MATH340 Project---UBC transportation optimization

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Introduction (Background & Aim)
UBC exchange bus loop is always crowded by students every weekday in rush hours(5-6 pm). However, for some bus lines, people have to wait in a 50-meter-long line for a long time, while for others, it's not that crowded. That means the schedule of buses can be modified and optimized to relieve the transportation burden and thus reduce carbon emissions and waiting time. Our goal is to make the most of each bus in peak hours, which means we aim to make sure all students can be transported out of campus in one hour, and in the meanwhile, total carbon emission by all buses is minimum.
Method we use: linear programming
Key packages we use in python: pulp ,gurobipy

Data collection & simulation

1. Data Collection

bus size

3 types of buses and their capacity: Mini-Bus(14 seats) ,Standard Bus(50) ,Articulated Bus(75)

bus line

We consider 9 bus lines coming from UBC: 4, 9, 14, 25, 33, 68, 84, 99, R4
to the 2019 UBC Vancouver Transportation Status Report, 54% of commuters to UBC use public transit. Given there are 58,768 students (UBC, 2022) and on-campus housing accounts for over 13,000 students (UBC, n.d.), the upper bound for how many students use public transit is roughly 24,715.

students data

People use public transit to commute to UBC: 49%(UBC,2021)[1]
Total number of students: 58,768(UBC,2022)[4]
Students living on campus: more than 13000(UBC)[2]

carbon emission

Small, medium and large size buses need approximately 20L, 30L, and 45L of gasoline to run 100km, respectively. Each liter of gasoline generates 2.7kg of carbon dioxide. As such, we can get their carbon emission per km:
Mini-Bus: 20L * 2.7kg/L / 100km = 0.54kg/km
Standard Bus: 30L * 2.7kg/L / 100km = 0.81kg/km
Articulated Bus: 45L * 2.7kg/L / 100km = 1.215kg/km(2019)[2]

2.Data Simulation

population around each bus stop:

surrounding population: We define it as the total number of UBC students living around a specific bus stop. (e.g., the surrounding population for bus stop#15 is 100 means: there are 100 UBC students living around bus stop#15) Some bus stops conjunct several bus lines, while some are only for one bus line. What we have to make sure of is that the total number of people transported to one bus stop>=the surrounding population of that bus stop.

- We suppose: As the UBC bus loop is the terminal bus stop in the westernmost part of Vancouver, which is far from the city center, the main factor leading to the crowded bus is UBC students. Thus, we assume that buses can only be fully loaded at the UBC bus loop, which means there are always enough seats for passengers at all the other stops.
- As data on the amount of UBC students living around each stop is missing, we randomly simulated the number of students in a reasonable range based on certain facts:
  Busy area: the area near downtown is a busy area of transportation. As such, we assume more people are living around this area.
  Campus surrounding area: since students tend to rent houses around campus, we assume more people live around campus as well.

stops on each bus line

We have 59(index from 0 to 58) stops in total. Here shows what stops are on each bus line:
4:[0, 30, 31, 32, 44, 45, 47, 49, 50, 52, 53, 40],
14:[0, 29, 31, 33, 35, 37, 39, 40],
25:[0, 16, 18, 19, 20, 21, 22, 23],
33:[0, 15, 17, 18, 24, 25, 26, 27, 28],
49:[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10],
68:[0, 54, 55, 56, 57, 58],
R4:[0, 11, 4, 12, 13, 14],
84:[0, 41, 42, 43, 44, 46, 48, 51, 40],
99:[0, 29, 31, 34, 36, 37, 38, 40]

	bus line	total length(km)	bus type
0	4	18.00	Standard Bus
1	14	18.00	Standard Bus
2	25	23.19	Standard Bus
3	33	17.63	Standard Bus
4	49	20.88	Articulated Bus
5	68	7.00	Mini Bus
6	R4	13.00	Articlated Bus
7	84	14.00	Standard Bus
8	99	13.42	Articlated Bus

- surrounding population for 59 stops are:[0, 418, 394, 375, 265, 303, 380, 164, 285, 522, 518, 373, 331, 591, 511, 528, 588, 340, 325, 385, 343, 307, 575, 570, 299, 335, 290, 506, 570, 525, 483, 414, 178, 187, 225, 330, 159, 349, 161, 417, 414, 441, 452, 515, 240, 198, 313, 348, 268, 201, 356, 347, 568, 568, 368, 331, 178, 280, 160]

