Advanced DS

Data Structure and Algorithms

Fall 2024

Binary Search Tree

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University of Engineering and Technology
Lahore Pakistan

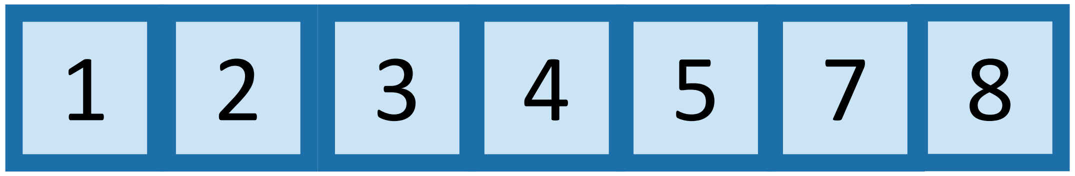
Binary Search Tree

Course Reference: CS161 by Stanford

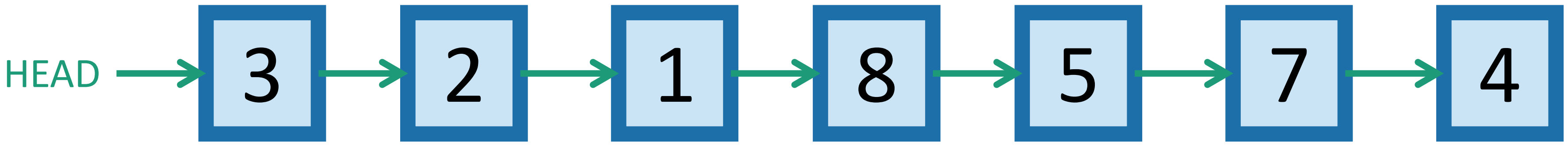
Some data structures

for storing objects like $\boxed{5}$ (aka, nodes with keys)

- (Sorted) arrays:



- Linked lists:



- Some basic operations:
  - INSERT, DELETE, SEARCH

Sorted Arrays

1	2	3	4	5	7	8

- O(n) INSERT/DELETE:
  - First, find the relevant element (we'll see how below), and then move a bunch elements in the array:

1	2	3	4	4.5	7	8

eg, insert 4.5

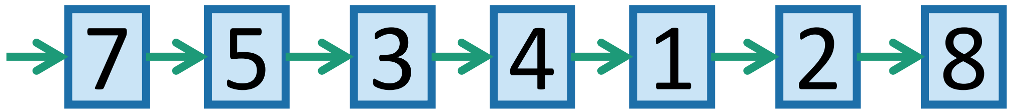
- O(log(n)) SEARCH:

1	2	3	4	5	7	8

eg, Binary search to see if 3 is in A.

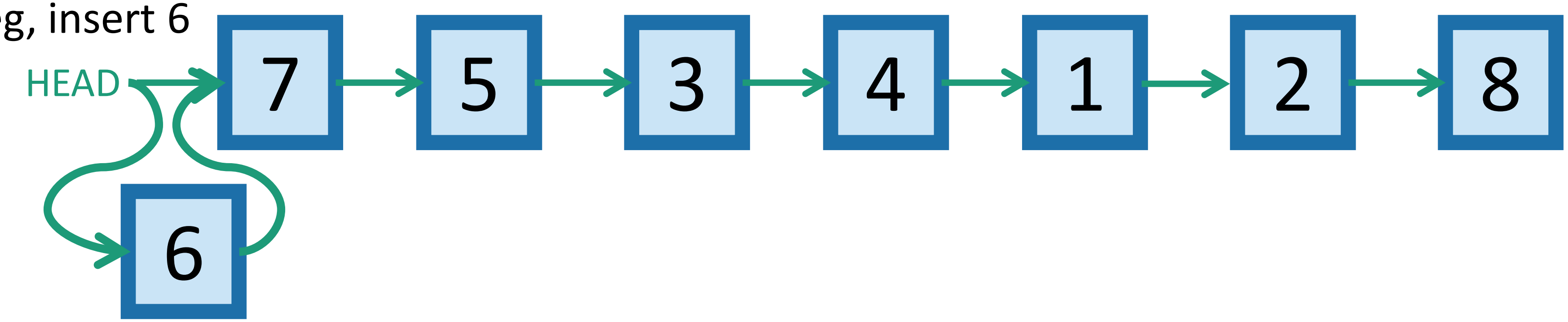
(Not necessarily sorted)

Linked lists

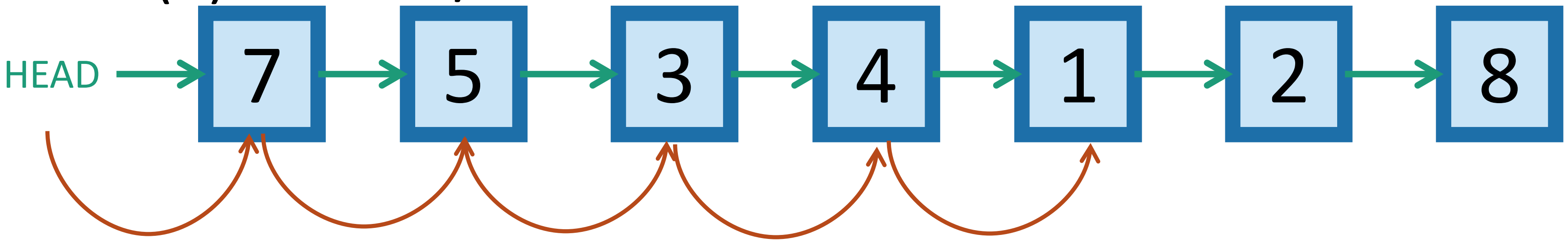


- O(1) INSERT:

eg, insert 6



- O(n) SEARCH/DELETE:



eg, search for 1 (and then you could delete it by manipulating pointers).

Motivation for Binary Search Trees

TODAY!

	Sorted Arrays	**Linked Lists**	(Balanced) **Binary Search Trees**
Search	O(log(n)) 😃	O(n) 🙁	O(log(n)) 😃
Delete	O(n) 🙁	O(n) 🙁	O(log(n)) 😃
Insert	O(n) 🙁	O(1) 😃	O(log(n)) 😃

For today all keys are distinct.

Binary tree terminology

Each node has two children.

The left child of 3 is 2

The right child of 3 is 4

The parent of 3 is 5

2 is a descendant of 5

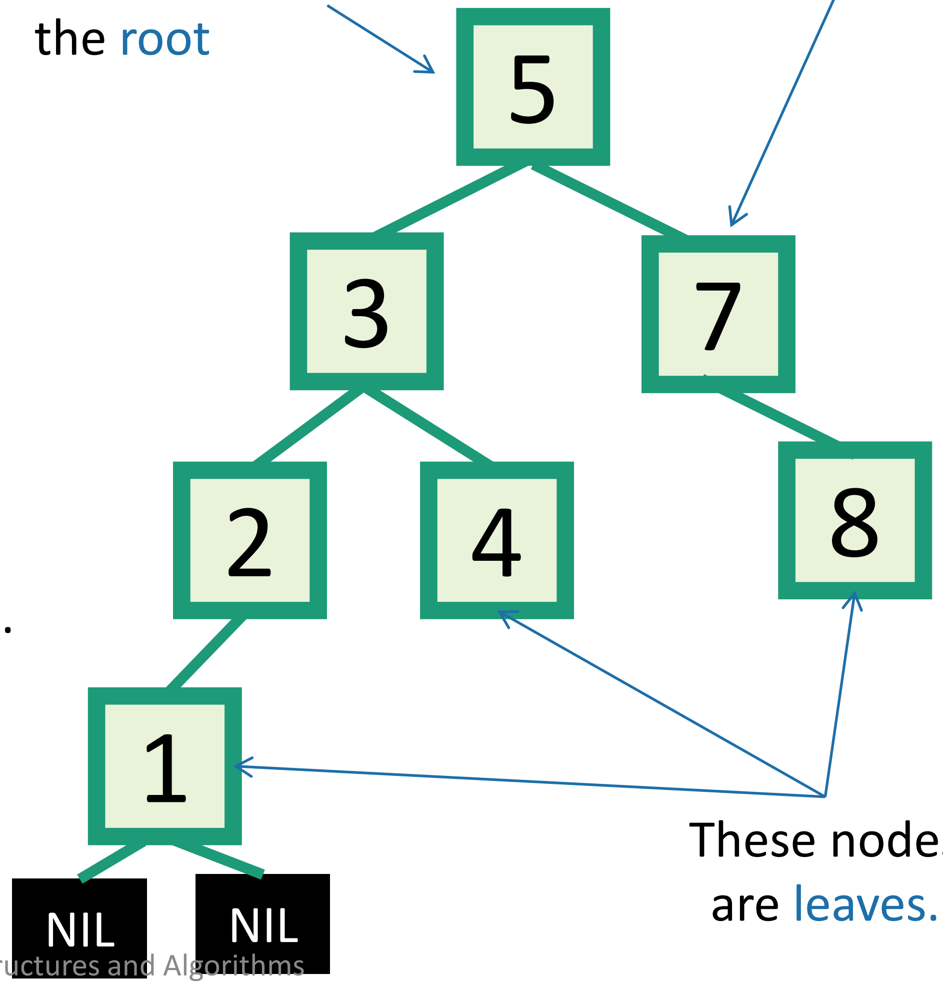
Each node has a pointer to its left child, right child, and parent.

Both children of 1 are NIL.
(I won't usually draw them).

The height of this tree is 3.
(Max number of edges from the root to a leaf).

This is a node.
It has a key (7).

