⇒ 3 Linear Transformation Examples from $R^2 \to R^3$

- **Example: 1**

Define $T : R^2 \to R^3$ by

$$T(x, y) = (x, 2y, x-y)$$

- **Sol:**

Let $\vec{u} = (x_1, x_2)$ and $\vec{v} = (y_1, y_2)$

$$(\vec{u}+\vec{v}) = (x_1+y_1, x_2+y_2)$$

$$(r\vec{u}) = (rx_1, rx_2)$$

T is said to be linear transformation if:

1. $T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$
2. $T(r\vec{u}) = rT(\vec{u})$ where r is scalar and $\vec{u}, \vec{v} \in V$.

$$T(u) = T(x_1, x_2) = (x_1, 2x_2, x_1 - x_2)$$

$$T(v) = T(y_1, y_2) = (y_1, 2y_2, y_1 - y_2)$$

$$\boxed{T(u)+T(v) = (x_1+y_1, 2x_2+2y_2, x_1-x_2+y_1-y_2)}$$

$$T(u+v) = T(x_1+y_1, x_2+y_2)$$

$$\boxed{T(u+v) = (x_1+y_1, 2x_2+y_2, x_1+y_1-x_2-y_2)}$$

$rT(u) = r(x_1, 2x_2, x_1 - x_2)$

$$\boxed{rT(u) = (rx_1, 2rx_2, rx_1 - rx_2)}$$

$T(ru) = T(rx_1, rx_2)$

$$\boxed{T(ru) = T(rx_1, 2rx_2, rx_1 - rx_2)}$$

As

1. $T(u+v) = T(u) + T(v)$ and
2. $T(ru) = rT(u)$

Hence T is linear transformation. **Proved:**

- **Example : 2**

Define $T : R^2 \to R^3$ by

$T(x, y) = (3x + 2y, -x + y, 4x - y)$

- **Sol:**

Let $u = (x_1, x_2)$ and $v = (y_1, y_2)$

$(u+v) = (x_1 + y_1, x_2 + y_2)$

$(ru) = (rx_1, rx_2)$

$T(u) = T(x_1, x_2) = (3x_1 + 2x_2, -x_1 + x_2, 4x_1 - x_2)$

$T(v) = T(y_1, y_2) = (3y_1 + 2y_2, -y_1 + y_2, 4y_1 - y_2)$

$T(u) + T(v) = (3x_1 + 2x_2 + 3y_1 + 2y_2, -x_1 + x_2 - y_1 + y_2,$
$4x_1 - x_2 + 4y_1 - y_2)$ —— (i)

$T(u+v) = T(x_1+y_1, x_2+y_2)$

$T(u+v) = (3x_1+3y_1+2x_2+2y_2, -x_1-y_1+x_2+y_2,$
$4x_1+4y_1-x_2-y_2)$ —— (ii)

$rT(u) = (3rx_1+2rx_2, -rx_1+rx_2, 4rx_1-rx_2)$ —— (iii)

$T(ru) = T(rx_1, rx_2)$

$T(ru) = (3rx_1+2rx_2, -rx_1+rx_2, 4rx_1-rx_2)$ —— (iv)

As,

1. $T(u+v) = T(u)+T(v)$
2. $T(ru) = rT(u)$

Hence T is Linear Transformation. **Proved**

- **Example: 3**

Define $T: R^2 \to R^3$ by:

$T(x,y) = (0, x+y, 2x-3y)$

- **Sol:**

Let $u = (x_1, x_2)$ and $v = (y_1, y_2)$

$(u+v) = (x_1+y_1, x_2+y_2)$

$(ru) = (rx_1, rx_2)$

$T(u) = T(x_1, x_2) = (0, x_1+x_2, 2x_1-3x_2)$

$T(v) = T(y_1, y_2) = (0, y_1+y_2, 2y_1-3y_2)$

$T(u)+T(v) = (0, x_1+x_2+y_1+y_2, 2x_1-3x_2+2y_1-3y_2)$ —— (i)

