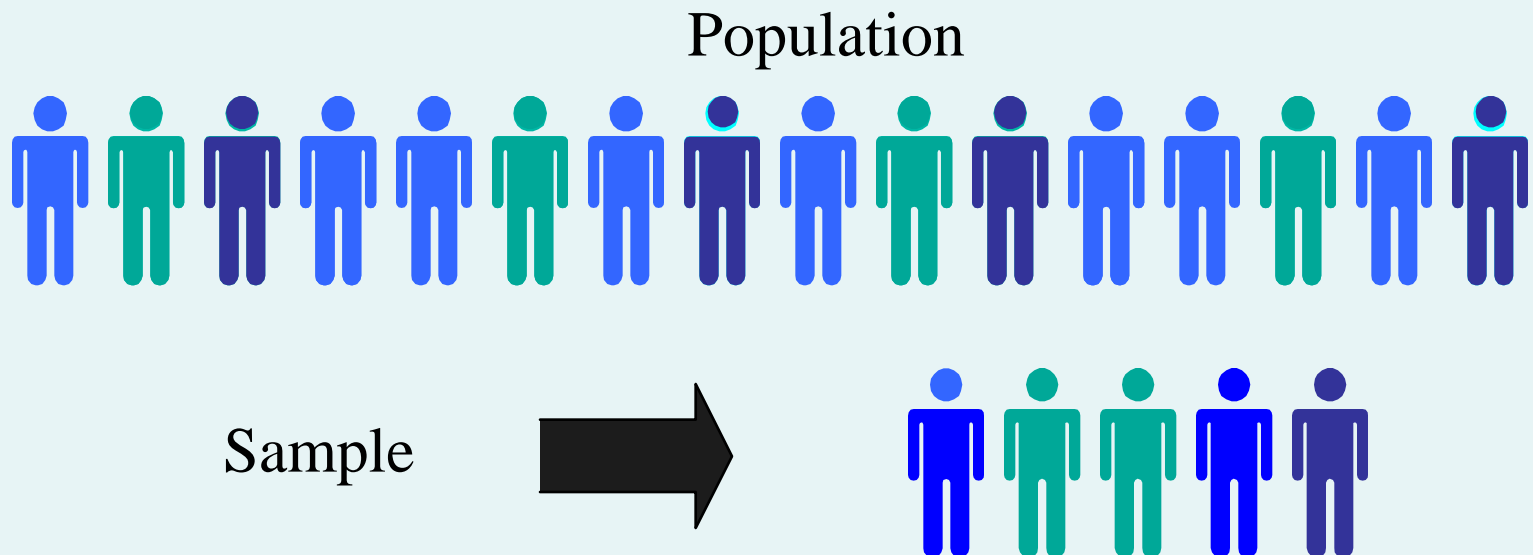


# The Hypothesis Testing Process

## 仮説テストのプロセス

DCOVA

- Claim: The population mean age is 50. 母集団の平均年齢は50才である。
  - $H_0: \mu = 50$ ,  $H_1: \mu \neq 50$
- Sample the population and find sample mean.



# The Hypothesis Testing Process

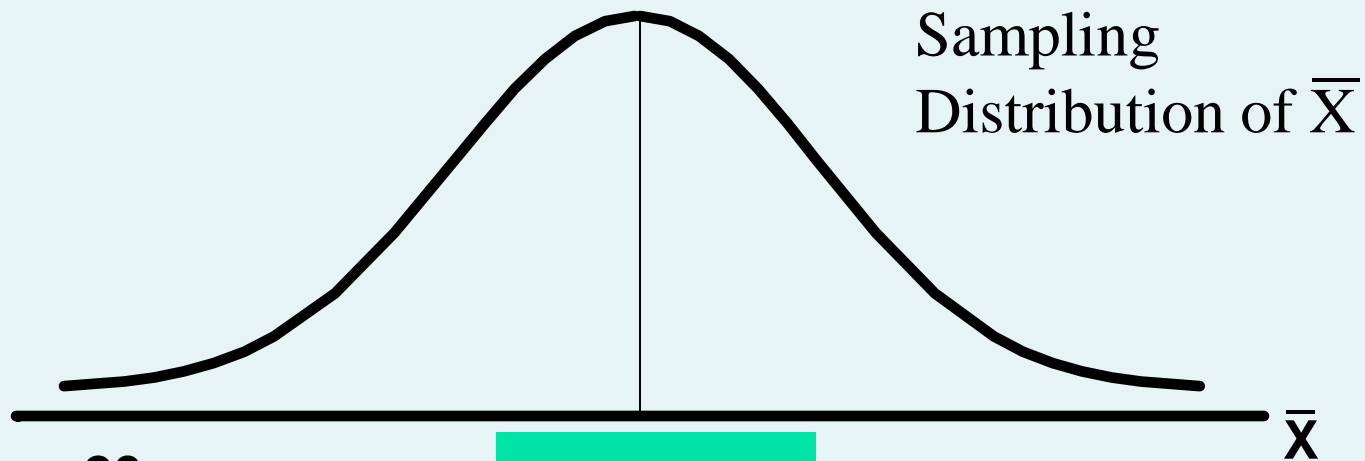
DCOVA<sup>A</sup>  
(continued)

- Suppose the sample mean age was  $\bar{X} = 20$ .  
サンプルの平均年齢が20と仮説する。
- This is significantly lower than the claimed mean population age of 50. サンプルの平均年齢は母集団の平均年齢とかなり差のある。
- If the null hypothesis were true, the probability of getting such a different sample mean would be very small, so you reject the null hypothesis. 帰無仮説が成り立つ確率が小さいので、拒否する。
- In other words, getting a sample mean of 20 is so unlikely if the population mean was 50, you conclude that the population mean must not be 50. 母集団の平均が50であれば、あるサンプルの平均が20である可能性が低い。



# The Hypothesis Testing Process

DCOVA<sup>A</sup>  
(continued)



If it is unlikely that you  
would get a sample  
mean of this value ...

$\mu = 50$   
If  $H_0$  is true

... When in fact this were  
the population mean...

... then you reject  
the null hypothesis  
that  $\mu = 50$ .

# The Test Statistic and Critical Values : 検定統計量と臨界値

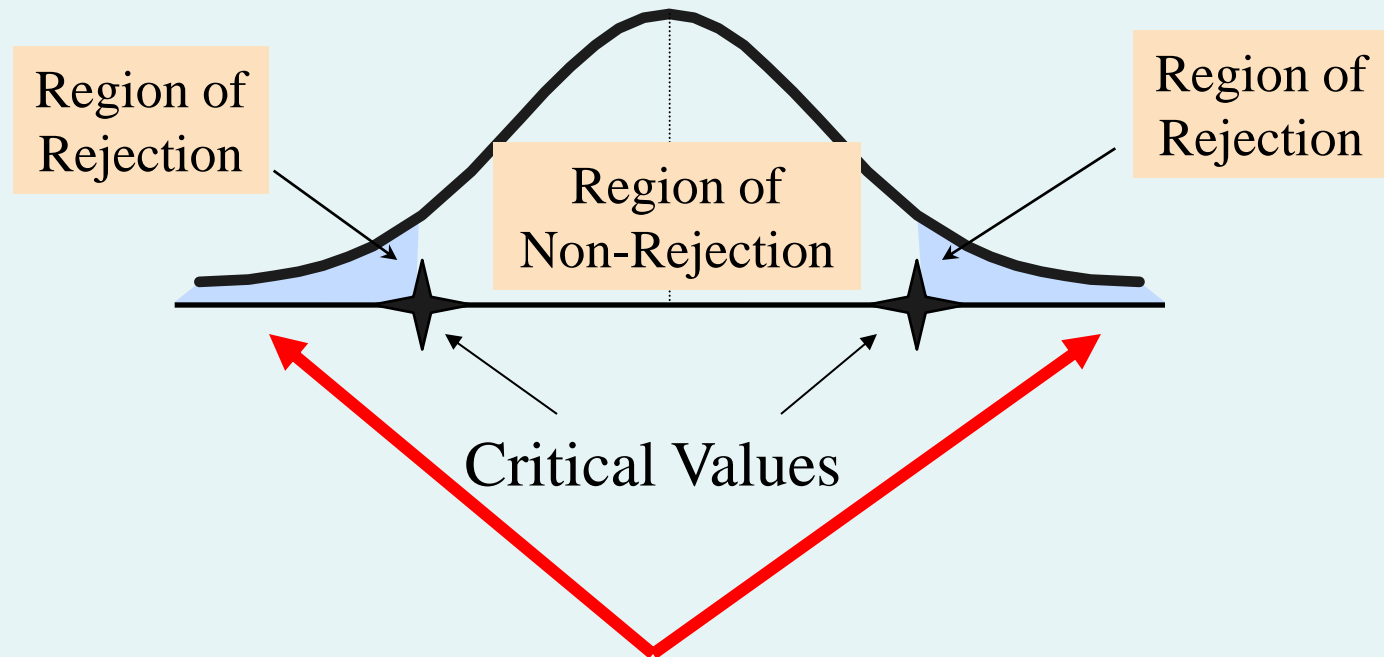
DCOVA

- If the sample mean is close to the stated population mean, the null hypothesis is not rejected.  
標本平均が述べた母集団の平均に近い場合、帰無仮説は棄却されない。
- If the sample mean is far from the stated population mean, the null hypothesis is rejected.  
サンプルの平均は母集団の平均から遥かに離れる場合、帰無仮説は棄却される。
- How far is “far enough” to reject  $H_0$ ?  
どのくらい離れるとき、帰無仮説は棄却されるのか。
- The critical value of a test statistic creates a “line in the sand” for decision making -- it answers the question of how far is far enough.  
検定統計量の臨界値は、意思決定のための「砂の中の行」を作成する - それはどこまでの質問に答える。

# The Test Statistic and Critical Values 臨界值

DCOVA

Sampling Distribution of the test statistic



# Possible Errors in Hypothesis Test Decision Making

DCOVA

## ■ Type I Error : 第 1 種の誤り

- Reject a true null hypothesis 真の帰無仮説が棄却された
- Considered a serious type of error
- The probability of a Type I Error is  $\alpha$ 
  - Called level of significance of the test
  - Set by researcher in advance

## ■ Type II Error : 第 2 種の誤り

- Failure to reject a false null hypothesis  
偽の帰無仮説を棄却に失敗した
- The probability of a Type II Error is  $\beta$

# Possible Errors in Hypothesis Test Decision Making

仮説検定の意思決定における可能なエラー DCOVA  
(continued)

Possible Hypothesis Test Outcomes		
	Actual Situation	
Decision	$H_0$ True	$H_0$ False
Do Not Reject $H_0$	No Error Probability $1 - \alpha$	Type II Error Probability $\beta$
Reject $H_0$	Type I Error Probability $\alpha$	No Error Probability $1 - \beta$



# Possible Errors in Hypothesis Test Decision Making

DCOVA  
(continued)


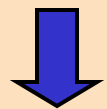
- The **confidence coefficient 信頼係数**  $(1-\alpha)$  is the probability of not rejecting  $H_0$  when it is true.
- The **confidence level 信頼水準** of a hypothesis test is  $(1-\alpha)*100\%$ .
- The **power of a statistical test**  $(1-\beta)$  is the probability of rejecting  $H_0$  when it is false.