This node is the root

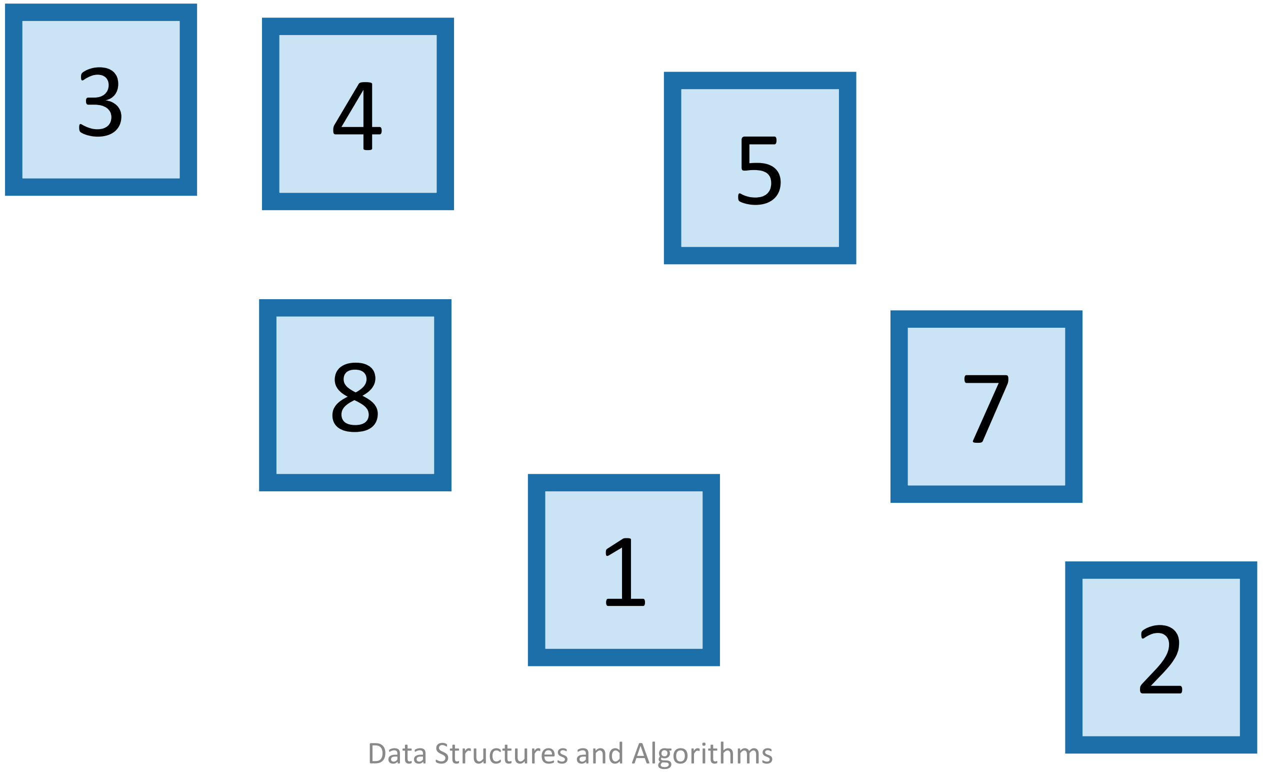


These nodes are leaves.

From your pre-lecture exercise…

Binary Search Trees

- A BST is a binary tree so that:
  - Every LEFT descendant of a node has key less than that node.
  - Every RIGHT descendant of a node has key larger than that node.
- Example of building a binary search tree:



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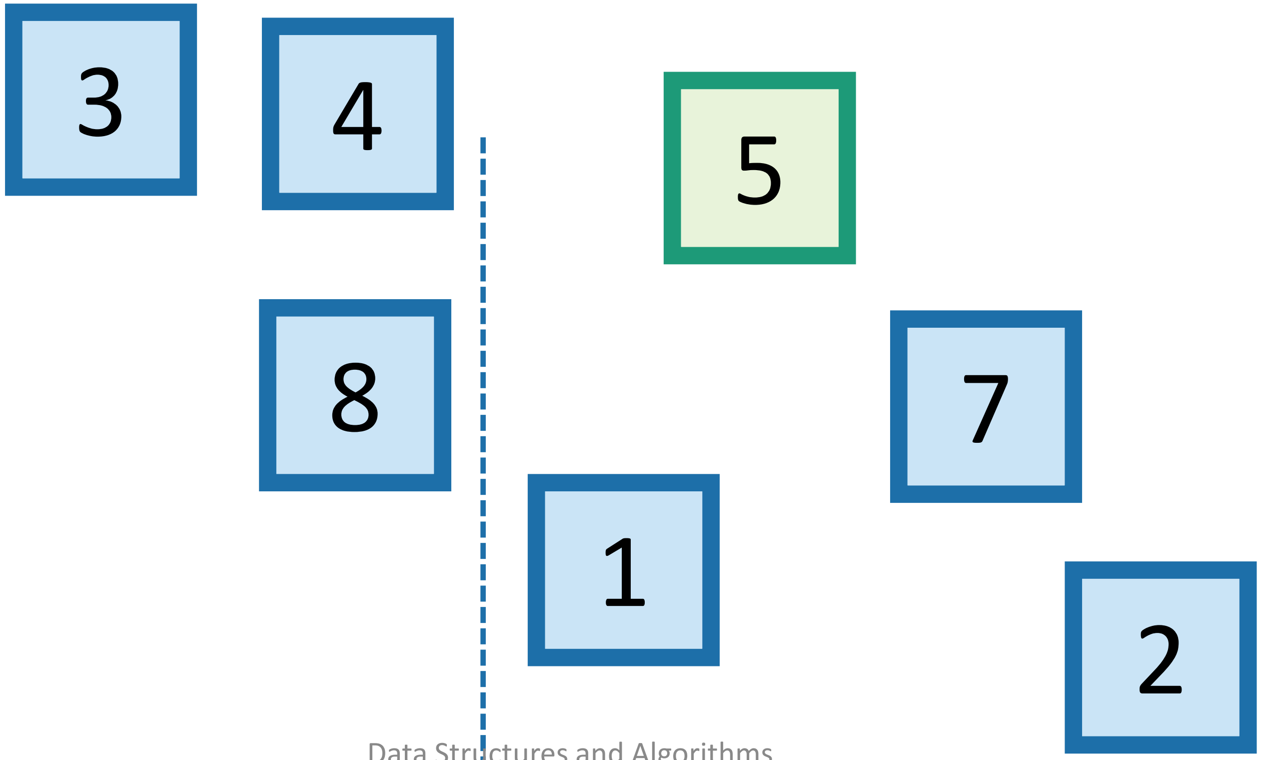

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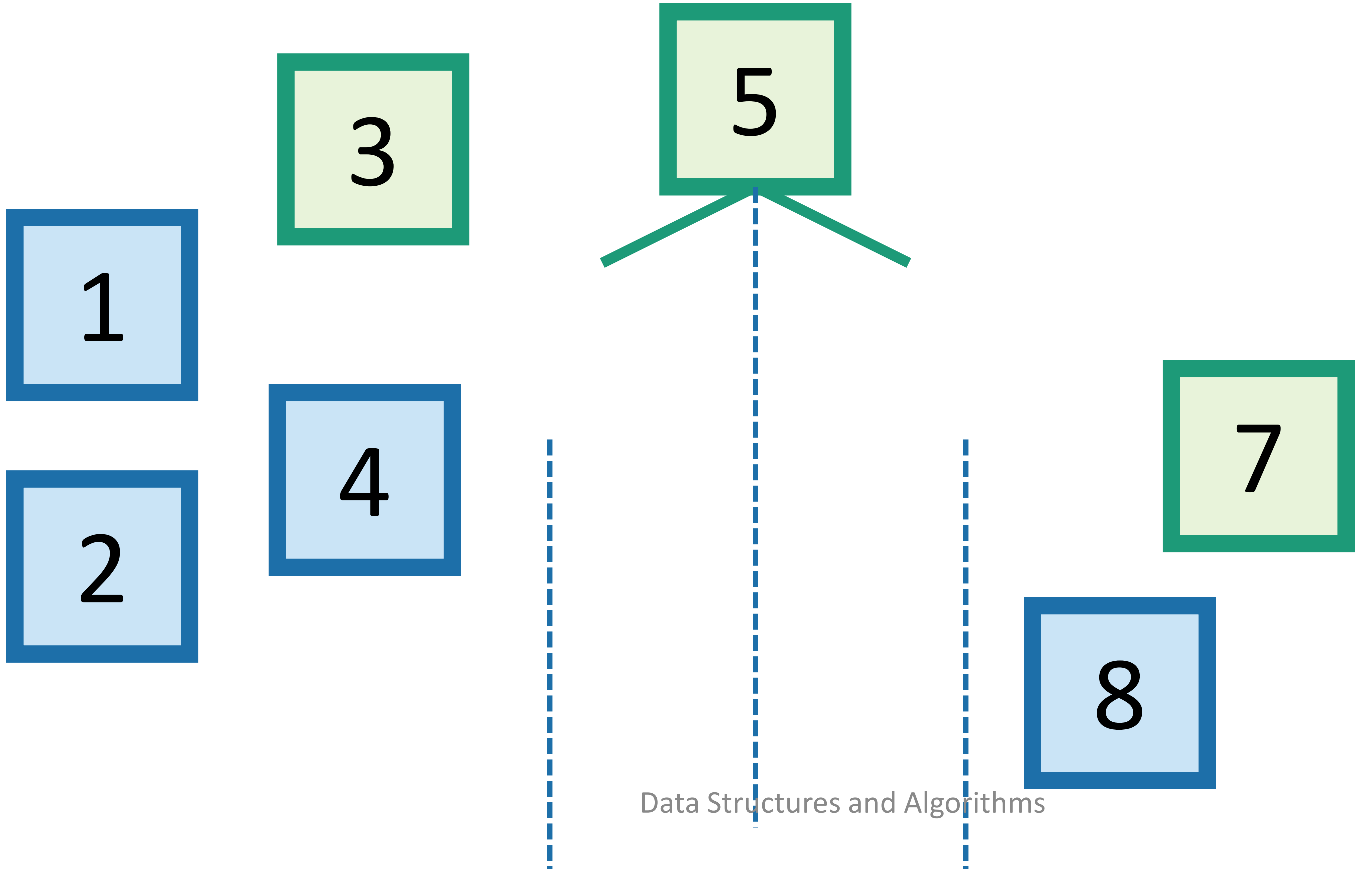

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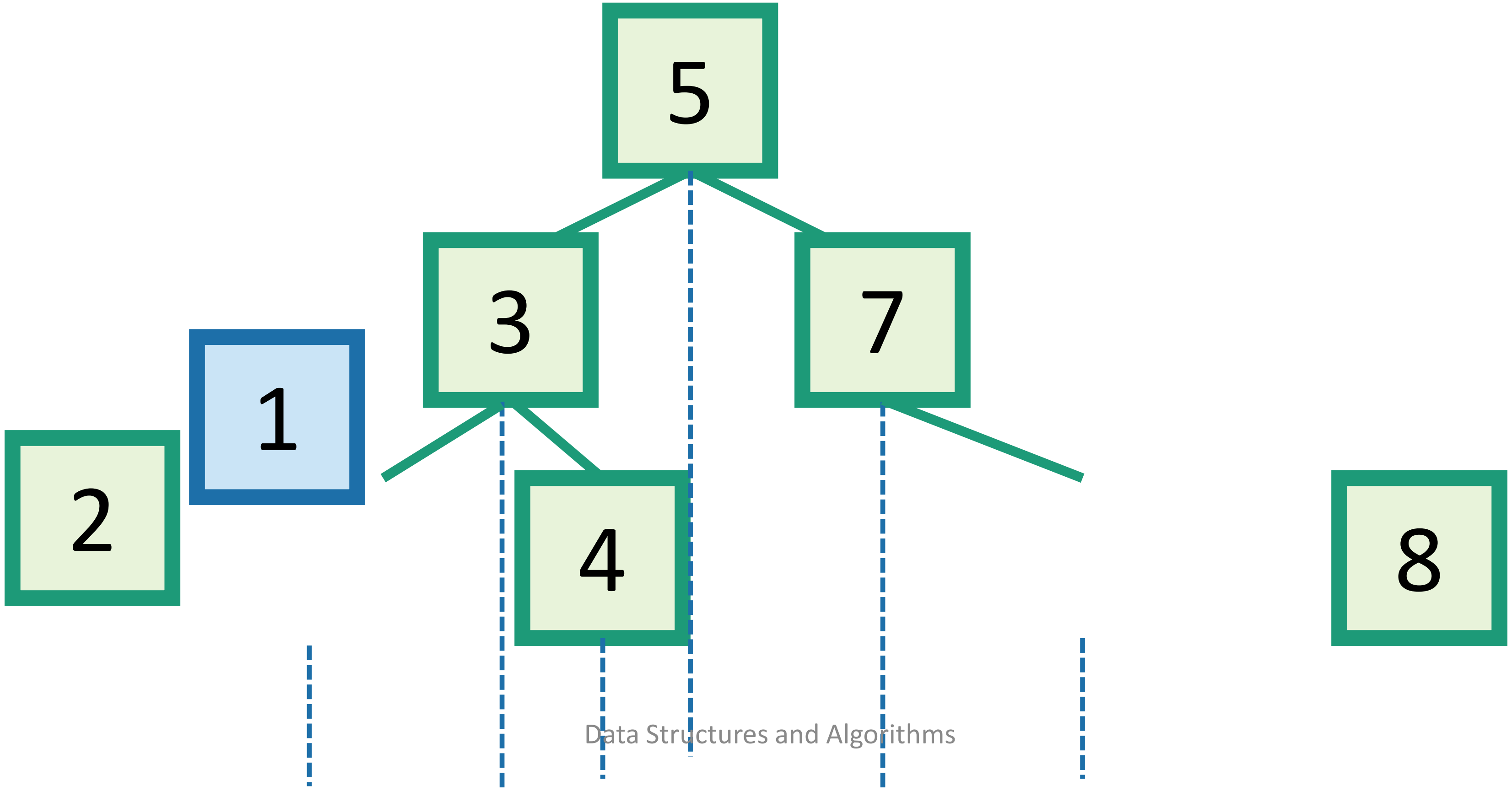

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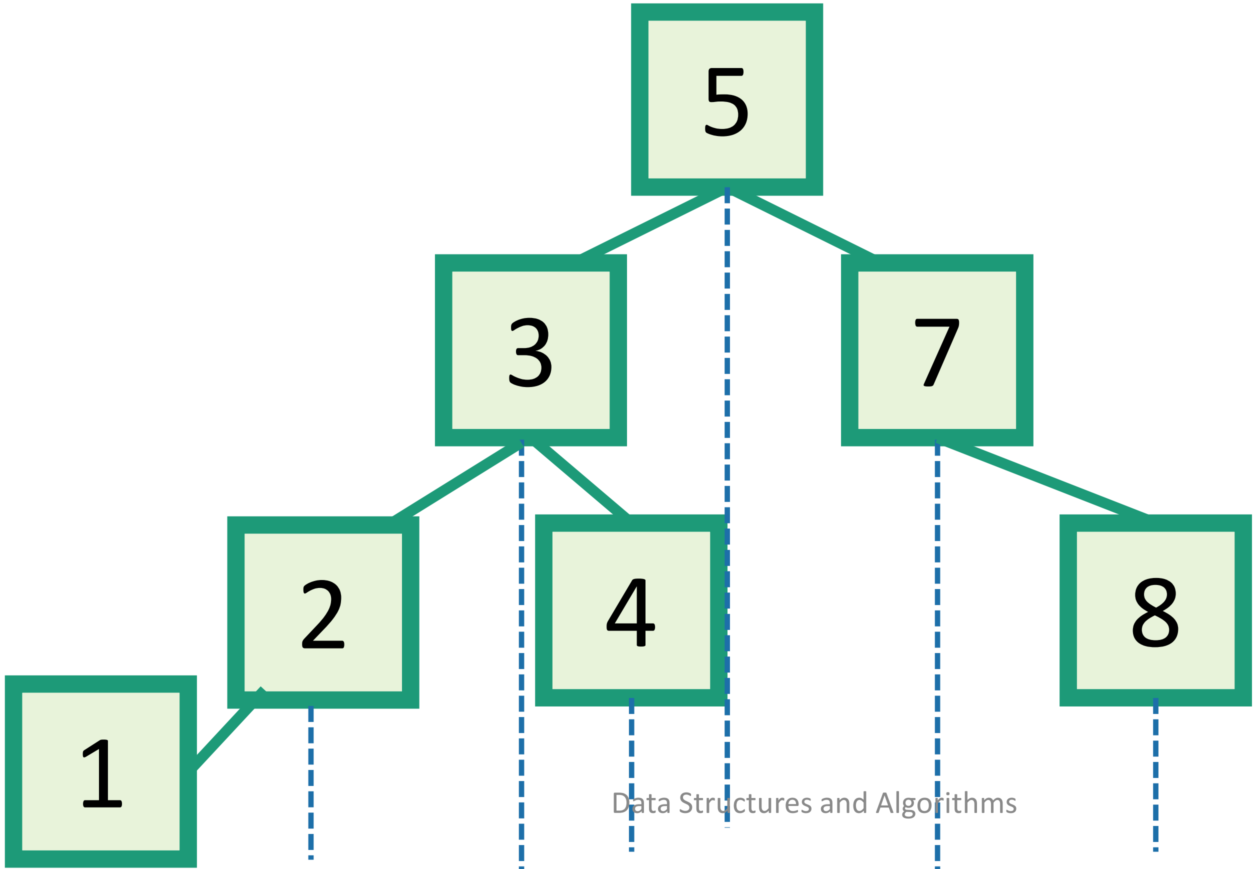
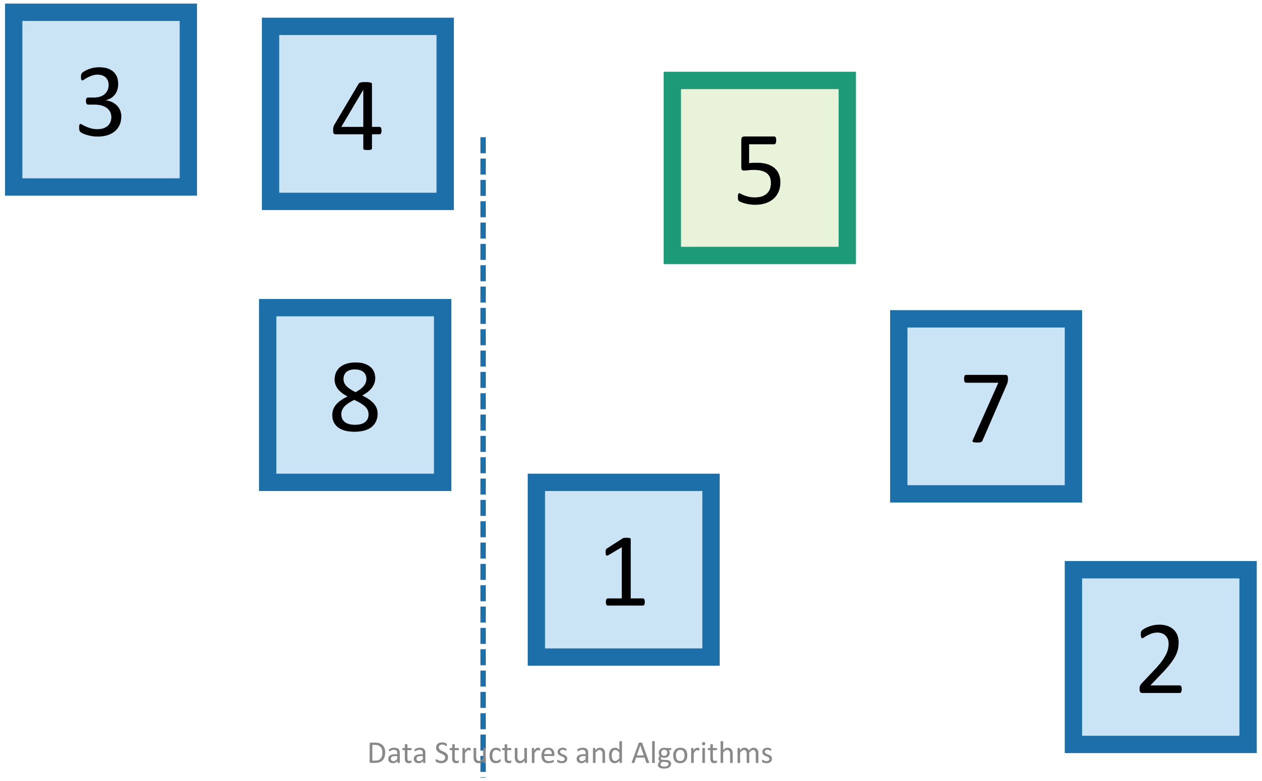