$T(u+v) = T(x_1+y_1, x_2+y_2)$

$T(u+v) = (0, x_1+y_1+x_2+y_2, 2x_1+2y_1-3x_2-3y_2)$ —— (ii)

$$rT(u) = (0, rx_1 + rx_2, 2rx_1 - 3rx_2) \text{ — (iii)}$$

$$T(ru) = T(rx_1, rx_2)$$

$$T(ru) = (0, rx_1 + rx_2, 2rx_1 - 3rx_2) \text{ — (iv)}$$

As,

$$T(u+v) = T(u) + T(v)$$

$$T(ru) = rT(u)$$

Hence T is Linear Transformation. **Proved**

⇒ 3 Non-Linear Transformation Examples from $R^3 \to R^2$

- **Example 1:**

Define $T: R^3 \to R^2$ by

$$T(x, y, z) = (x + y, yz)$$

- **Sol:**

Let $u = (x_1, x_2, x_3)$ and $v = (y_1, y_2, y_3)$

$$u + v = (x_1 + y_1, x_2 + y_2, x_3 + y_3)$$

$$ru = (rx_1, rx_2, rx_3)$$

$$T(u) = (x_1 + x_2, x_2 \cdot x_3)$$

$$T(v) = (y_1 + y_2, y_2 \cdot y_3)$$

$$T(u) + T(v) = (x_1 + x_2 + y_1 + y_2, x_2 x_3 + y_2 y_3) \text{ — (i}$$

$$T(u+v) = T(x_1 + y_1, x_2 + y_2, x_3 + y_3)$$

$$T(u+v) = (x_1 + y_1 + x_2 + y_2, (x_2 + y_2)(x_3 + y_3))$$

$$T(u+v) = (x_1 + y_1 + x_2 + y_2, x_2 x_3 + x_2 y_3 + y_2 x_3 + y_2$$

$rT(u) = (rx_1 + rx_2, rx_2x_3)$ —— (iii)

$T(ru) = T(rx_1, rx_2, rx_3)$

$T(ru) = (rx_1 + rx_2, (rx_2)(rx_3))$

$T(ru) = (rx_1 + rx_2, r^2x_2x_3)$ —— (iv)

As

1. $T(u+v) \neq T(u) + T(v)$ and
2. $T(ru) \neq rT(u)$

Hence, T is Non-Linear Transformation.

Proved.

• **Example 2:**

Define $T: R^3 \to R^2$ by

$T(x, y, z) = (x^2, y+z)$

• **Sol:**

Let $u = (x_1, x_2, x_3)$ and $v = (y_1, y_2, y_3)$

$u+v = (x_1+y_1, x_2+y_2, x_3+y_3)$

$ru = (rx_1, rx_2, rx_3)$

$T(u) = (x_1^2, x_2+x_3)$

$T(v) = (y_1^2, y_2+y_3)$

$T(u)+T(v) = (x_1^2+y_1^2, x_2+x_3+y_2+y_3)$ —— (i)

$T(u+v) = T(x_1+y_1, x_2+y_2, x_3+y_3)$

$T(u+v) = ((x_1+y_1)^2, x_2+y_2+x_3+y_3)$

$T(u+v) = (x_1^2+y_1^2+2x_1y_1, x_2+y_2+x_3+y_3)$ —— (ii)

$T(u) = r(x_1^2, x_2+x_3)$

$T(u) = (rx_1^2, rx_2+rx_3)$ —— (iii)

$T(r(w)) = T(rx_1, rx_2, rx_3)$

$T(r(w)) = (rx_1^2, rx_2 + rx_3)$ ——— (iv)

As,

$T(u+v) \neq T(u) + T(v)$

Hence, T is Non-Linear Transformation.

Proved.

• **Example:03**

Define $T_3 : R^3 \to R^2$ by

$T_3(x, y, z) = (x + y^2, y - z)$

• **Sol:**

Let $u = (x_1, x_2, x_3)$ and $v = (y_1, y_2, y_3)$

$(u+v) = (x_1 + y_1, x_2 + y_2, x_3 + y_3)$

$ru = (rx_1, rx_2, rx_3)$

$T(u) = T(x_1, x_2, x_3) = (x_1 + x_2^2, x_2 - x_3)$

$T(v) = T(y_1, y_2, y_3) = (y_1 + y_2^2, y_2 - y_3)$

$T(u) + T(v) = (x_1 + x_2^2 + y_1 + y_2^2, x_2 - x_3 + y_2 - y_3)$ ——— (i)

$T(u+v) = T(x_1 + y_1, x_2 + y_2, x_3 + y_3)$

$T(u+v) = (x_1 + y_1 + (x_2 + y_2)^2, x_2 + y_2 - x_3 - y_3)$

$T(u+v) = (x_1 + y_1 + x_2^2 + y_2^2 + 2x_2y_2, x_2 + y_2 - x_3 - y_3)$ ——— (ii)

$rT(u) = r(x_1 + x_2^2, x_2 - x_3)$

$rT(u) = (rx_1 + rx_2^2, rx_2 - rx_3)$ —— (iii)

$T(ru) = T(rx_1, rx_2, rx_3)$

$T(ru) = (rx_1 + rx_2^2, rx_2 - rx_3)$ —— (iv)

As,

$T(u+v) \neq T(u) + T(v)$

Hence, T is Non-Linear Transformation.

Proved: