Equivalence Hierarchy

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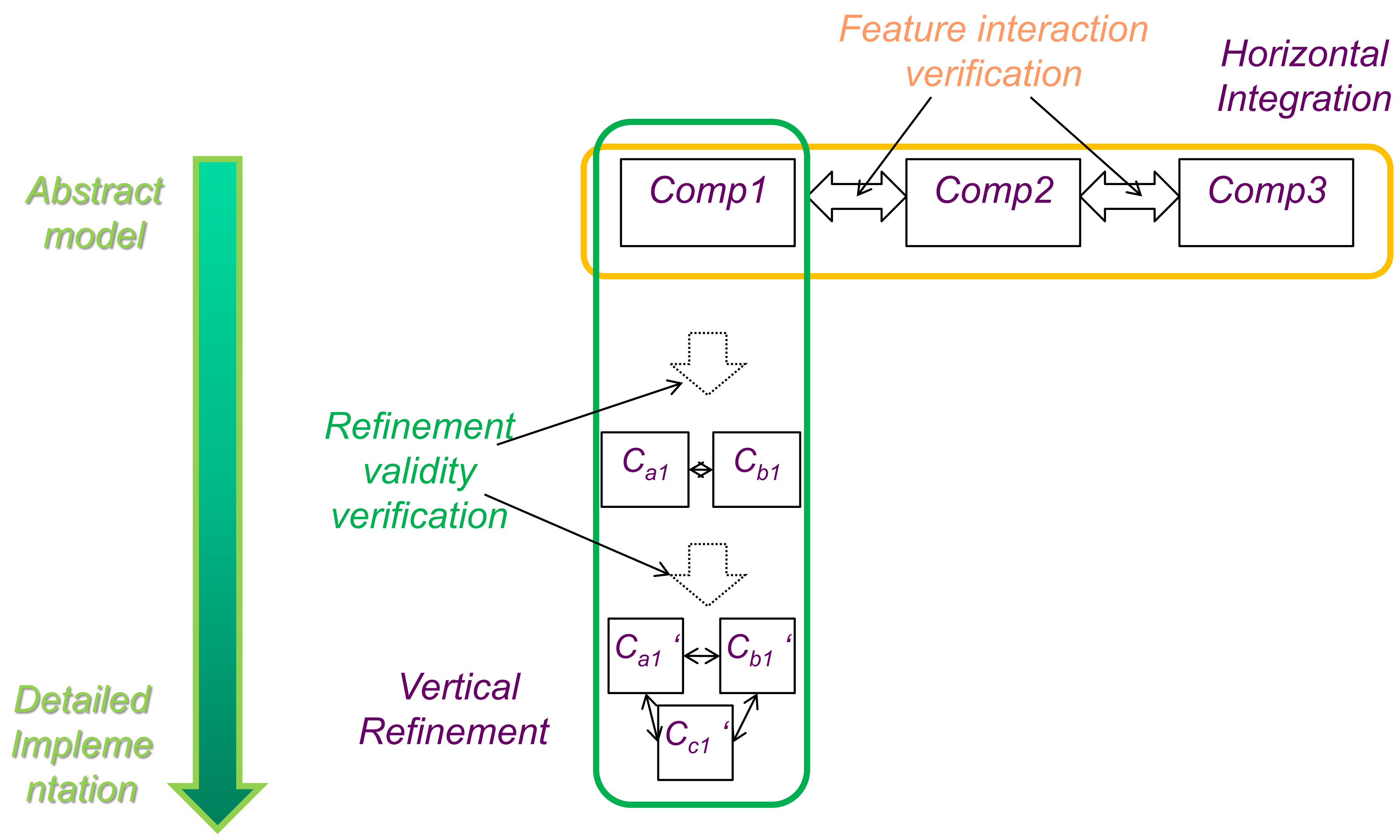
Outline

- Equivalence semantics and SW design
- Preliminary
- Hierarchy Diagram
- Trace-based Semantics
  - Trace EQ
  - Complete Trace EQ
  - Failure EQ
- Branching-based Semantics
  - Simulation EQ
  - Bisimulation EQ

Equivalence Preserving Refinement and SW Design

- Design can start with a very abstract specification, representing the requirements
- Then, using equivalence-preserving transformations, this specification can be gradually refined into an implementation-oriented specification.
- Maintenance may require to replace some components with others, while maintaining the same system behavior (congruence property)

