Chapter 3
Conway's *Life*

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The rules of Conway's *Life*, a two dimensional cellular automaton, are explained. Some of its characteristics and typical behavior are described. An algorithm to calculate periodic states using de Bruijn diagrams is explained.[1]

3.1 Introduction

The game of *Life* is an invention of John Horton Conway, a British mathematician, which was widely publicized in the early seventies through Martin Gardner's monthly column "Mathematical Recreations" in *Scientific American*. Programmers at numerous computer centers set out to explore the game with the machines at their disposal, those with access to visual display equipment having somewhat of an advantage, particularly if their equipment was interactive. Nevertheless many others apparently found pencil and paper adequate to obtain significant results.

Sufficient interest existed to maintain a quarterly newsletter, which was published by Robert T. Wainwright, for almost three years. The third issue was certainly one of the highlights of the series, reporting a wide variety of constructions, of glider guns, glider collisions, puffer trains and so on. A sufficient number of artifacts were found to enable Conway to demonstrate a universal constructor, which has been one of the principal goals of the theory ever since John von Neumann became interested in automatic factories.

The factories which he envisioned don't just make particular things; they work from a description of whatever they are supposed to build. Presumably one of them could be given its own description, making it universal in accordance with Alan Turing's sense of the theory of computability.

Gradually computer capacity and the theoretical understanding of the subject became saturated, although there were attempts in later years to apply some of the precepts of information theory to the game. A new surge of interest was provoked

[1] The paper is written in July 4, 1988.

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from another direction when detailed computer experiments with nonlinear differential equations began to illuminate some of the more abstruse theories of topological dynamics. Quite independently of the fact that cellular automata define the continuous mappings within one esoteric approach to the subject, Stephen Wolfram thought that he saw an application of Stephen Smale's theory of strange attractors to one dimensional cellular automata. He created a classification into four categories for the occasion.

The early eighties saw intensive activity on Wolfram's part, exploring the evolution of linear cellular automata of one kind or another, with the intention of relating their behavior to the theory of computational complexity as well as the topologist's classification of the asymptotic behavior of dynamical systems. Some insights that were gained from this approach, such as the use of the Bruijn diagrams to discover the periods of cycles of evolution, can be applied to the original game of *Life*, and so resolve some of the questions which were left unanswered during the original excitement.

It is also true that the computers with which *Life* can be studied have improved over the years, both in terms of capacity and of availability. Results which were previously out of reach can now be computed, with the prospect of obtaining still further results as the performance of computers improves. Nevertheless one of the lessons to be learned is the exponential growth of all such computations; what can be calculated will always remain relatively modest.

Attempts to modify Conway's rule have not met with much success, although variants have been considered. The apparent 2^{2^9} different choices of a rule is illusory; rules without rotational and reflective symmetry are not acceptable, nor are rules without a quiescent state. Given restrictions such as totality, thousands or millions of candidates remain—many, but not entirely impossible, to sort through.

Some promising three dimensional analogues of *Life* have been found, but working with higher dimensions is clearly a formidable proposition.

3.2 The Rules of the Game

Automata are characterized by having states, and rules for deciding whether and how to change their states from time to time. The cells making up cellular automata all change their own states simultaneously, according to the states of their neighbors in the lattice in which they are situated. Consequently cellular automata are classified by the basic number of states their cells can display, and the exact form of their neighborhood.

With just two states per cell, binary automata are the simplest. The distinction between "living" and "dead" or "inactive" gave the published version of *Life* a readily understandable ecological interpretation, then a very popular concern.

As for their neighborhoods, the first consideration is dimension and the second radius. One dimensional automata are very restricted in the way that connections can be eventually be established between the cells; perhaps a reason that they were generally disregarded before Wolfram's interest. Two dimensional automata are more satisfactory, although von Neumann had to exercise considerable ingenuity to be able to construct crossing wires which would not short circuit one another. Two dimensional automata seem to suffice; more dimensions would increase the complexity of all computations without necessarily providing any better insight, granted that the conclusions to be drawn from them are highly symbolic anyway.

Even though a two dimensional network has been chosen, there are choices between square, triangular and hexagonal lattices, as well as the exact constitution of the neighborhood. Von Neumann used a square lattice, taking single neighbors on each side in both the horizontal and the vertical direction, but he needed a large number of states to accomplish his purposes (28 plus a quiescent state). Conway settled for fuller neighborhoods, restricted to binary cells. Thus *Life* uses diagonal neighbors as well as lateral neighbors; a neighborhood contains altogether nine cells, eight of them bordering the central cell.

Finally, the rule of evolution has to be chosen; Conway's final choice was

- a live cell survives if it has two or three live neighbors
- a new cell is born whenever there are three live neighbors
- all other cells either die or remain inactive

Fanciful interpretations assuming some strange kind of microbe provide a useful metaphor for discussing the game, although not to be taken too seriously.

Conway's criterion was that the rule should neither lead to populations which quickly died out, nor which expanded without end from limited beginnings. Through Martin Gardner, he challenged the readers of *Scientific American* to find out whether long term growth was nevertheless possible. Two mechanisms were suggested.