# Type I & II Error Relationship

DCOVAA

- Type I and Type II errors cannot happen at the same time 第一種の誤りと第二種の誤りが同時に発生することがない
  - A Type I error can only occur if  $H_0$  is **true**
  - A Type II error can only occur if  $H_0$  is **false**

If Type I error probability ( $\alpha$ )  , then  
Type II error probability ( $\beta$ ) 

# Factors Affecting Type II Error

## タイプIIのエラーに影響する要因

DCOVA

- All else equal,
  - $\beta$  ↑ when the difference between hypothesized parameter and its true value ↓
  - $\beta$  ↑ when  $\alpha$  ↓
  - $\beta$  ↑ when  $\sigma$  ↑
  - $\beta$  ↑ when  $n$  ↓



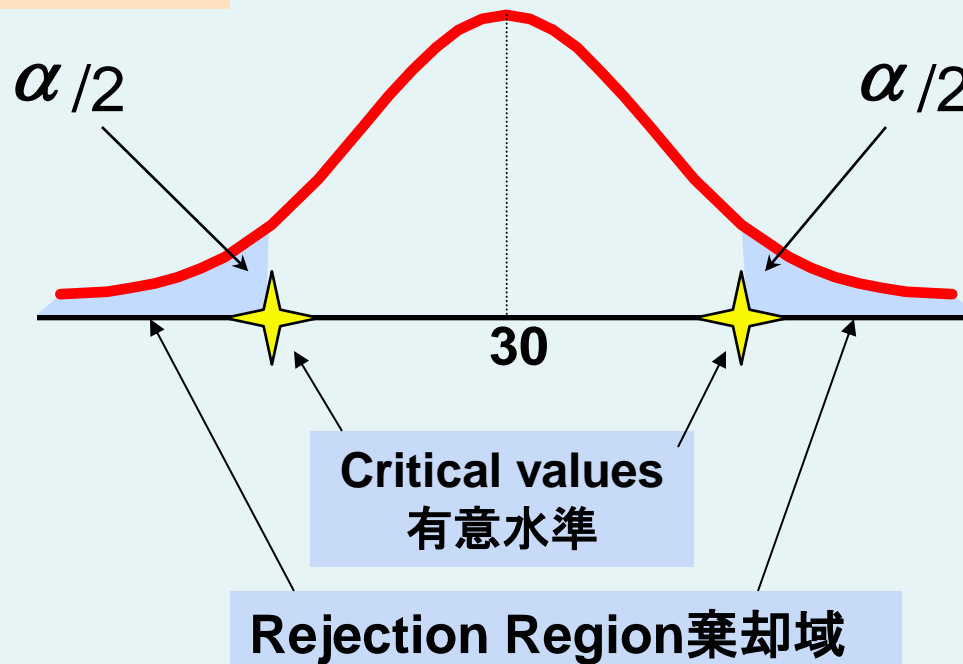
# Level of Significance and the Rejection Region 有意水準と棄却域

DCOVA

$$H_0: \mu = 30$$

$$H_1: \mu \neq 30$$

Level of significance =  $\alpha$

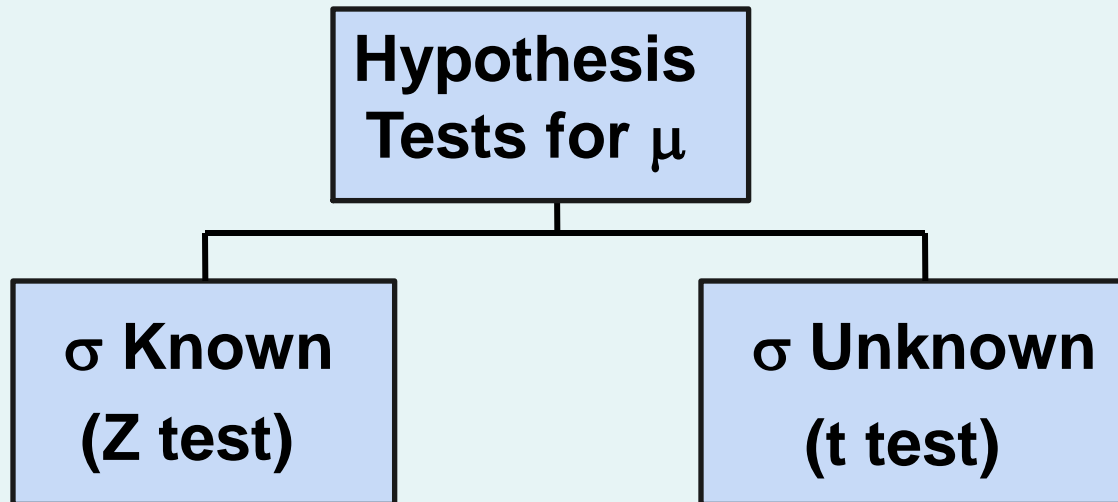


This is a **two-tail test** because there is a rejection region in both tails



# Hypothesis Tests for the Mean

DCOVA



# Z Test of Hypothesis for the Mean ( $\sigma$ Known)

DCOVA

- Convert sample statistic ( $\bar{X}$ ) to a  $Z_{\text{STAT}}$  test statistic

## Hypothesis Tests for $\mu$

$\sigma$  Known  
(Z test)

$\sigma$  Unknown  
(t test)

The test statistic is:

$$Z_{\text{STAT}} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

# Critical Value 臨界値 Approach to Testing テストへのアプローチ

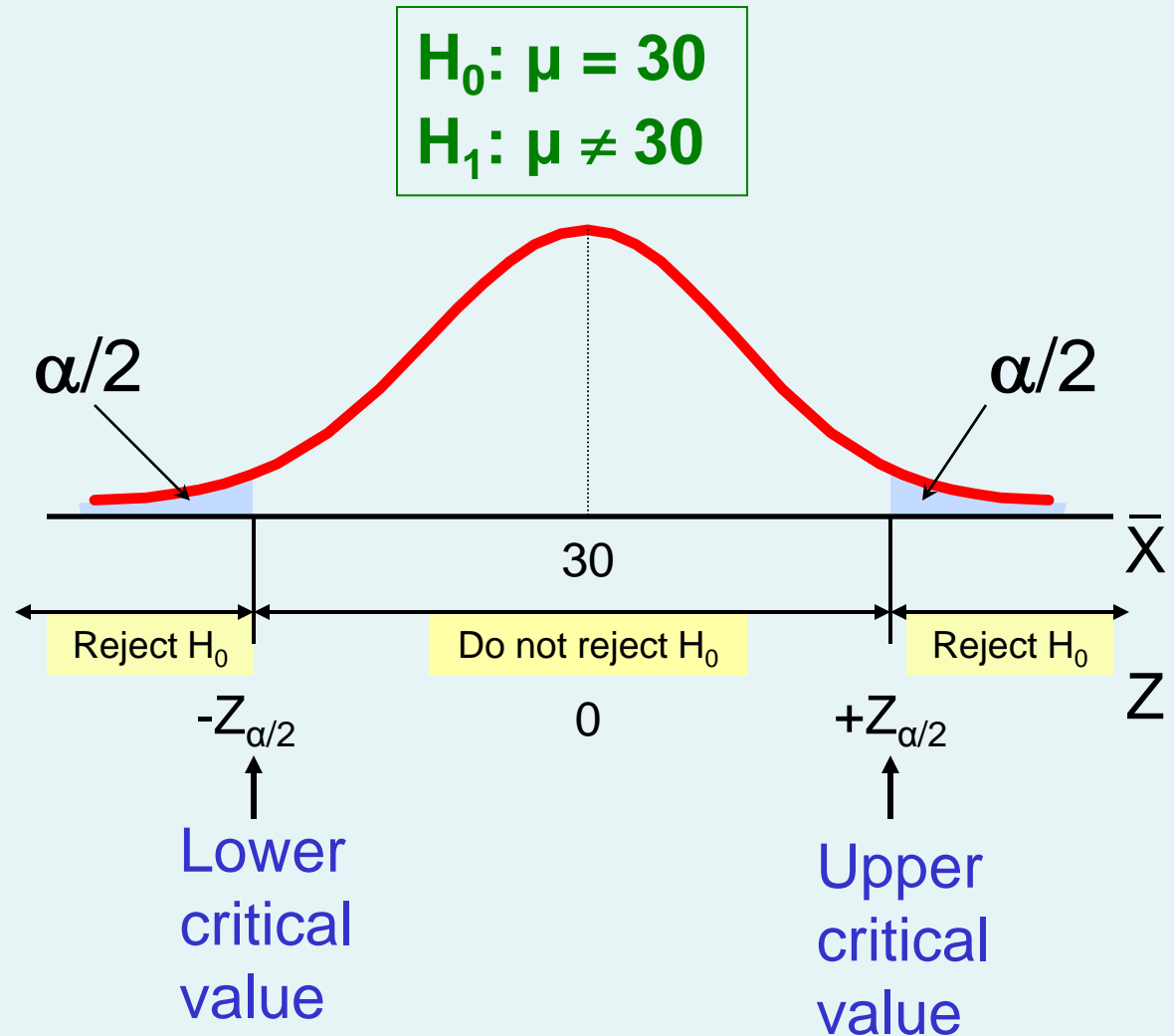
DCOVA

- For a two-tail test (両側検定) for the mean,  $\sigma$  known:
- Convert sample statistic ( $\bar{X}$ ) to test statistic ( $Z_{STAT}$ )  
サンプルの統計量  $\bar{X}$  をテストの統計量( $Z_{STAT}$ )に変換する
- Determine the critical Z values for a specified level of significance  $\alpha$  from a table or computer  
表或いはコンピューターを調べ、確定した有意水準の下、z 臨界値を決定する。
- **Decision Rule:** If the test statistic falls in the rejection region, reject  $H_0$ ; otherwise do not reject  $H_0$
- 検定統計量が棄却域に入る場合、 $H_0$ を棄却する。

# Two-Tail Tests : 両側検定

DCOVA

- There are two cutoff values (critical values), defining the regions of rejection





# 6 Steps in Hypothesis Testing

DCOVA

1. State the null hypothesis,  $H_0$  and the alternative hypothesis,  $H_1$
  2. Choose the level of significance,  $\alpha$ , and the sample size,  $n$
  3. Determine the appropriate test statistic and sampling distribution
  4. Determine the critical values that divide the rejection and nonrejection regions
- 
- 1、帰無仮説を立てる、その対立仮説を述べる
  - 2、有意水準を決め、サンプルのサイズを確定する
  - 3、適切な検定統計量と標本分布を選ぶ
  - 4、臨界値、棄却域を確定する



# 6 Steps in Hypothesis Testing

DCOVA

(continued)

5. Collect data and compute the value of the test statistic
6. Make the statistical decision and state the managerial conclusion. If the test statistic falls into the nonrejection region, do not reject the null hypothesis  $H_0$ . If the test statistic falls into the rejection region, reject the null hypothesis. Express the managerial conclusion in the context of the problem
- 7、データを収集し、検定統計量の値を計算する
- 8、結論をおとす

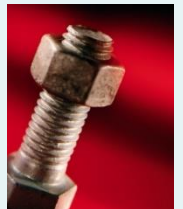
# Hypothesis Testing Example

## 仮説検定の例

DCOVA

**Test the claim that the true mean diameter of a manufactured bolt is 30mm.  
(Assume  $\sigma = 0.8$ )**

1. State the appropriate null and alternative hypotheses 適切な帰無仮説と対立仮説を立てる
  - $H_0: \mu = 30$      $H_1: \mu \neq 30$  (This is a two-tail test)
2. Specify the desired level of significance and the sample size 有意水準と標本空間を決める
  - Suppose that  $\alpha = 0.05$  and  $n = 100$  are chosen for this test





# Hypothesis Testing Example

DCOVA

(continued)

3. Determine the appropriate technique 統計量と分布を選ぶ

- $\sigma$  is assumed known so this is a Z test.