Model-driven SW Design Framework



Semantic Mapping

- An example of small language
  - Syntax
    - F := 0 | 1 | F + 1 | 1 + F
    - Ex. 0, 0+1+1, 1+0+1, but not 0+0
  - Possible semantics
    - 1 + 1 == 1 + 1 + 0 ?
      - Yes (interpreting formula as a natural #),
        - $[1 + 1]_{N1} = 2$, $[1 + 1 + 0]_{N1} = 2 \rightarrow 1 + 1 =_{N1} 1 + 1 + 0$
      - No (interpreting formula as string),
        - $[1+1]_S$="1+1",$[1+1+0]_S$="1+1+0"$\rightarrow$ $1+1 \,!=_S 1+1+0$
      - No (interpreting formula as a natural # of string length)
        - $[1 + 1]_{N2} = 3$, $[1 + 1 + 0]_{N2} = 5 \rightarrow 1 + 1 \,!=_{N2} 1 + 1 + 0$

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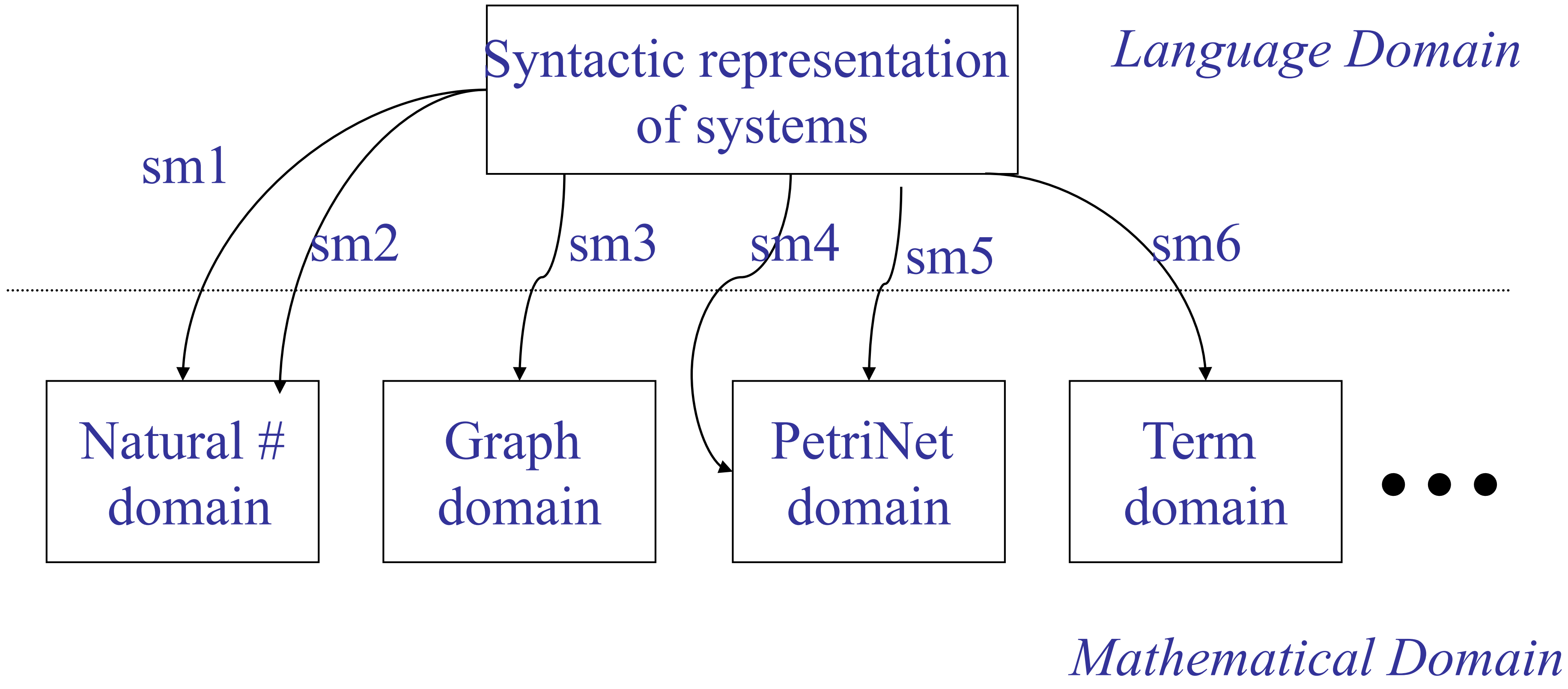
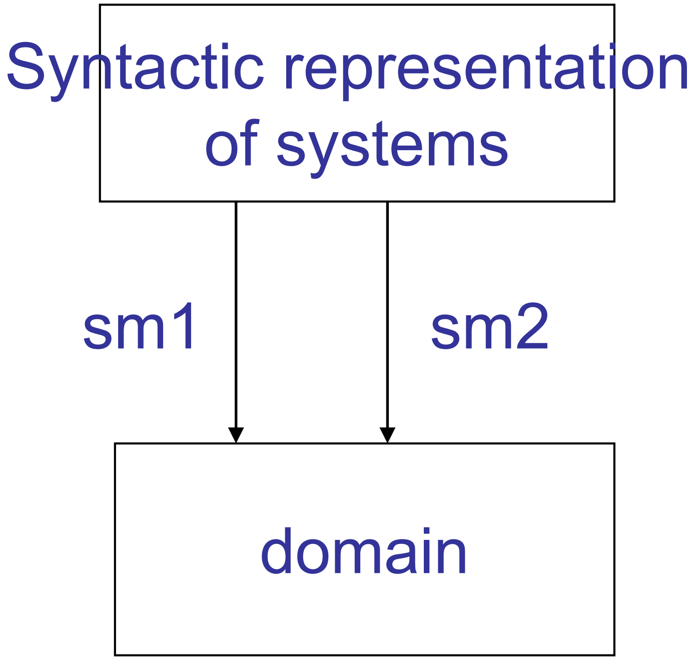
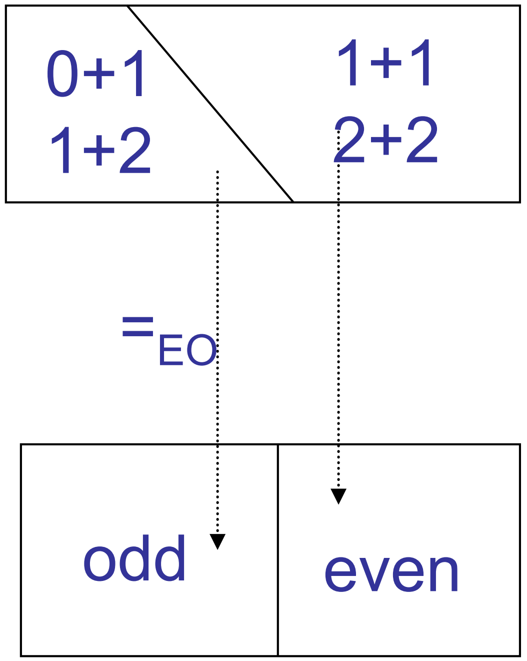
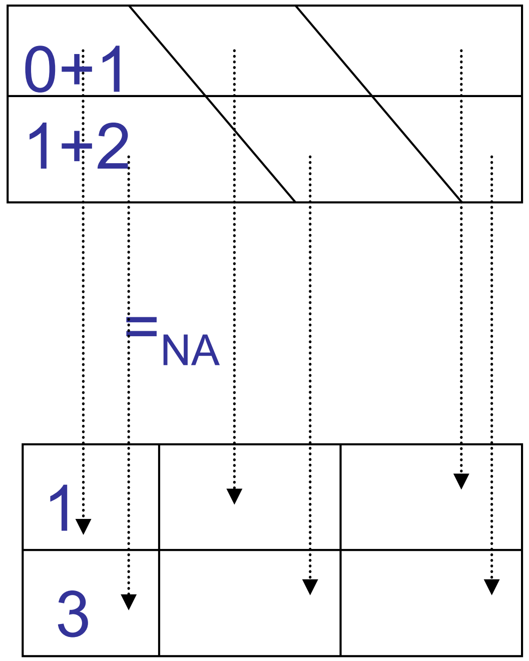
Semantic Mapping (cont.)