The first was by the construction of a "glider gun." Early experimentation with the rule had revealed an aggregate of five cells which moved across the *Life* field in a diagonal direction, advancing one square every four generations. If some mass of live cells could be found which emitted gliders periodically, it would produce the growth he was seeking. Evidence for the existence of some such configuration was the fact that gliders were frequent evolutionary products in random fields that had been observed. William Gosper at MIT soon found one glider gun; then another, structurally different one, was found. Since then no more have been reported.

The second possibility was that there were more complicated objects than gliders which could also move about a *Life* field, leaving stable debris behind. Belief in their existence was encouraged by the fact that certain larger structures, which were christened "space ships," had been found which moved horizontally or vertically, and that numerous small stable structures had been observed in the evolution of random fields.

Interestingly enough, a variety of "puffer trains" were also found, but more interesting still was the discovery that there were puffer trains that left gliders as debris in just such a fashion that they could collide to form glider guns.

This last combination, a mass whose periphery increases with time, violated Conway's concept of a stable ecology in the worst possible way; nevertheless it was a carefully engineered and delicate construct, and not a combination likely to be found by chance. The behavior of the rules which he had discarded was evidently much cruder.

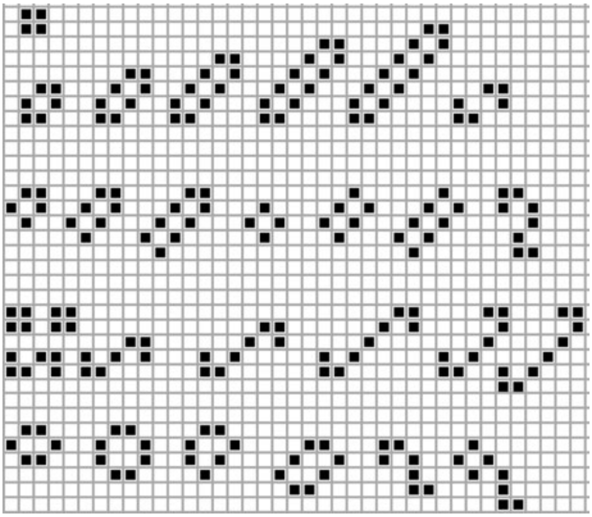


Fig. 3.1 Some small still lifes

3.3 Still Lifes

Since finite automata eventually evolve into a cycle of states which repeat indefinitely thereafter, cycles are one of the first things to look for; the same principle holds for infinite automata as well, even though such repetition is not obligatory. The shortest possible cycle, with period 1, would be a configuration of states which never changes; they are called still lifes.

A field in which all cells are inactive remains so; such quiescence was deliberately included in the definition of *Life*. Examination of the evolution of random *Life* fields reveals many still lifes formed from small clusters of live cells; the simplest is a 2×2 square in which each cell has exactly three neighbors, and thus persists from one generation to the next. Cells outside the cluster have at most two live neighbors, which is never enough for any additional cells to be formed. Thus the conditions for a still life are met.

An empirical search of random fields reveals configurations that are easily formed, especially if a very limited sample is taken. It needs to be supplemented by a more thorough procedure, so it is not surprising that much of the response to Gardner's article consisted in carefully recording the evolution of all the small objects that it was possible to form. Gradually a consensus was built up concerning the catalog of still lifes up to twelve pixels, including the existence of several families of still lifes and rules for generating them.

Figure 3.1 shows the 21 symmetry classes of connected still lifes containing eight live cells or less. The symmetry of *Life's* evolutionary rule ensures that objects differing by rotation or reflection undergo a similarly rotated or reflected evolution, so only one representative of each class has to be shown.

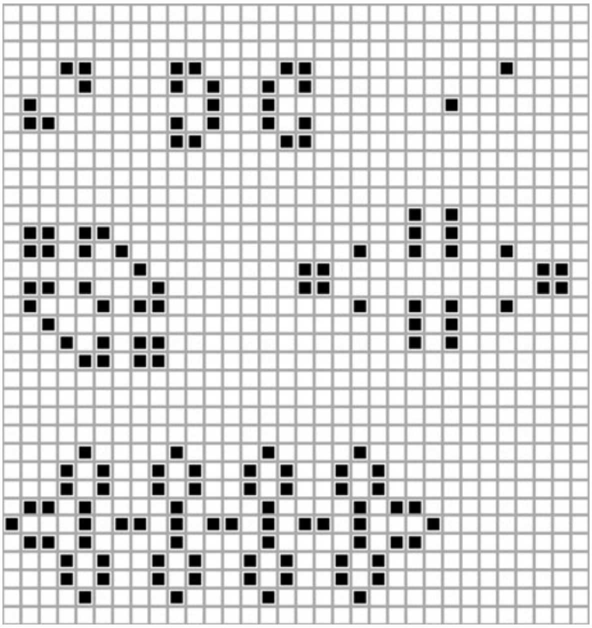


Fig. 3.2 Alternators due to overcrowding

3.4 Period Two

The structures called blinkers, consisting of three live cells in a row, alternate in form between three horizontal cells and three vertical cells. Next to square clusters of four blocks, they are the most common debris remaining after evolution has run its course. They exemplify structures with period two, which, like the smaller still lifes, often arise during the evolution of random fields.

