Equivalence Semantics of CCS

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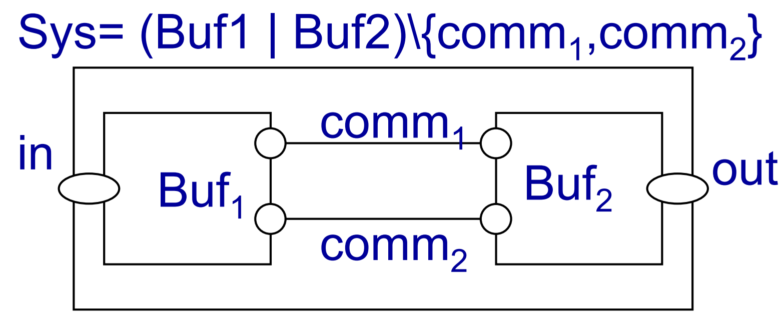
Oultline

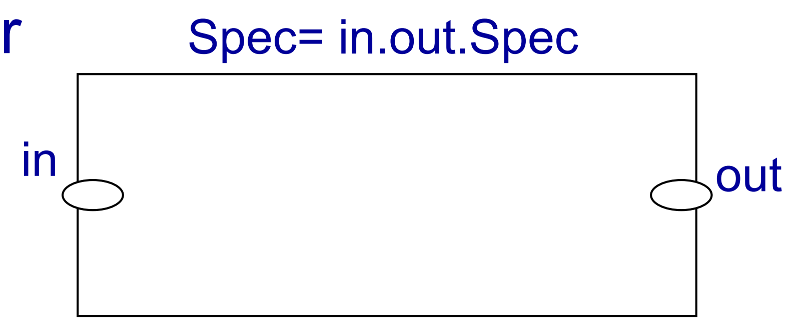
- Trace Equivalence
- Observational Trace Equivalence
- Bisimulation Equivalence
- Observational Bisimulation Equivalence
- May Preorder and Must Preorder
- Example
- Usage of Concurrent Workbench

Trace Equivalence

- Sys is a design for buffer with separated input/output ports
  - $Sys = (Buf_1 \mid Buf_2)\backslash\{comm_1, comm_2\}$
    - $Buf_1 = in.comm_1'.Buf_1'$, $Buf_1' = comm_2.Buf_1$
    - $Buf_2 = comm_1.Buf_2'$, $Buf_2' = out'.comm_2'.Buf_2$



- Spec is a requirement for the buffer design
  - Spec = in.Spec', Spec'=out'.Spec

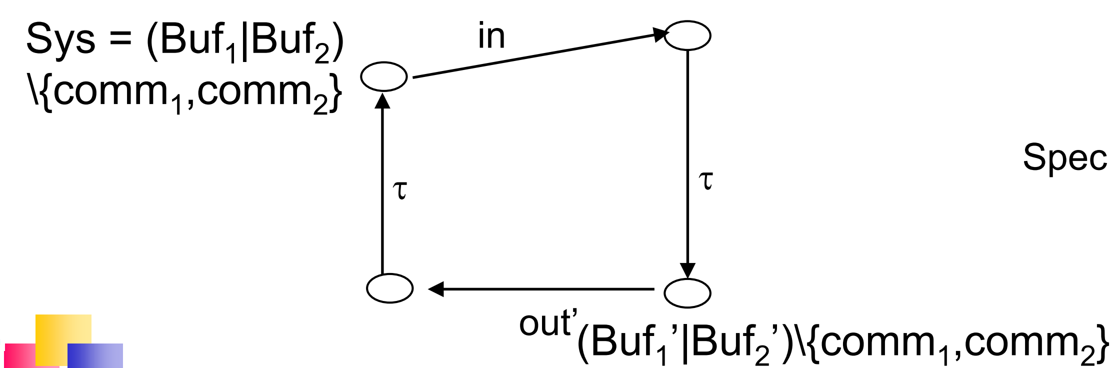


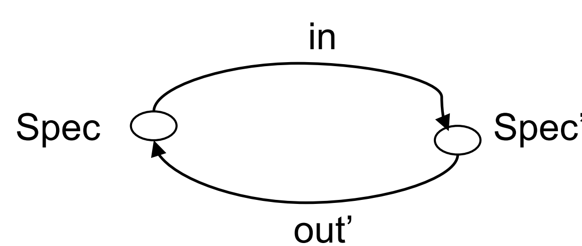
- Question: Sys == Spec?
  - Let us consider trace equivalence (i.e. language equivalence) $=_T$
    - $T(P) = \{ s \in Act^* \mid s$ is an execution trace of $P\}$
    - $P =_T Q$ iff $T(P) = T(Q)$

Observational Trace Equivalence

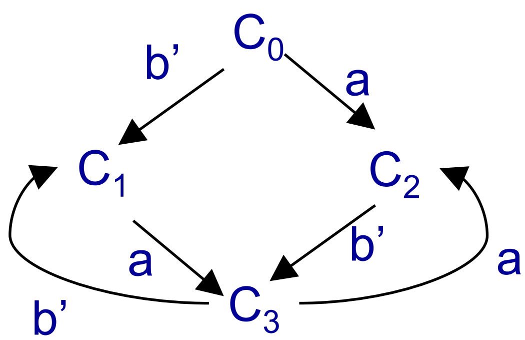
- Sys $=_T$ Spec?
  - No. Sys has τ which Spec does not
    - T(Sys) = {in, in.τ, in.τ.out' , in.τ.out'.τ,…}
    - T(Spec) = {in , in.out' …}
  - Yes. τ is an internal hidden action not visible outside (not observable). Thus, τ should not be included in an execution
    - If s$\in$Act*, then ŝ $\in$(Act –{τ})* is the action sequence obtained by deleting all occurrences of τ from s.
      - Ex> s = a.τ.b.τ.c, then ŝ = a.b.c
    - A set of observable execution traces: T'(P) = {ŝ | s $\in$ T(P)}
    - P $=_{OT}$ Q iff T'(P) = T'(Q)
    - Sys $=_{OT}$ Spec because T'(Sys) = {in, in.out',…}, T'(Spec) = {in, in.out', …}

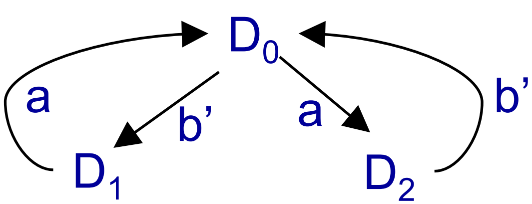
Sys= (Buf1 | Buf2)\{comm1,comm2}
Buf1 = in.comm1.Buf1', Buf1' = comm2.Buf1
Buf2 = comm1'.Buf2',Buf2'=out.comm2'.Buf2
Spec= in.out.Spec





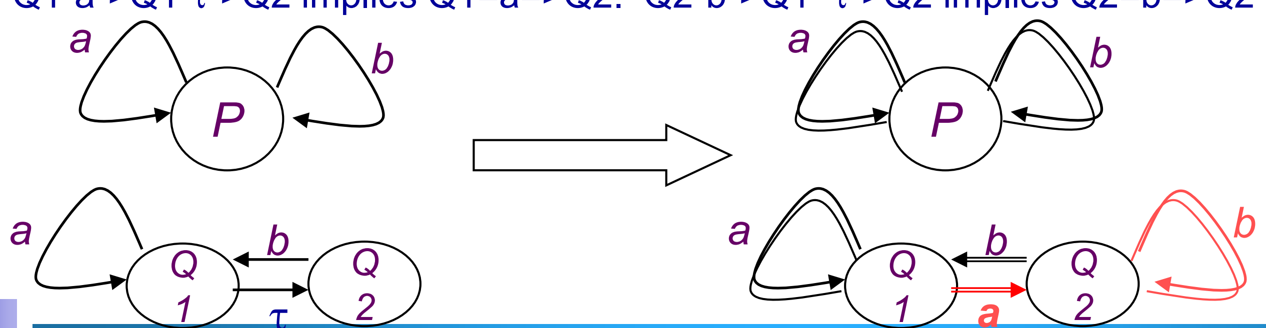
- $P =_{BS} Q$ iff for all $\alpha \in Act$
  - Whenever $P \xrightarrow{\alpha} P'$, then for some Q', $Q \xrightarrow{\alpha} Q'$ and $P' =_{BS} Q'$
  - Whenever $Q \xrightarrow{\alpha} Q'$, then for some P', $P \xrightarrow{\alpha} P'$ and $P' =_{BS} Q'$
- Note
  - $=_{BS}$ is an equivalence relation (reflexive, transitive, symmetric)
  - $P =_{BS} Q$ implies $P =_T Q$, but not vice versa
- Example>
  - $C_0 = b'.C_1+a.C_2$, $C_1=a.C_3$, $C_2=b'.C_3$, $C_3 = b'.C_1+a.C_2$
  - $D_0 = b'.D_1 + a.D_2$, $D_1=a.D_0$, $D_2=b'.D_0$
  - A binary relation R proves that $C_0 =_{BS} D_0$
    - $R = \{(C_0,D_0), (C_1,D_1), (C_2,D_2), (C_3,D_0)\}$