Q: Is this the only binary search tree I could possibly build with these values?

A: **No.** I made choices about which nodes to choose when. Any choices would have been fine.

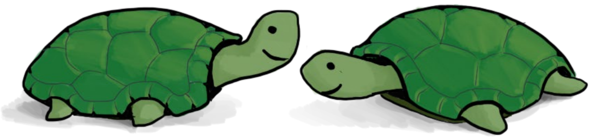
Aside: this should look familiar

kinda like QuickSort



Binary Search Trees

Which of these is a BST?
1 minute Think-Pair-Share



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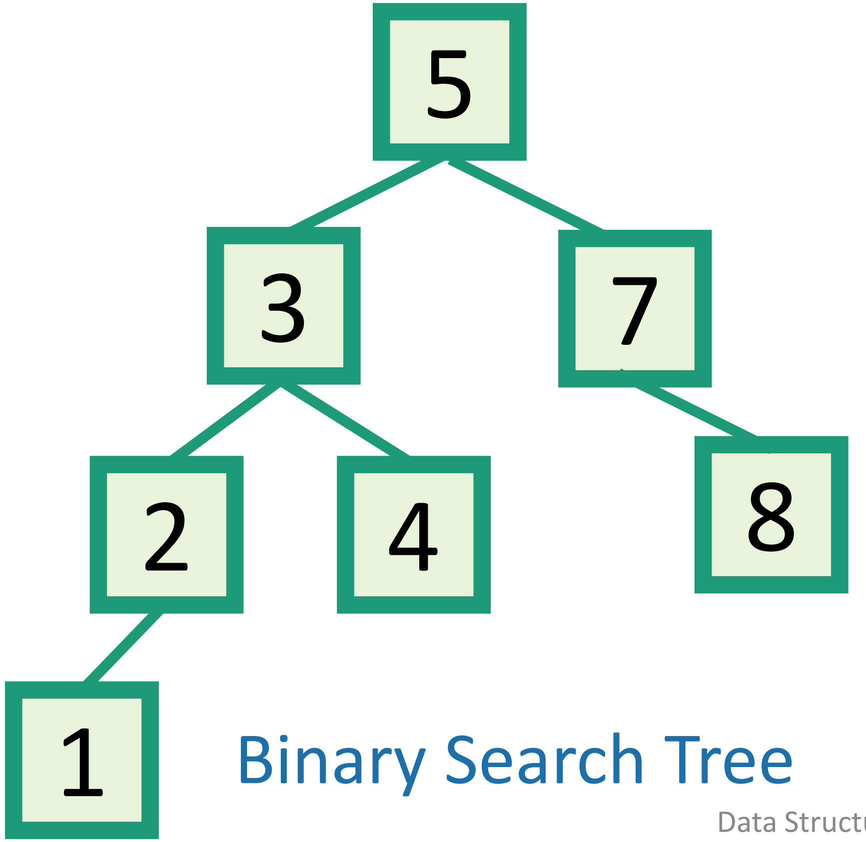
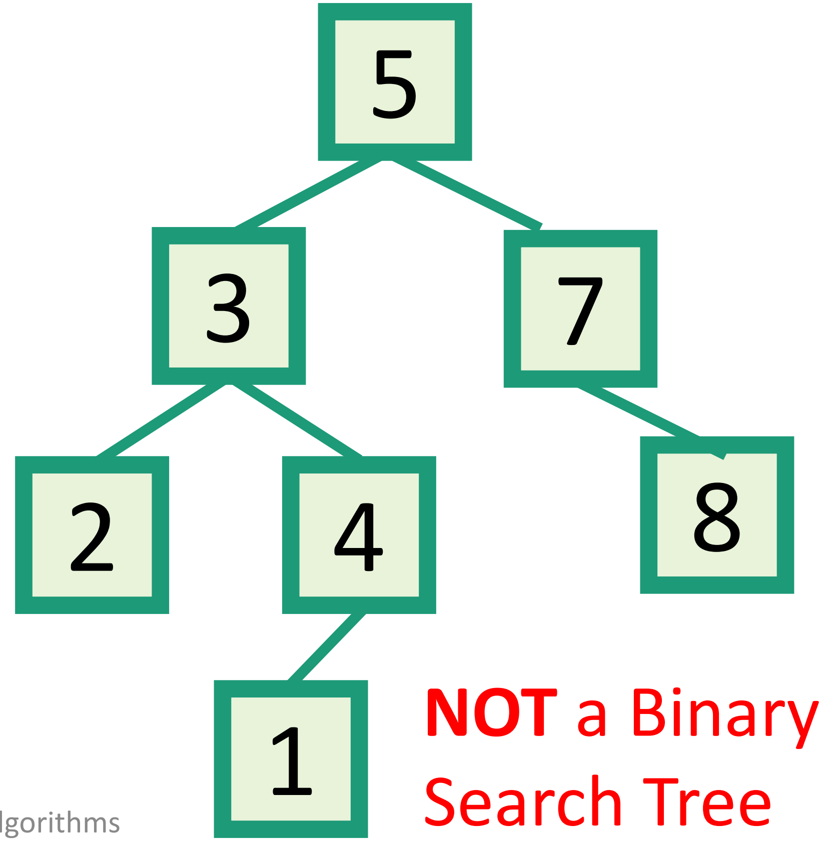


Binary Search Tree

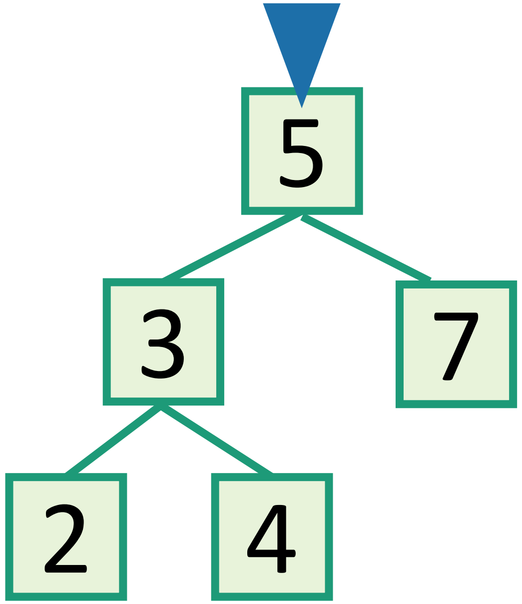


NOT a Binary Search Tree

Aside: In-Order Traversal of BSTs

- Output all the elements in sorted order!

- inOrderTraversal(x):
    - if x!= NIL:
        - inOrderTraversal(x.left)
        - print(x.key)
        - inOrderTraversal(x.right)



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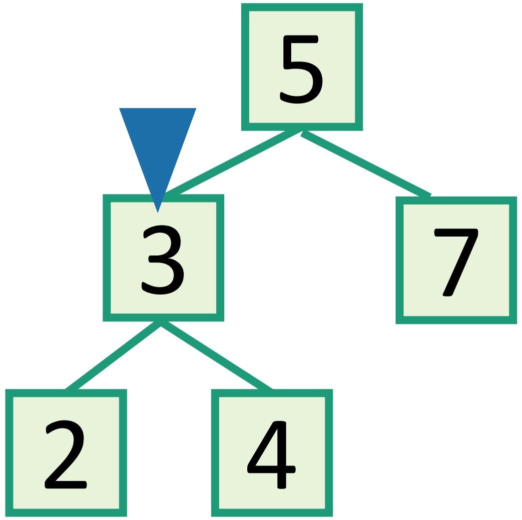

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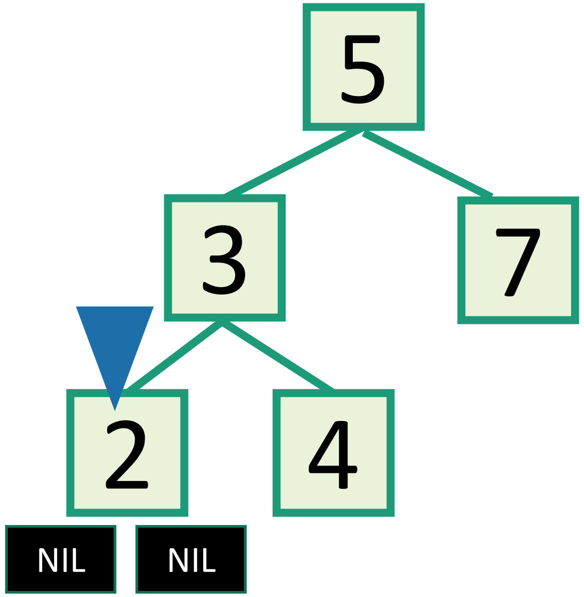