Alternators, or period two oscillators, tend to be rather fragile. Nevertheless, they are easily constructed, once the basic technique has been grasped. There are really two different kinds of alternators, depending on whether cells disappear from isolation or from overcrowding. Of course, it is possible for a large configuration to incorporate both kinds of alternators.

Figure 3.2 shows an assortment of alternators which depend on overcrowding. The neighbors of the deceased remain in place, giving birth to a new cell which they promptly crush, repeating the cycle indefinitely.

Figure 3.3 shows some typical examples of the converse situation, in which the field contains only isolated cells or pairs of cells, which will surely disappear. However, their mutual placement is sufficient to reproduce a similar configuration in the next generation. Then, under the proper conditions of symmetry, this second generation recreates the first, establishing a permanent cycle. Extremely large and intricate networks can be built up from a few basic configurations.

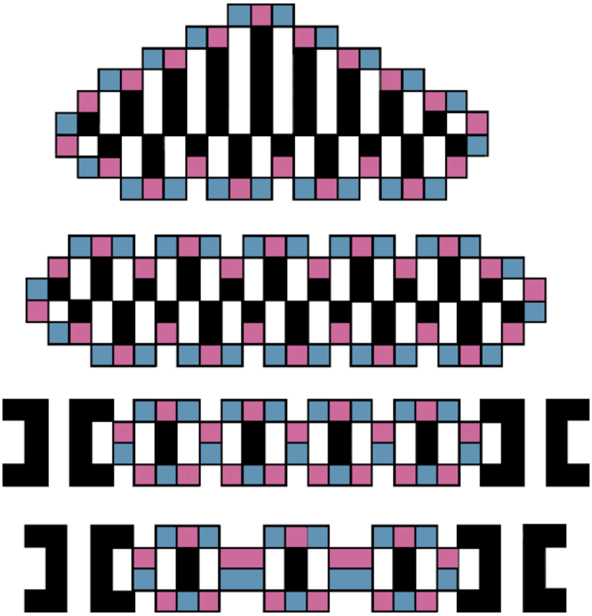


Fig. 3.3 Alternators due to isolation

3.5 Gliders

Gliders were probably the most unexpected characteristic of Conway's *Life*, and in the end the one responsible for its great complexity and utility. Conway quickly realized that gliders were the substitute for von Neumann's wires as the mechanism for transporting information from one place to another in the *Life* field. They form an essential ingredient of his demonstration of a self-replicating configuration for *Life*, which does not detract from their interest for other applications.

Figure 3.4 shows the four phases of a glider, which would constitute a cycle except for the displacement. Consequently gliders can never have a finite period in an infinite *Life* field; an exception to the theorem that finite automata must eventually retrace a cycle of states, except for the finiteness condition. Allowed to evolve on a finite torus, gliders will have periods sufficiently long for them to traverse the length of the torus and return to their starting position.

A variety of glider precursors were reported in Wainwright's newsletter; in view of the asymmetry of the glider itself, it would seem that the principal requirement for the precursor is that it or significant segments of it also be asymmetric. For all of their interest and importance, gliders are not hard to find in random *Life* fields; it is reported that they were first observed wiggling their way across a video display.

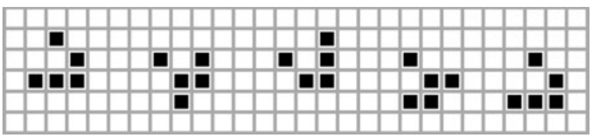


Fig. 3.4 The cycle of evolution of a glider

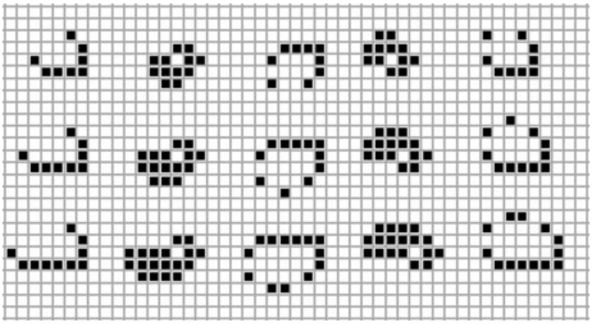


Fig. 3.5 The three primitive space ships

Their prevalence is consistent with the fact that they are formed from only five live cells, since the smaller stable figures are by far the most common residues of long term evolution.

The search for other moving configurations eventually turned up the three space ships shown in Fig. 3.5. They and their symmetric images are capable of horizontal or vertical motion. In principle the central section of the space ships could be stretched out to arbitrary lengths, but in practice the designs shown are not quite correct; they are always accompanied by some additional "sparks," which promptly die out.

The sparks are too big to die out in the longer space ships, but they can be suppressed by stacking a series of smaller space ships alongside the longer ones to produce a flotilla which will travel together smoothly.

Certain other combinations produce the puffer trains which Conway had foreseen. However, gliders and the three small space ships are the only primitive structures which have ever been found capable of self propulsion.