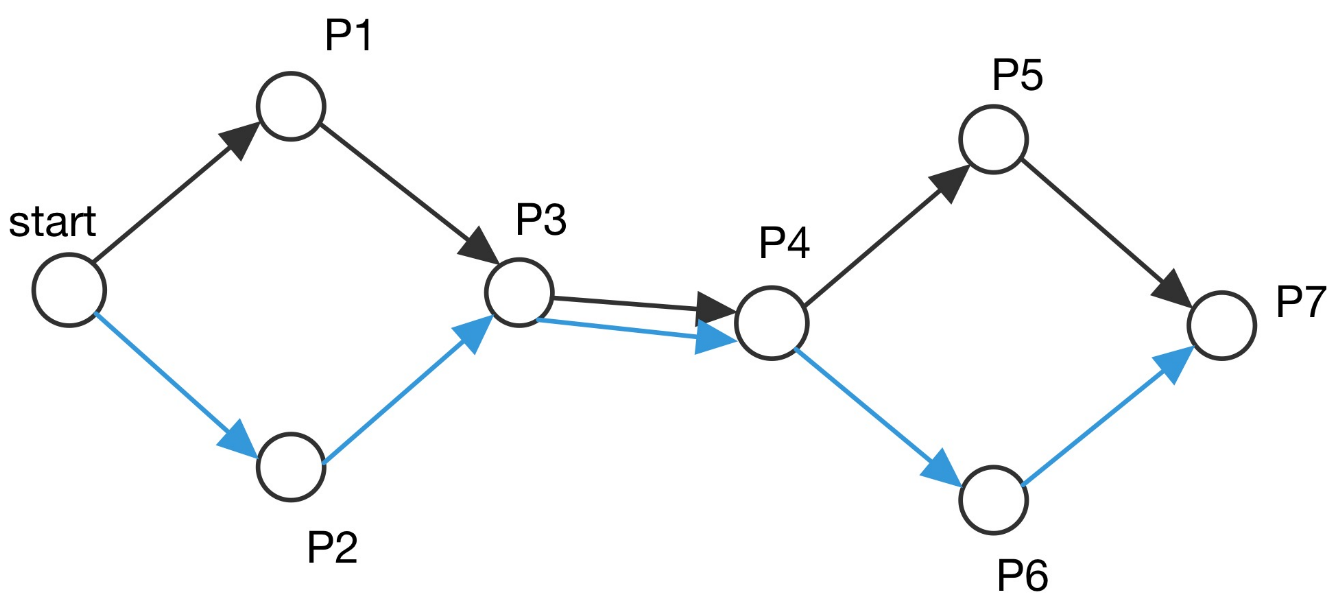
each bus line has different color; each circle represents a bus stop;circle with dashed line represents an intersection:

population capacity of each bus stop

Let the minimum needed population capacity of each bus stop be $sp_1, sp_2 \dots sp_m$. We can calculate this sp array by using p array. For each bus stop, we track all bus lines that can get to this bus stop, and sum up all the surrounding populations which the bus lines have to reach before this bus stop. Then we get the sp array, which means the capacity needed from the start to this bus stop to satisfy this bus stop's surrounding population.

$SP_1 = P_1$
$SP_2 = P2$
$SP_3 = P_1 + P_2 + P_3$
$SP_4 = P_1 + P_2 + P_3 + P_4$
$SP_5 = P_1 + P_3 + P_4 + P_5$
$SP_6 = P_2 + P_3 + P_4 + P_6$
$SP_7 = P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7$

For each color line is a bus line, and each Circle means a bus stop.