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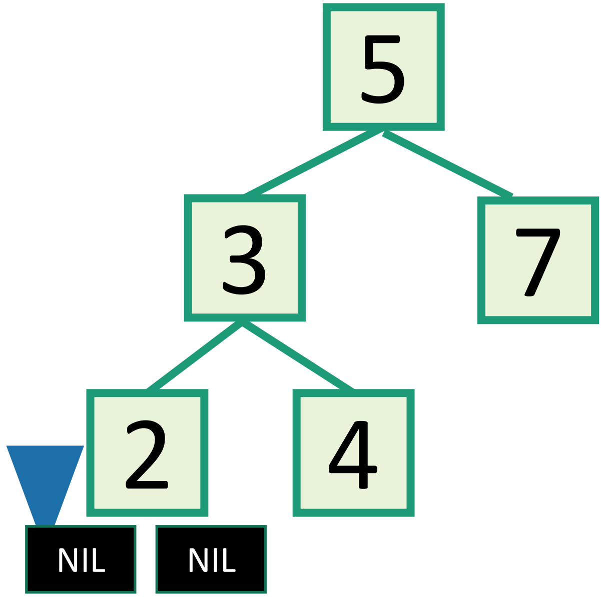

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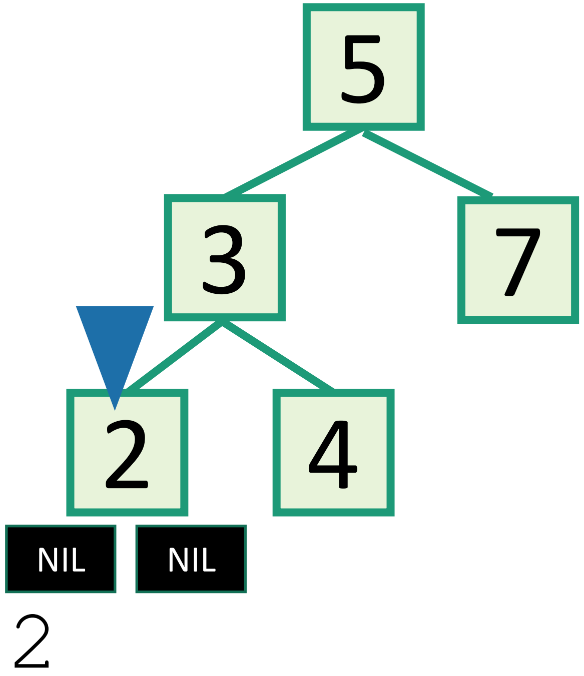

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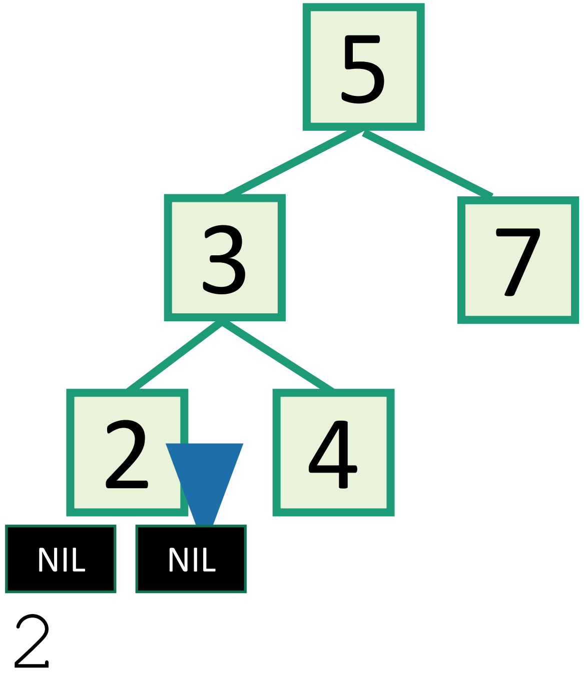

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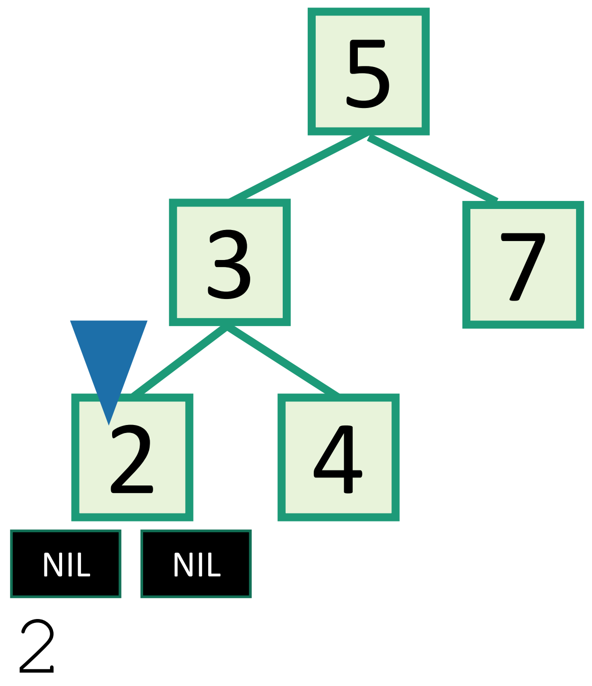

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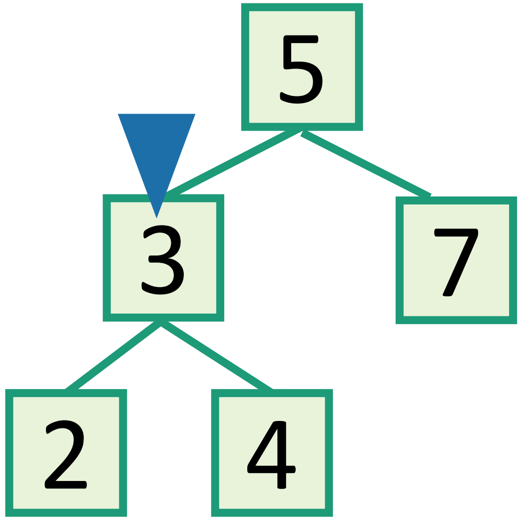

2 3

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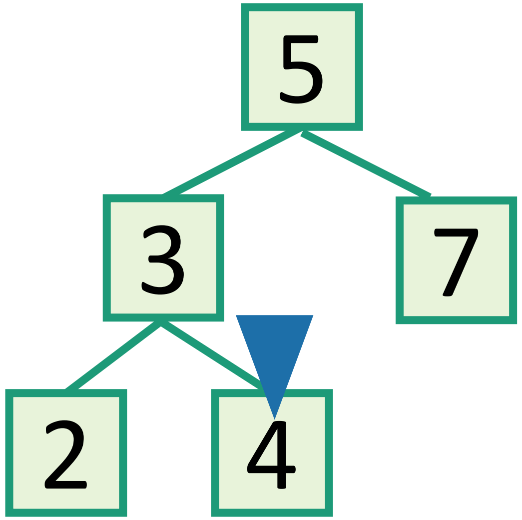

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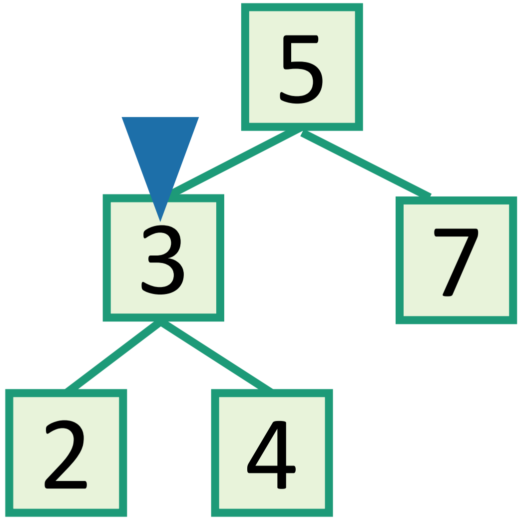

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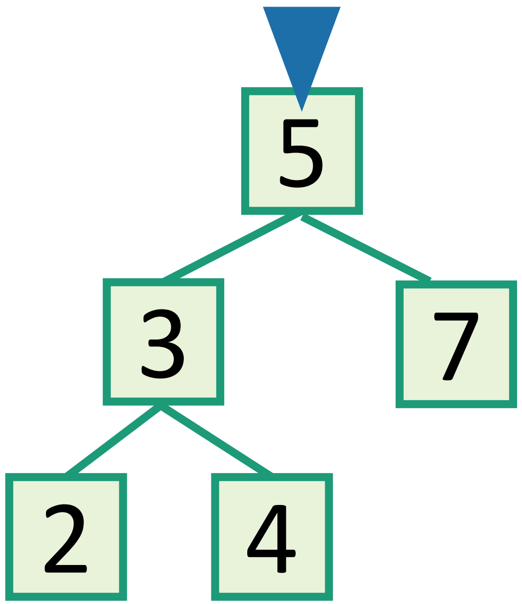