4. Determine the critical values 臨界値を決める

- For  $\alpha = 0.05$  the critical Z values are  $\pm 1.96$

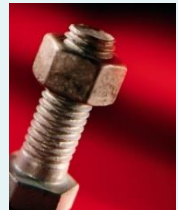
5. Collect the data and compute the test statistic

- Suppose the sample results are

$n = 100$ ,  $\bar{X} = 29.84$  ( $\sigma = 0.8$  is assumed known)

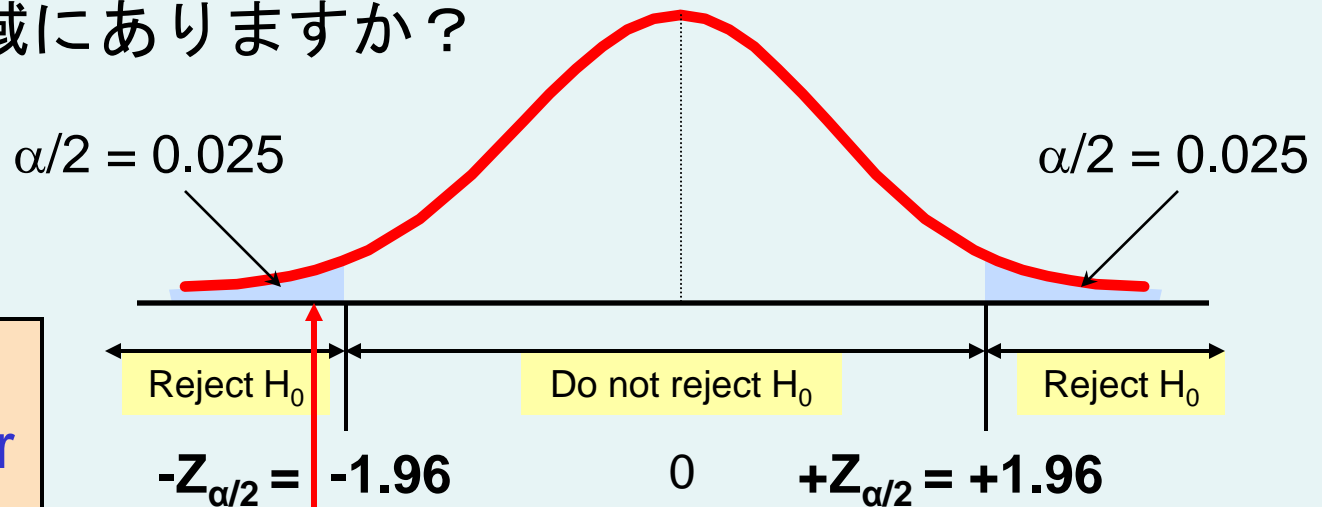
So the test statistic is:

$$Z_{\text{STAT}} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{29.84 - 30}{\frac{0.8}{\sqrt{100}}} = \frac{-0.16}{0.08} = -2.0$$



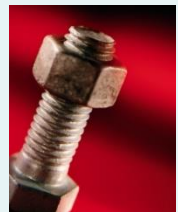
# Hypothesis Testing Example DCOVA<sub>A</sub> (continued)

- 6. Is the test statistic in the rejection region? 統計量は棄却域にありますか？



Reject  $H_0$  if  
 $Z_{STAT} < -1.96$  or  
 $Z_{STAT} > 1.96$ ;  
otherwise do  
not reject  $H_0$

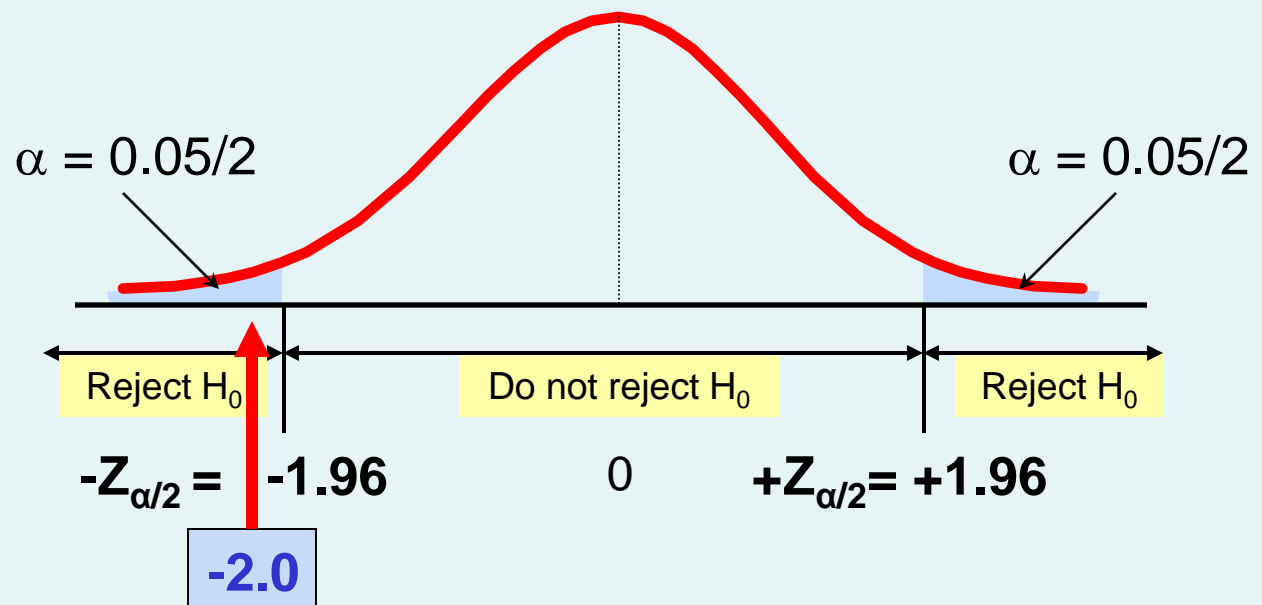
Here,  $Z_{STAT} = -2.0 < -1.96$ , so the test statistic is in the rejection region



# Hypothesis Testing Example DCOVA<sub>A</sub>

(continued)

6 (continued). Reach a decision and interpret the result



Since  $Z_{\text{STAT}} = -2.0 < -1.96$ , reject the null hypothesis and conclude there is sufficient evidence that the mean diameter of a manufactured bolt is not equal to 30



# p-Value Approach to Testing

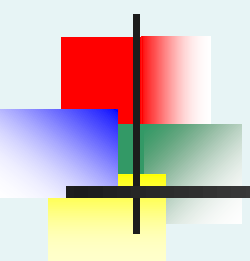
## P値

DCOVA

- p-value: Probability of obtaining a test statistic equal to or more extreme than the observed sample value given  $H_0$  is true

帰無仮説の下で実際にデータから計算された統計量よりも極端な統計量が観測される確率を、p値(p-value)という。

- The p-value is also called the observed level of significance
- $H_0$  can be rejected if the p-value is less than  $\alpha$



# p-Value Approach to Testing: Interpreting the p-value

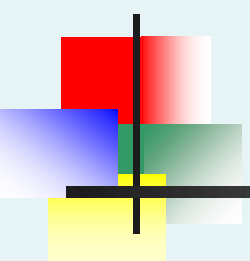
DCOVA

- Compare the p-value with  $\alpha$

- If  $\text{p-value} < \alpha$ , reject  $H_0$
- If  $\text{p-value} \geq \alpha$ , do not reject  $H_0$

- Remember

- If the p-value is low then  $H_0$  must go



# The 5 Step p-value approach to Hypothesis Testing

DCOVA

1. State the null hypothesis,  $H_0$  and the alternative hypothesis,  $H_1$   
帰無仮説とその対立仮説を立てる。
2. Choose the level of significance,  $\alpha$ , and the sample size,  $n$  有意水準と標本空間を決める。
3. Determine the appropriate test statistic and sampling distribution  
適切な統計量と標本分布を選ぶ。
4. Collect data and compute the value of the test statistic and the p-value  
データを収集し、統計量と p 値を計算する。
5. Make the statistical decision and state the managerial conclusion.  
If the p-value is  $< \alpha$  then reject  $H_0$ , otherwise do not reject  $H_0$ .  
State the managerial conclusion in the context of the problem  
結論を出す

# p-value Hypothesis Testing Example

DCOVA

**Test the claim that the true mean diameter of a manufactured bolt is 30mm.  
(Assume  $\sigma = 0.8$ )**

1. State the appropriate null and alternative hypotheses
  - $H_0: \mu = 30$       $H_1: \mu \neq 30$  (This is a two-tail test)
2. Specify the desired level of significance and the sample size
  - Suppose that  $\alpha = 0.05$  and  $n = 100$  are chosen for this test



# p-value Hypothesis Testing Example

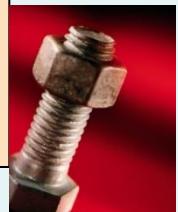
DCOVA

(continued)

3. Determine the appropriate technique
  - $\sigma$  is assumed known so this is a Z test.
4. Collect the data, compute the test statistic and the p-value
  - Suppose the sample results are  
 $n = 100$ ,  $\bar{X} = 29.84$  ( $\sigma = 0.8$  is assumed known)