3.6 Oscillators

Compared with the case with which still lifes or alternators can be formed, it is quite difficult to form configurations with longer periods, so that they are not often found as transients during the evolution of random fields nor among the debris which remains when they have stabilized. Nevertheless their construction is not impossible, and oscillators of every small period have been found. As with alternators, they tend to fall into different categories depending upon the mechanism involved.

One scheme is to create vacancies and then replacements for them, a common aspect of the two classes of alternators. If the replacement can be deferred, a longer cycle that of period two can be created. Figure 3.6 shows a cycle of length 3 created in this manner.

Another mechanism is to note that some configurations are naturally expansive, such as those which evolve from a long row of live cells. However, as the center of the row expands laterally, the end of the row contracts longitudinally; the competition between these two tendencies can result in an alternation which leads to a cycle

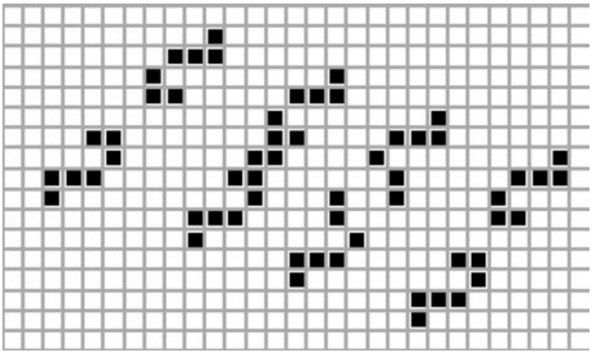


Fig. 3.6 A period 3 oscillator

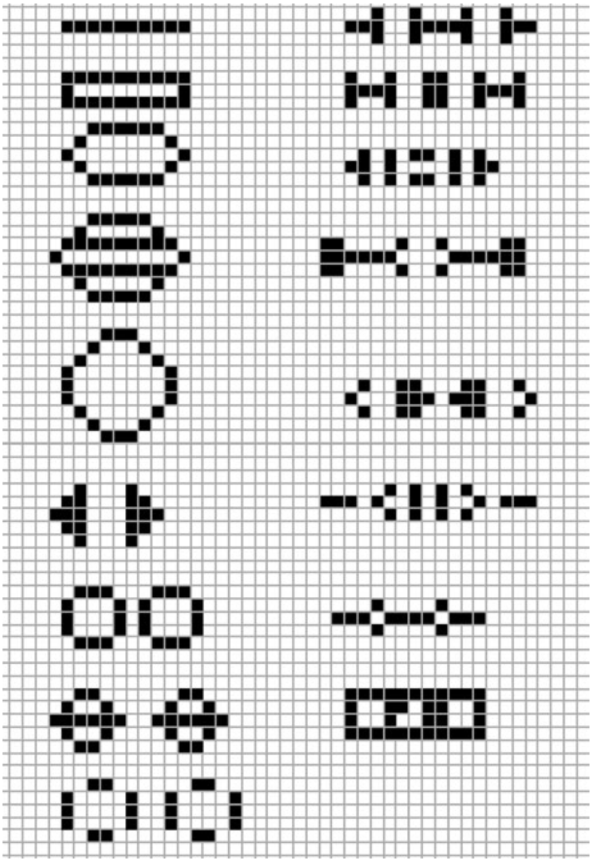


Fig. 3.7 A period 15 oscillator based on expansion and contraction

of long period. Experimentation when *Life* was still new led to the discovery that a row of ten live cells would reproduce itself after fifteen generations, as shown in Fig. 3.7.

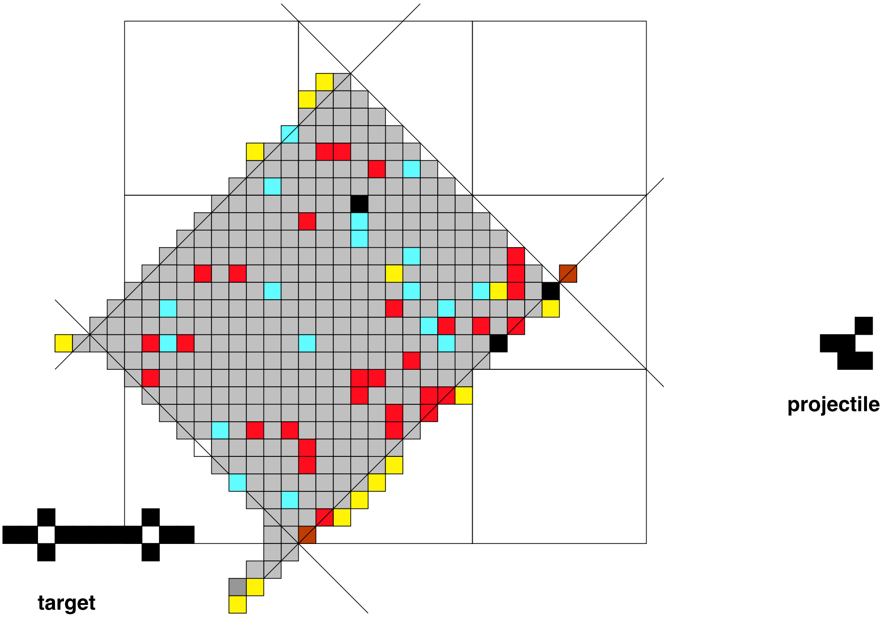


Fig. 3.8 Map of period 15 oscillator collisions with a glider

Another configuration which behaves similarly is formed by two solid 3×3 squares staggered diagonally, touching at their corners.