- population capacity(SP_i) for 59 bus stops(from index 0 to 59):
  [21365, 418, 812, 1187, 1825, 1755, 2135, 2299, 2584, 3106, 3624, 373, 969, 1560, 2071, 528, 588, 868, 1781, 1298, 1641, 1948, 2523, 3093, 1492, 1827, 2117, 2623, 3193, 525, 483, 1422, 1075, 1126, 1164, 1456, 1323, 2189, 1833, 2222, 8657, 441, 893, 1408, 2723, 1513, 1961, 1861, 2229, 2062, 2418, 2576, 2986, 3554, 368, 699, 877, 1157, 1317]

LP Model

objective function:

minimize:

$\vec{x} =< x_1, x_2, x_3 ... x_n >$ where x_i stands for the number of buses of bus line i getting out of UBC bus loop

$\vec{c} =< e_1, e_2, e_3 ... e_n >$ where e_i stands for carbon emission for each bus to travel from starting station to terminal station.

*meaning in reality: total amount of carbon emission for all buses in rush hours coming out of UBC bus loop.

subject to:

1.for starting station:$\vec{w} * \vec{x} >= U$

where U stands for total amount of UBC students.w_i stands for capacity for one bus of line i. x_i stands for the number of buses of bus line i getting out of UBC bus loop.

2.for other stops: $A * \vec{x} >= \vec{b}$

where each entry in A a_{ij} stands for carrying capacity for bus x_{ij}, and b_i stands for population capacity of bus stop i(same as SP_i above).

*meaning in reality: total amount of students that can be transported to stop i should >= the total amount of students living aroung this stop.So that we can make sure all students can be transported out of UBC.\ In this case, we consider the scenario where buses fully load,instead of making sure that each person has a seat. 3.non-negative constraints:$\vec{x} >= 0$

Our variables(bus0-9) correspond to these bus lines:

bus0:bus line4.bus1:bus line14. bus2:bus line25. bus3:bus line33. bus4:bus line49.bus5:bus line68.bus6:bus lineR4.bus7:bus line84.bus8:bus line99

LP Model:(*Only first few constraints are displayed as an example)*

MINIMIZE

*3.24*bus0 + 9.72*bus1 + 15.187500000000002*bus2 + 15.795000000000002*bus3 + 15.187500000000002*bus4 + 5.4*bus5 + 5.130000000000001*bus6 + 8.100000000000001*bus7 + 13.354*bus8

SUBJECT TO

stop0: 18 bus0 + 25 bus1 + 40 bus2 + 40 bus3 + 40 bus4 + 18 bus5 + 18 bus6 + 25 bus7 + 40 bus8 >= 10805

stop1: 25 bus7 + 40 bus8 >= 391

stop2: 18 bus0 + 25 bus1 + 40 bus2 + 40 bus4 >= 331

stop3: 18 bus0 + 25 bus1 + 40 bus2 + 40 bus3 >= 647

stop4: 18 bus0 + 25 bus1 + 40 bus2 >= 973

......

bus0 ,bus1,bus2...bus8>=0

Primal optimal solution & Result visualization:

status : 1 Optimal

objective : 7285.0485

bus0 : 254.0

bus1 : 77.0

bus2 : 65.0

bus3 : 67.0

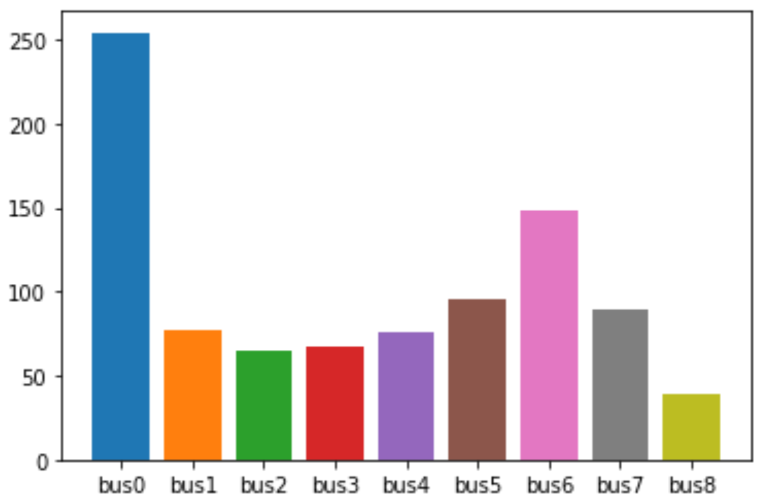
bus4 : 76.0

bus5 : 95.0

bus6 : 148.0

bus7 : 89.0

bus8 : 39.0



Dual Problem

dual:(only first few constraints are given as an example)