So the test statistic is:

$$Z_{\text{STAT}} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{29.84 - 30}{\frac{0.8}{\sqrt{100}}} = \frac{-0.16}{0.08} = -2.0$$



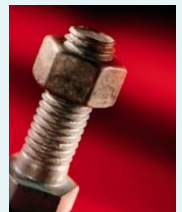
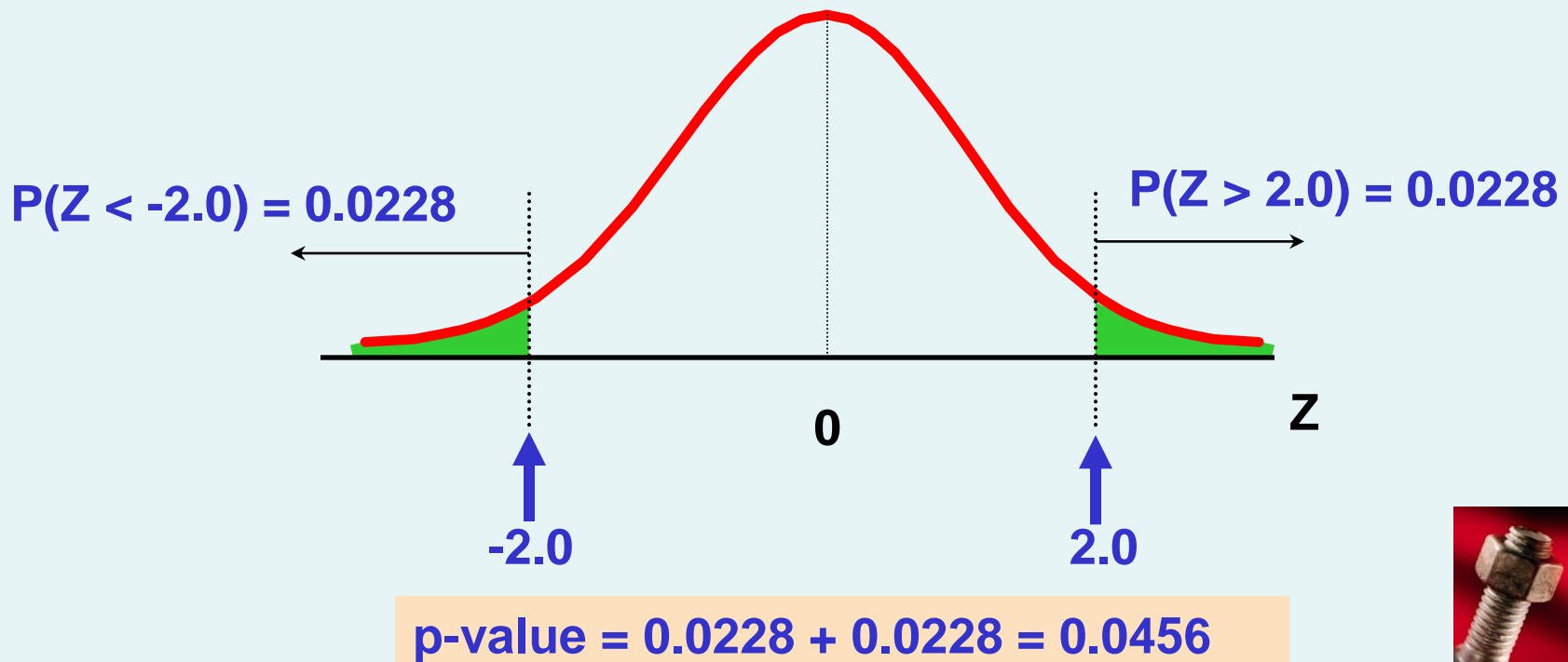


# p-Value Hypothesis Testing Example: Calculating the p-value

DCOVA

4. (continued) Calculate the p-value.

- How likely is it to get a  $Z_{\text{STAT}}$  of -2 (or something farther from the mean (0), in either direction) if  $H_0$  is true?

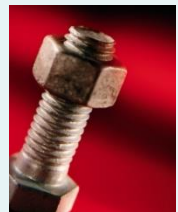


# p-value Hypothesis Testing Example

DCOVA

(continued)

- 5. Is the p-value  $< \alpha$ ?
  - Since p-value = 0.0456  $< \alpha = 0.05$  Reject  $H_0$
  - State the managerial conclusion in the context of the situation.
  - There is sufficient evidence to conclude the average diameter of a manufactured bolt is not equal to 30mm.



# Connection Between Two-Tail Tests and Confidence Intervals 両側検定と信頼空間

DCOVA

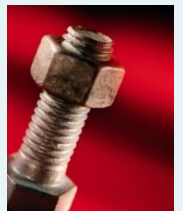
- For  $\bar{X} = 29.84$ ,  $\sigma = 0.8$  and  $n = 100$ , the 95% confidence interval is:

$$29.84 - (1.96) \frac{0.8}{\sqrt{100}} \quad \text{to} \quad 29.84 + (1.96) \frac{0.8}{\sqrt{100}}$$

$$29.6832 \leq \mu \leq 29.9968$$

- Since this interval does not contain the hypothesized mean (30), we reject the null hypothesis at  $\alpha = 0.05$

- 仮設した平均は信頼空間に含まれていないため、帰無仮説を棄却する。





# Do You Ever Truly Know $\sigma$ ?

DCOVAA

- Probably not!
- In virtually all real world business situations,  $\sigma$  is not known.
- If there is a situation where  $\sigma$  is known then  $\mu$  is also known (since to calculate  $\sigma$  you need to know  $\mu$ .)
- If you truly know  $\mu$  there would be no need to gather a sample to estimate it.



# Do You Ever Truly Know $\sigma$ ?

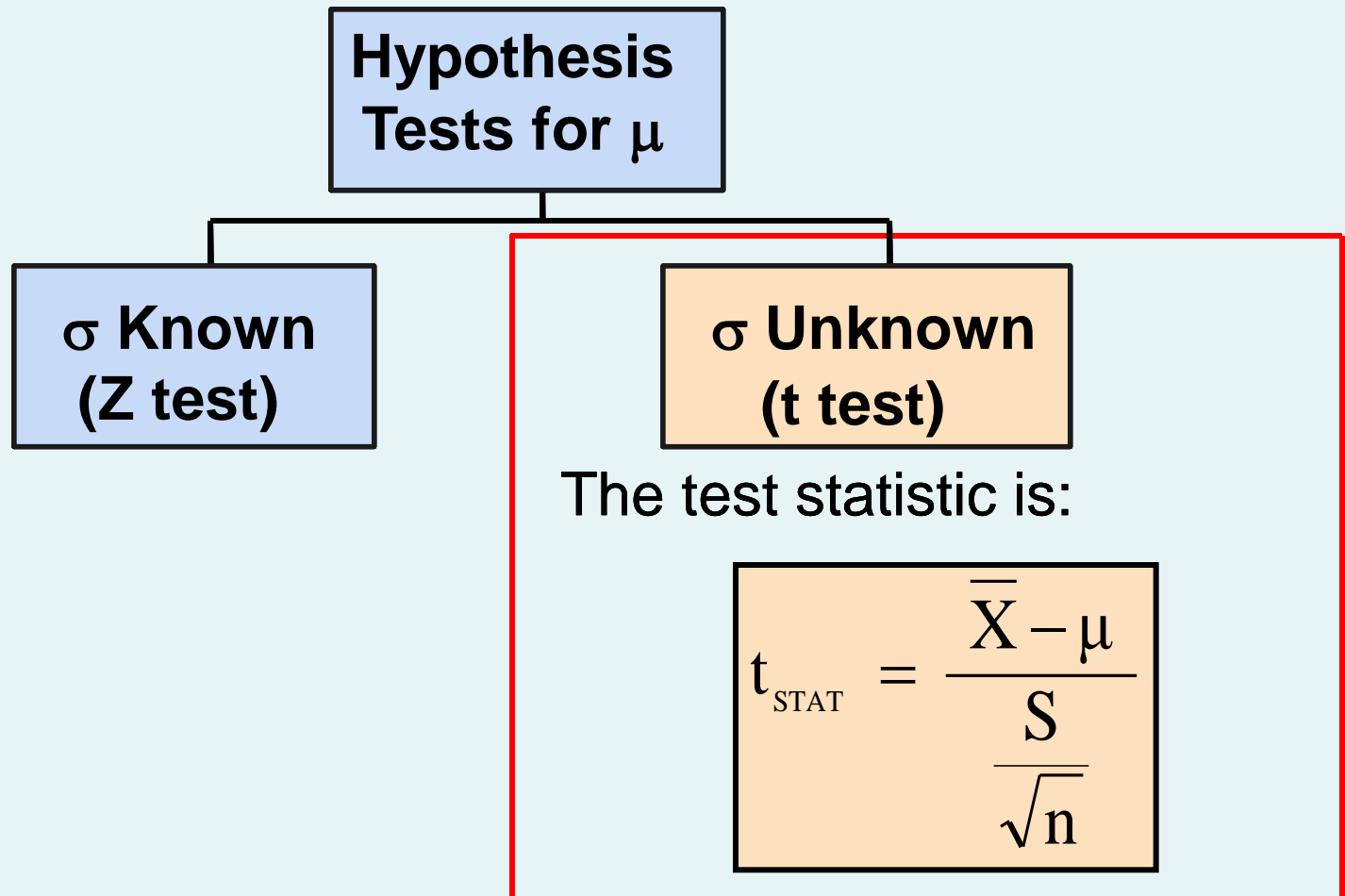
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- Probably not!
- In virtually all real world business situations,  $\sigma$  is not known. 事実上すべてのビジネス状況において、 $\sigma$ は知られていない。
- If there is a situation where  $\sigma$  is known, then  $\mu$  is also known (since to calculate  $\sigma$  you need to know  $\mu$ .)  $\sigma$ が既知の場合、 $\mu$ も知られている。
- If you truly know  $\mu$  there would be no need to gather a sample to estimate it. 本当に $\mu$ を知っていれば、それを推定するためのサンプルを収集する必要はないであろう。

# t Test of Hypothesis for the Mean ( $\sigma$ Unknown)

DCOVA

- Convert sample statistic ( $\bar{X}$ ) to a  $t_{\text{STAT}}$  test statistic



# Example: Two-Tail Test ( $\sigma$ Unknown)

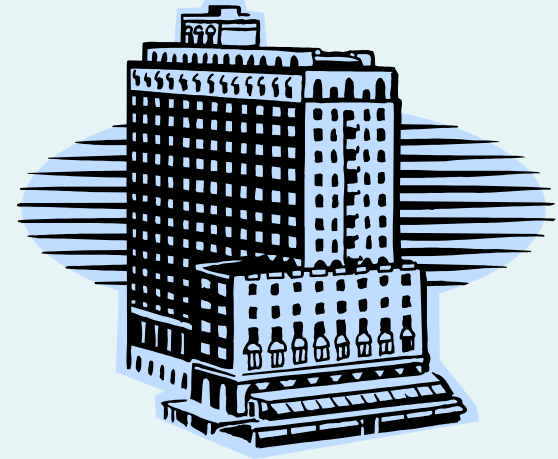
DCOVA

The average cost of a hotel room in New York is said to be \$168 per night. To determine if this is true, a random sample of 25 hotels is taken and resulted in an  $\bar{X}$  of \$172.50 and an S of \$15.40. Test the appropriate hypotheses at  $\alpha = 0.05$ .