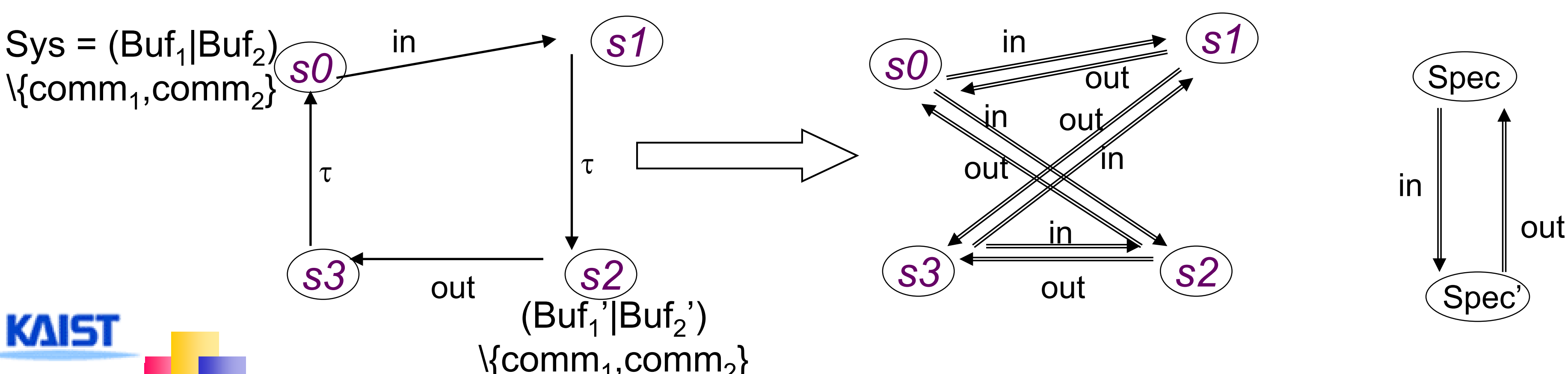
Observational Bisimulation Equivalence

- We cannot simply ignore τ for observational bisimulation equivalence. Thus, we define a new observational transition $=\alpha=>$
- P $=_{OBS}$ Q iff for all $\alpha \in$ Act
  - Whenever P $=\alpha=>$ P', then for some Q', Q $=\alpha=>$ Q' and P' $=_{OBS}$ Q'
  - Whenever Q $=\alpha=>$ Q', then for some P', P $=\alpha=>$ P' and P' $=_{OBS}$ Q'
- P $=\alpha=>$ Q iff P $(-\tau->)^*-\alpha->(-\tau->)^*$ Q where $\alpha \in Act$-$\{\tau\}$
  - Let s$\in$(Act-$\{\tau\}$)*. Then q =s=> q' if there exists s' s.t. q-s'->q' and s=ŝ'
  - P = a.P + b.P, Q1=a.Q1 + τ.Q2, Q2=b.Q1
    - Suppose that 'a' means pushing button 'a'. Similarly for 'b'
      - P always allows a user to push any buttons.
      - Q1 allows a user to push button 'a' sometimes, button 'b' sometimes.
    - Thus, we need to distinguish P from Q1 (P and Q1 are not observationally bisimilar), which can be done using $=\alpha=>$ instead of $-\alpha->$
      - Q1-a->Q1 implies Q1=a=>Q1. Similary Q2-b->Q1 implies Q2=b=>Q1
      - Q1-a->Q1-τ->Q2 implies Q1=a=>Q2. Q2-b->Q1- τ->Q2 implies Q2=b=>Q2



Observational Bisimulation Equivalence (cont)

- Sys $=_{BS}$ Spec? (see slide 3)
  - No. Sys has τ which Spec does not (i.e. not strongly bisimilar)
- Sys $=_{OBS}$ Spec?
  - Yes. Sys is observationally bismilar to Spec
    - Proof: R = { (s0,Spec), (s1,Spec'),(s3,Spec),(s2,Spec')}
      - s0 –in->s1 implies s0=in=> s1. Similarly, s2-out->s3 implies s2=out=>s3
      - s0 -in->s1 -τ->s2 implies s0=in=>s2.
      - s2-out->s3-τ-> s0 implies s2=out=>s0



CWB-NC Commands

- load <ccs filename>
- help <command>
- ls
- cat <process>
- compile <process>
- es <script file> <output file>
- eq –S <trace|bisim|obseq> <proc1> <proc2>
- le –S may <proc1> <proc2> /* Trace subset relation */
- sim <process>
  - semantics <bisim|obseq>
  - random <n>
  - back <n>
  - break <act list>
  - history
  - quit
- quit

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Example: Faulty Mutual Exclusion Protocol

```
byte cnt, byte x,y,z;
active[2] proctype user()
{       byte me = _pid +1; /* me is 1 or 2*/
again:
        x = me;
        If
        :: (y ==0 || y== me) -> skip
        :: else -> goto again
        fi;

        z =me;
        If
        :: (x == me) -> skip
        :: else -> goto again
        fi;

        y=me;
        If
        :: (z==me) -> skip
        :: else -> goto again
        fi;

        /* enter critical section */
        cnt++
        assert( cnt ==1);
        cnt --;
        goto again
}
```

```
proc Sys = (P1|P2|X0|Y0|Z0|CNT0)\{x_[0-2],y_[0-2],z_[0-2],
test_x_[0-2],test_y_[0-2],test_z_[0-2], inc_cnt,dec_cnt}

proc P1    = x_1.(test_y_0.P1' + test_y_1.P1' + test_y_2.P1)
proc P1'   = z_1.(test_x_0.P1  + test_x_1.P1'' + test_x_2.P1)
proc P1''  = y_1.(test_z_0.P1  + test_z_1.P1''' + test_z_2.P1)
proc P1''' = inc_cnt.dec_cnt.P1

proc P2    = x_2.(test_y_0.P2' + test_y_1.P2 + test_y_2.P2')
proc P2'   = z_2.(test_x_0.P2  + test_x_1.P2 + test_x_2.P2'')
proc P2''  = y_2.(test_z_0.P2  + test_z_1.P2 + test_z_2.P2''')
proc P2''' = inc_cnt.dec_cnt.P2

* Variable x, y,z, and cnt
proc UpdateX = 'x_0.X0 + 'x_1.X1 + 'x_2.X2
proc X0 = 'test_x_0.X0 + UpdateX
proc X1 = 'test_x_1.X1 + UpdateX
proc X2 = 'test_x_2.X2 + UpdateX

proc UpdateY = 'y_0.Y0 + 'y_1.Y1 + 'y_2.Y2
proc Y0 = 'test_y_0.Y0 + UpdateY
proc Y1 = 'test_y_1.Y1 + UpdateY
proc Y2 = 'test_y_2.Y2 + UpdateY

proc UpdateZ = 'z_0.Z0 + 'z_1.Z1 + 'z_2.Z2
proc Z0 = 'test_z_0.Z0 + UpdateZ
proc Z1 = 'test_z_1.Z1 + UpdateZ
proc Z2 = 'test_z_2.Z2 + UpdateZ

proc CNT0 = 'inc_cnt.cnt_1.CNT1
proc CNT1 = 'inc_cnt.cnt_2.CNT2 + 'dec_cnt.cnt_0.CNT0
proc CNT2 = 'dec_cnt.cnt_1.CNT1

proc Spec = cnt_1.cnt_0.Spec
```

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Action and Process Def.

a_i: start $task_i$

b_i: stop $task_i$

Requirements:

- $a_1,\ldots,a_n$ to occur cyclically
- a_i/b_i to occur alternately beginning with a_i

$Sched_{i,X}$ for $X \subseteq \{1,\ldots,n\}$

- i to be scheduled
- X pending completion

$$Scheduler = Sched_{i,\varnothing}$$

$$\begin{aligned} Sched_{i,X} &= \Sigma_{j\in X}\, b_j.Sched_{i,X-\{j\}}, \text{ if } i \in X \\ &= \Sigma_{j\in X}\, b_j.Sched_{i,X-\{j\}} \\ &\quad + a_i.Sched_{i+1,X\cup\{i\}}, \text{ if } i \notin X \end{aligned}$$