MAXIMIZE

10805*stop0 + 391*stop1 + 242*stop10 + 914*stop11 + 1150*stop12 + 924*stop13 + 488*stop14 + 641*stop15 + 1170*stop16 + 891*stop17 + 1408*stop18 + 1361*stop19 + 331*stop2 + 1315*stop20 + 1431*stop21 + 1166*stop22 + 1469*stop23 + 699*stop24 + 1100*stop25 + 1657*stop26 + 2642*stop27 + 1741*stop28 + 1505*stop29 + 647*stop3 + 1237*stop30 + 840*stop31 + 2229*stop32 + 2876*stop33 + 3054*stop34 + 670*stop35 + 1289*stop36 + 1615*stop37 + 1769*stop38 + 1893*stop39 + 973*stop4 + 1826*stop40 + 1972*stop41 + 746*stop42 + 890*stop43 + 3470*stop44 + 3588*stop45 + 3499*stop46 + 852*stop47 + 997*stop48 + 1704*stop49 + 1295*stop5 + 1245*stop50 + 2796*stop51 + 4342*stop52 + 1622*stop6 + 2002*stop7 + 2745*stop8 + 995*stop9

SUBJECT TO
bus0: 18 stop0 + 18 stop2 + 18 stop3 + 18 stop4 + 18 stop5 + 18 stop6

- 18 stop7 + 18 stop8 <= 3.24

bus1: 25 stop0 + 25 stop12 + 25 stop19 + 25 stop2 + 25 stop26 + 25 stop3

- 25 stop38 + 25 stop4 + 25 stop44 + 25 stop45 <= 9.72

bus2: 40 stop0 + 40 stop12 + 40 stop2 + 40 stop20 + 40 stop27 + 40 stop3

- 40 stop33 + 40 stop34 + 40 stop4 + 40 stop40 + 40 stop46 <= 15.1875

......
stop0,stop1...stop58>=0

Dual optimal solution

stop1 : 0.0; stop10 : 0.31640625; stop11 : 0.0; stop12 : 0.0; stop13 : 0.0; stop14 : 0.36642857; stop15 : 0.0; stop16 : 0.0; stop17 : 0.0; stop18 : 0.0; stop19 : 0.0; stop2 : 0.0; stop20 : 0.0; stop21 : 0.0; stop22 : 0.0; stop23 : 0.31640625; stop24 : 0.0; stop25 : 0.0; stop26 : 0.0; stop27 : 0.0; stop28 : 0.3290625; stop29 : 0.0; stop3 : 0.0; stop30 : 0.0; stop31 : 0.0; stop32 : 0.0; stop33 : 0.0; stop34 : 0.0; stop35 : 0.0; stop36 : 0.0; stop37 : 0.0; stop38 : 0.27820833; stop39 : 0.33517241; stop4 : 0.0; stop40 : 0.0; stop41 : 0.0; stop42 : 0.0; stop43 : 0.0; stop44 : 0.0; stop45 : 0.0; stop46 : 0.0; stop47 : 0.0; stop48 : 0.0; stop49 : 0.0; stop5 : 0.0; stop50 : 0.0; stop51 : 0.27931034; stop52 : 0.0; stop53 : 0.23142857; stop54 : 0.0; stop55 : 0.0; stop56 : 0.0; stop57 : 0.0; stop58 : 0.38571429; stop6 : 0.0; stop7 : 0.0; stop8 : 0.0; stop9 : 0.0;

Further analysis & Discussion

our further analysis&discussion contains 3 parts:
1.release bus burden(because we do not want buses to always fully load)
2.sensitivity analysis 3.comparison between dual and primal optimal solution