ニューヨークにあるホテルの平均値段は一泊168ドルと言われている、実の状況を検証しましょう。

25軒のホテルをサンプルとして、無作為的に抽出する。  
平均は172.50、標準偏差は15.40。有意水準は0.05。

(Assume the population distribution is normal)



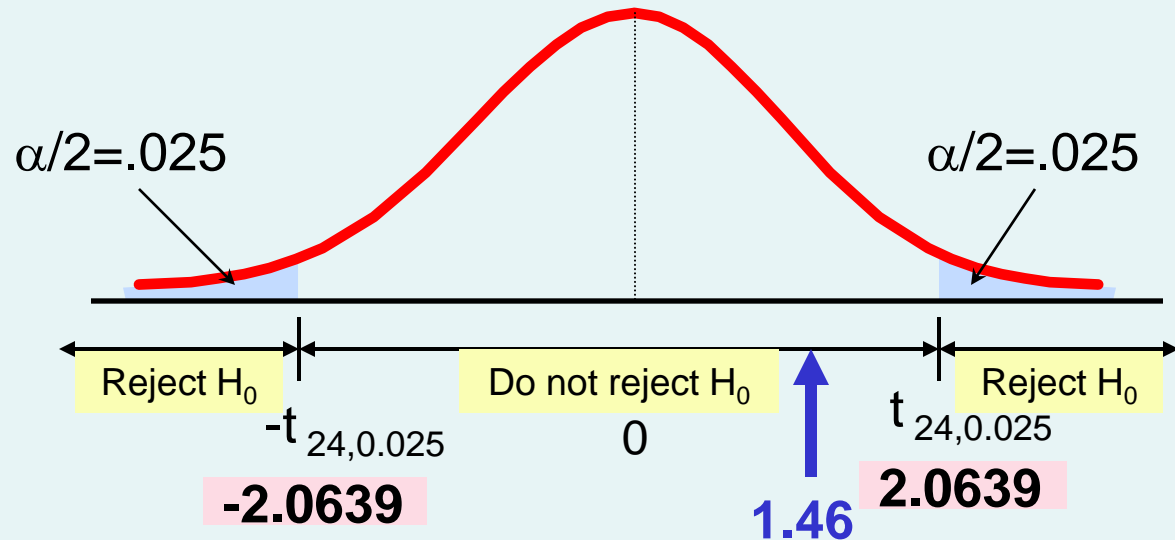
$H_0$ : \_\_\_\_\_  
 $H_1$ : \_\_\_\_\_

# Example Solution: Two-Tail t Test

DCOVA

$$H_0: \mu = 168$$
$$H_1: \mu \neq 168$$

- $\alpha = 0.05$
- $n = 25, df = 25-1=24$
- $\sigma$  is unknown, so use a **t statistic**
- Critical Value:



$$t_{\text{STAT}} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} = \frac{172.50 - 168}{\frac{15.40}{\sqrt{25}}} = 1.46$$

$$\pm t_{24,0.025} = \pm 2.0639$$

**Do not reject  $H_0$ :** insufficient evidence that true mean cost is different from \$168



# Example Two-Tail t Test Using A p-value from Excel

DCOVA

- Since this is a t-test we cannot calculate the p-value without some calculation aid.
- The Excel output below does this:

t Test for the Hypothesis of the Mean

Data	
Null Hypothesis $\mu =$	\$ 168.00
Level of Significance	0.05
Sample Size	25
Sample Mean	\$ 172.50
Sample Standard Deviation	\$ 15.40

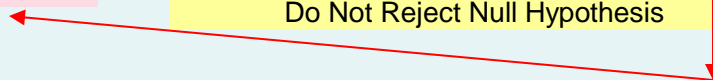
Intermediate Calculations

Standard Error of the Mean	\$	3.08	=B8/SQRT(B6)
Degrees of Freedom		24	=B6-1
<b>t test statistic</b>		<b>1.46</b>	=(B7-B4)/B11

Two-Tail Test

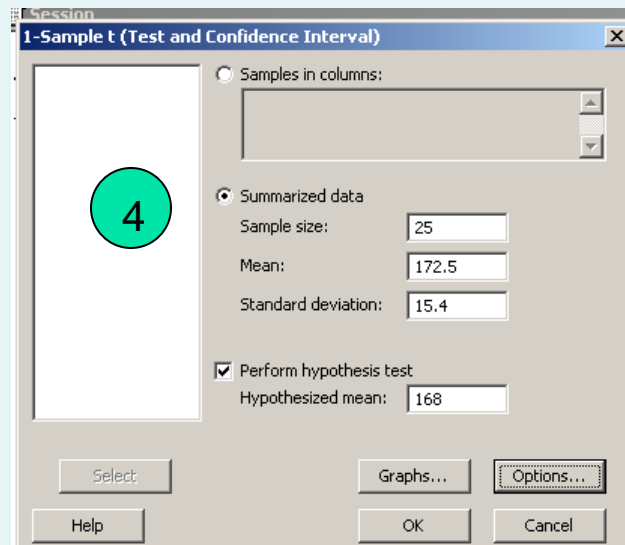
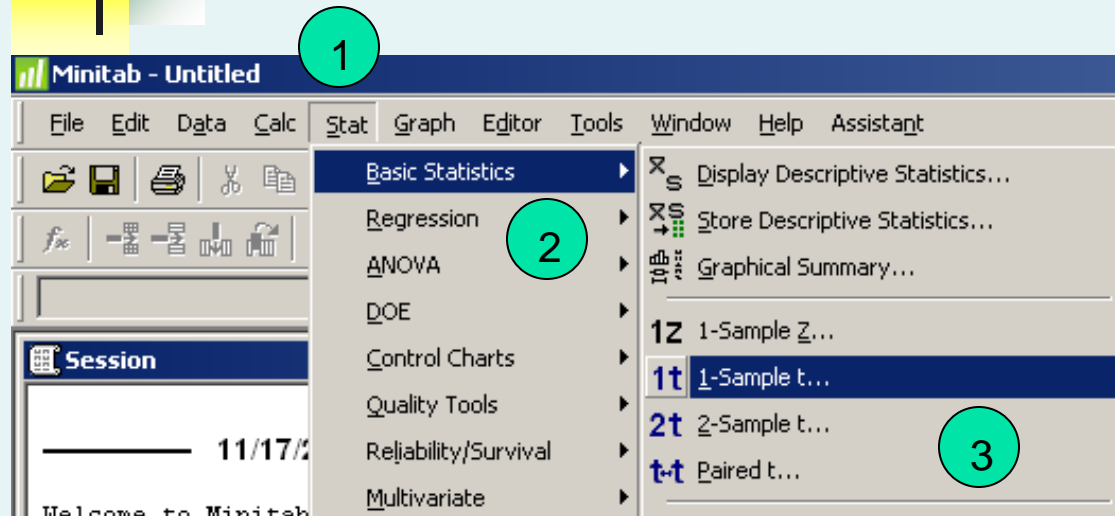
Lower Critical Value	-2.0639	=TINV(B5,B12)
Upper Critical Value	2.0639	=TINV(B5,B12)
p-value	0.157	=TDIST(ABS(B13),B12,2)
Do Not Reject Null Hypothesis		=IF(B18<B5, "Reject null hypothesis", "Do not reject null hypothesis")

p-value >  $\alpha$   
So do not reject  $H_0$



# Example Two-Tail t Test Using A p-value from Minitab

DCOVA



p-value >  $\alpha$   
So do not reject  $H_0$

## One-Sample T

Test of  $\mu = 168$  vs not = 168

N	Mean	StDev	SE Mean	95% CI	T	P
25	172.50	15.40	3.08	(166.14, 178.86)	1.46	0.157





# Connection of Two-Tail Tests to Confidence Intervals

DCOVA

- For  $\bar{X} = 172.5$ ,  $S = 15.40$  and  $n = 25$ , the 95% confidence interval for  $\mu$  is:

$$172.5 - (2.0639) 15.4/\sqrt{25} \quad \text{to} \quad 172.5 + (2.0639) 15.4/\sqrt{25}$$

$$166.14 \leq \mu \leq 178.86$$

- Since this interval contains the hypothesized mean (168), we do not reject the null hypothesis at  $\alpha = 0.05$

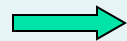
# One-Tail Tests : 片側検定

DCOVA

- In many cases, the alternative hypothesis focuses on a particular direction 多くの場合では、対立仮説は特定の方向に注目する。

$$H_0: \mu \geq 3$$

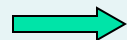
$$H_1: \mu < 3$$



This is a **lower**-tail test since the alternative hypothesis is focused on the lower tail below the mean of 3