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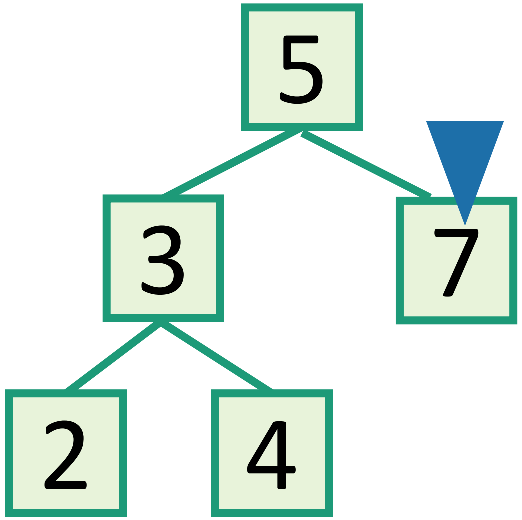

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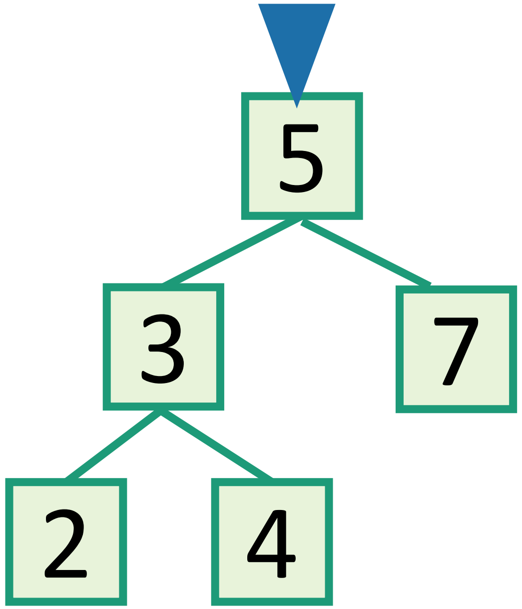


- Runs in time O(n).

2 3 4 5 7 Sorted!

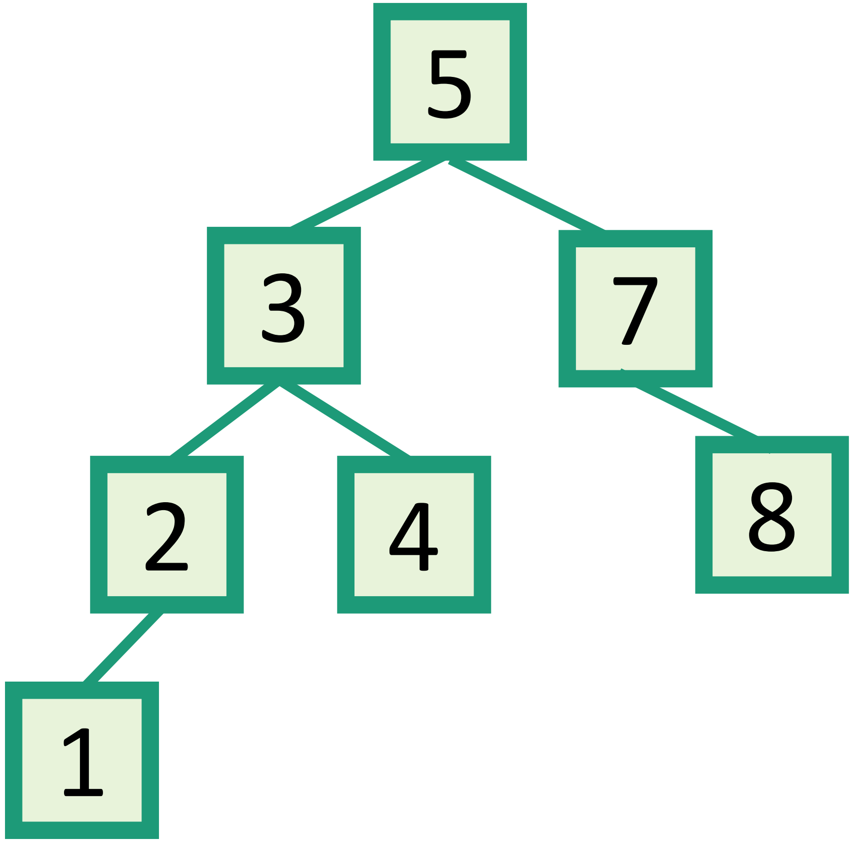
Tree Minimum and Tree Maximum

TREE-MINIMUM(x)

```
while x.left ≠ NIL
    x = x.left
return x
```

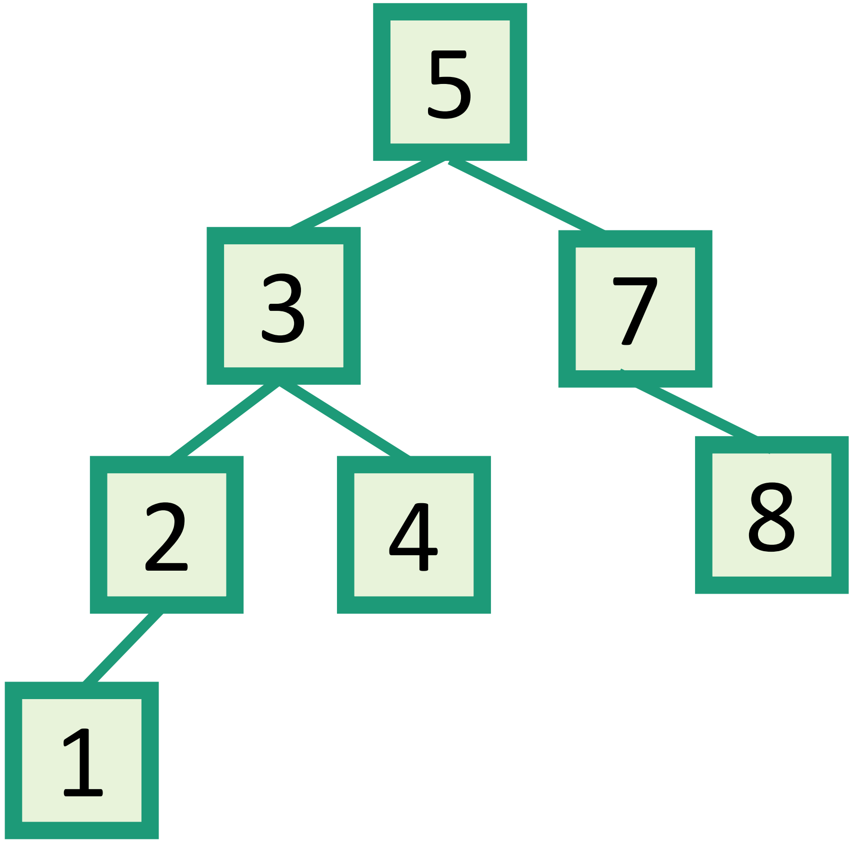
TREE-MAXIMUM(x)

```
while x.right ≠ NIL
    x = x.right
return x
```

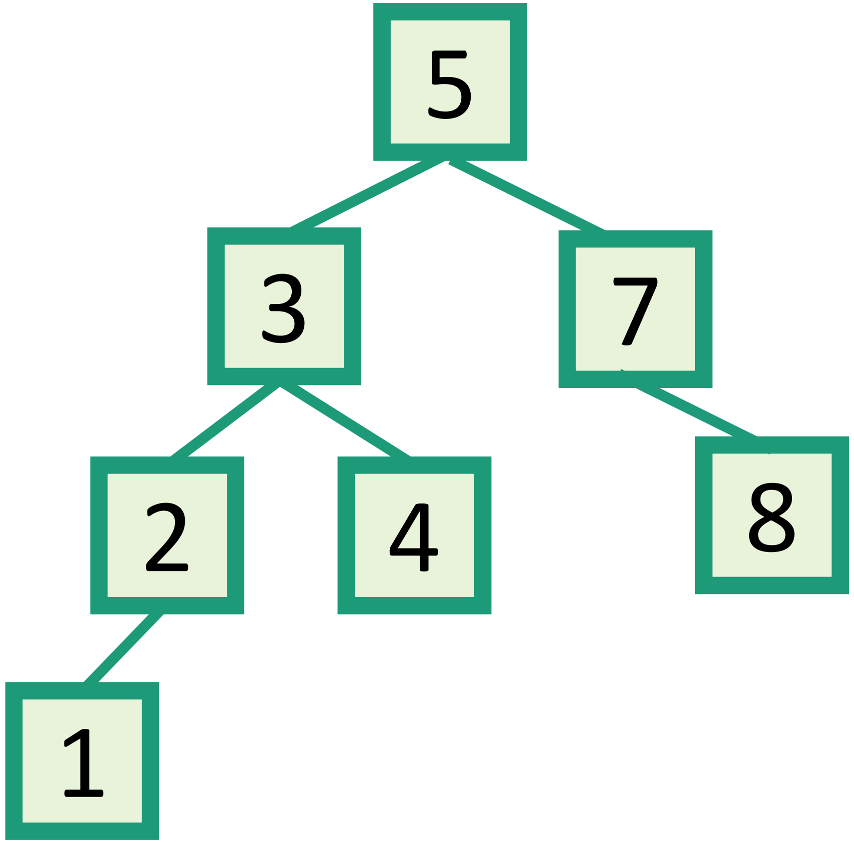


Tree Successor

```
TREE-SUCCESSOR(x)
if x.right ≠ NIL
    return TREE-MINIMUM(x.right)
y = x.p
while y ≠ NIL and x == y.right
    x = y
    y = y.p
return y
```



Tree Predecessor



Back to the goal

Fast SEARCH/INSERT/DELETE

Can we do these?