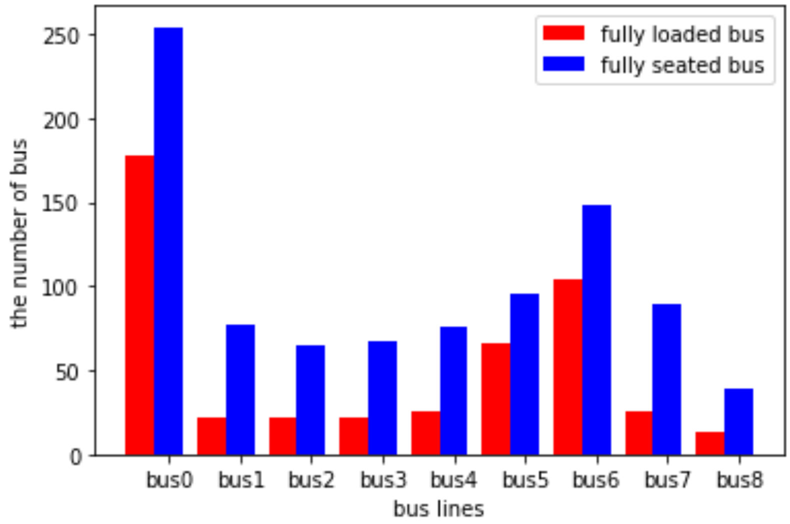
1. release bus burden

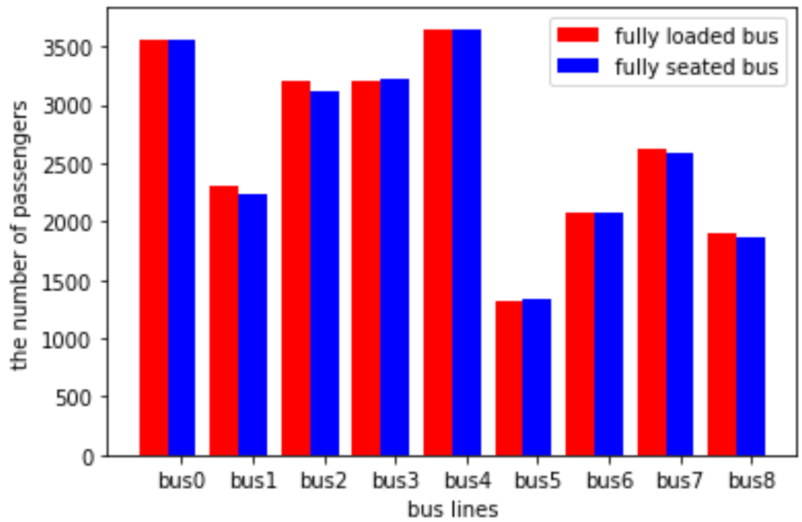
Compared with the case where everyone has a seat

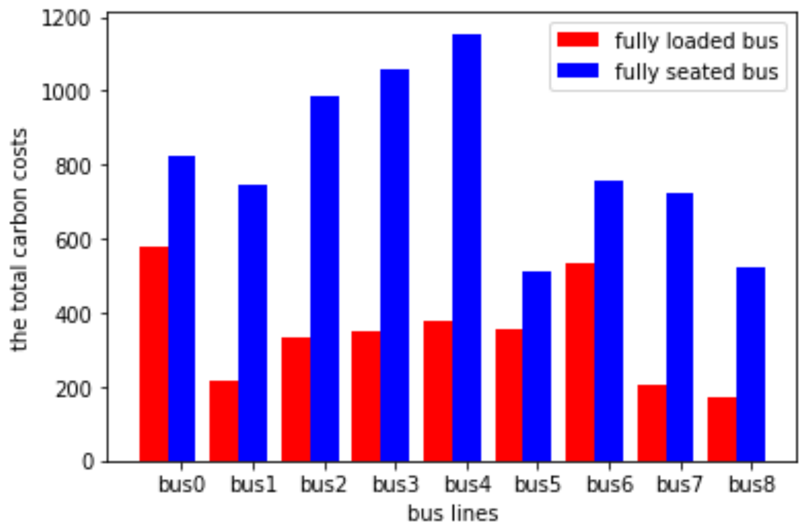
In our original problem, we suppose that buses are fully loaded, which means buses are very crowded and many passengers have to stand all the way home. Here we plan to find a way to release the bus burden. Thus, we suppose bus capacity equals the number of seats. That is to say; all passengers are guaranteed to have seats in this case.