$$H_0: \mu \leq 3$$

$$H_1: \mu > 3$$

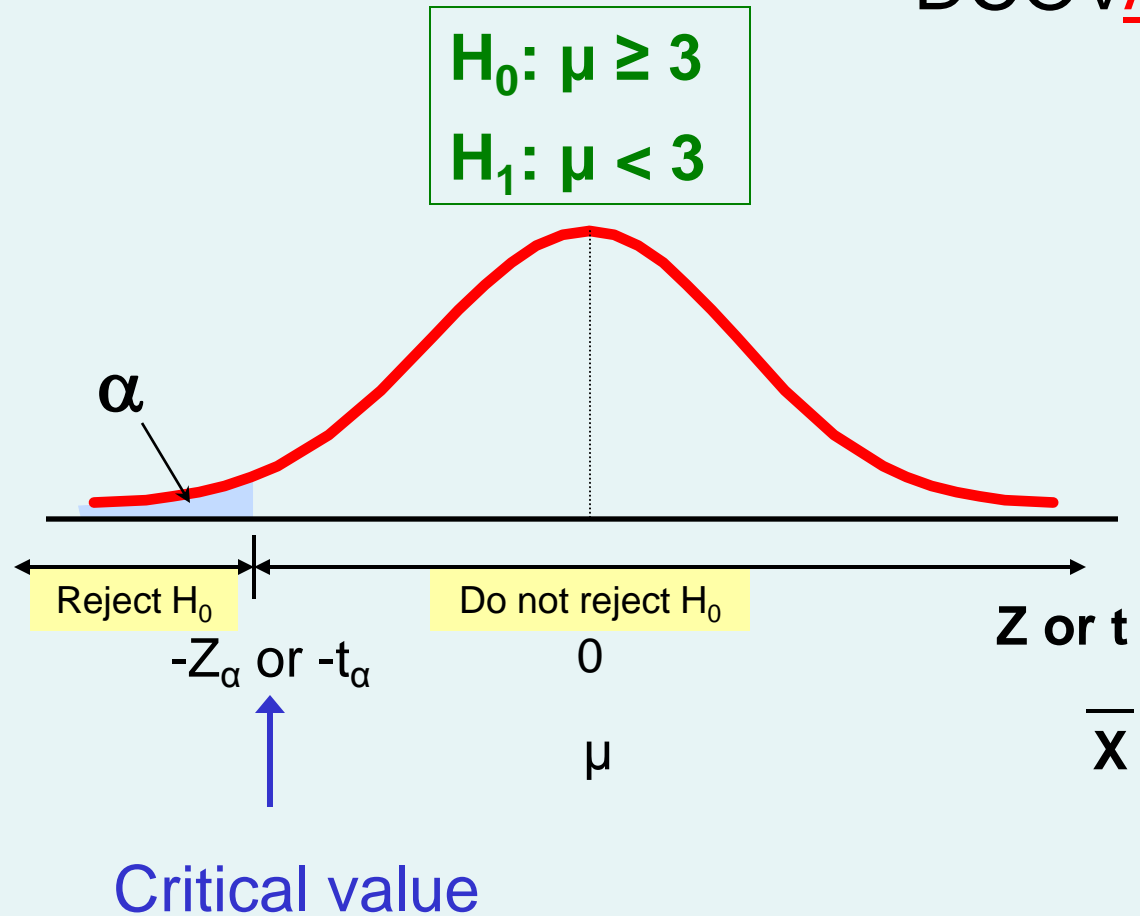


This is an **upper**-tail test since the alternative hypothesis is focused on the upper tail above the mean of 3

# Lower-Tail Tests : 左片側

DCOVA

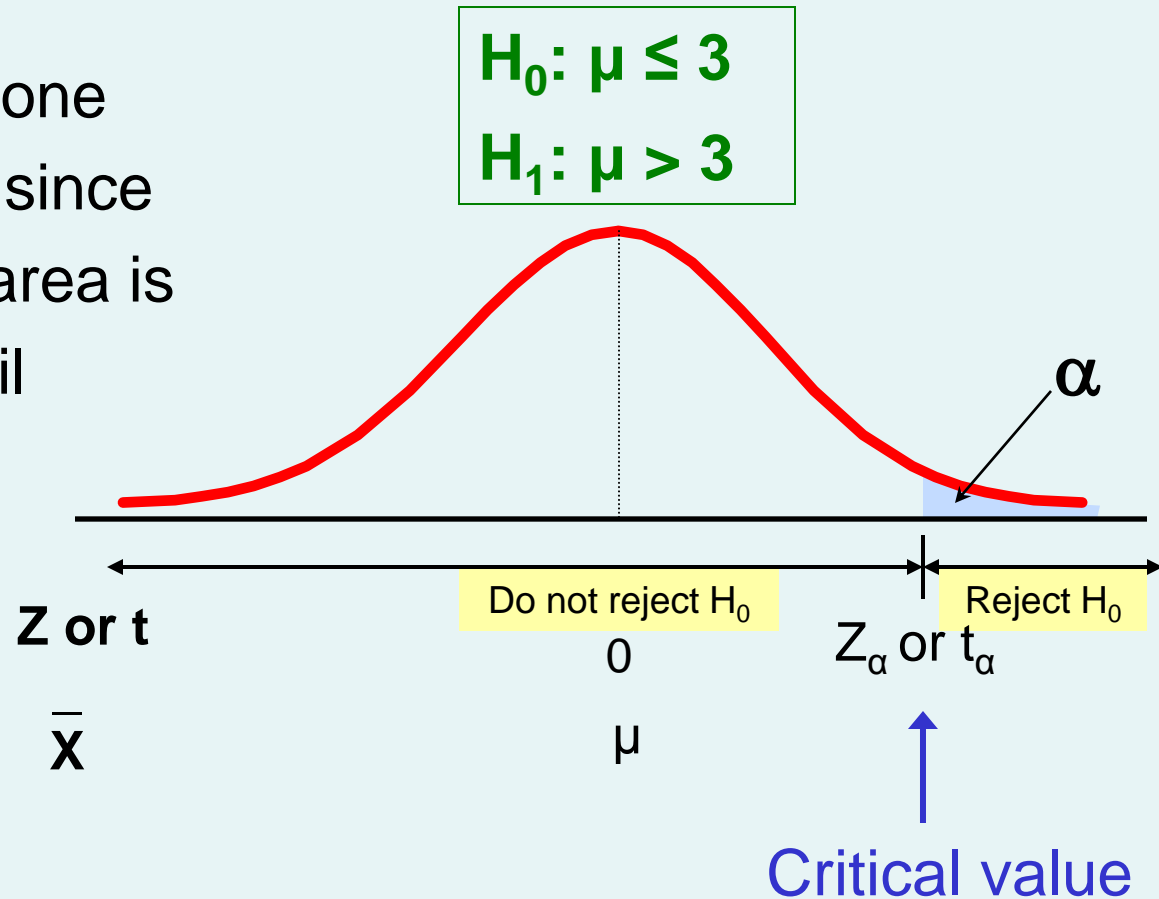
- There is only one critical value, since the rejection area is in only one tail



# Upper-Tail Tests : 右片側

DCOVA

- There is only one critical value, since the rejection area is in only one tail



# Example: Upper-Tail t Test for Mean ( $\sigma$ unknown)

DCOVA

A phone industry manager thinks that customer monthly cell phone bills have increased, and now average over \$52 per month. The company wishes to test this claim. (Assume a normal population)

ある通信業界のマネジャーはカスタマーの携帯電話の月間コストが増加して、52ドルに達したと思う。会社はこの観点を仮説検証しようと思っている。

## Form hypothesis test:

$H_0: \mu \leq 52$     the average is not over \$52 per month

$H_1: \mu > 52$     the average **is** greater than \$52 per month  
(i.e., sufficient evidence exists to support the manager's claim)



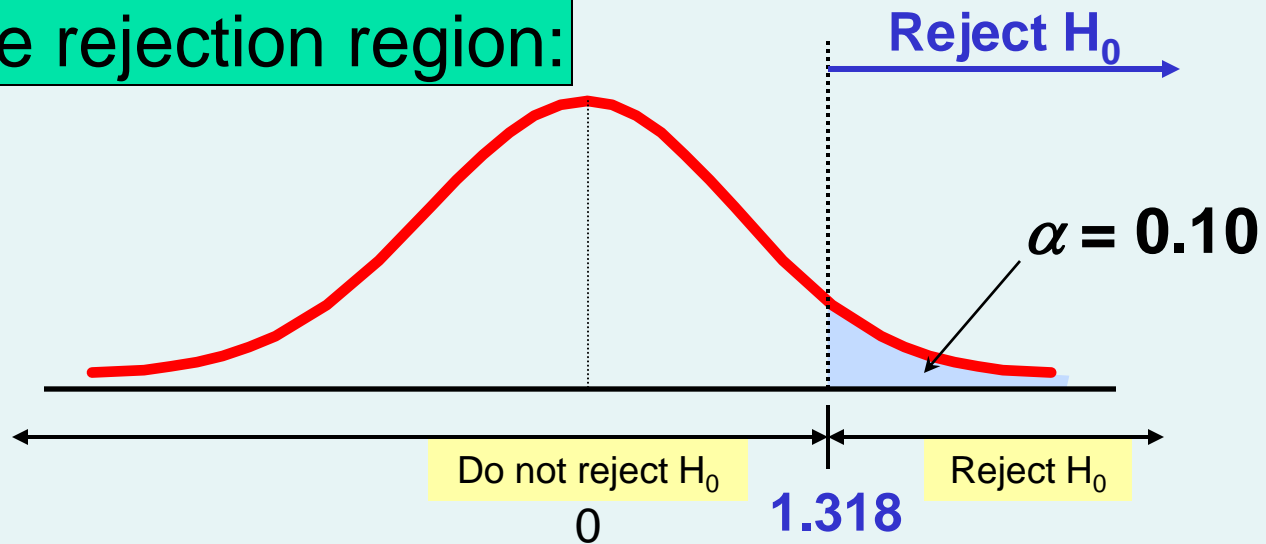
# Example: Find Rejection Region

DCOVA

(continued)

- Suppose that  $\alpha = 0.10$  is chosen for this test and  $n = 25$ .

Find the rejection region:



Reject  $H_0$  if  $t_{\text{STAT}} > 1.318$







# Example: Test Statistic


DCOVA

(continued)

Obtain sample and compute the test statistic

Suppose a sample is taken with the following results:  $n = 25$ ,  $\bar{X} = 53.1$ , and  $S = 10$

- Then the test statistic is:

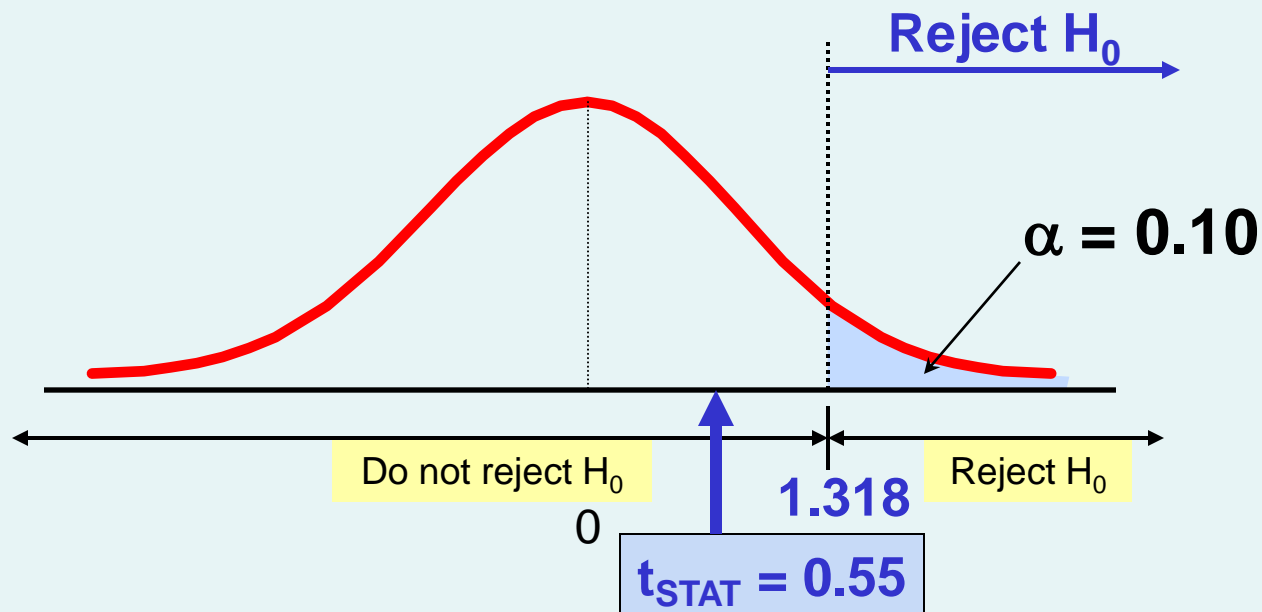

$$t_{\text{STAT}} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} = \frac{53.1 - 52}{\frac{10}{\sqrt{25}}} = 0.55$$