Yet another arrangement which can be exploited for arbitrarily large assemblages of live cells is to surround figures which are known to be highly expansive with other highly stable figures which are known to dampen expansion. Under favorable circumstances the central figures will reconstitute themselves from the debris, phoenix like, to initiate a new cycle. By repeating itself periodically the combination becomes an oscillator whose period depends roughly on the distance from the center to the periphery.

Many experiments have been performed to observe collisions of moving objects with each other, still lifes, or oscillators. Figure 3.8 shows the possible initial positions for collisions between a glider and the period fifteen oscillator. Most are messy, but dark squares show where the glider will simply be eaten, having struck inessential trash in the oscillator. For just one arrangement, the glider reflects. By bouncing in between two oscillators, a composite oscillator with an arbitrarily long period is possible. Collisions which turn a glider by 90 degrees rather than 180 degrees are extremely rare.

3.7 Glider Guns

The approach of combining "eaters" with naturally expansive configurations led a group of students in Ontario to large numbers of interesting oscillators of diverse periods; over a hundred of them according to reports. However, a slightly different approach led the group at MIT to two different glider guns.

A variant on the line of ten cells is a pair of lines of fifteen cells, both are similar in evolving through a cycle of expansion and contraction, but the fifteen cell configuration leaves behind enough debris to destroy itself rather than continuing through its second cycle. The clever realization that the debris could be broken up by a strategically placed square block to act as an eater led to another shuttle based oscillator.

Thanks to a curiosity as to the consequences of collisions and near collisions, a pair of shuttles were set in motion in a way that led them to just barely graze each other. Altogether many distances of approach and relative phases were tested before a combination was found which produced pure gliders every time the shuttles met. That became the glider gun which Conway had conjectured. It won the prize offered in Martin Gardner's column, and led the way to quite an interesting series of further discoveries.

Besides investigating all kinds of shuttle collisions, there has always been a search for additional shuttles. Although it would seem that there is an impossibly large number of possibilities to sort through, a certain faith that relatively small structures will be significant can be combined with the fact that the evolution of all the different small clusters of live cells has been tracked. Thus there is a certain data base from which one can work.

No other reasonable explanation seems to exist for the fact that a second glider gun was discovered, consisting of another self reversing shuttle in which eaters could be inserted to clean up unwanted scraps of its field.

Pairs of shuttles of the second type, this time colliding at right angles, when correctly combined with each other, again produced a glider gun; it is generally referred to as the "new gun." It is indeed fortunate that the new gun exists and was discovered, because it turns out to be essential for some of the advanced constructions which have been made.

It is difficult to say whether any additional shuttles or glider guns remain to be found. The intense search which revealed the ones that are now known should have discovered others, if they existed; but there are many starting points.

Figure 3.9 shows the glider gun in one of its phases, together with the stream of emitted gliders. Variations on the theme are possible, in which the confining blocks are removed and a cascade of shuttles is allowed to generate parallel glider streams. However their separation is not convenient for certain applications, which is why it is fortunate that the new gun was discovered.

Figure 3.10 shows the new gun in one of its phases, together with the stream of emitted gliders. It has a period of 46, in contrast to the regular glider gun whose period is 30. Consequently the gliders which it produces are spaced further apart; every second glider produced by the period 46 gun is 23 cells. Successive gliders are out of phase by two generations, having been produced by shuttles moving in directions opposite to those of the previous pass.