SEARCH in a Binary Search Tree

definition by example

```
TREE-SEARCH(x, k)
    if x == NIL or k == x.key
        return x
    if k < x.key
        return TREE-SEARCH(x.left, k)
    else return TREE-SEARCH(x.right, k)
```

1

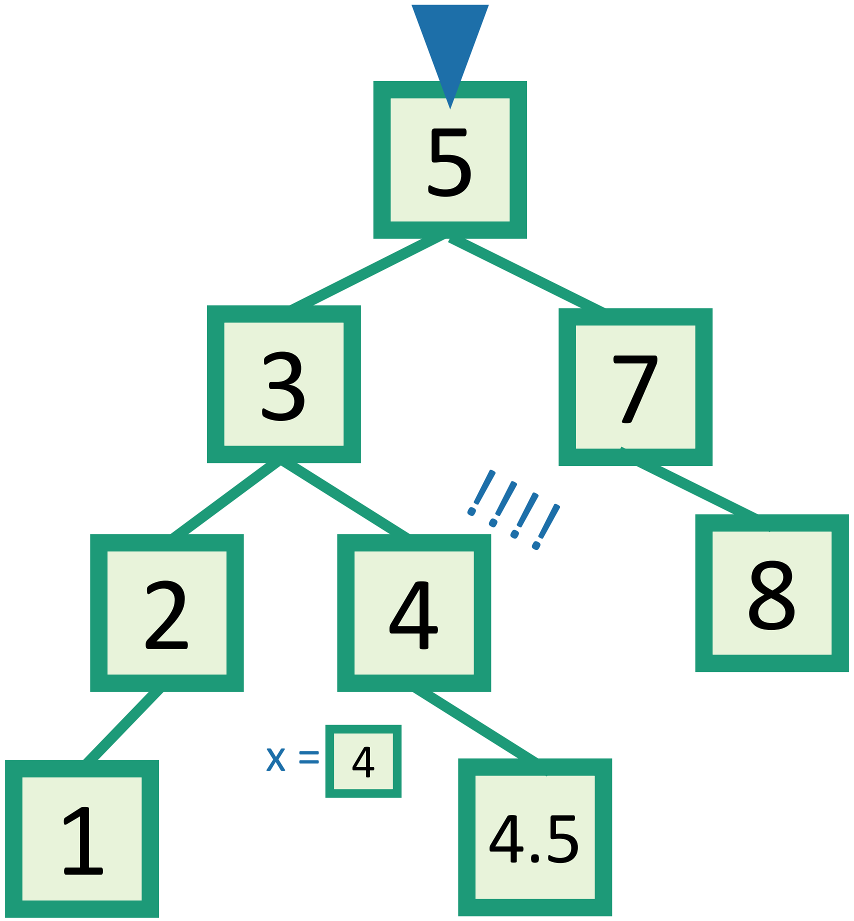
How long does this take?

O(length of longest path) = O(height)

Write pseudocode (or actual code) to implement this!

Ollie the over-achieving ostrich

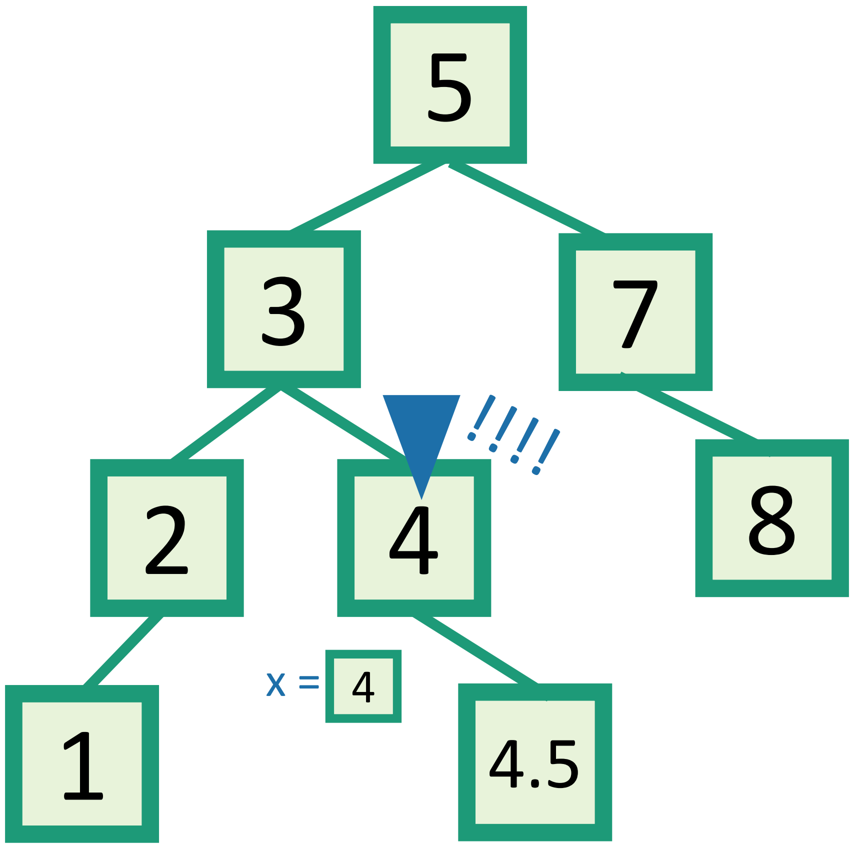
INSERT in a Binary Search Tree



EXAMPLE: Insert 4.5

- INSERT(key):
  - x = SEARCH(key)
  - **Insert** a new node with desired key at x…

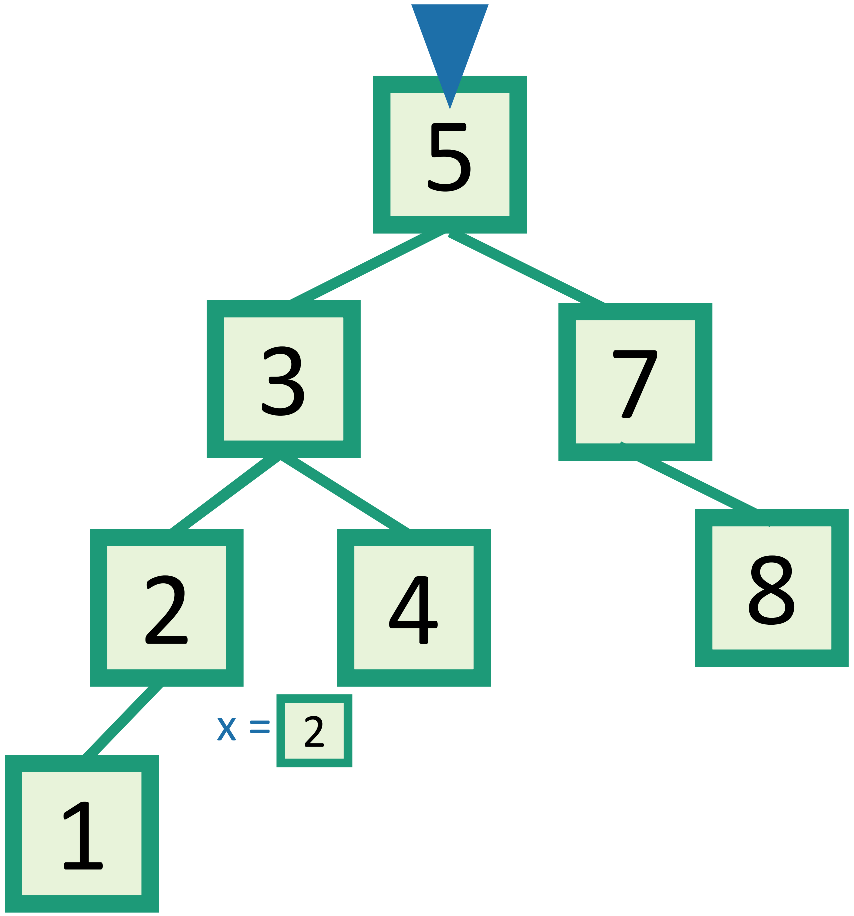
INSERT in a Binary Search Tree



EXAMPLE: Insert 4.5

- INSERT(key):
  - x = SEARCH(key)
  - **if** key > x.key:
    - Make a new node with the correct key, and put it as the right child of x.
  - **if** key < x.key:
    - Make a new node with the correct key, and put it as the left child of x.
  - **if** x.key == key:
    - **return**

DELETE in a Binary Search Tree



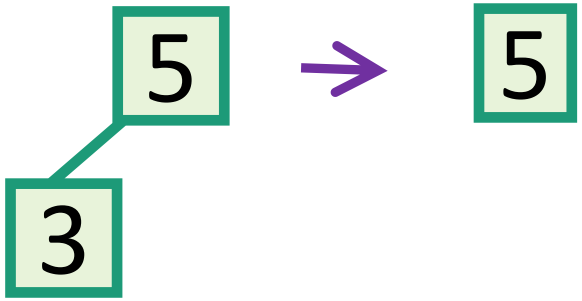
EXAMPLE: Delete 2

- DELETE(key):
  - x = SEARCH(key)
  - **if** x.key == key:
    - ….delete x….

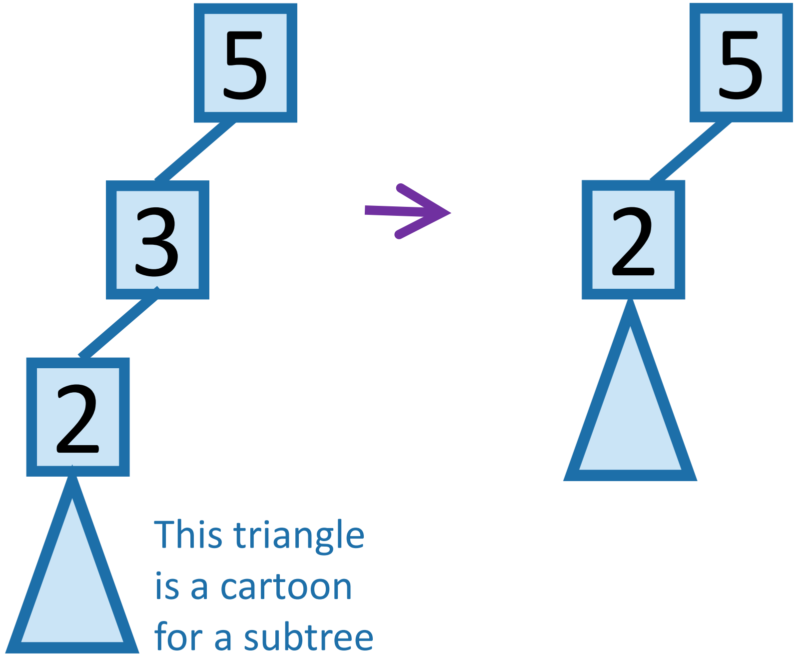
DELETE in a Binary Search Tree

several cases (by example)

say we want to delete 3



Case 1: if 3 is a leaf,
just delete it.