Result Visualization

We visualize the total number of passengers each bus line transports in these 2 scenarios: bus fully loaded or all passengers have seats. We can quickly tell that bus line 0 transports much more passengers than most of other bus lines.







Based on number of passengers each bus transport,we can tell the top3 busy lines are bus line 4, 0 and 2

To reduce burden of buses, we may consider:

- solution 1.add more bus lines to the area where busy bus lines passes by

- solution 2.add more buses for bus busy bus line Based on our comparison of 2 scenarios, our original one gives how many buses we need at least(lower bound), our second one gives us how many buses we need at most(upper bound). As such,if we choose solution 2, the number of buses added to each bus line should be between the lower bound and upper bound, depending on to what extend we want to lease bus burden.

2.sensitivity analysis

Constraint	Sense	Slack	Pi	RHS	SARHSLow	SARHSUp
stop0	>	-2118	0	21365	-inf	23483
stop1	>	-3206	0	418	-inf	3624
stop2	>	-2812	0	812	-inf	3624
stop3	>	-2437	0	1187	-inf	3624
stop4	>	-3870	0	1825	-inf	5695
stop5	>	-1869	0	1755	-inf	3624
stop6	>	-1489	0	2135	-inf	3624
stop7	>	-1325	0	2299	-inf	3624
stop8	>	-1040	0	2584	-inf	3624
stop9	>	-518	0	3106	-inf	3624
stop10	>	0	0.316406	3624	3106	inf
stop11	>	-1698	0	373	-inf	2071
stop12	>	-1102	0	969	-inf	2071
stop13	>	-511	0	1560	-inf	2071
stop14	>	0	0.366429	2071	1560	inf

Here shows the first 15 rows of our sensitivity analysis.
The key columns we use in our analysis are:

- 1.Slack:describes how close we are to satisfying a constraint as an equality
- 2.SARHSLow:smallest right-hand-side value at which the current optimal solution remains unchanged
- 3.SARHSUp:largest right-hand-side value at which the current optimal solution remains unchanged

2.1 What constraints affect results

$$stop5 : 18bus0 >= 1295$$

$$stop6 : 18bus0 >= 1622$$

$$stop7 : 18bus0 >= 2002$$

Above is a typical example we found in our LP model that makes us interested. Here we can see that these 3 constraints share the same coefficient and variable with different right-hand side b_i . In this case, what actually affects our optimal result is the constraint of stop7 as the constraint of stop7 is more strict than stop5 and 6. There are many such cases in our LP model, which means the problem could be simplified.

2.2 What causes constraints to be ineffective

Along one bus line, as SP (defined in simulated data) accumulates, the right-hand side b_i accumulates. Thus, in all constraints with the same coefficients, only the one with the largest b_i is effective. What's more, when bus lines have intersections, both coefficients and b_i change compared to constraints with only 1 bus. As such, constraints representing intersections are also effective.
To add, not all constraints representing intersections are effective. For instance.

$$stop44 : 25bus1 + 40bus3 >= 3470$$

$$stop45 : 25bus1 + 40bus3 >= 3588$$

stop44 is the first time bus1 and bus3 intersect, and after that, they share the same route till stop45. In this case, only the constraint of stop 45 is effective. Essentially, the reason why the constraint of stop44 is ineffective is the same as the reason why constraints of stop5 and 6 are ineffective. Due to the accumulation of the amount of passengers, b_i increases along the bus line. So among all constraints with the same right-hand side, only the one with the largest b_i is effective.

2.3 explanation of our sensitivity analysis results

- From our full result data frame(59 rows in total with 15 rows shown above), we can see that all the terminal stops have Slack as 0(eg.stop10 and stop14 as above). Additionally, along a single bus line, stops that are close to the terminal stop tend to have Slacks closer to 0 compared with other stops. It means our optimality is reached only at terminal stops which makes sense as our RHS value Sp_i in constraints about passenger amount are accumulative. As such, accumulative passenger amounts are the largest at terminal stops. It also explains why at terminal stops SARHSUps are all inf while SARHSLows are some constants. It is because we already reach optimality when satisfying this constraint as equality.If the RHS value of this constraint gets larger we still need to satisfy equality to reach optimality. However, if the RHS values of constraints of terminal stops get too small, say, smaller than that of another stop along the same bus line. Then we need to satisfy that new stop's constraint as equality,which changes the optimal solution.
- For intersections of bus lines, we can notice their Slacks decrease compared with their previous stops. Slacks always increase, getting closer to 0 along a single bus line apart from bus line intersections. An intersection can generate a looser constraint compared with its previous stop. Here is an example of an explanation:
  Suppose the bus line1 and bus line 3 intersect at some point. Adding the first 2 constraints, we will get the third one, which represents intersection:
  $25bus1 >= 1500$
  $40bus3 >= 1500$
  $25bus1 + 40bus3 >= 3000$
  But the third one is looser than the first 2 constraints together, as in the third one