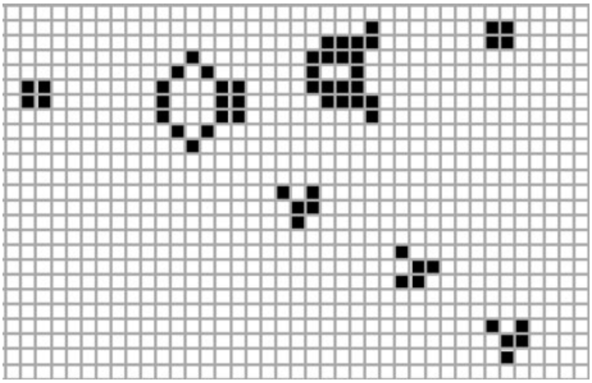


Fig. 3.9 Glider gun, together with a stream of gliders

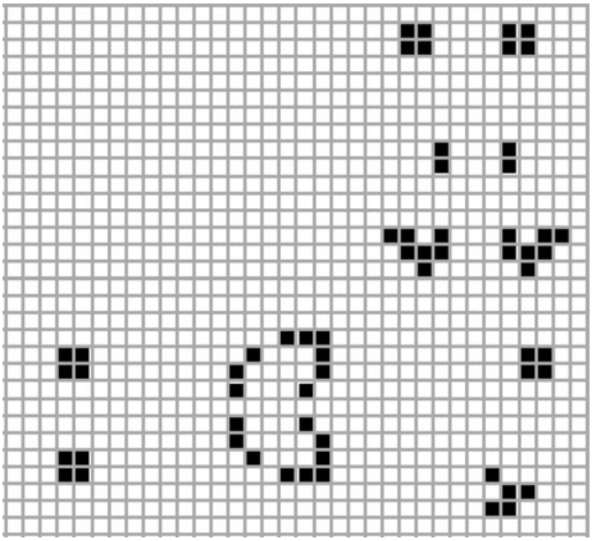


Fig. 3.10 The new gun, with a freshly generated glider

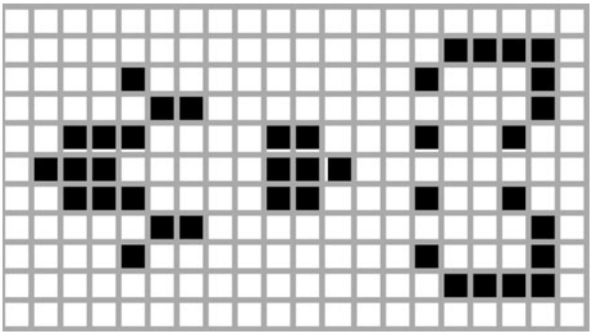
3.8 Puffer Trains

Shuttle precursors tend to turn back on themselves, so the natural tendency is to place eaters at strategic locations to absorb the debris which prevents them from becoming oscillators. There are other small configurations which advance steadily, reproducing portions of themselves, until their leading edge is overwhelmed by advancing debris from behind. There are two different approaches to salvaging the wavefront which have been found effective.

One of them is to place eaters behind them in the hope that they will both control the debris, and somehow reproduce themselves. Two versions of one rather slow diagonal puffer train have been found by using this technique.

The other approach uses space ships as escorts, feasible if the velocity of the advancing front in the same as the velocity of the space ship. The periodic sparks cast off by the space ship will sometimes interact to control the expanding debris

Fig. 3.11 A "smokeless" puffer train escorted by a space ship



without affecting the integrity of the space ship. Figure 3.11 shows a clean example, in which the pair of space ships and the heptomino advance one space every second generation, but with an overall period of twelve.

Gliders frequently inhabit the cloud lagging behind a puffer train, making it worthwhile to adjust the escort to allow them to escape, yet cleaning up the rest of the cloud. Given that there are known glider collisions producing all the smaller artifacts, the way is open to use multiple puffer trains to create gliders whose collisions produce other desirable objects in their turn.

3.9 Life on a Torus

Although *Life* was intended for an infinite square lattice, it is possible to deform the lattice into a torus by identifying cells which have a fixed separation from one another. This procedure is equivalent to working with configurations which have a spatial periodicity. Since it transforms the infinite lattice into a finite structure, there is some hope of discovering its properties by exhaustive enumeration.

Limitations of both computer space, but principally speed, restrict the dimensions of the cases which can be treated, the present limit being an area encompassing approximately 32 cells. Even so, much data can be gathered, allowing definitive, conclusions about some of the simpler configurations to be reached, and revealing some interesting patterns which were not previously known.

The simplest mapping of the infinite plane onto a torus is to identify all cells differing by a constant horizontal or vertical distance. Conversely, a rectangle with opposite edges identified may be mapped onto the plane by making a new copy of the rectangle whenever crossing one of its boundaries, instead of reentering the opposite side.

More extensive planar areas may be explored by making alternative identifications of the opposite edges of a rectangle, the equivalent of forming a Möbius strip or a Klein bottle. However, it is easy enough to accumulate voluminous data from the simplest mapping, raising the doubt that collecting still further data would contribute further insight. It is true that the feasible dimensions reveal gliders but none of the space ships, much less shuttles, glider guns, puffer trains, or any of the other

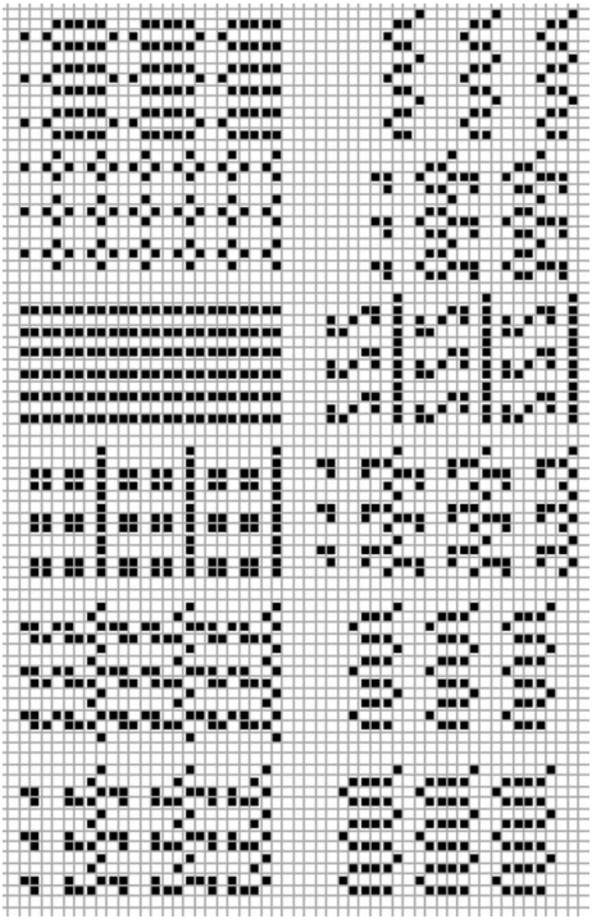


Fig. 3.12 A sampling of still lifes on a 4×8 torus

interesting objects. However, their study provides sufficient insight to form fairly general conclusions.

Figure 3.12 shows some of the still lifes for a 4×8 tours, found through an exhaustive search. Some of them belong to recognized families.

The extreme case of an $1 \times n$ torus does not seem very interesting at first, but it has two advantages. Values of n as large as 34 have been completely analyzed, some of the most interesting detail emerging in the vicinity of $n = 30$. So, it would have been missed for even a $2 \times n$ torus. Better still, the $1 \times n$ automaton is really one dimensional, so subdiagrams of the de Bruijn diagrams used in shift register theory suffice to calculate explicitly all the configurations of any given period.

Once the relevance of the de Bruijn diagrams has been seen, they can be applied to $2 \times n$ tori, $3 \times n$ tori, and so on. The amount of calculation they require rapidly becomes overwhelming, but their theoretical implications are always valid.

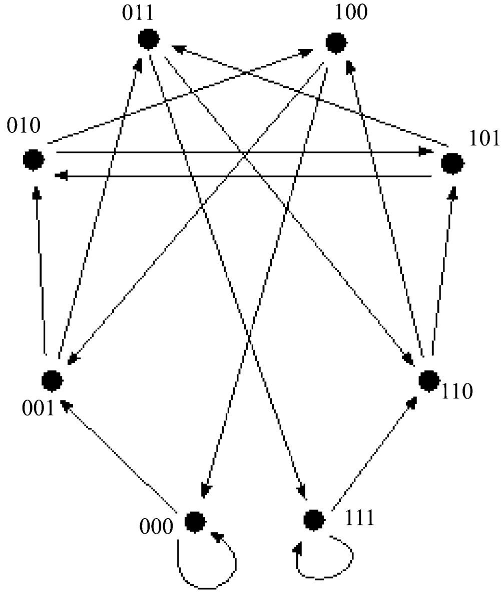


Fig. 3.13 A three stage binary de Bruijn diagram

3.10 Cycles of Finite Periodicity

One of the advantages of working with *Life* on a finite torus is that there is an algorithm by which the cycles of finite periodicity can be determined. In one dimension the procedure actually finds all the periods on an infinite lattice, but in applying the procedure twice to get to two dimensions the first stage has to remain finite. This is probably a fundamental limitation, given the fact that it is known that many properties of a two dimensional lattice are recursively insoluble, in Turing's sense. Thus the most to be expected are results valid for structures of finite periodicity.

A de Bruijn diagram is a very simple thing, merely a diagrammatic technique to keep track of sequences of digits of a given length. A binary de Bruijn diagram of three stages contains all possible sequences xyz of three bits, eight sequences altogether. Suppose that they are placed around the circumference of the unit circle, forming an octagon. Arrows are drawn to link the sequences according to how they are initial or terminal segments of one another, arrows are to be drawn from all vertices wxy to vertices xyz; for example 001 links to 010, also to 011.

In arithmetic terms, links always run from vertex i to vertices $2i$ and $2i + 1$, modulo 8. Figure 3.13 shows the octagon with its links. With respect to the rules of *Life*, the relevance of the de Bruijn diagram is that it shows how to build up long chains of cells out of overlapping neighborhoods; precisely, it shows which neighborhoods can overlap and which can not.

If the number of stages is just one less than the length of the neighborhoods, links in the diagram match the neighborhoods exactly, classifiable as "good" or "bad" according to the neighborhood. For still lifes, a "good" neighborhood evolves into its central cell. If all the "bad" links are discarded, paths along the surviving links describe chains of cells whose cores remain unchanged in the next generation.

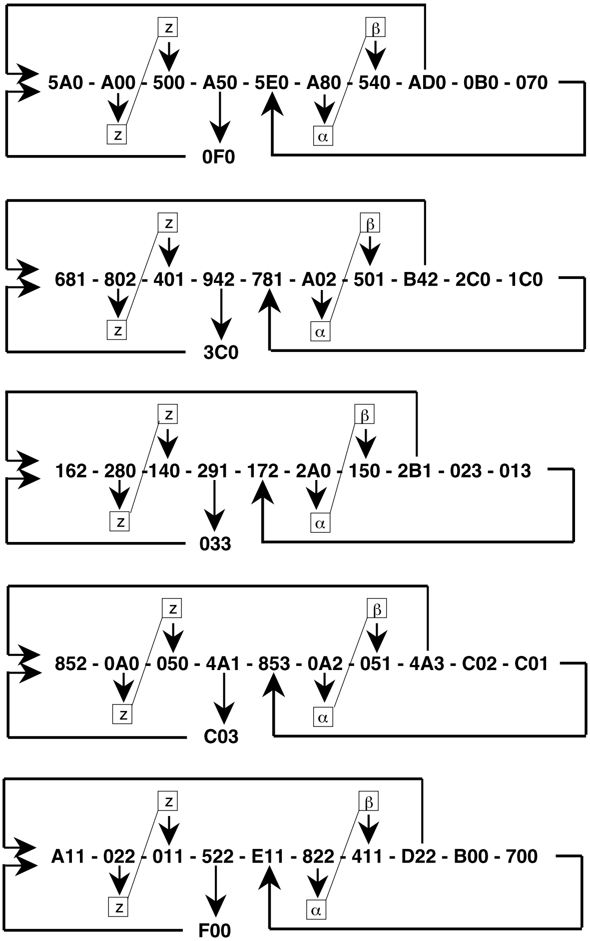


Fig. 3.14 Strip overlap for all the .1(1,0) configurations on a $5 \times n$ torus

Shrinkage due to the loose ends can be avoided by discarding all links which are not parts of closed loops. Every possible still life can be read off from the remaining diagram. No ingenuity is involved; the procedure is an algorithm. Long and cumbersome it may be, but less so than testing every configuration; indeed the only configurations actually tested are just the neighborhoods by which the evolutionary rule is defined.

Cycles with longer periods require the longer neighborhoods of the iterated rule of evolution; non-binary automata require the de Bruijn diagram of the corresponding number base. That is the secret to working with *Life* on $2 \times n$ tori; a vertical section of two cells is taken to be a single cell of four states. Then the de Bruijn diagram for a four state linear cellular automaton provides the information; similarly a $3 \times n$ torus becomes an eight state linear automaton.

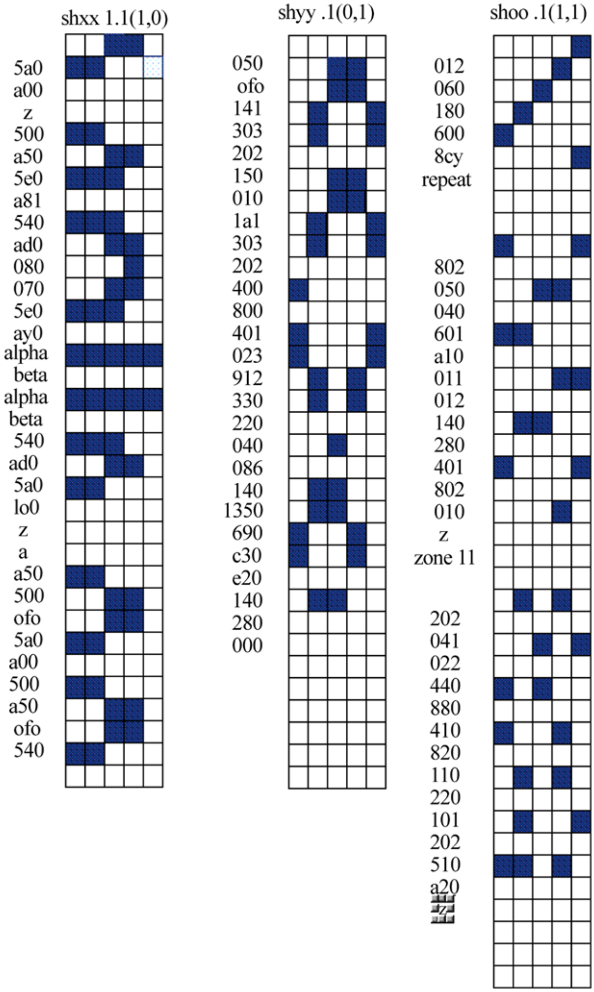


Fig. 3.15 Sample shifting configurations on a $5 \times n$ torus

Fortunately there is an alternative to treating vertical slices as a single cell with a huge number of states. A vertical strip of three cells is sufficiently wide to calculate the central cell of the next generation, and even to compare it with any of the nine members of its neighborhood. Not only still lifes, but shifting patterns can be detected.The first step is to generate the diagram for this single vertical strip. Next, the vertical strips have to be fitted together to fill up the plane.

General fitting seems to be a difficult proposition, but it is easy enough to work out sequences three cells wide which fit into a common horizontal strip, by selecting closed loops in the diagram whose length is equal to the selected width. All such loops are used as nodes in a new diagram whose links indicate that the two left cells of one match the two right cells of the others when they are overlapped. Following a path through the second diagram builds up vertical sections which eventually fill out the plane.

Figure 3.14 shows the strip overlap diagram for the some configurations on a $5 \times n$ torus which correspond to configurations which move horizontally by one cell in each generation.

Figure 3.15 shows some examples of creepers and crawlers on a $5 \times n$ torus, gotten from strip diagrams similar to those of Fig. 3.14.