Relation between (Equivalence) Semantics



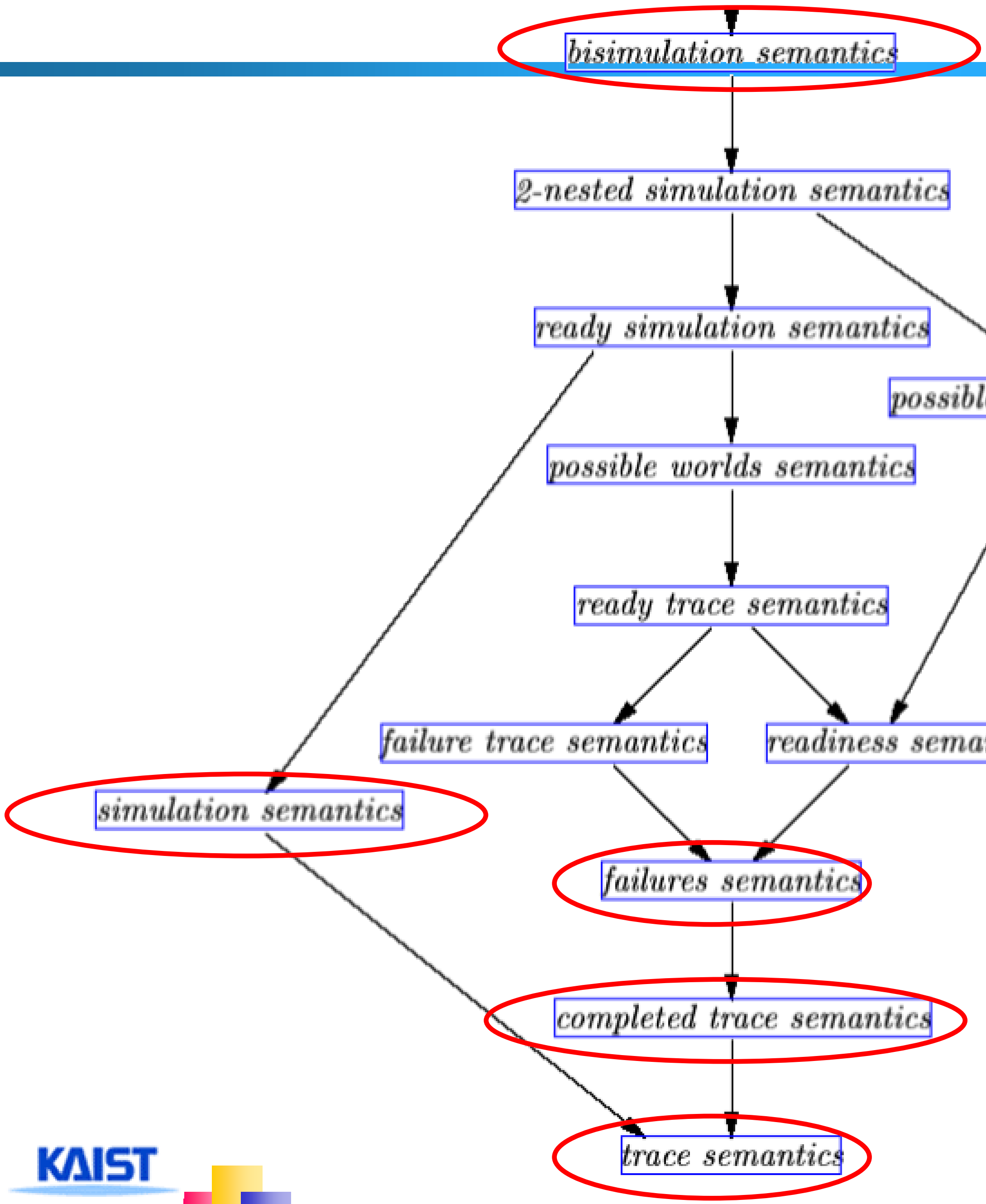




$0+1 =_{EO} 1+2$
$1+1 =_{EO} 2+2$

$0+1 \;!=_{NA} 1+2$

$P =_{NA} Q \rightarrow P =_{EO} Q$ but not vice versa
Therefore, $=_{EO} \; < \; =_{NA}$



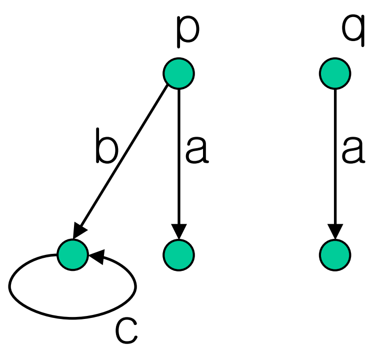
$$\begin{tikzcd}
 & \text{bisimulation} \arrow[dl] \arrow[d] \\
\text{simulation} \arrow[ddr] & \text{failure} \arrow[d] \\
 & \text{complete trace} \arrow[d] \\
 & \text{trace}
\end{tikzcd}$$