# Example: Decision

DCOVA

(continued)

Reach a decision and interpret the result:



**Do not reject  $H_0$  since  $t_{\text{STAT}} = 0.55 \leq 1.318$**

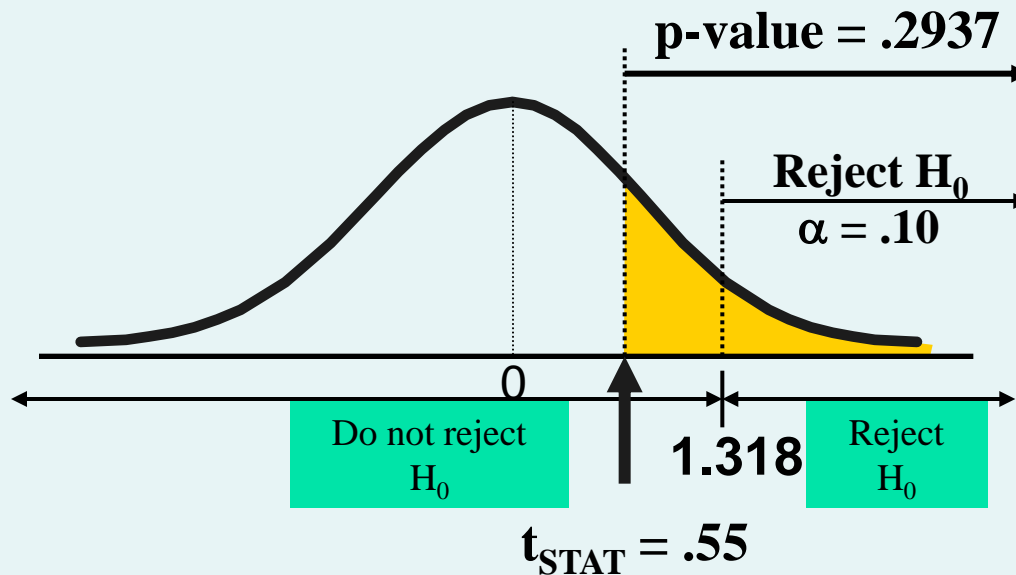
There is **insufficient** evidence that the mean bill is over \$52.



# Example: Utilizing The p-value for The Test P値の応用

DCOVA

- Calculate the p-value and compare to  $\alpha$  (p-value below calculated using Excel spreadsheet on next page)



**Do not reject  $H_0$  since p-value = .2937 >  $\alpha = .10$**

# Excel Spreadsheet Calculating The p-value for The Upper Tail t Test

DCOVA

## t Test for the Hypothesis of the Mean

Data	
Null Hypothesis $\mu =$	52.00
Level of Significance	0.1
Sample Size	25
Sample Mean	53.10
Sample Standard Deviation	10.00

Intermediate Calculations	
Standard Error of the Mean	2.00
Degrees of Freedom	24
<b>t test statistic</b>	<b>0.55</b>
Upper Tail Test	
Upper Critical Value	1.318
p-value	0.2937
Do Not Reject Null Hypothesis	

=B8/SQRT(B6)

=B6-1

=(B7-B4)/B11

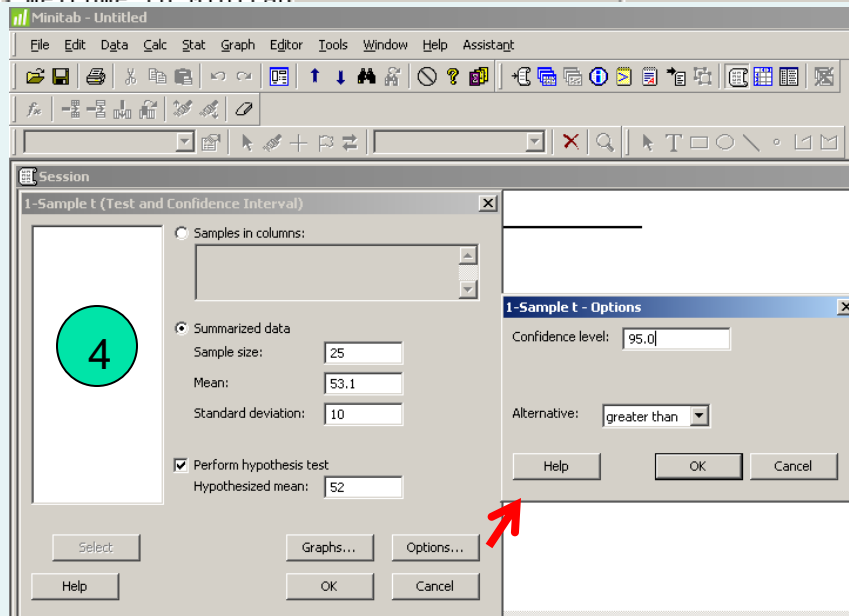
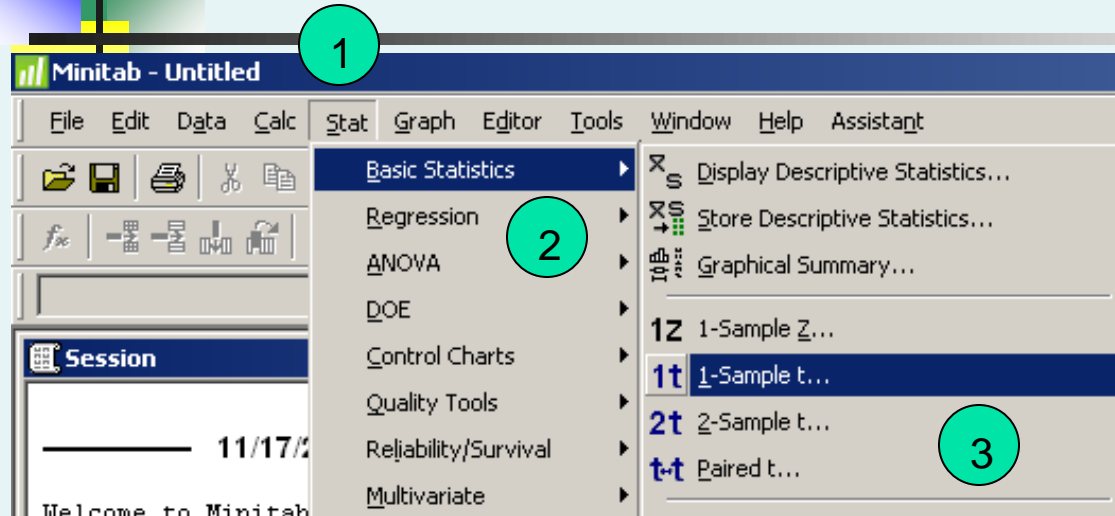
=TINV(2\*B5,B12)

=TDIST(ABS(B13),B12,1)

=IF(B18<B5, "Reject null hypothesis",  
"Do not reject null hypothesis")

# Using Minitab to calculate The p-value for The Upper Tail t Test

DCOVA



$p\text{-value} > \alpha$   
So do not reject  $H_0$

## One-Sample T

Test of  $\mu = 52$  vs  $> 52$

N	Mean	StDev	SE Mean
25	53.10	10.00	2.00

95% Lower Bound	T	P
49.68	0.55	0.294

# Hypothesis Tests for Proportions

## 割合の仮説検定

DCOVA

- Involves categorical variables    カテゴリ変数
- Two possible outcomes    二つの可能な結果
  - Possesses characteristic of interest
  - Does not possess characteristic of interest
- Fraction or proportion of the population in the category of interest is denoted by  $\pi$



# Proportions 割合

DCOVA

(continued)

- Sample proportion in the category of interest is denoted by  $p$

$$p = \frac{X}{n} = \frac{\text{number in category of interest in sample}}{\text{sample size}}$$

- When both  $X$  and  $n - X$  are at least 5,  $p$  can be approximated by a normal distribution with mean and standard deviation

■

$$\mu_p = \pi$$

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}$$

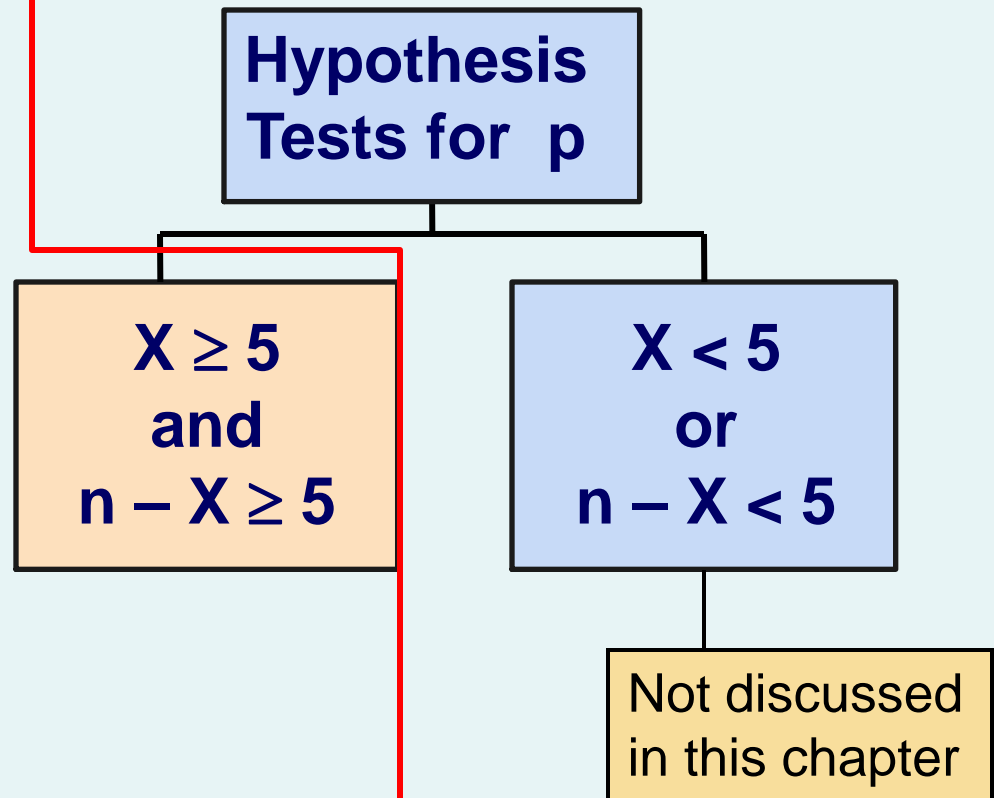
# Hypothesis Tests for Proportions

## 割合の仮説検定

DCOVA

- The sampling distribution of  $p$  is approximately normal, so the test statistic is a  $Z_{\text{STAT}}$  value:

$$Z_{\text{STAT}} = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}}$$



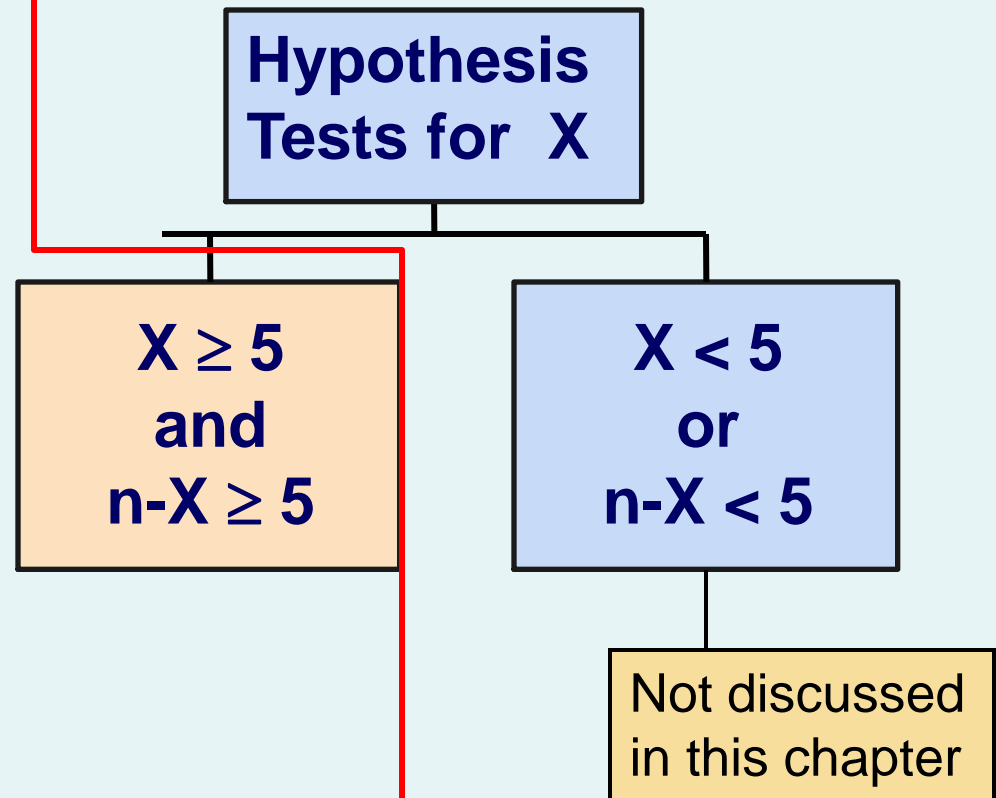


# Z Test for Proportion in Terms of Number in Category of Interest

DCOVA

- An equivalent form to the last slide, but in terms of the number in the category of interest,  $X$ :

$$Z_{\text{STAT}} = \frac{X - n\pi}{\sqrt{n\pi(1-\pi)}}$$

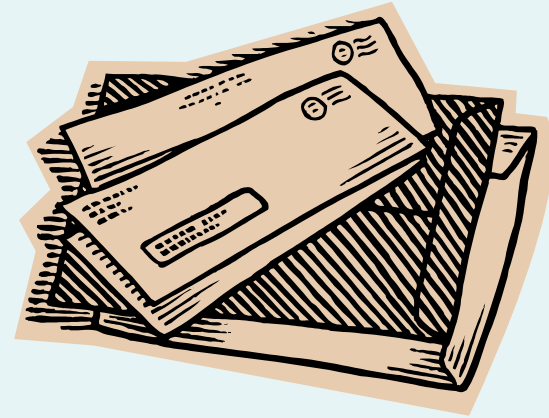


# Example: Z Test for Proportion

DCOVA

A marketing company claims that it receives responses from 8% of those surveyed. To test this claim, a random sample of 500 were surveyed with 25 responses. Test at the  $\alpha = 0.05$  significance level.

マーケティング会社は、調査対象者の8%からの応答を受信することを主張している。この主張を検証するため、調査を行った。無作為的に500サンプルを抽出し、25個の応答を返した。有意水準は0.05である。



Check:

$$X = 25$$

$$n - X = 475$$



# Z Test for Proportion: Solution

DCOVA

$$H_0: \pi = 0.08$$

$$H_1: \pi \neq 0.08$$

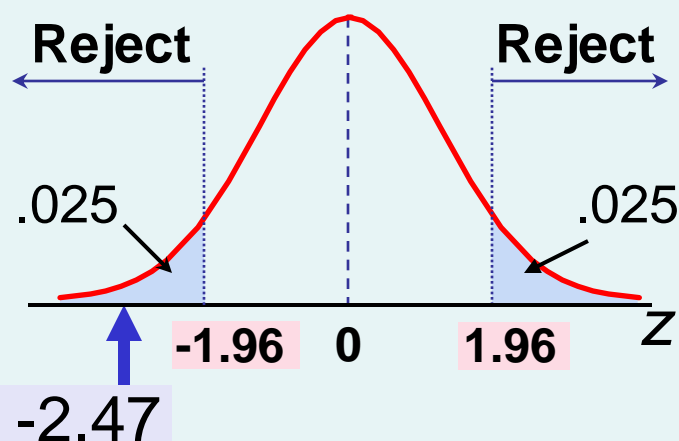
$$\alpha = 0.05$$

$$n = 500, \quad p = 0.05$$

**Test Statistic:**

$$Z_{\text{STAT}} = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}} = \frac{.05 - .08}{\sqrt{\frac{.08(1 - .08)}{500}}} = -2.47$$

**Critical Values:  $\pm 1.96$**



**Decision:**

Reject  $H_0$  at  $\alpha = 0.05$

**Conclusion:**

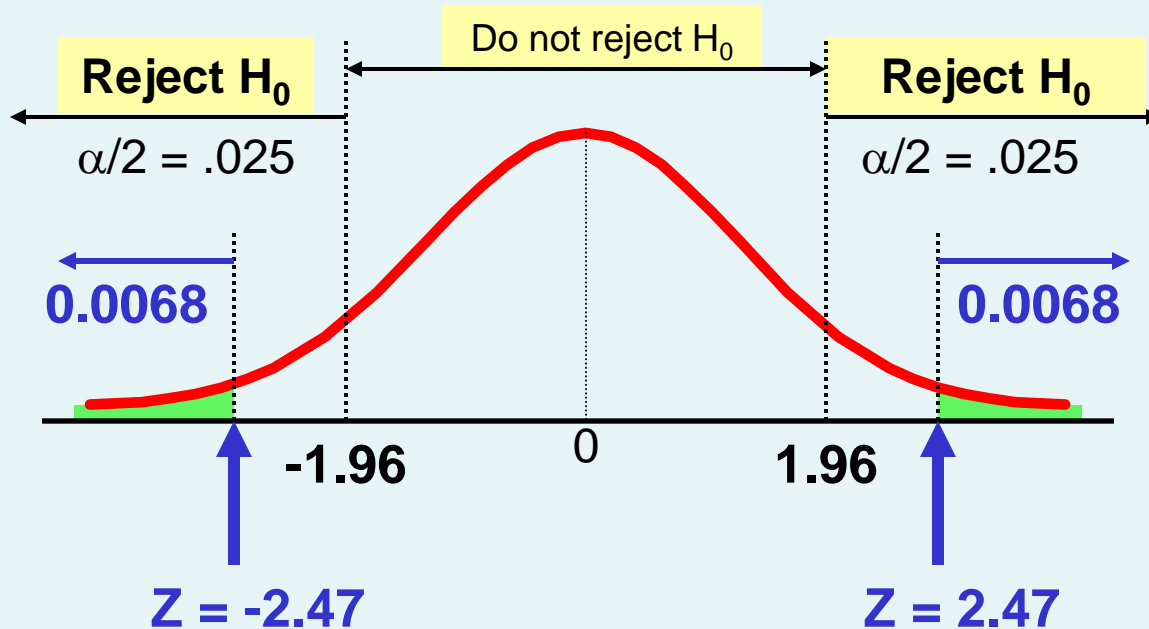
There is sufficient evidence to reject the company's claim of 8% response rate.

# p-Value Solution

DCOVA

(continued)

Calculate the p-value and compare to  $\alpha$   
(For a two-tail test the p-value is always two-tail)



**p-value = 0.0136:**

$$P(Z \leq -2.47) + P(Z \geq 2.47) \\ = 2(0.0068) = 0.0136$$

**Reject  $H_0$  since p-value = 0.0136 <  $\alpha$  = 0.05**



# Potential Pitfalls and Ethical Considerations

## 潜在的な落とし穴と倫理的考慮事項

---

- Use randomly collected data to reduce selection biases 選択バイアスを低減するために、ランダムに収集したデータを使用する
- Do not use human subjects without informed consent インフォームドコンセントなしにヒトを対象とする使用しないでください
- Choose the level of significance,  $\alpha$ , and the type of test (one-tail or two-tail) before data collection データ収集の前に有意水準 $\alpha$ 、およびテスト（片側検定或いは両側検定）の種類を選択してください
- Do not employ “data snooping” to choose between one-tail and two-tail test, or to determine the level of significance 有意水準を選択し、両側検定或いは片側検定を選択し、有意水準を決定する。
- Do not practice “data cleansing” to hide observations that do not support a stated hypothesis 述べた仮説をサポートしていないの観測を非表示にしないこと。
- Report all pertinent findings including both statistical significance and practical importance 統計的有意性と実用的な重要性の両方を含む調査結果を報告する



# Chapter Summary

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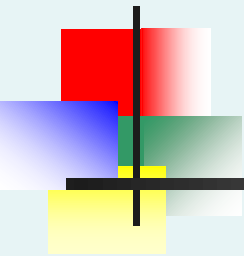
- Addressed hypothesis testing methodology  
仮説検定の方法論
- Performed Z Test for the mean ( $\sigma$  known)
- Discussed critical value and p-value approaches to hypothesis testing 臨界値とp値のアプローチ
- Performed one-tail and two-tail tests 両側検定と片側検定



# Chapter Summary

*(continued)*

- Performed t test for the mean ( $\sigma$  unknown)
- Performed Z test for the proportion
- Discussed pitfalls and ethical issues



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