Case 2: if 3 has just one child,
move that up.

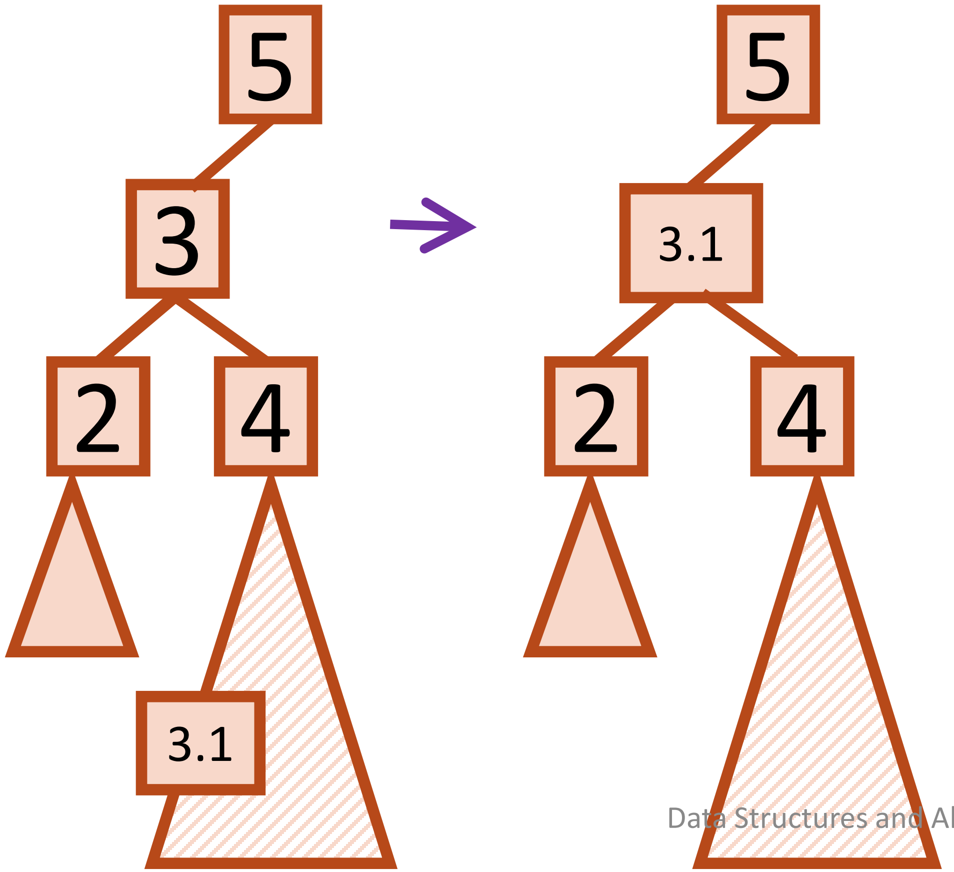
Write pseudocode for all of these!

Siggi the Studious Stork

DELETE in a Binary Search Tree

ctd.

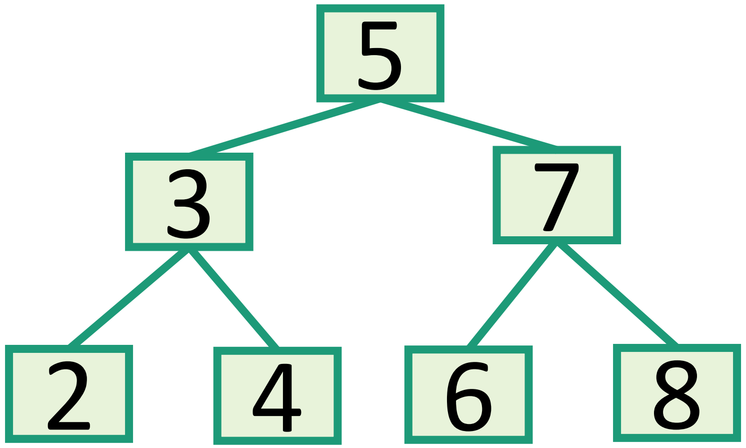
Case 3: if 3 has two children,
replace 3 with it's immediate successor.
(aka, next biggest thing after 3)



- Does this maintain the BST property?
  - Yes.
- How do we find the immediate successor?
  - SEARCH for 3 in the subtree under 3.right
- How do we remove it when we find it?
  - If [3.1] has 0 or 1 children, do one of the previous cases.
- What if [3.1] has two children?
  - It doesn't.

How long do these operations take?

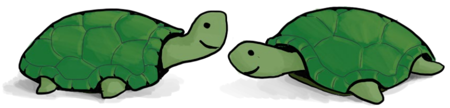
- SEARCH is the big one.
  - Everything else just calls SEARCH and then does some small O(1)-time operation.



Time = O(height of tree)

How long does search take?

1 minute think; 1 minute pair+share



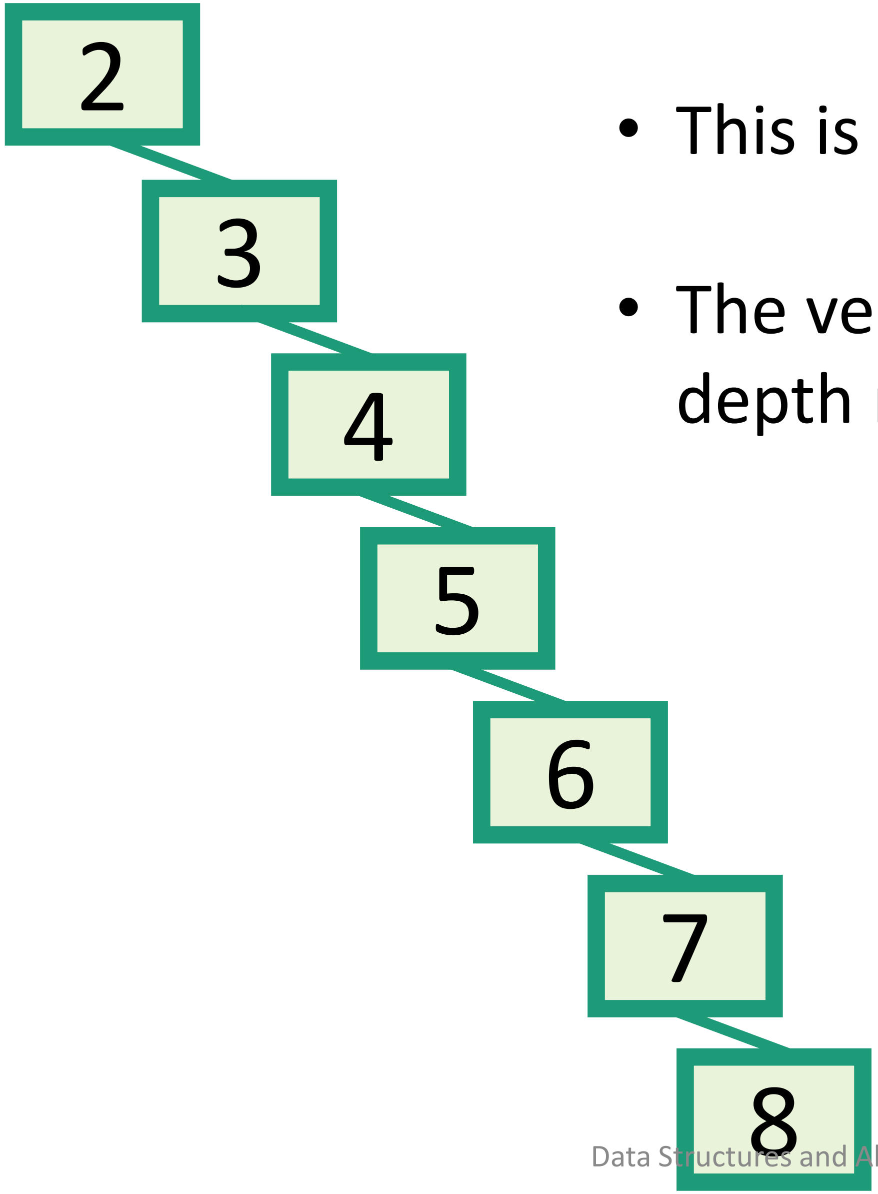
Trees have depth O(log(n)). **Done!**

Lucky the lackadaisical lemur.

Wait a second…

Plucky the Pedantic Penguin

Search might take time O(n).



- This is a valid binary search tree.
- The version with n nodes has depth n, **not** O(log(n)).

What to do?

How often is "every so often" in the worst case? It's actually pretty often!

Ollie the over-achieving ostrich

- Goal: Fast SEARCH/INSERT/DELETE
- All these things take time O(height)
- And the height might be big!!! ☹

- Idea 0:
  - Keep track of how deep the tree is getting.
  - If it gets too tall, re-do everything from scratch.
    - At least $\Omega(n)$ every so often....