Labeled Transition System

- Process Theory
  - A *process* represents behavior of a system
  - Two main activities of process theory are *modeling* and *verification*
    - The semantics of equalities is required to verify system
    - Determine which semantics is suitable for which applications
- Labeled Transition System (LTS)
  - *Act*: a set of actions which process performs
  - LTS: $(P, \rightarrow)$
    - Where P is a set of processes and $\rightarrow \subseteq P \times Act \times P$
  - In this presentation, we deal with only *finitely branching, concrete, sequential processes*
- Useful notations
  - Equivalence notation for each semantics
    - $=_T, =_{CT}, =_F, =_R, =_{FT}, =_{RT}, =_S, =_{RS}, =_B$
    - I(p) is $\{a \in Act \mid \exists q.\ p \text{ -a->} q\}$

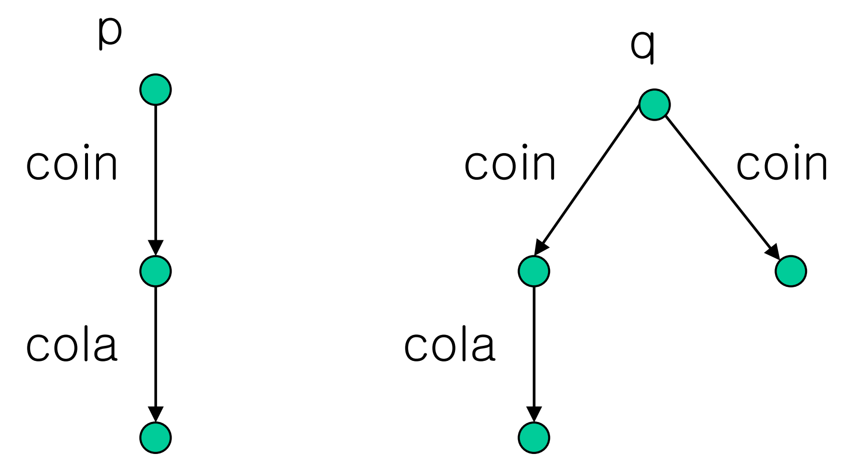
Trace v.s. Complete Trace

- Trace semantics (T)
  - $\sigma \in Act^*$ is a *trace* of a process p if there is a process q s.t. p -σ-> p
  - T(p) is a set of traces of a process p
  - $p =_T q$ iff T(p) = T(q)
- Complete trace semantics (CT)
  - $\sigma \in Act^*$ is a *complete trace* of a process p if there is a process q s.t. p -σ-> q and I(q) = $\emptyset$
  - CT(p) is a set of complete traces of a process p
  - $p =_{CT} q$ iff T(p) = T(q) and CT(p) = CT(q)
  - Note that CT(p) = CT(q) does not imply T(p) = T(q)



- $=_T < =_{CT}$
  - $p =_{CT} q$ implies $p =_T q$, but not vice versa

Counter Example 1



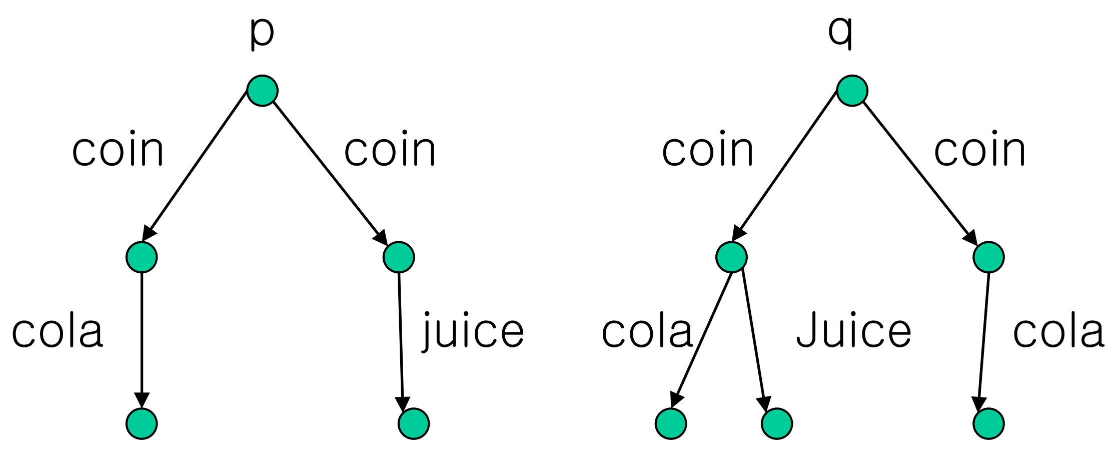
- $p =_T q$
  - T(p) = {coin.cola, coin}
  - T(q) = {coin.cola, coin}
- $p \neq_{CT} q$
  - CT(p) = {coin.cola}
  - CT(q) = {coin.cola, coin}