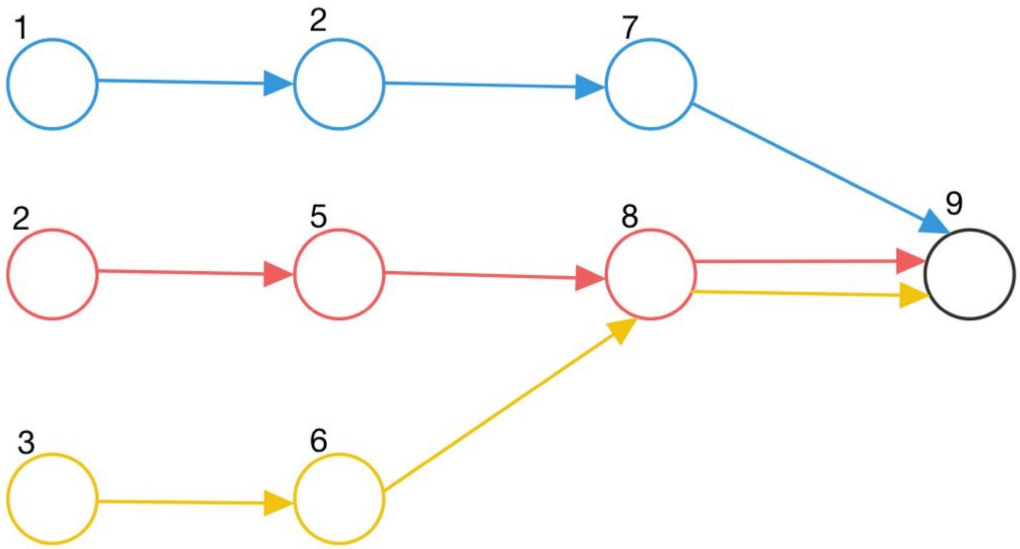
$25bus1 < 1500$ but $40bus3 > 1500$ also satisfies the constraint. It is the reason why intersection makes constraints looser.
In this way, although we do not reach equality at intersections in this case, they affect constraints of later stops.

2.4 What kind of stops affect optimal solutions directly

From our discussion above, only 2 kinds of constraints are effective:

- on each bus route with a single bus line: constraints with largest b_i
  For a bus map with a given amount of bus lines, simply enlarging stop numbers or bus line length does not increase the number of effective constraints. Instead, a map with more bus line intersections has more effective constraints.
- Exception: most constraints of terminal stops reach equality in our optimal solution but may NOT for terminal stops that are also intersections. As we discussed above, intersections generate looser constraints, which may affect which constraints are satisfied as equality.

For example:
In the below case, due to intersections, constraints of stops 8 and 9 may be looser than those of 5,6,7. Thus constraints of stops 5,6,7 may be satisfied as equality, instead of later stops 8 and 9.



3.Optimal value to Primal and Dual

In reality, the number of buses should be an integer. However, if we set variables to the primal and dual problem as integers, we would get 0 as the optimal value to the dual problem and

4946.602 to the primal problem. In this case, when we want to double-check if our solution is optimal, we should not set variables as an integer.

References

[1]"Transportation Status Report Fall 2021."The University of British Columbia website.The University of British Columbia.June 2022.https://planning.ubc.ca/sites/default/files/2022-06/UBC2021-TransportationStatusReport-FINAL.pdf

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[4]"UBC Overview & Facts" The University of British Columbia website.The University of British Columbia.2022.https://www.ubc.ca/about/facts.html