Failure Semantics

- Failure Semantics (F)
  - $<\sigma,X> \in Act^* \times$ ⍰ (Act) is a failure pair of p if $\exists$q s.t. p $-\sigma\text{->}$ q and I(q) $\cap$ $X = \varnothing$
  - F(p) is a set of failure pairs of p
  - $p =_F q$ iff F(p) = F(q)

- $=_{CT} < =_F$
  - $p =_F q$ implies $p =_{CT} q$
    - $\sigma \in$ CT(p) iff $<\sigma,Act> \in$ F(p)
    - $\sigma \in$ T(p) iff $<\sigma,X> \in$ F(p) for some X s.t. X $\cap$ I(q) $= \varnothing$ Where $p-\sigma\text{->}q$
  - not vice versa

Counter Example 2



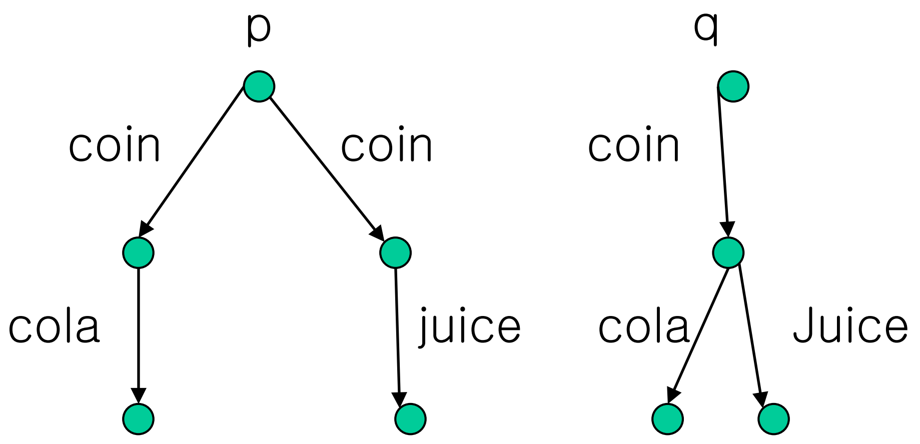
- $p =_{CT} q$
  - CT(p)={coin.cola, coin.juice}
  - CT(q)={coin.cola, coin.juice}
- $p \neq_F q$
  - $\{\langle coin,\{coin,cola\}\rangle\} \in F(p)$
  - $\{\langle coin,\{coin,cola\}\rangle\} \notin F(q)$

Simulation Semantics

- The set F_s of simulation formulas over Act is defined inductively by
  - $\mathit{True} \in F_s$
  - If $\Phi, \Psi \in F_s$ then $\Phi \wedge \Psi \in F_s$
  - If $\Phi \in F_s$ and $a \in \mathit{Act}$, then $a.\Phi \in F_s$
- The satisfaction relation $\models \subseteq P \times F_s$ is defined inductively by
  - $p \models \mathit{True}$ for all $p \in P$
  - $p \models \Phi \wedge \Psi$ if $p \models \Phi$ and $p \models \Psi$
  - $p \models a.\Phi$ if for some $q \in P$: $p \xrightarrow{a} q$ and $q \models \Phi$
- $p =_S q$ iff $S(p) = S(q)$ where $S(p) = \{\Phi \in F_S \mid p \models \Phi\}$

$=_T < =_S$

- $=_T < =_S$
  - $p =_S q$ implies $p =_T q$
    - $=_T < =_S$ by $\sigma \in T(p)$ iff $\sigma.True \in S(p)$
  - not vice versa



- $p \neq_S q$
  - $S(p)= \{True$, coin.$True$, coin.cola.$True$, coin.juice.$True$, …, coin.cola.$True \wedge$ coin.juice.$True\}$
  - $S(q) = \{True$, coin.$True$, coin.cola.$True$, coin.cola.$True$, …, coin.cola.$True \wedge$ coin.juice.$True$, coin.(cola.$True \wedge$ juice.$True$) $\}$

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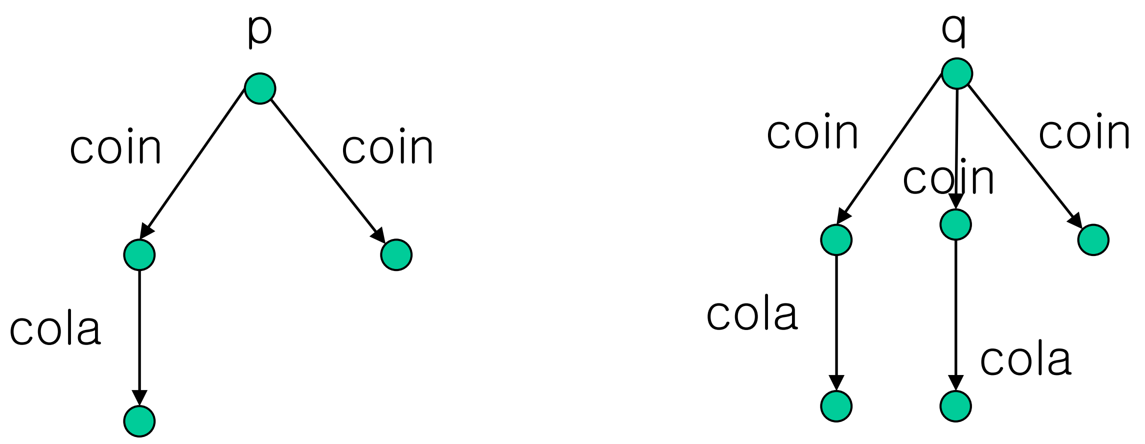
Simulation v.s. Bisimulation

- A simulation is a binary relation R on processes satisfying for a $\in$ *Act*
  - If pRq and p-a->p' , then $\exists q'$:q-a->q' and p'Rq'
- $p =_S q$ iff there exist simulation relations R_1 and R_2 such that pR_1q and qR_2p
- A bisimulation is a binary relation R on processes satisfying for a $\in$ *Act*
  - If pRq and p-a->p' , then $\exists q'$:q-a->q' and p'Rq'
  - If pRq and q-a->q' , then $\exists p'$:p-a->p' and p'Rq'
- $P =_B q$ if there exists a bisimulation R with pRq

Counter Example 3

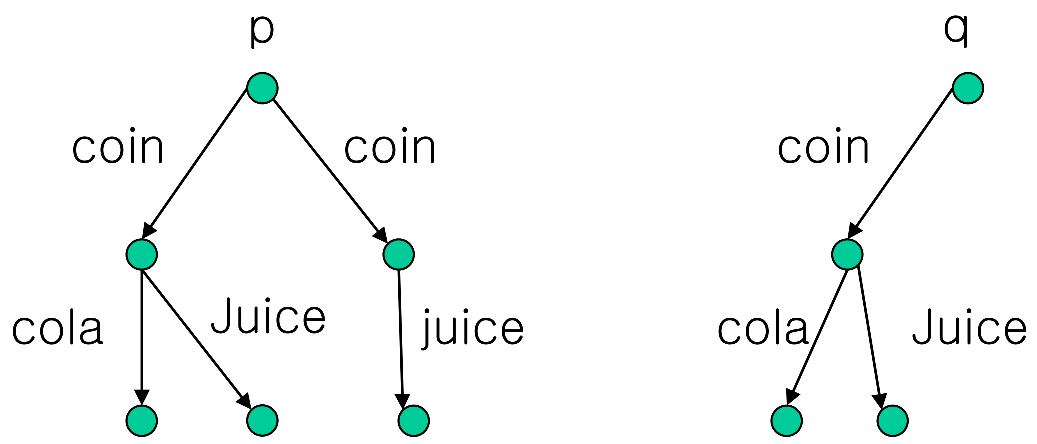
$p =_B q$

$p =_s q$



$p \neq_B q$

$p =_s q$


p
coin
coin
cola
Juice
juice
q
coin
cola
Juice