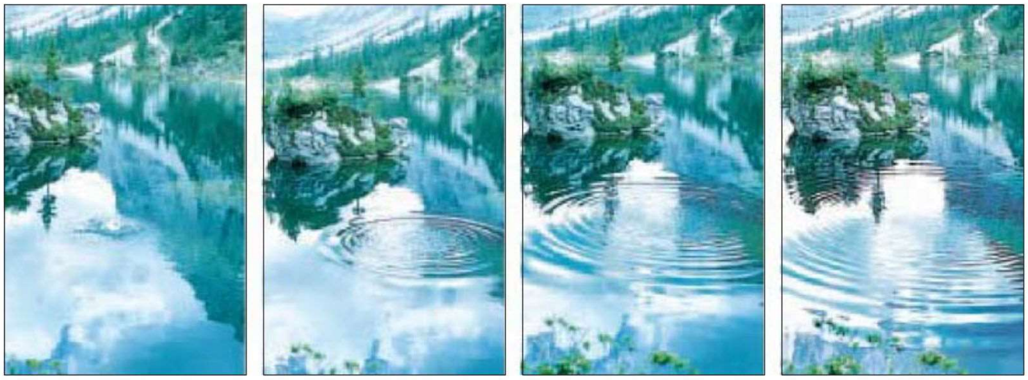


Waves—such as these water waves—spread outward from a source. The source in this case is a small spot of water oscillating up and down briefly where a rock was thrown in (left photo). Other kinds of waves include waves on a cord or string, which also are produced by a vibration. Waves move away from their source, but we also study waves that seem to stand still ("standing waves"). Waves reflect, and they can interfere with each other when they pass through any point at the same time.

CHAPTER 15

# Wave Motion

**CHAPTER-OPENING QUESTION—Guess now!**

You drop a rock into a pond, and water waves spread out in circles.

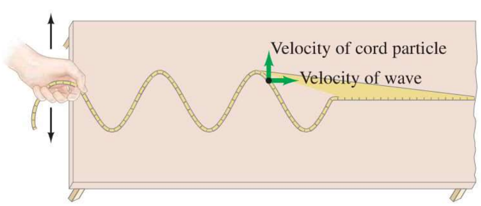
**(a)** The waves carry water outward, away from where the rock hit. That moving water carries energy outward.

**(b)** The waves only make the water move up and down. No energy is carried outward from where the rock hit.

**(c)** The waves only make the water move up and down, but the waves do carry energy outward, away from where the rock hit.

When you throw a stone into a lake or pool of water, circular waves form and move outward, as shown in the photos above. Waves will also travel along a cord that is stretched out flat on a table if you vibrate one end back and forth as shown in Fig. 15–1. Water waves and waves on a cord are two common examples of **mechanical waves**, which propagate as oscillations of matter. We will discuss other kinds of waves in later Chapters, including electromagnetic waves and light.

**FIGURE 15–1** Wave traveling on a cord. The wave travels to the right along the cord. Particles of the cord oscillate back and forth on the tabletop.

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If you have ever watched ocean waves moving toward shore before they break, you may have wondered if the waves were carrying water from far out at sea onto the beach. They don't.† Water waves move with a recognizable velocity. But each particle (or molecule) of the water itself merely oscillates about an equilibrium point. This is clearly demonstrated by observing leaves on a pond as waves move by. The leaves (or a cork) are not carried forward by the waves, but simply oscillate about an equilibrium point because this is the motion of the water itself.

**CONCEPTUAL EXAMPLE 15–1** **Wave vs. particle velocity.** Is the velocity of a wave moving along a cord the same as the velocity of a particle of the cord? See Fig. 15–1.

**RESPONSE** No. The two velocities are different, both in magnitude and direction. The wave on the cord of Fig. 15–1 moves to the right along the tabletop, but each piece of the cord only vibrates to and fro. (The cord clearly does not travel in the direction that the wave on it does.)

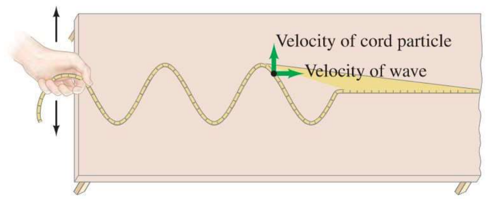


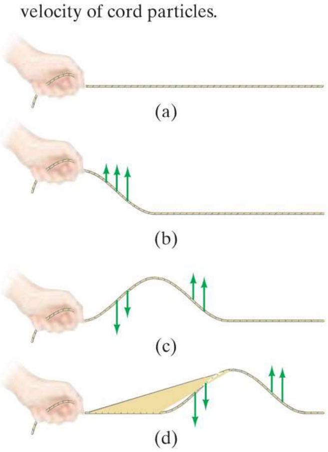

**FIGURE 15–1** (repeated) Wave traveling on a cord. The wave travels to the right along the cord. Particles of the cord oscillate back and forth on the tabletop.

Waves can move over large distances, but the medium (the water or the cord) itself has only a limited movement, oscillating about an equilibrium point as in simple harmonic motion. Thus, although a wave is not matter, the wave pattern can travel in matter. A wave consists of oscillations that move without carrying matter with them.

Waves carry energy from one place to another. Energy is given to a water wave, for example, by a rock thrown into the water, or by wind far out at sea. The energy is transported by waves to the shore. The oscillating hand in Fig. 15–1 transfers energy to the cord, and that energy is transported down the cord and can be transferred to an object at the other end. All forms of traveling waves transport energy.

**EXERCISE A** Return to the Chapter-Opening Question, page 395, and answer it again now. Try to explain why you may have answered differently the first time.

**FIGURE 15–2** Motion of a wave pulse to the right. Arrows indicate velocity of cord particles.




## 15–1 Characteristics of Wave Motion

Let us look a little more closely at how a wave is formed and how it comes to "travel." We first look at a single wave bump, or **pulse**. A single pulse can be formed on a cord by a quick up-and-down motion of the hand, Fig. 15–2. The hand pulls up on one end of the cord. Because the end section is attached to adjacent sections, these also feel an upward force and they too begin to move upward. As each succeeding section of cord moves upward, the wave crest moves outward along the cord. Meanwhile, the end section of cord has been returned to its original position by the hand. As each succeeding section of cord reaches its peak position, it too is pulled back down again by tension from the adjacent section of cord. Thus the source of a traveling wave pulse is a disturbance, and cohesive forces between adjacent sections of cord cause the pulse to travel. Waves in other media are created

†Do not be confused by the "breaking" of ocean waves, which occurs when a wave interacts with the ground in shallow water and hence is no longer a simple wave.

and propagate outward in a similar fashion. A dramatic example of a wave pulse is a tsunami or tidal wave that is created by an earthquake in the Earth's crust under the ocean. The bang you hear when a door slams is a sound wave pulse.

A **continuous** or **periodic wave**, such as that shown in Fig. 15–1, has as its source a disturbance that is continuous and oscillating; that is, the source is a *vibration* or *oscillation*. In Fig. 15–1, a hand oscillates one end of the cord. Water waves may be produced by any vibrating object at the surface, such as your hand; or the water itself is made to vibrate when wind blows across it or a rock is thrown into it. A vibrating tuning fork or drum membrane gives rise to sound waves in air. And we will see later that oscillating electric charges give rise to light waves. Indeed, almost any vibrating object sends out waves.

The source of any wave, then, is a vibration. And it is a *vibration* that propagates outward and thus constitutes the wave. If the source vibrates sinusoidally in SHM, then the wave itself—if the medium is perfectly elastic—will have a sinusoidal shape both in space and in time. (1) In space: if you take a picture of the wave in space at a given instant of time, the wave will have the shape of a sine or cosine as a function of position. (2) In time: if you look at the motion of the medium at one place over a long period of time—for example, if you look between two closely spaced posts of a pier or out of a ship's porthole as water waves pass by—the up-and-down motion of that small segment of water will be simple harmonic motion. The water moves up and down sinusoidally in time.

Some of the important quantities used to describe a periodic sinusoidal wave are shown in Fig. 15–3. The high points on a wave are called *crests*; the low points, *troughs*. The **amplitude**, $A$, is the maximum height of a crest, or depth of a trough, relative to the normal (or equilibrium) level. The total swing from a crest to a trough is twice the amplitude. The distance between two successive crests is called the **wavelength**, $\lambda$ (the Greek letter lambda). The wavelength is also equal to the distance between *any* two successive identical points on the wave. The **frequency**, $f$, is the number of crests—or complete cycles—that pass a given point per unit time. The **period**, $T$, equals $1/f$ and is the time elapsed between two successive crests passing by the same point in space.

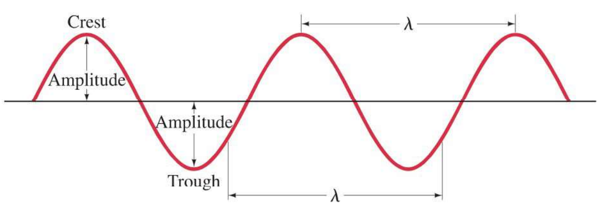



**FIGURE 15–3** Characteristics of a single-frequency continuous wave moving through space.

The **wave velocity**, $v$, is the velocity at which wave crests (or any other part of the waveform) move forward. The wave velocity must be distinguished from the velocity of a particle of the medium itself as we saw in Example 15–1.

A wave crest travels a distance of one wavelength, $\lambda$, in a time equal to one period, $T$. Thus the wave velocity is $v = \lambda/T$. Then, since $1/T = f$,

$$v = \lambda f. \tag{15–1}$$

For example, suppose a wave has a wavelength of 5 m and a frequency of 3 Hz. Since three crests pass a given point per second, and the crests are 5 m apart, the first crest (or any other part of the wave) must travel a distance of 15 m during the 1 s. So the wave velocity is 15 m/s.

**EXERCISE B** You notice a water wave pass by the end of a pier with about 0.5 s between crests. Therefore (*a*) the frequency is 0.5 Hz; (*b*) the velocity is 0.5 m/s; (*c*) the wavelength is 0.5 m; (*d*) the period is 0.5 s.

# 15–2 Types of Waves: Transverse and Longitudinal

When a wave travels down a cord—say, from left to right as in Fig. 15–1—the particles of the cord vibrate up and down in a direction transverse (that is, perpendicular) to the motion of the wave itself. Such a wave is called a **transverse wave** (Fig. 15–4a). There exists another type of wave known as a **longitudinal wave**. In a longitudinal wave, the vibration of the particles of the medium is *along* the direction of the wave's motion. Longitudinal waves are readily formed on a stretched spring or Slinky by alternately compressing and expanding one end. This is shown in Fig. 15–4b, and can be compared to the transverse wave in Fig. 15–4a.

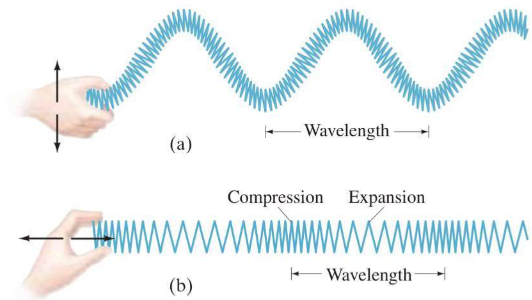
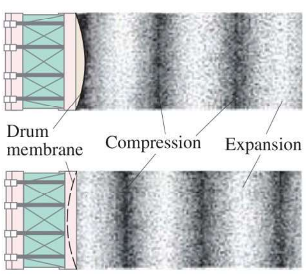



**FIGURE 15–4** (a) Transverse wave; (b) longitudinal wave.

A series of compressions and expansions propagate along the spring. The *compressions* are those areas where the coils are momentarily close together. *Expansions* (sometimes called *rarefactions*) are regions where the coils are momentarily far apart. Compressions and expansions correspond to the crests and troughs of a transverse wave.

**FIGURE 15–5** Production of a sound wave, which is longitudinal, shown at two moments in time about a half period $\left(\frac{1}{2}T\right)$ apart.




An important example of a longitudinal wave is a sound wave in air. A vibrating drumhead, for instance, alternately compresses and rarefies the air in contact with it, producing a longitudinal wave that travels outward in the air, as shown in Fig. 15–5.

As in the case of transverse waves, each section of the medium in which a longitudinal wave passes oscillates over a very small distance, whereas the wave itself can travel large distances. Wavelength, frequency, and wave velocity all have meaning for a longitudinal wave. The wavelength is the distance between successive compressions (or between successive expansions), and frequency is the number of compressions that pass a given point per second. The wave velocity is the velocity with which each compression appears to move; it is equal to the product of wavelength and frequency, $v = \lambda f$ (Eq. 15–1).

A longitudinal wave can be represented graphically by plotting the density of air molecules (or coils of a Slinky) versus position at a given instant, as shown in Fig. 15–6. Such a graphical representation makes it easy to illustrate what is happening. Note that the graph looks much like a transverse wave.

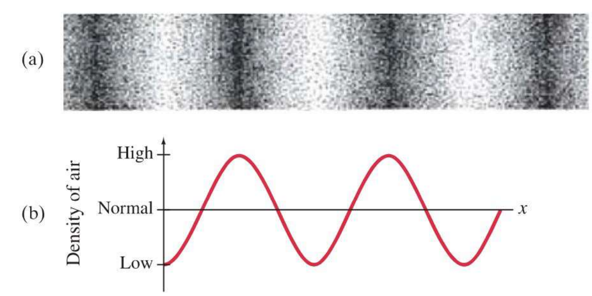



**FIGURE 15–6** (a) A longitudinal wave with (b) its graphical representation at a particular instant in time.

## Velocity of Transverse Waves

The velocity of a wave depends on the properties of the medium in which it travels. The velocity of a transverse wave on a stretched string or cord, for example, depends on the tension in the cord, $F_T$, and on the mass per unit length of the cord, $\mu$ (the Greek letter mu, where here $\mu = m/\ell$). For waves of small amplitude, the relationship is

$$v = \sqrt{\frac{F_T}{\mu}}. \qquad \left[\begin{array}{l}\text{transverse}\\ \text{wave on a cord}\end{array}\right] \quad \textbf{(15–2)}$$

Before giving a derivation of this formula, it is worth noting that at least qualitatively it makes sense on the basis of Newtonian mechanics. That is, we do expect the tension to be in the numerator and the mass per unit length in the denominator. Why? Because when the tension is greater, we expect the velocity to be greater since each segment of cord is in tighter contact with its neighbor. And, the greater the mass per unit length, the more inertia the cord has and the more slowly the wave would be expected to propagate.

**EXERCISE C** A wave starts at the left end of a long cord (see Fig. 15–1) when someone shakes the cord back and forth at the rate of 2.0 Hz. The wave is observed to move to the right at 4.0 m/s. If the frequency is increased from 2.0 to 3.0 Hz, the new speed of the wave is (*a*) 1.0 m/s, (*b*) 2.0 m/s, (*c*) 4.0 m/s, (*d*) 8.0 m/s, (*e*) 16.0 m/s.

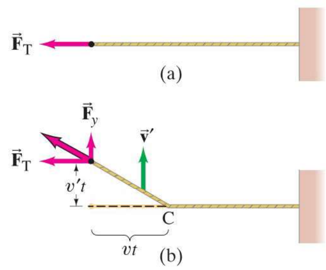



**FIGURE 15–7** Diagram of simple wave pulse on a cord for derivation of Eq. 15–2. The vector shown in (b) as the resultant of $\vec{F}_T + \vec{F}_y$ has to be directed along the cord because the cord is flexible. (Diagram is not to scale: we assume $v' \ll v$; the upward angle of the cord is exaggerated for visibility.)

We can make a simple derivation of Eq. 15–2 using a simple model of a cord under a tension $F_T$ as shown in Fig. 15–7a. The cord is pulled upward at a speed $v'$ by the force $F_y$. As shown in Fig. 15–7b all points of the cord to the left of point C move upward at the speed $v'$, and those to the right are still at rest. The speed of propagation, $v$, of this wave pulse is the speed of point C, the leading edge of the pulse. Point C moves to the right a distance $vt$ in a time $t$, whereas the end of the cord moves upward a distance $v't$. By similar triangles we have the approximate relation

$$\frac{F_T}{F_y} = \frac{vt}{v't} = \frac{v}{v'},$$

which is accurate for small displacements ($v't \ll vt$) so that $F_T$ does not change appreciably. As we saw in Chapter 9, the impulse given to an object is equal to its change in momentum. During the time $t$ the total upward impulse is $F_y t = (v'/v)F_T t$. The change in momentum of the cord, $\Delta p$, is the mass of cord moving upward times its velocity. Since the upward moving segment of cord has mass equal to the mass per unit length $\mu$ times its length $vt$ we have

$$\begin{aligned} F_y t &= \Delta p \\ \frac{v'}{v} F_T t &= (\mu v t)v'. \end{aligned}$$

Solving for $v$ we find $v = \sqrt{F_T/\mu}$ which is Eq. 15–2. Although it was derived for a special case, it is valid for any wave shape since other shapes can be considered to be made up of many tiny such lengths. But it is valid only for small displacements (as was our derivation). Experiment is in accord with this result derived from Newtonian mechanics.

**EXAMPLE 15–2** **Pulse on a wire.** An 80.0-m-long, 2.10-mm-diameter copper wire is stretched between two poles. A bird lands at the center point of the wire, sending a small wave pulse out in both directions. The pulses reflect at the ends and arrive back at the bird's location 0.750 seconds after it landed. Determine the tension in the wire.

**APPROACH** From Eq. 15–2, the tension is given by $F_T = \mu v^2$. The speed $v$ is distance divided by the time. The mass per unit length $\mu$ is calculated from the density of copper and the dimensions of the wire.

**SOLUTION** Each wave pulse travels 40.0 m to the pole and back again (= 80.0 m) in 0.750 s. Thus their speed is $v = (80.0\,\mathrm{m})/(0.750\,\mathrm{s}) = 107\,\mathrm{m/s}$. We take (Table 13–1) the density of copper as $8.90 \times 10^3\,\mathrm{kg/m^3}$. The volume of copper in the wire is the cross-sectional area $(\pi r^2)$ times the length $\ell$, and the mass of the wire is the volume times the density: $m = \rho(\pi r^2)\ell$ for a wire of radius $r$ and length $\ell$. Then $\mu = m/\ell$ is

$$\mu = \rho\pi r^2\ell/\ell = \rho\pi r^2 = (8.90 \times 10^3\,\mathrm{kg/m^3})\pi(1.05 \times 10^{-3}\,\mathrm{m})^2 = 0.0308\,\mathrm{kg/m}.$$

Thus, the tension is $F_T = \mu v^2 = (0.0308\,\mathrm{kg/m})(107\,\mathrm{m/s})^2 = 353\,\mathrm{N}$.

## Velocity of Longitudinal Waves

The velocity of a longitudinal wave has a form similar to that for a transverse wave on a cord (Eq. 15–2); that is,

$$v = \sqrt{\frac{\text{elastic force factor}}{\text{inertia factor}}}.$$

In particular, for a longitudinal wave traveling down a long solid rod,

$$v = \sqrt{\frac{E}{\rho}}, \qquad \begin{bmatrix}\text{longitudinal}\\ \text{wave in a long rod}\end{bmatrix} \qquad \textbf{(15–3)}$$

where $E$ is the elastic modulus (Section 12–4) of the material and $\rho$ is its density. For a longitudinal wave traveling in a liquid or gas,

$$v = \sqrt{\frac{B}{\rho}}, \qquad \begin{bmatrix}\text{longitudinal wave}\\ \text{in a fluid}\end{bmatrix} \qquad \textbf{(15–4)}$$

where $B$ is the bulk modulus (Section 12–4) and $\rho$ again is the density.

**PHYSICS APPLIED**
*Space perception by animals, using sound waves*

**EXAMPLE 15–3** **Echolocation.** Echolocation is a form of sensory perception used by animals such as bats, toothed whales, and dolphins. The animal emits a pulse of sound (a longitudinal wave) which, after reflection from objects, returns and is detected by the animal. Echolocation waves can have frequencies of about 100,000 Hz. (*a*) Estimate the wavelength of a sea animal's echolocation wave. (*b*) If an obstacle is 100 m from the animal, how long after the animal emits a wave is its reflection detected?

**APPROACH** We first compute the speed of longitudinal (sound) waves in sea water, using Eq. 15–4 and Tables 12–1 and 13–1. The wavelength is $\lambda = v/f$.

**SOLUTION** (*a*) The speed of longitudinal waves in sea water, which is slightly more dense than pure water, is

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.0 \times 10^9\,\mathrm{N/m^2}}{1.025 \times 10^3\,\mathrm{kg/m^3}}} = 1.4 \times 10^3\,\mathrm{m/s}.$$

Then, using Eq. 15–1, we find

$$\lambda = \frac{v}{f} = \frac{(1.4 \times 10^3\,\mathrm{m/s})}{(1.0 \times 10^5\,\mathrm{Hz})} = 14\,\mathrm{mm}.$$

(*b*) The time required for the round-trip between the animal and the object is

$$t = \frac{\text{distance}}{\text{speed}} = \frac{2(100\,\mathrm{m})}{1.4 \times 10^3\,\mathrm{m/s}} = 0.14\,\mathrm{s}.$$

**NOTE** We shall see later that waves can be used to "resolve" (or detect) objects only if the wavelength is comparable to or smaller than the object. Thus, a dolphin can resolve objects on the order of a centimeter or larger in size.

## *Deriving Velocity of Wave in a Fluid

We now derive Eq. 15–4. Consider a wave pulse traveling in a fluid in a long tube, so that the wave motion is one dimensional. The tube is fitted with a piston at the end and is filled with a fluid which, at $t = 0$, is of uniform density $\rho$ and at uniform pressure $P_0$, Fig. 15–8a. At this moment the piston is abruptly made to start moving to the right with speed $v'$, compressing the fluid in front of it. In the (short) time $t$ the piston moves a distance $v't$. The compressed fluid itself also moves with speed $v'$, but the leading edge of the compressed region moves to the right at the characteristic speed $v$ of compression waves in that fluid; we assume the wave speed $v$ is much larger than the piston speed $v'$. The leading edge of the compression (which at $t = 0$ was at the piston face) thus moves a distance $vt$ in time $t$ as shown in Fig. 15–8b. Let the pressure in the compression be $P_0 + \Delta P$, which is $\Delta P$ higher than in the uncompressed fluid. To move the piston to the right requires an external force $(P_0 + \Delta P)S$ acting to the right, where $S$ is the cross-sectional area of the tube. ($S$ for "surface area"; we save $A$ for amplitude.) The *net* force on the compressed region of the fluid is

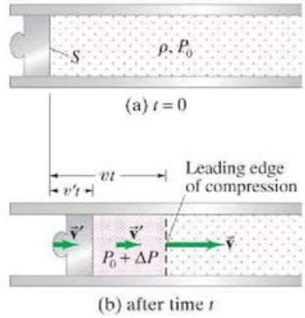



**FIGURE 15–8** Determining the speed of a one-dimensional longitudinal wave in a fluid contained in a long narrow tube.

$$F_{\text{net}} = (P_0 + \Delta P)S - P_0 S = S\,\Delta P$$

since the uncompressed fluid exerts a force $P_0 S$ to the left at the leading edge. Hence the impulse given to the compressed fluid, which equals its change in momentum, is

$$\begin{aligned} F_{\text{net}}\, t &= \Delta m v' \\ S\,\Delta P\, t &= (\rho S v t) v', \end{aligned}$$

where $(\rho S v t)$ represents the mass of fluid which is given the speed $v'$ (the compressed fluid of area $S$ moves a distance $vt$, Fig. 15–8, so the volume moved is $Svt$). Hence we have

$$\Delta P = \rho v v'.$$

From the definition of the bulk modulus, $B$ (Eq. 12–7):

$$B = -\frac{\Delta P}{\Delta V/V_0} = -\frac{\rho v v'}{\Delta V/V_0},$$

where $\Delta V/V_0$ is the fractional change in volume due to compression. The original volume of the compressed fluid is $V_0 = Svt$ (see Fig. 15–8), and it has been compressed by an amount $\Delta V = -Sv't$ (Fig. 15–8b). Thus

$$B = -\frac{\rho v v'}{\Delta V/V_0} = -\rho v v'\left(\frac{Svt}{-Sv't}\right) = \rho v^2,$$

and so

$$v = \sqrt{\frac{B}{\rho}},$$

which is what we set out to show, Eq. 15–4.

The derivation of Eq. 15–3 follows similar lines, but takes into account the expansion of the sides of a rod when the end of the rod is compressed.

## Other Waves

PHYSICS APPLIED
*Earthquake waves*

Both transverse and longitudinal waves are produced when an **earthquake** occurs. The transverse waves that travel through the body of the Earth are called S waves (S for shear), and the longitudinal waves are called P waves (P for pressure) or *compression* waves. Both longitudinal and transverse waves can travel through a solid since the atoms or molecules can vibrate about their relatively fixed positions in any direction. But only longitudinal waves can propagate through a fluid, because any transverse motion would not experience any restoring force since a fluid is readily deformable. This fact was used by geophysicists to infer that a portion of the Earth's core must be liquid: after an earthquake, longitudinal waves are detected diametrically across the Earth, but not transverse waves.

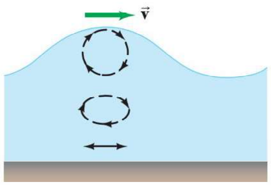
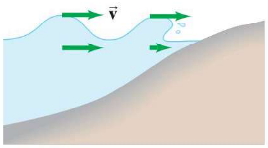


**FIGURE 15–9** A water wave is an example of a *surface wave*, which is a combination of transverse and longitudinal wave motions.

**FIGURE 15–10** How a wave breaks. The green arrows represent the local velocity of water molecules.



Besides these two types of waves that can pass through the body of the Earth (or other substance), there can also be *surface waves* that travel along the boundary between two materials. A wave on water is actually a surface wave that moves on the boundary between water and air. The motion of each particle of water at the surface is circular or elliptical (Fig. 15–9), so it is a combination of transverse and longitudinal motions. Below the surface, there is also transverse plus longitudinal wave motion, as shown. At the bottom, the motion is only longitudinal. (When a wave approaches shore, the water drags at the bottom and is slowed down, while the crests move ahead at higher speed (Fig. 15–10) and "spill" over the top.)

Surface waves are also set up on the Earth when an earthquake occurs. The waves that travel along the surface are mainly responsible for the damage caused by earthquakes.

Waves which travel along a line in one dimension, such as transverse waves on a stretched string, or longitudinal waves in a rod or fluid-filled tube, are *linear* or *one-dimensional waves*. Surface waves, such as the water waves pictured at the start of this Chapter, are *two-dimensional waves*. Finally, waves that move out from a source in all directions, such as sound from a loudspeaker or earthquake waves through the Earth, are *three-dimensional waves*.

## 15–3 Energy Transported by Waves

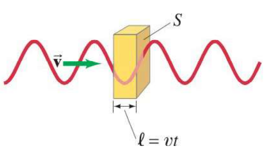
Waves transport energy from one place to another. As waves travel through a medium, the energy is transferred as vibrational energy from particle to particle of the medium. For a sinusoidal wave of frequency $f$, the particles move in simple harmonic motion (Chapter 14) as a wave passes, and each particle has energy $E = \frac{1}{2}kA^2$ where $A$ is the maximum displacement (amplitude) of its motion, either transversely or longitudinally (Eq. 14–10a). Using Eq. 14–7a we can write $k = 4\pi^2 mf^2$, where $m$ is the mass of a particle (or small volume) of the medium. Then in terms of the frequency $f$ and amplitude $A$,

$$E = \frac{1}{2}kA^2 = 2\pi^2 mf^2A^2.$$

For three-dimensional waves traveling in an elastic medium, the mass $m = \rho V$, where $\rho$ is the density of the medium and $V$ is the volume of a small slice of the medium. The volume $V = S\ell$ where $S$ is the cross-sectional area through which the wave travels (Fig. 15–11), and we can write $\ell$ as the distance the wave travels in a time $t$ as $\ell = vt$, where $v$ is the speed of the wave. Thus $m = \rho V = \rho S\ell = \rho Svt$ and

$$E = 2\pi^2\rho Svtf^2A^2. \tag{15–5}$$

**FIGURE 15–11** Calculating the energy carried by a wave moving with velocity $v$.




From this equation we have the important result that *the energy transported by a wave is proportional to the square of the amplitude, and to the square of the frequency*. The average *rate* of energy transferred is the average power $\overline{P}$:

$$\overline{P} = \frac{E}{t} = 2\pi^2\rho Svf^2A^2. \tag{15–6}$$

Finally, the **intensity**, $I$, of a wave is defined as the average power transferred across unit area perpendicular to the direction of energy flow:

$$I = \frac{\overline{P}}{S} = 2\pi^2 v\rho f^2A^2. \tag{15–7}$$

If a wave flows out from the source in all directions, it is a three-dimensional wave. Examples are sound traveling in the open air, earthquake waves, and light waves.

If the medium is isotropic (same in all directions), the wave from a point source is a *spherical wave* (Fig. 15–12). As the wave moves outward, the energy it carries is spread over a larger and larger area since the surface area of a sphere of radius $r$ is $4\pi r^2$. Thus the intensity of a wave is

$$I = \frac{\overline{P}}{S} = \frac{\overline{P}}{4\pi r^2}.$$

If the power output $\overline{P}$ is constant, then the intensity decreases as the inverse square of the distance from the source:

$$I \propto \frac{1}{r^2}. \qquad \text{[spherical wave]} \quad \textbf{(15–8a)}$$

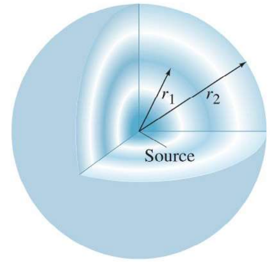



**FIGURE 15–12** Wave traveling outward from a point source has spherical shape. Two different crests (or compressions) are shown, of radius $r_1$ and $r_2$.

If we consider two points at distances $r_1$ and $r_2$ from the source, as in Fig. 15–12, then $I_1 = \overline{P}/4\pi r_1^2$ and $I_2 = \overline{P}/4\pi r_2^2$, so

$$\frac{I_2}{I_1} = \frac{\overline{P}/4\pi r_2^2}{\overline{P}/4\pi r_1^2} = \frac{r_1^2}{r_2^2}. \qquad \textbf{(15–8b)}$$

Thus, for example, when the distance doubles $(r_2/r_1 = 2)$, then the intensity is reduced to $\frac{1}{4}$ of its earlier value: $I_2/I_1 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

The amplitude of a wave also decreases with distance. Since the intensity is proportional to the square of the amplitude (Eq. 15–7), $I \propto A^2$, the amplitude $A$ must decrease as $1/r$, so that $I$ can be proportional to $1/r^2$ (Eq. 15–8a). Hence

$$A \propto \frac{1}{r}.$$

To see this directly from Eq. 15–6, consider again two different distances from the source, $r_1$ and $r_2$. For constant power output, $S_1 A_1^2 = S_2 A_2^2$ where $A_1$ and $A_2$ are the amplitudes of the wave at $r_1$ and $r_2$, respectively. Since $S_1 = 4\pi r_1^2$ and $S_2 = 4\pi r_2^2$, we have $\left(A_1^2 r_1^2\right) = \left(A_2^2 r_2^2\right)$, or

$$\frac{A_2}{A_1} = \frac{r_1}{r_2}.$$

When the wave is twice as far from the source, the amplitude is half as large, and so on (ignoring damping due to friction).

**EXAMPLE 15–4** **Earthquake intensity.** The intensity of an earthquake P wave traveling through the Earth and detected 100 km from the source is $1.0 \times 10^6\ \mathrm{W/m^2}$. What is the intensity of that wave if detected 400 km from the source?

**APPROACH** We assume the wave is spherical, so the intensity decreases as the square of the distance from the source.

**SOLUTION** At 400 km the distance is 4 times greater than at 100 km, so the intensity will be $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$ of its value at 100 km, or $\left(1.0 \times 10^6\ \mathrm{W/m^2}\right)/16 = 6.3 \times 10^4\ \mathrm{W/m^2}$.

**NOTE** Using Eq. 15–8b directly gives

$$I_2 = I_1 r_1^2/r_2^2 = \left(1.0 \times 10^6\ \mathrm{W/m^2}\right)(100\ \mathrm{km})^2/(400\ \mathrm{km})^2 = 6.3 \times 10^4\ \mathrm{W/m^2}.$$

The situation is different for a one-dimensional wave, such as a transverse wave on a string or a longitudinal wave pulse traveling down a thin uniform metal rod. The area remains constant, so the amplitude $A$ also remains constant (ignoring friction). Thus the amplitude and the intensity do not decrease with distance.

In practice, frictional damping is generally present, and some of the energy is transformed into thermal energy. Thus the amplitude and intensity of a one-dimensional wave decrease with distance from the source. For a three-dimensional wave, the decrease will be greater than that discussed above, although the effect may often be small.

# 15–4 Mathematical Representation of a Traveling Wave

Let us now consider a one-dimensional wave traveling along the $x$ axis. It could be, for example, a transverse wave on a cord or a longitudinal wave traveling in a rod or in a fluid-filled tube. Let us assume the wave shape is sinusoidal and has a particular wavelength $\lambda$ and frequency $f$. At $t = 0$, suppose the wave shape is given by

$$D(x) = A \sin \frac{2\pi}{\lambda} x, \tag{15–9}$$

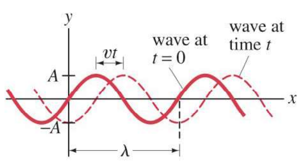



**FIGURE 15–13** A traveling wave. In time $t$, the wave moves a distance $vt$.

as shown by the solid curve in Fig. 15–13: $D(x)$ is the **displacement**† of the wave (be it a longitudinal or transverse wave) at position $x$, and $A$ is the **amplitude** (maximum displacement) of the wave. This relation gives a shape that repeats itself every wavelength, which is needed so that the displacement is the same at $x = 0,\ x = \lambda,\ x = 2\lambda,$ and so on (since $\sin 4\pi = \sin 2\pi = \sin 0$).

Now suppose the wave is moving to the right with velocity $v$. Then, after a time $t$, each part of the wave (indeed, the whole wave "shape") has moved to the right a distance $vt$; see the dashed curve in Fig. 15–13. Consider any point on the wave at $t = 0$: say, a crest which is at some position $x$. After a time $t$, that crest will have traveled a distance $vt$ so its new position is a distance $vt$ greater than its old position. To describe this same point on the wave shape, the argument of the sine function must be the same, so we replace $x$ in Eq. 15–9 by $(x - vt)$:

*1-D wave moving in positive x direction*

$$D(x,t) = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right]. \tag{15–10a}$$

Said another way, if you are riding on a crest, the argument of the sine function, $(2\pi/\lambda)(x - vt)$, remains the same $(= \pi/2, 5\pi/2$, and so on); as $t$ increases, $x$ must increase at the same rate so that $(x - vt)$ remains constant.

Equation 15–10a is the mathematical representation of a sinusoidal wave traveling along the $x$ axis to the right (increasing $x$). It gives the displacement $D(x,t)$ of the wave at any chosen point $x$ at any time $t$. The function $D(x,t)$ describes a curve that represents the actual shape of the wave in space at time $t$. Since $v = \lambda f$ (Eq. 15–1) we can write Eq. 15–10a in other ways that are often convenient:

*1-D wave moving in positive x direction*

$$D(x,t) = A \sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right), \tag{15–10b}$$

where $T = 1/f = \lambda/v$ is the period; and

$$D(x,t) = A \sin(kx - \omega t), \tag{15–10c}$$

where $\omega = 2\pi f = 2\pi/T$ is the angular frequency and

$$k = \frac{2\pi}{\lambda} \tag{15–11}$$

**CAUTION**
*Don't confuse wave number k with spring constant k*

is called the **wave number**. (Do not confuse the wave number $k$ with the spring constant $k$; they are very different quantities.) All three forms, Eqs. 15–10a, b, and c, are equivalent; Eq. 15–10c is the simplest to write and is perhaps the most common. The quantity $(kx - \omega t)$, and its equivalent in the other two equations, is called the **phase** of the wave. The velocity $v$ of the wave is often called the **phase velocity**, since it describes the velocity of the phase (or shape) of the wave and it can be written in terms of $\omega$ and $k$:

$$v = \lambda f = \left(\frac{2\pi}{k}\right)\left(\frac{\omega}{2\pi}\right) = \frac{\omega}{k}. \tag{15–12}$$

†Some books use $y(x)$ in place of $D(x)$. To avoid confusion, we reserve $y$ (and $z$) for the coordinate positions of waves in two or three dimensions. Our $D(x)$ can stand for pressure (in longitudinal waves), position displacement (transverse mechanical waves), or—as we will see later—electric or magnetic fields (for electromagnetic waves).

For a wave traveling along the $x$ axis to the left (decreasing values of $x$), we start again with Eq. 15–9 and note that the velocity is now $-v$. A particular point on the wave changes position by $-vt$ in a time $t$, so $x$ in Eq. 15–9 must be replaced by $(x + vt)$. Thus, for a wave traveling to the left with velocity $v$,

$$D(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x + vt)\right] \qquad \textbf{(15–13a)}$$

$$= A\sin\left(\frac{2\pi x}{\lambda} + \frac{2\pi t}{T}\right) \qquad \textbf{(15–13b)}$$

$$= A\sin(kx + \omega t). \qquad \textbf{(15–13c)}$$

*1-D wave moving in negative x direction*

In other words, we simply replace $v$ in Eqs. 15–10 by $-v$.

Let us look at Eq. 15–13c (or, just as well, at Eq. 15–10c). At $t = 0$ we have

$$D(x,0) = A\sin kx,$$

which is what we started with, a sinusoidal wave shape. If we look at the wave shape in space at a particular later time $t_1$, then we have

$$D(x,t_1) = A\sin(kx + \omega t_1).$$

That is, if we took a picture of the wave at $t = t_1$, we would see a sine wave with a phase constant $\omega t_1$. Thus, for fixed $t = t_1$, the wave has a sinusoidal shape in space. On the other hand, if we consider a fixed point in space, say $x = 0$, we can see how the wave varies in time:

$$D(0,t) = A\sin\omega t$$

where we used Eq. 15–13c. This is just the equation for simple harmonic motion (Section 14–2). For any other fixed value of $x$, say $x = x_1$, $D = A\sin(\omega t + kx_1)$ which differs only by a phase constant $kx_1$. Thus, at any fixed point in space, the displacement undergoes the oscillations of simple harmonic motion in time. Equations 15–10 and 15–13 combine both these aspects to give us the representation for a **traveling sinusoidal wave** (also called a **harmonic wave**).

The argument of the sine in Eqs. 15–10 and 15–13 can in general contain a phase angle $\phi$, which for Eq. 15–10c is

$$D(x,t) = A\sin(kx - \omega t + \phi),$$

to adjust for the position of the wave at $t = 0$, $x = 0$, just as in Section 14–2 (see Fig. 14–7). If the displacement is zero at $t = 0$, $x = 0$, as in Fig. 14–6 (or Fig. 15–13), then $\phi = 0$.

Now let us consider a general wave (or wave pulse) of any shape. If frictional losses are small, experiment shows that the wave maintains its shape as it travels. Thus we can make the same arguments as we did right after Eq. 15–9. Suppose our wave has some shape at $t = 0$, given by

$$D(x,0) = D(x)$$

where $D(x)$ is the displacement of the wave at $x$ and is not necessarily sinusoidal. At some later time, if the wave is traveling to the right along the $x$ axis, the wave will have the same shape but all parts will have moved a distance $vt$ where $v$ is the phase velocity of the wave. Hence we must replace $x$ by $x - vt$ to obtain the amplitude at time $t$:

$$D(x,t) = D(x - vt). \qquad \textbf{(15–14)}$$

Similarly, if the wave moves to the left, we must replace $x$ by $x + vt$, so

$$D(x,t) = D(x + vt). \qquad \textbf{(15–15)}$$

Thus, any wave traveling along the $x$ axis must have the form of Eq. 15–14 or 15–15.

**EXERCISE D** A wave is given by $D(x,t) = (5.0\,\text{mm})\sin(2.0x - 20.0t)$ where $x$ is in meters and $t$ is in seconds. What is the speed of the wave? (*a*) 10 m/s, (*b*) 0.10 m/s, (*c*) 40 m/s, (*d*) 0.005 m/s, (*e*) $2.5 \times 10^{-4}$ m/s.

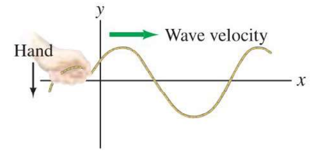



**FIGURE 15–14** Example 15–5. The wave at $t = 0$ (the hand is falling). Not to scale.

**EXAMPLE 15–5** **A traveling wave.** The left-hand end of a long horizontal stretched cord oscillates transversely in SHM with frequency $f = 250\,\text{Hz}$ and amplitude 2.6 cm. The cord is under a tension of 140 N and has a linear density $\mu = 0.12\,\text{kg/m}$. At $t = 0$, the end of the cord has an upward displacement of 1.6 cm and is falling (Fig. 15–14). Determine (*a*) the wavelength of waves produced and (*b*) the equation for the traveling wave.

**APPROACH** We first find the phase velocity of the transverse wave from Eq. 15–2; then $\lambda = v/f$. In (*b*), we need to find the phase $\phi$ using the initial conditions.

**SOLUTION** (*a*) The wave velocity is

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{140\,\text{N}}{0.12\,\text{kg/m}}} = 34\,\text{m/s}.$$

Then

$$\lambda = \frac{v}{f} = \frac{34\,\text{m/s}}{250\,\text{Hz}} = 0.14\,\text{m} \quad \text{or} \quad 14\,\text{cm}.$$

(*b*) Let $x = 0$ at the left-hand end of the cord. The phase of the wave at $t = 0$ is not zero in general as was assumed in Eqs. 15–9, 10, and 13. The general form for a wave traveling to the right is

$$D(x, t) = A\sin(kx - \omega t + \phi),$$

where $\phi$ is the phase angle. In our case, the amplitude $A = 2.6\,\text{cm}$; and at $t = 0$, $x = 0$, we are given $D = 1.6\,\text{cm}$. Thus

$$1.6 = 2.6\sin\phi,$$

so $\phi = \sin^{1}(1.6/2.6) = 38° = 0.66\,\text{rad}$. We also have $\omega = 2\pi f = 1570\,\text{s}^{1}$ and $k = 2\pi/\lambda = 2\pi/0.14\,\text{m} = 45\,\text{m}^{-1}$. Hence

$$D = (0.026\,\text{m})\sin\left[(45\,\text{m}^{-1})x - (1570\,\text{s})t + 0.66\right]$$

which we can write more simply as

$$D = 0.026\sin(45x - 1570t + 0.66),$$

and we specify clearly that $D$ and $x$ are in meters and $t$ in seconds.

## *15–5 The Wave Equation

Many types of waves satisfy an important general equation that is the equivalent of Newton's second law of motion for particles. This "equation of motion for a wave" is called the **wave equation**, and we derive it now for waves traveling on a stretched horizontal string.

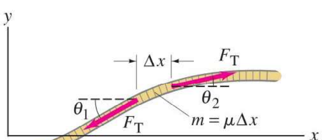



**FIGURE 15–15** Deriving the wave equation from Newton's second law: a segment of string under tension $F_T$.

We assume the amplitude of the wave is small compared to the wavelength so that each point on the string can be assumed to move only vertically and the tension in the string, $F_T$, does not vary during a vibration. We apply Newton's second law, $\Sigma F = ma$, to the vertical motion of a tiny section of the string as shown in Fig. 15–15. The amplitude of the wave is small, so the angles $\theta_1$ and $\theta_2$ that the string makes with the horizontal are small. The length of this section is then approximately $\Delta x$, and its mass is $\mu\,\Delta x$, where $\mu$ is the mass per unit length of the string. The net vertical force on this section of string is $F_T\sin\theta_2 - F_T\sin\theta_1$. So Newton's second law applied to the vertical ($y$) direction gives

$$\Sigma F_y = ma_y$$

$$F_T\sin\theta_2 - F_T\sin\theta_1 = (\mu\,\Delta x)\frac{\partial^2 D}{\partial t^2}. \qquad \textbf{(i)}$$

We have written the acceleration as $a_y = \partial^2 D/\partial t^2$ since the motion is only vertical, and we use the partial derivative notation because the displacement $D$ is a function of both $x$ and $t$.

Because the angles $\theta_1$ and $\theta_2$ are assumed small, $\sin\theta \approx \tan\theta$ and $\tan\theta$ is equal to the slope $s$ of the string at each point:

$$\sin\theta \approx \tan\theta = \frac{\partial D}{\partial x} = s.$$

Thus our equation (i) at the bottom of the previous page becomes

$$F_{\mathrm{T}}(s_2 - s_1) = \mu\,\Delta x\,\frac{\partial^2 D}{\partial t^2}$$

or

$$F_{\mathrm{T}}\frac{\Delta s}{\Delta x} = \mu\frac{\partial^2 D}{\partial t^2}, \qquad \textbf{(ii)}$$

where $\Delta s = s_2 - s_1$ is the difference in the slope between the two ends of our tiny section. Now we take the limit of $\Delta x \to 0$, so that

$$F_{\mathrm{T}}\lim_{\Delta x\to 0}\frac{\Delta s}{\Delta x} = F_{\mathrm{T}}\frac{\partial s}{\partial x}$$
$$= F_{\mathrm{T}}\frac{\partial}{\partial x}\left(\frac{\partial D}{\partial x}\right) = F_{\mathrm{T}}\frac{\partial^2 D}{\partial x^2}$$

since the slope $s = \partial D/\partial x$, as we wrote above. Substituting this into the equation labeled (ii) above gives

$$F_{\mathrm{T}}\frac{\partial^2 D}{\partial x^2} = \mu\frac{\partial^2 D}{\partial t^2}$$

or

$$\frac{\partial^2 D}{\partial x^2} = \frac{\mu}{F_{\mathrm{T}}}\frac{\partial^2 D}{\partial t^2}.$$

We saw earlier in this Chapter (Eq. 15–2) that the velocity of waves on a string is given by $v = \sqrt{F_{\mathrm{T}}/\mu}$, so we can write this last equation as

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}. \qquad \textbf{(15–16)}$$

This is the **one-dimensional wave equation**, and it can describe not only small amplitude waves on a stretched string, but also small amplitude longitudinal waves (such as sound waves) in gases, liquids, and elastic solids, in which case $D$ can refer to the pressure variations. In this case, the wave equation is a direct consequence of Newton's second law applied to a continuous elastic medium. The wave equation also describes electromagnetic waves for which $D$ refers to the electric or magnetic field, as we shall see in Chapter 31. Equation 15–16 applies to waves traveling in one dimension only. For waves spreading out in three dimensions, the wave equation is the same, with the addition of $\partial^2 D/\partial y^2$ and $\partial^2 D/\partial z^2$ to the left side of Eq. 15–16.

The wave equation is a *linear* equation: the displacement $D$ appears singly in each term. There are no terms that contain $D^2$, or $D(\partial D/\partial x)$, or the like in which $D$ appears more than once. Thus, if $D_1(x,t)$ and $D_2(x,t)$ are two different solutions of the wave equation, then the linear combination

$$D_3(x,t) = aD_1(x,t) + bD_2(x,t),$$

where $a$ and $b$ are constants, is also a solution. This is readily seen by direct substitution into the wave equation. This is the essence of the *superposition principle*, which we discuss in the next Section. Basically it says that if two waves pass through the same region of space at the same time, the actual displacement is the sum of the separate displacements. For waves on a string, or for sound waves, this is valid only for small-amplitude waves. If the amplitude is not small enough, the equations for wave propagation may become nonlinear and the principle of superposition would not hold and more complicated effects may occur.

**EXAMPLE 15–6** **Wave equation solution.** Verify that the sinusoidal wave of Eq. 15–10c, $D(x,t) = A\sin(kx - \omega t)$, satisfies the wave equation.

**APPROACH** We substitute Eq. 15–10c into the wave equation, Eq. 15–16.

**SOLUTION** We take the derivative of Eq. 15–10c twice with respect to $t$:

$$\frac{\partial D}{\partial t} = -\omega A\cos(kx - \omega t)$$

$$\frac{\partial^2 D}{\partial t^2} = -\omega^2 A\sin(kx - \omega t).$$

With respect to $x$, the derivatives are

$$\frac{\partial D}{\partial x} = kA\cos(kx - \omega t)$$

$$\frac{\partial^2 D}{\partial x^2} = -k^2 A\sin(kx - \omega t).$$

If we now divide the second derivatives we get

$$\frac{\partial^2 D/\partial t^2}{\partial^2 D/\partial x^2} = \frac{-\omega^2 A\sin(kx - \omega t)}{-k^2 A\sin(kx - \omega t)} = \frac{\omega^2}{k^2}.$$

From Eq. 15–12 we have $\omega^2/k^2 = v^2$, so we see that Eq. 15–10 does satisfy the wave equation (Eq. 15–16).

# 15–6 The Principle of Superposition

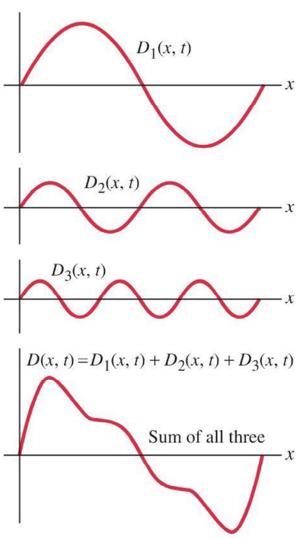
When two or more waves pass through the same region of space at the same time, it is found that for many waves *the actual displacement is the vector* (or *algebraic*) *sum of the separate displacements*. This is called the **principle of superposition**. It is valid for mechanical waves as long as the displacements are not too large and there is a linear relationship between the displacement and the restoring force of the oscillating medium.† If the amplitude of a mechanical wave, for example, is so large that it goes beyond the elastic region of the medium, and Hooke's law is no longer operative, the superposition principle is no longer accurate.‡ For the most part, we will consider systems for which the superposition principle can be assumed to hold.

One result of the superposition principle is that if two waves pass through the same region of space, they continue to move independently of one another. You may have noticed, for example, that the ripples on the surface of water (two-dimensional waves) that form from two rocks striking the water at different places will pass through each other.

Figure 15–16 shows an example of the superposition principle. In this case there are three waves present, on a stretched string, each of different amplitude and frequency. At any time, such as at the instant shown, the actual amplitude at any position $x$ is the algebraic sum of the amplitude of the three waves at that position. The actual wave is not a simple sinusoidal wave and is called a *composite* (or *complex*) *wave*. (Amplitudes are exaggerated in Fig. 15–16.)

It can be shown that any complex wave can be considered as being composed of many simple sinusoidal waves of different amplitudes, wavelengths, and frequencies. This is known as *Fourier's theorem*. A complex periodic wave of period $T$ can be represented as a sum of pure sinusoidal terms whose frequencies are integral multiples of $f = 1/T$. If the wave is not periodic, the sum becomes an integral (called a *Fourier integral*). Although we will not go into the details here, we see the importance of considering sinusoidal waves (and simple harmonic motion): because any other wave shape can be considered a sum of such pure sinusoidal waves.

**FIGURE 15–16** The superposition principle for one-dimensional waves. Composite wave formed from three sinusoidal waves of different amplitudes and frequencies $(f_0, 2f_0, 3f_0)$ at a certain instant in time. The amplitude of the composite wave at each point in space, at any time, is the algebraic sum of the amplitudes of the component waves. Amplitudes are shown exaggerated; for the superposition principle to hold, they must be small compared to the wavelengths.




†For electromagnetic waves in vacuum, Chapter 31, the superposition principle always holds.

‡Intermodulation distortion in high-fidelity equipment is an example of the superposition principle not holding when two frequencies do not combine linearly in the electronics.

**CONCEPTUAL EXAMPLE 15–7** **Making a square wave.** At $t = 0$, three waves are given by $D_1 = A\cos kx$, $D_2 = -\frac{1}{3}A\cos 3kx$, and $D_3 = \frac{1}{5}A\cos 5kx$, where $A = 1.0\,\text{m}$ and $k = 10\,\text{m}^{-1}$. Plot the sum of the three waves from $x = -0.4\,\text{m}$ to $+0.4\,\text{m}$. (These three waves are the first three Fourier components of a "square wave.")

**RESPONSE** The first wave, $D_1$, has amplitude of 1.0 m and wavelength $\lambda = 2\pi/k = (2\pi/10)\,\text{m} = 0.628\,\text{m}$. The second wave, $D_2$, has amplitude of 0.33 m and wavelength $\lambda = 2\pi/3k = (2\pi/30)\,\text{m} = 0.209\,\text{m}$. The third wave, $D_3$, has amplitude of 0.20 m and wavelength $\lambda = 2\pi/5k = (2\pi/50)\,\text{m} = 0.126\,\text{m}$. Each wave is plotted in Fig. 15–17a. The sum of the three waves is shown in Fig. 15–17b. The sum begins to resemble a "square wave," shown in blue in Fig. 15–17b.

PHYSICS APPLIED
*Square wave*

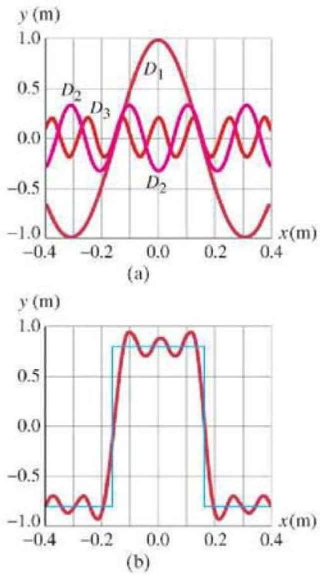



**FIGURE 15–17** Example 15–7. Making a square wave.

When the restoring force is not precisely proportional to the displacement for mechanical waves in some continuous medium, the speed of sinusoidal waves depends on the frequency. The variation of speed with frequency is called **dispersion**. The different sinusoidal waves that compose a complex wave will travel with slightly different speeds in such a case. Consequently, a complex wave will change shape as it travels if the medium is "dispersive." A pure sine wave will not change shape under these conditions, however, except by the influence of friction or dissipative forces. If there is no dispersion (or friction), even a complex linear wave does not change shape.

# 15–7 Reflection and Transmission

When a wave strikes an obstacle, or comes to the end of the medium in which it is traveling, at least a part of the wave is reflected. You have probably seen water waves reflect off a rock or the side of a swimming pool. And you may have heard a shout reflected from a distant cliff—which we call an "echo."

A wave pulse traveling down a cord is reflected as shown in Fig. 15–18. The reflected pulse returns inverted as in Fig. 15–18a if the end of the cord is fixed; it returns right side up if the end is free as in Fig. 15–18b. When the end is fixed to a support, as in Fig. 15–18a, the pulse reaching that fixed end exerts a force (upward) on the support. The support exerts an equal but opposite force downward on the cord (Newton's third law). This downward force on the cord is what "generates" the inverted reflected pulse.

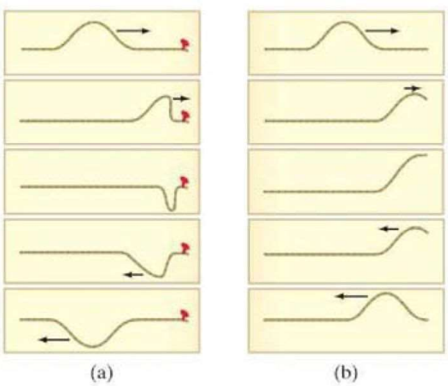
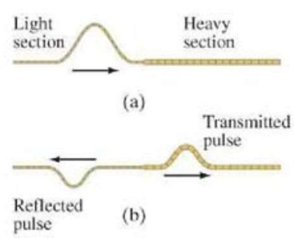



**FIGURE 15–18** Reflection of a wave pulse on a cord lying on a table. (a) The end of the cord is fixed to a peg. (b) The end of the cord is free to move.

**FIGURE 15–19** When a wave pulse traveling to the right along a thin cord (a) reaches a discontinuity where the cord becomes thicker and heavier, then part is reflected and part is transmitted (b).




Consider next a pulse that travels down a cord which consists of a light section and a heavy section, as shown in Fig. 15–19. When the wave pulse reaches the boundary between the two sections, part of the pulse is reflected and part is transmitted, as shown. The heavier the second section of the cord, the less the energy that is transmitted. (When the second section is a wall or rigid support, very little is transmitted and most is reflected, as in Fig. 15–18a.) For a periodic wave, the frequency of the transmitted wave does not change across the boundary since the boundary point oscillates at that frequency. Thus if the transmitted wave has a lower speed, its wavelength is also less $(\lambda = v/f)$.

For a two- or three-dimensional wave, such as a water wave, we are concerned with **wave fronts**, by which we mean all the points along the wave forming the wave crest (what we usually refer to simply as a "wave" at the seashore). A line drawn in the direction of motion, perpendicular to the wave front, is called a **ray**, as shown in Fig. 15–20. Wave fronts far from the source have lost almost all their curvature (Fig. 15–20b) and are nearly straight, as ocean waves often are; they are then called **plane waves**.

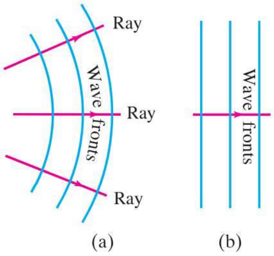



**FIGURE 15–20** Rays, signifying the direction of motion, are always perpendicular to the wave fronts (wave crests). (a) Circular or spherical waves near the source. (b) Far from the source, the wave fronts are nearly straight or flat, and are called plane waves.

For reflection of a two- or three-dimensional plane wave, as shown in Fig. 15–21, the angle that the incoming or *incident wave* makes with the reflecting surface is equal to the angle made by the reflected wave. This is the **law of reflection**:

**the angle of reflection equals the angle of incidence.**

The "angle of incidence" is defined as the angle ($\theta_i$) the incident ray makes with the perpendicular to the reflecting surface (or the wave front makes with a tangent to the surface), and the "angle of reflection" is the corresponding angle ($\theta_r$) for the reflected wave.

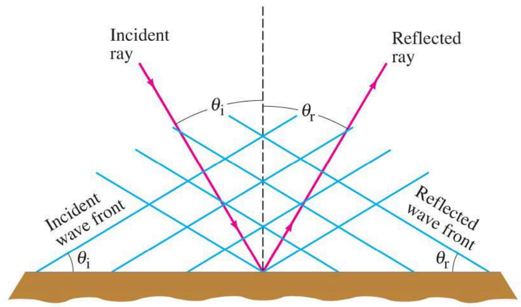



**FIGURE 15–21** Law of reflection: $\theta_r = \theta_i$.

## 15–8 Interference

**Interference** refers to what happens when two waves pass through the same region of space at the same time. Consider, for example, the two wave pulses on a cord traveling toward each other as shown in Fig. 15–22. In Fig. 15–22a the two pulses have the same amplitude, but one is a crest and the other a trough; in Fig. 15–22b they are both crests. In both cases, the waves meet and pass right by each other. However, in the region where they overlap, the resultant displacement is the *algebraic sum of their separate displacements* (a crest is considered positive and a trough negative). This is another example of the principle of superposition. In Fig. 15–22a, the two waves have opposite displacements at the instant they pass one another, and they add to zero. The result is called **destructive interference**. In Fig. 15–22b, at the instant the two pulses overlap, they produce a resultant displacement that is greater than the displacement of either separate pulse, and the result is **constructive interference**.

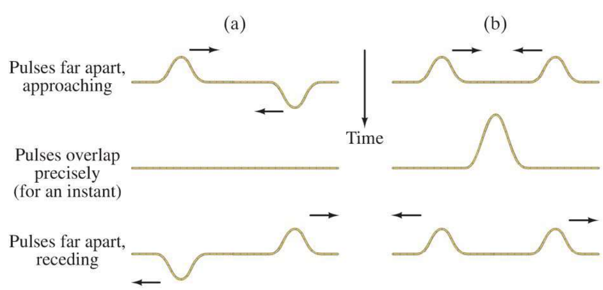
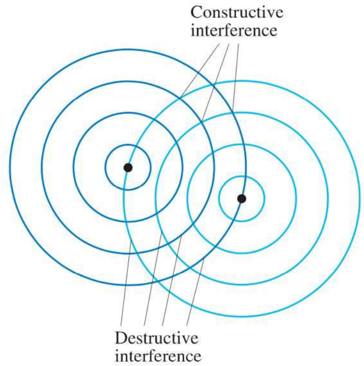



**FIGURE 15–22** Two wave pulses pass each other. Where they overlap, interference occurs: (a) destructive, and (b) constructive.

[You may wonder where the energy is at the moment of destructive interference in Fig. 15–22a; the cord may be straight at this instant, but the central parts of it are still moving up or down.]

(a)

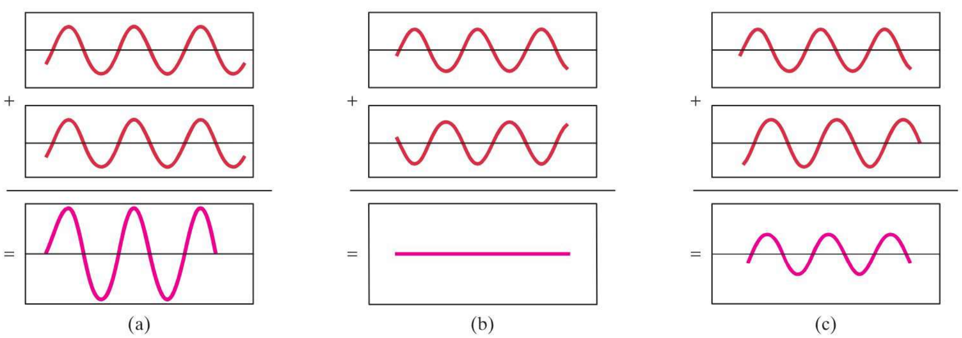



(b)

**FIGURE 15–23** (a) Interference of water waves. (b) Constructive interference occurs where one wave's maximum (a crest) meets the other's maximum. Destructive interference ("flat water") occurs where one wave's maximum (a crest) meets the other's miminum (a trough).

When two rocks are thrown into a pond simultaneously, the two sets of circular waves interfere with one another as shown in Fig. 15–23a. In some areas of overlap, crests of one wave repeatedly meet crests of the other (and troughs meet troughs), Fig. 15–23b. Constructive interference is occuring at these points, and the water continuously oscillates up and down with greater amplitude than either wave separately. In other areas, destructive interference occurs where the water does not move up and down at all over time. This is where crests of one wave meet troughs of the other, and vice versa. Figure 15–24a shows the displacement of two identical waves graphically as a function of time, as well as their sum, for the case of constructive interference. For constructive interference (Fig. 15–24a), the two waves are **in phase**. At points where destructive interference occurs (Fig. 15–24b) crests of one wave repeatedly meet troughs of the other wave and the two waves are **out of phase** by one-half wavelength or 180°. The crests of one wave occur a half wavelength behind the crests of the other wave. The relative phase of the two water waves in Fig. 15–23 in most areas is intermediate between these two extremes, resulting in *partially* destructive interference, as illustrated in Fig. 15–24c. If the amplitudes of two interfering waves are not equal, fully destructive interference (as in Fig. 15–24b) does not occur.

**FIGURE 15–24** Graphs showing two identical waves, and their sum, as a function of time at three locations. In (a) the two waves interfere constructively, in (b) destructively, and in (c) partially destructively.



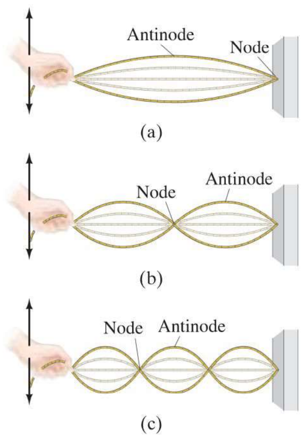



**FIGURE 15–25** Standing waves corresponding to three resonant frequencies.

## 15–9 Standing Waves; Resonance

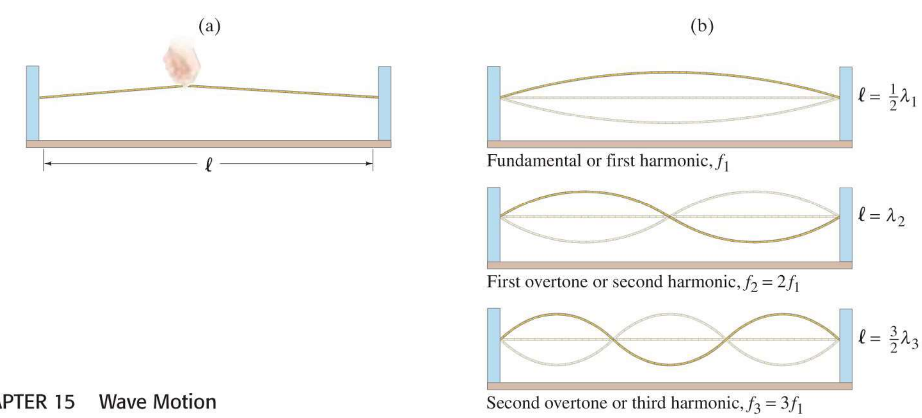
If you shake one end of a cord and the other end is kept fixed, a continuous wave will travel down to the fixed end and be reflected back, inverted, as we saw in Fig. 15–18a. As you continue to vibrate the cord, waves will travel in both directions, and the wave traveling along the cord, away from your hand, will interfere with the reflected wave coming back. Usually there will be quite a jumble. But if you vibrate the cord at just the right frequency, the two traveling waves will interfere in such a way that a large-amplitude **standing wave** will be produced, Fig. 15–25. It is called a "standing wave" because it does not appear to be traveling. The cord simply appears to have segments that oscillate up and down in a fixed pattern. The points of destructive interference, where the cord remains still at all times, are called **nodes**. Points of constructive interference, where the cord oscillates with maximum amplitude, are called **antinodes**. The nodes and antinodes remain in fixed positions for a particular frequency.

Standing waves can occur at more than one frequency. The lowest frequency of vibration that produces a standing wave gives rise to the pattern shown in Fig. 15–25a. The standing waves shown in Figs. 15–25b and 15–25c are produced at precisely twice and three times the lowest frequency, respectively, assuming the tension in the cord is the same. The cord can also vibrate with four loops (four antinodes) at four times the lowest frequency, and so on.

The frequencies at which standing waves are produced are the **natural frequencies** or **resonant frequencies** of the cord, and the different standing wave patterns shown in Fig. 15–25 are different "resonant modes of vibration." A standing wave on a cord is the result of the interference of two waves traveling in opposite directions. A standing wave can also be considered a vibrating object at resonance. Standing waves represent the same phenomenon as the resonance of a vibrating spring or pendulum, which we discussed in Chapter 14. However, a spring or pendulum has only one resonant frequency, whereas the cord has an infinite number of resonant frequencies, each of which is a whole-number multiple of the lowest resonant frequency.

Consider a string stretched between two supports that is plucked like a guitar or violin string, Fig. 15–26a. Waves of a great variety of frequencies will travel in both directions along the string, will be reflected at the ends, and will travel back in the opposite direction. Most of these waves interfere with each other and quickly die out. However, those waves that correspond to the resonant frequencies of the string will persist. The ends of the string, since they are fixed, will be nodes. There may be other nodes as well. Some of the possible resonant modes of vibration (standing waves) are shown in Fig. 15–26b. Generally, the motion will be a combination of these different resonant modes, but only those frequencies that correspond to a resonant frequency will be present.

**FIGURE 15–26** (a) A string is plucked. (b) Only standing waves corresponding to resonant frequencies persist for long.



To determine the resonant frequencies, we first note that the wavelengths of the standing waves bear a simple relationship to the length $\ell$ of the string. The lowest frequency, called the **fundamental frequency**, corresponds to one antinode (or loop). And as can be seen in Fig. 15–26b, the whole length corresponds to one-half wavelength. Thus $\ell = \frac{1}{2}\lambda_1$, where $\lambda_1$ stands for the wavelength of the fundamental frequency. The other natural frequencies are called **overtones**; for a vibrating string they are whole-number (integral) multiples of the fundamental, and then are also called **harmonics**, with the fundamental being referred to as the **first harmonic**.† The next mode of vibration after the fundamental has two loops and is called the **second harmonic** (or first overtone), Fig. 15–26b. The length of the string $\ell$ at the second harmonic corresponds to one complete wavelength: $\ell = \lambda_2$. For the third and fourth harmonics, $\ell = \frac{3}{2}\lambda_3$, and $\ell = 2\lambda_4$, respectively, and so on. In general, we can write

$$\ell = \frac{n\lambda_n}{2}, \qquad \text{where } n = 1, 2, 3, \cdots.$$

The integer $n$ labels the number of the harmonic: $n = 1$ for the fundamental, $n = 2$ for the second harmonic, and so on. We solve for $\lambda_n$ and find

$$\lambda_n = \frac{2\ell}{n}, \qquad n = 1, 2, 3, \cdots. \qquad \left[\begin{array}{c}\text{string fixed}\\ \text{at both ends}\end{array}\right] \qquad \textbf{(15–17a)}$$

To find the frequency $f$ of each vibration we use Eq. 15–1, $f = v/\lambda$, and we see that

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2\ell} = nf_1, \qquad n = 1, 2, 3, \cdots, \qquad \textbf{(15–17b)}$$

where $f_1 = v/\lambda_1 = v/2\ell$ is the fundamental frequency. We see that each resonant frequency is an integer multiple of the fundamental frequency.

Because a standing wave is equivalent to two traveling waves moving in opposite directions, the concept of wave velocity still makes sense and is given by Eq. 15–2 in terms of the tension $F_T$ in the string and its mass per unit length $(\mu = m/\ell)$. That is, $v = \sqrt{F_T/\mu}$ for waves traveling in both directions.

**EXAMPLE 15–8** **Piano string.** A piano string is 1.10 m long and has a mass of 9.00 g. (*a*) How much tension must the string be under if it is to vibrate at a fundamental frequency of 131 Hz? (*b*) What are the frequencies of the first four harmonics?

**APPROACH** To determine the tension, we need to find the wave speed using Eq. 15–1 $(v = \lambda f)$, and then use Eq. 15–2, solving it for $F_T$.

**SOLUTION** (*a*) The wavelength of the fundamental is $\lambda = 2\ell = 2.20\text{ m}$ (Eq. 15–17a with $n = 1$). The speed of the wave on the string is $v = \lambda f = (2.20\text{ m})(131\text{ s}^{-1}) = 288\text{ m/s}$. Then we have (Eq. 15–2)

$$F_T = \mu v^2 = \frac{m}{\ell}v^2 = \left(\frac{9.00 \times 10^{-3}\text{ kg}}{1.10\text{ m}}\right)(288\text{ m/s})^2 = 679\text{ N}.$$

(*b*) The frequencies of the second, third, and fourth harmonics are two, three, and four times the fundamental frequency: 262, 393, and 524 Hz, respectively.

**NOTE** The speed of the wave on the string is *not* the same as the speed of the sound wave that the piano string produces in the air (as we shall see in Chapter 16).

A standing wave does appear to be standing in place (and a traveling wave appears to move). The term "standing" wave is also meaningful from the point of view of energy. Since the string is at rest at the nodes, no energy flows past these points. Hence the energy is not transmitted down the string but "stands" in place in the string.

Standing waves are produced not only on strings, but on any object that is struck, such as a drum membrane or an object made of metal or wood. The resonant frequencies depend on the dimensions of the object, just as for a string they depend on its length. Large objects have lower resonant frequencies than small objects. All musical instruments, from stringed instruments to wind instruments (in which a column of air vibrates as a standing wave) to drums and other percussion instruments, depend on standing waves to produce their particular musical sounds, as we shall see in Chapter 16.

†The term "harmonic" comes from music, because such integral multiples of frequencies "harmonize."

## Mathematical Representation of a Standing Wave

As we saw, a standing wave can be considered to consist of two traveling waves that move in opposite directions. These can be written (see Eqs. 15–10c and 15–13c)

$$D_1(x,t) = A\sin(kx - \omega t) \quad \text{and} \quad D_2(x,t) = A\sin(kx + \omega t)$$

since, assuming no damping, the amplitudes are equal as are the frequencies and wavelengths. The sum of these two traveling waves produces a standing wave which can be written mathematically as

$$D = D_1 + D_2 = A[\sin(kx - \omega t) + \sin(kx + \omega t)].$$

From the trigonometric identity $\sin\theta_1 + \sin\theta_2 = 2\sin\frac{1}{2}(\theta_1 + \theta_2)\cos\frac{1}{2}(\theta_1 - \theta_2)$, we can rewrite this as

$$D = 2A\sin kx \cos\omega t. \tag{15–18}$$

If we let $x = 0$ at the left-hand end of the string, then the right-hand end is at $x = \ell$ where $\ell$ is the length of the string. Since the string is fixed at its two ends (Fig. 15–26), $D(x,t)$ must be zero at $x = 0$ and at $x = \ell$. Equation 15–18 already satisfies the first condition ($D = 0$ at $x = 0$) and satisfies the second condition if $\sin k\ell = 0$ which means

$$k\ell = \pi, 2\pi, 3\pi, \cdots, n\pi, \cdots$$

where $n =$ an integer. Since $k = 2\pi/\lambda$, then $\lambda = 2\ell/n$, which is just Eq. 15–17a.

Equation 15–18, with the condition $\lambda = 2\ell/n$, is the mathematical representation of a standing wave. We see that a particle at any position $x$ vibrates in simple harmonic motion (because of the factor $\cos\omega t$). All particles of the string vibrate with the same frequency $f = \omega/2\pi$, but the amplitude depends on $x$ and equals $2A\sin kx$. (Compare this to a traveling wave for which all particles vibrate with the same amplitude.) The amplitude has a maximum, equal to $2A$, when $kx = \pi/2, 3\pi/2, 5\pi/2$, and so on—that is, at

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \cdots.$$

These are, of course, the positions of the antinodes (see Fig. 15–26).

**EXAMPLE 15–9** **Wave forms.** Two waves traveling in opposite directions on a string fixed at $x = 0$ are described by the functions

$$D_1 = (0.20\text{ m})\sin(2.0x - 4.0t) \quad \text{and} \quad D_2 = (0.20\text{ m})\sin(2.0x + 4.0t)$$

(where $x$ is in m, $t$ is in s), and they produce a standing wave pattern. Determine (*a*) the function for the standing wave, (*b*) the maximum amplitude at $x = 0.45$ m, (*c*) where the other end is fixed ($x > 0$), (*d*) the maximum amplitude, and where it occurs.

**APPROACH** We use the principle of superposition to add the two waves. The given waves have the form we used to obtain Eq. 15–18, which we thus can use.

**SOLUTION** (*a*) The two waves are of the form $D = A\sin(kx \pm \omega t)$, so

$$k = 2.0\text{ m}^{-1} \quad \text{and} \quad \omega = 4.0\text{ s}^{-1}.$$

These combine to form a standing wave of the form of Eq. 15–18:

$$D = 2A\sin kx\cos\omega t = (0.40\text{ m})\sin(2.0x)\cos(4.0t),$$

where $x$ is in meters and $t$ in seconds.

(*b*) At $x = 0.45$ m,

$$D = (0.40\text{ m})\sin(0.90)\cos(4.0t) = (0.31\text{ m})\cos(4.0t).$$

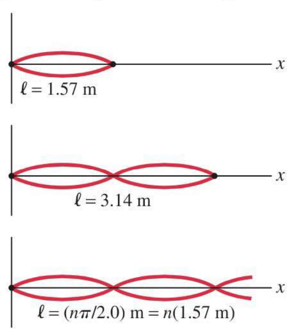
The maximum amplitude at this point is $D = 0.31$ m and occurs when $\cos(4.0t) = 1$.

(*c*) These waves make a standing wave pattern, so both ends of the string must be nodes. Nodes occur every half wavelength, which for our string is

$$\frac{\lambda}{2} = \frac{1}{2}\frac{2\pi}{k} = \frac{\pi}{2.0}\text{ m} = 1.57\text{ m}.$$

If the string includes only one loop, its length is $\ell = 1.57$ m. But without more information, it could be twice as long, $\ell = 3.14$ m, or any integral number times 1.57 m, and still provide a standing wave pattern, Fig. 15–27.

**FIGURE 15–27** Example 15–9: possible lengths for the string.



($d$) The nodes occur at $x = 0$, $x = 1.57\,\text{m}$, and, if the string is longer than $\ell = 1.57\,\text{m}$, at $x = 3.14\,\text{m}$, $4.71\,\text{m}$, and so on. The maximum amplitude (antinode) is $0.40\,\text{m}$ [from part ($b$) above] and occurs midway between the nodes. For $\ell = 1.57\,\text{m}$, there is only one antinode, at $x = 0.79\,\text{m}$.

# *15–10 Refraction†

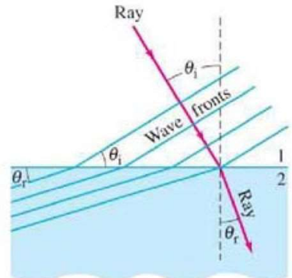



**FIGURE 15–28** Refraction of waves passing a boundary.

When any wave strikes a boundary, some of the energy is reflected and some is transmitted or absorbed. When a two- or three-dimensional wave traveling in one medium crosses a boundary into a medium where its speed is different, the transmitted wave may move in a different direction than the incident wave, as shown in Fig. 15–28. This phenomenon is known as **refraction**. One example is a water wave; the velocity decreases in shallow water and the waves refract, as shown in Fig. 15–29 below. [When the wave velocity changes gradually, as in Fig. 15–29, without a sharp boundary, the waves change direction (refract) gradually.]

In Fig. 15–28, the velocity of the wave in medium 2 is less than in medium 1. In this case, the wave front bends so that it travels more nearly parallel to the boundary. That is, the *angle of refraction*, $\theta_r$, is less than the *angle of incidence*, $\theta_i$. To see why this is so, and to help us get a quantitative relation between $\theta_r$ and $\theta_i$, let us think of each wave front as a row of soldiers. The soldiers are marching from firm ground (medium 1) into mud (medium 2) and hence are slowed down after the boundary. The soldiers that reach the mud first are slowed down first, and the row bends as shown in Fig. 15–30a. Let us consider the wave front (or row of soldiers) labeled A in Fig. 15–30b. In the same time $t$ that $A_1$ moves a distance $\ell_1 = v_1 t$, we see that $A_2$ moves a distance $\ell_2 = v_2 t$. The two right triangles in Fig. 15–30b, shaded yellow and green, have the side labeled $a$ in common. Thus

$$\sin\theta_1 = \frac{\ell_1}{a} = \frac{v_1 t}{a}$$

since $a$ is the hypotenuse, and

$$\sin\theta_2 = \frac{\ell_2}{a} = \frac{v_2 t}{a}.$$

Dividing these two equations, we obtain the **law of refraction**:

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}. \qquad \textbf{(15–19)}$$

Since $\theta_1$ is the angle of incidence $(\theta_i)$, and $\theta_2$ is the angle of refraction $(\theta_r)$, Eq. 15–19 gives the quantitative relation between the two. If the wave were going in the opposite direction, the geometry would not change; only $\theta_1$ and $\theta_2$ would change roles: $\theta_1$ would be the angle of refraction and $\theta_2$ the angle of incidence. Clearly then, if the wave travels into a medium where it can move faster, it will bend the opposite way, $\theta_r > \theta_i$. We see from Eq. 15–19 that if the velocity increases, the angle increases, and vice versa.

Earthquake waves refract within the Earth as they travel through rock layers of different densities (and therefore the velocity is different) just as water waves do. Light waves refract as well, and when we discuss light, we shall find Eq. 15–19 very useful.

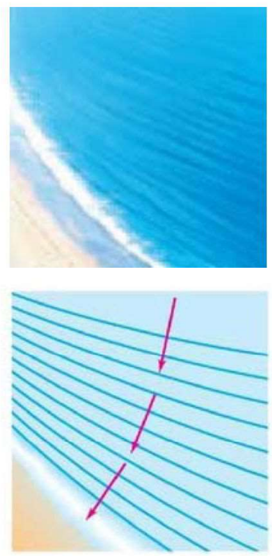


**FIGURE 15–29** Water waves refract gradually as they approach the shore, as their velocity decreases. There is no distinct boundary, as in Fig. 15–28, because the wave velocity changes gradually.

†This Section and the next are covered in more detail in Chapters 32 to 35 on optics.

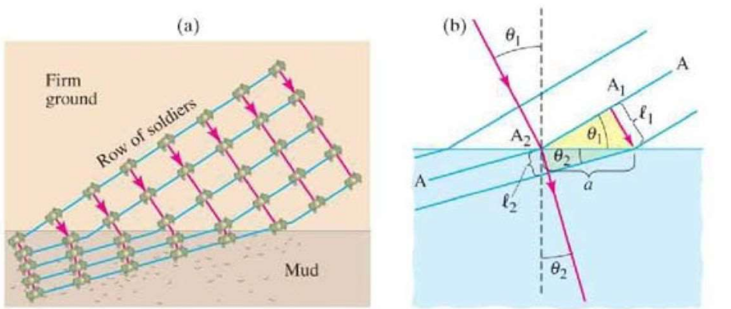



**FIGURE 15–30** (a) Soldier analogy to derive (b) law of refraction for waves.

PHYSICS APPLIED
*Earthquake wave refraction*

**EXAMPLE 15–10** **Refraction of an earthquake wave.** An earthquake P wave passes across a boundary in rock where its velocity increases from 6.5 km/s to 8.0 km/s. If it strikes this boundary at 30°, what is the angle of refraction?

**APPROACH** We apply the law of refraction, Eq. 15–19, $\sin\theta_2/\sin\theta_1 = v_2/v_1$.

**SOLUTION** Since $\sin 30° = 0.50$, Eq. 15–19 yields

$$\sin\theta_2 = \frac{(8.0\ \text{m/s})}{(6.5\ \text{m/s})}(0.50) = 0.62.$$

So $\theta_2 = \sin^{-1}(0.62) = 38°$.

**NOTE** Be careful with angles of incidence and refraction. As we discussed in Section 15–7 (Fig. 15–21), these angles are between the wave front and the boundary line, or—equivalently—between the ray (direction of wave motion) and the line perpendicular to the boundary. Inspect Fig. 15–30b carefully.

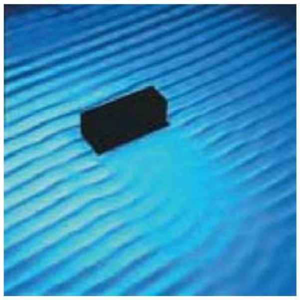


**FIGURE 15–31** Wave diffraction. The waves are coming from the upper left. Note how the waves, as they pass the obstacle, bend around it, into the "shadow region" behind the obstacle.

## *15–11 Diffraction

Waves spread as they travel. When they encounter an obstacle, they bend around it somewhat and pass into the region behind it, as shown in Fig. 15–31 for water waves. This phenomenon is called **diffraction**.

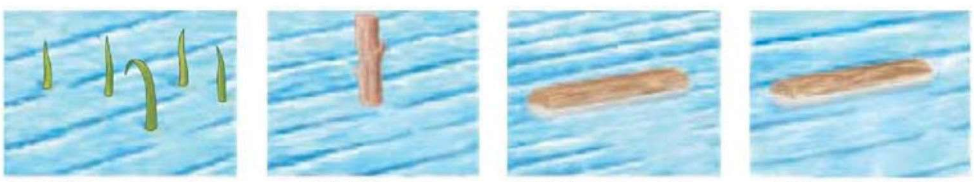
The amount of diffraction depends on the wavelength of the wave and on the size of the obstacle, as shown in Fig. 15–32. If the wavelength is much larger than the object, as with the grass blades of Fig. 15–32a, the wave bends around them almost as if they are not there. For larger objects, parts (b) and (c), there is more of a "shadow" region behind the obstacle where we might not expect the waves to penetrate—but they do, at least a little. Then notice in part (d), where the obstacle is the same as in part (c) but the wavelength is longer, that there is more diffraction into the shadow region. As a rule of thumb, *only if the wavelength is smaller than the size of the object will there be a significant shadow region*. This rule applies to *reflection* from an obstacle as well. Very little of a wave is reflected unless the wavelength is smaller than the size of the obstacle.

A rough guide to the amount of diffraction is

$$\theta(\text{radians}) \approx \frac{\lambda}{\ell},$$

where $\theta$ is roughly the angular spread of waves after they have passed through an opening of width $\ell$ or around an obstacle of width $\ell$.

**FIGURE 15–32** Water waves passing objects of various sizes. Note that the longer the wavelength compared to the size of the object, the more diffraction there is into the "shadow region."



(a) Water waves passing blades of grass
(b) Stick in water
(c) Short-wavelength waves passing log
(d) Long-wavelength waves passing log

That waves can bend around obstacles, and thus can carry energy to areas behind obstacles, is very different from energy carried by material particles. A clear example is the following: if you are standing around a corner on one side of a building, you cannot be hit by a baseball thrown from the other side, but you can hear a shout or other sound because the sound waves diffract around the edges of the building.

# Summary

Vibrating objects act as sources of **waves** that travel outward from the source. Waves on water and on a string are examples. The wave may be a **pulse** (a single crest) or it may be continuous (many crests and troughs).

The **wavelength** of a continuous wave is the distance between two successive crests (or any two identical points on the wave shape).

The **frequency** is the number of full wavelengths (or crests) that pass a given point per unit time.

The **wave velocity** (how fast a crest moves) is equal to the product of wavelength and frequency,

$$v = \lambda f. \tag{15–1}$$

The **amplitude** of a wave is the maximum height of a crest, or depth of a trough, relative to the normal (or equilibrium) level.

In a **transverse wave**, the oscillations are perpendicular to the direction in which the wave travels. An example is a wave on a string.

In a **longitudinal wave**, the oscillations are along (parallel to) the line of travel; sound is an example.

The velocity of both longitudinal and transverse waves in matter is proportional to the square root of an elastic force factor divided by an inertia factor (or density).

Waves carry energy from place to place without matter being carried. The **intensity** of a wave (energy transported across unit area per unit time) is proportional to the square of the amplitude of the wave.

For a wave traveling outward in three dimensions from a point source, the intensity (ignoring damping) decreases with the square of the distance from the source,

$$I \propto \frac{1}{r^2}. \tag{15–8a}$$

The amplitude decreases linearly with distance from the source.

A one-dimensional transverse wave traveling in a medium to the right along the $x$ axis ($x$ increasing) can be represented by a formula for the displacement of the medium from equilibrium at any point $x$ as a function of time as

$$D(x,t) = A\sin\left[\left(\frac{2\pi}{\lambda}\right)(x - vt)\right] \tag{15–10a}$$

$$= A\sin(kx - \omega t) \tag{15–10c}$$

where

$$k = \frac{2\pi}{\lambda} \tag{15–11}$$

and

$$\omega = 2\pi f.$$

If a wave is traveling toward decreasing values of $x$,

$$D(x,t) = A\sin(kx + \omega t). \tag{15–13c}$$

[*Waves can be described by the **wave equation**, which in one dimension is $\partial^2 D/\partial x^2 = (1/v^2)\,\partial^2 D/\partial t^2$, Eq. 15–16.]

When two or more waves pass through the same region of space at the same time, the displacement at any given point will be the vector sum of the displacements of the separate waves. This is the **principle of superposition**. It is valid for mechanical waves if the amplitudes are small enough that the restoring force of the medium is proportional to displacement.

Waves reflect off objects in their path. When the **wave front** of a two- or three-dimensional wave strikes an object, the *angle* of *reflection* equals the *angle* of *incidence*, which is the **law of reflection**. When a wave strikes a boundary between two materials in which it can travel, part of the wave is reflected and part is transmitted.

When two waves pass through the same region of space at the same time, they **interfere**. From the superposition principle, the resultant displacement at any point and time is the sum of their separate displacements. This can result in **constructive interference**, **destructive interference**, or something in between depending on the amplitudes and relative phases of the waves.

Waves traveling on a cord of fixed length interfere with waves that have reflected off the end and are traveling back in the opposite direction. At certain frequencies, **standing waves** can be produced in which the waves seem to be standing still rather than traveling. The cord (or other medium) is vibrating as a whole. This is a resonance phenomenon, and the frequencies at which standing waves occur are called **resonant frequencies**. The points of destructive interference (no vibration) are called **nodes**. Points of constructive interference (maximum amplitude of vibration) are called **antinodes**. On a cord of length $\ell$ fixed at both ends, the wavelengths of standing waves are given by

$$\lambda_n = 2\ell/n \tag{15–17a}$$

where $n$ is an integer.

[*Waves change direction, or **refract**, when traveling from one medium into a second medium where their speed is different. Waves spread, or **diffract**, as they travel and encounter obstacles. A rough guide to the amount of diffraction is $\theta \approx \lambda/\ell$, where $\lambda$ is the wavelength and $\ell$ the width of an opening or obstacle. There is a significant "shadow region" only if the wavelength $\lambda$ is smaller than the size of the obstacle.]

# Questions

1. Is the frequency of a simple periodic wave equal to the frequency of its source? Why or why not?
2. Explain the difference between the speed of a transverse wave traveling down a cord and the speed of a tiny piece of the cord.
3. You are finding it a challenge to climb from one boat up onto a higher boat in heavy waves. If the climb varies from 2.5 m to 4.3 m, what is the amplitude of the wave? Assume the centers of the two boats are a half wavelength apart.
4. What kind of waves do you think will travel down a horizontal metal rod if you strike its end (*a*) vertically from above and (*b*) horizontally parallel to its length?
5. Since the density of air decreases with an increase in temperature, but the bulk modulus $B$ is nearly independent of temperature, how would you expect the speed of sound waves in air to vary with temperature?
6. Describe how you could estimate the speed of water waves across the surface of a pond.
7. The speed of sound in most solids is somewhat greater than in air, yet the density of solids is much greater ($10^3$ to $10^4$ times). Explain.
8. Give two reasons why circular water waves decrease in amplitude as they travel away from the source.

9. Two linear waves have the same amplitude and speed, and otherwise are identical, except one has half the wavelength of the other. Which transmits more energy? By what factor?

10. Will any function of $(x - vt)$—see Eq. 15–14—represent a wave motion? Why or why not? If not, give an example.

11. When a sinusoidal wave crosses the boundary between two sections of cord as in Fig. 15–19, the frequency does not change (although the wavelength and velocity do change). Explain why.

12. If a sinusoidal wave on a two-section cord (Fig. 15–19) is inverted upon reflection, does the transmitted wave have a longer or shorter wavelength?

13. Is energy always conserved when two waves interfere? Explain.

14. If a string is vibrating as a standing wave in three segments, are there any places you could touch it with a knife blade without disturbing the motion?

15. When a standing wave exists on a string, the vibrations of incident and reflected waves cancel at the nodes. Does this mean that energy was destroyed? Explain.

16. Can the amplitude of the standing waves in Fig. 15–25 be greater than the amplitude of the vibrations that cause them (up and down motion of the hand)?

17. When a cord is vibrated as in Fig. 15–25 by hand or by a mechanical oscillator, the "nodes" are not quite true nodes (at rest). Explain. [*Hint*: Consider damping and energy flow from hand or oscillator.]

*18. AM radio signals can usually be heard behind a hill, but FM often cannot. That is, AM signals bend more than FM. Explain. (Radio signals, as we shall see, are carried by electromagnetic waves whose wavelength for AM is typically 200 to 600 m and for FM about 3 m.)

*19. If we knew that energy was being transmitted from one place to another, how might we determine whether the energy was being carried by particles (material objects) or by waves?

# Problems

## 15–1 and 15–2 Characteristics of Waves

1. (I) A fisherman notices that wave crests pass the bow of his anchored boat every 3.0 s. He measures the distance between two crests to be 8.0 m. How fast are the waves traveling?

2. (I) A sound wave in air has a frequency of 262 Hz and travels with a speed of 343 m/s. How far apart are the wave crests (compressions)?

3. (I) Calculate the speed of longitudinal waves in (*a*) water, (*b*) granite, and (*c*) steel.

4. (I) AM radio signals have frequencies between 550 kHz and 1600 kHz (kilohertz) and travel with a speed of $3.0 \times 10^8$ m/s. What are the wavelengths of these signals? On FM the frequencies range from 88 MHz to 108 MHz (megahertz) and travel at the same speed. What are their wavelengths?

5. (I) Determine the wavelength of a 5800-Hz sound wave traveling along an iron rod.

6. (II) A cord of mass 0.65 kg is stretched between two supports 8.0 m apart. If the tension in the cord is 140 N, how long will it take a pulse to travel from one support to the other?

7. (II) A 0.40-kg cord is stretched between two supports, 7.8 m apart. When one support is struck by a hammer, a transverse wave travels down the cord and reaches the other support in 0.85 s. What is the tension in the cord?

8. (II) A sailor strikes the side of his ship just below the surface of the sea. He hears the echo of the wave reflected from the ocean floor directly below 2.8 s later. How deep is the ocean at this point?

9. (II) A ski gondola is connected to the top of a hill by a steel cable of length 660 m and diameter 1.5 cm. As the gondola comes to the end of its run, it bumps into the terminal and sends a wave pulse along the cable. It is observed that it took 17 s for the pulse to return. (*a*) What is the speed of the pulse? (*b*) What is the tension in the cable?

10. (II) P and S waves from an earthquake travel at different speeds, and this difference helps locate the earthquake "epicenter" (where the disturbance took place). (*a*) Assuming typical speeds of 8.5 km/s and 5.5 km/s for P and S waves, respectively, how far away did the earthquake occur if a particular seismic station detects the arrival of these two types of waves 1.7 min apart? (*b*) Is one seismic station sufficient to determine the position of the epicenter? Explain.

11. (II) The wave on a string shown in Fig. 15–33 is moving to the right with a speed of 1.10 m/s. (*a*) Draw the shape of the string 1.00 s later and indicate which parts of the string are moving up and which down at that instant. (*b*) Estimate the vertical speed of point A on the string at the instant shown in the Figure.

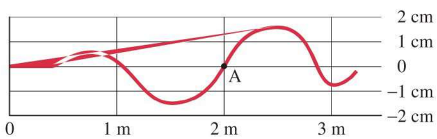



**FIGURE 15–33** Problem 11.

12. (II) A 5.0 kg ball hangs from a steel wire 1.00 mm in diameter and 5.00 m long. What would be the speed of a wave in the steel wire?

13. (II) Two children are sending signals along a cord of total mass 0.50 kg tied between tin cans with a tension of 35 N. It takes the vibrations in the string 0.50 s to go from one child to the other. How far apart are the children?

*14. (II) **Dimensional analysis.** Waves on the surface of the ocean do not depend significantly on the properties of water such as density or surface tension. The primary "return force" for water piled up in the wave crests is due to the gravitational attraction of the Earth. Thus the speed $v$ (m/s) of ocean waves depends on the acceleration due to gravity $g$. It is reasonable to expect that $v$ might also depend on water depth $h$ and the wave's wavelength $\lambda$. Assume the wave speed is given by the functional form $v = Cg^{\alpha}h^{\beta}\lambda^{\gamma}$, where $\alpha, \beta, \gamma$, and $C$ are numbers without dimension. (*a*) In deep water, the water deep below does not affect the motion of waves at the surface. Thus $v$ should be independent of depth $h$ (i.e., $\beta = 0$). Using only dimensional analysis (Section 1–7), determine the formula for the speed of surface waves in deep water. (*b*) In shallow water, the speed of surface waves is found experimentally to be independent of the wavelength (i.e., $\gamma = 0$). Using only dimensional analysis, determine the formula for the speed of waves in shallow water.

## 15–3 Energy Transported by Waves

**15.** (I) Two earthquake waves of the same frequency travel through the same portion of the Earth, but one is carrying 3.0 times the energy. What is the ratio of the amplitudes of the two waves?

**16.** (II) What is the ratio of (*a*) the intensities, and (*b*) the amplitudes, of an earthquake P wave passing through the Earth and detected at two points 15 km and 45 km from the source.

**17.** (II) Show that if damping is ignored, the amplitude $A$ of circular water waves decreases as the square root of the distance $r$ from the source: $A \propto 1/\sqrt{r}$.

**18.** (II) The intensity of an earthquake wave passing through the Earth is measured to be $3.0 \times 10^6\,\mathrm{J/m^2 \cdot s}$ at a distance of 48 km from the source. (*a*) What was its intensity when it passed a point only 1.0 km from the source? (*b*) At what rate did energy pass through an area of $2.0\,\mathrm{m^2}$ at 1.0 km?

**19.** (II) A small steel wire of diameter 1.0 mm is connected to an oscillator and is under a tension of 7.5 N. The frequency of the oscillator is 60.0 Hz and it is observed that the amplitude of the wave on the steel wire is 0.50 cm. (*a*) What is the power output of the oscillator, assuming that the wave is not reflected back? (*b*) If the power output stays constant but the frequency is doubled, what is the amplitude of the wave?

**20.** (II) Show that the intensity of a wave is equal to the energy density (energy per unit volume) in the wave times the wave speed.

**21.** (II) (*a*) Show that the average rate with which energy is transported along a cord by a mechanical wave of frequency $f$ and amplitude $A$ is

$$\overline{P} = 2\pi^2 \mu v f^2 A^2,$$

where $v$ is the speed of the wave and $\mu$ is the mass per unit length of the cord. (*b*) If the cord is under a tension $F_T = 135\,\mathrm{N}$ and has mass per unit length 0.10 kg/m, what power is required to transmit 120-Hz transverse waves of amplitude 2.0 cm?

## 15–4 Mathematical Representation of Traveling Wave

**22.** (I) A transverse wave on a wire is given by $D(x,t) = 0.015\sin(25x - 1200t)$ where $D$ and $x$ are in meters and $t$ is in seconds. (*a*) Write an expression for a wave with the same amplitude, wavelength, and frequency but traveling in the opposite direction. (*b*) What is the speed of either wave?

**23.** (I) Suppose at $t = 0$, a wave shape is represented by $D = A\sin(2\pi x/\lambda + \phi)$; that is, it differs from Eq. 15–9 by a constant phase factor $\phi$. What then will be the equation for a wave traveling to the left along the $x$ axis as a function of $x$ and $t$?

**24.** (II) A transverse traveling wave on a cord is represented by $D = 0.22\sin(5.6x + 34t)$ where $D$ and $x$ are in meters and $t$ is in seconds. For this wave determine (*a*) the wavelength, (*b*) frequency, (*c*) velocity (magnitude and direction), (*d*) amplitude, and (*e*) maximum and minimum speeds of particles of the cord.

**25.** (II) Consider the point $x = 1.00\,\mathrm{m}$ on the cord of Example 15–5. Determine (*a*) the maximum velocity of this point, and (*b*) its maximum acceleration. (*c*) What is its velocity and acceleration at $t = 2.50\,\mathrm{s}$?

**26.** (II) A transverse wave on a cord is given by $D(x,t) = 0.12\sin(3.0x - 15.0t)$, where $D$ and $x$ are in m and $t$ is in s. At $t = 0.20\,\mathrm{s}$, what are the displacement and velocity of the point on the cord where $x = 0.60\,\mathrm{m}$?

**27.** (II) A transverse wave pulse travels to the right along a string with a speed $v = 2.0\,\mathrm{m/s}$. At $t = 0$ the shape of the pulse is given by the function

$$D = 0.45\cos(2.6x + 1.2),$$

where $D$ and $x$ are in meters. (*a*) Plot $D$ vs. $x$ at $t = 0$. (*b*) Determine a formula for the wave pulse at any time $t$ assuming there are no frictional losses. (*c*) Plot $D(x,t)$ vs. $x$ at $t = 1.0\,\mathrm{s}$. (*d*) Repeat parts (*b*) and (*c*) assuming the pulse is traveling to the left. Plot all 3 graphs on the same axes for easy comparison.

**28.** (II) A 524-Hz longitudinal wave in air has a speed of 345 m/s. (*a*) What is the wavelength? (*b*) How much time is required for the phase to change by 90° at a given point in space? (*c*) At a particular instant, what is the phase difference (in degrees) between two points 4.4 cm apart?

**29.** (II) Write the equation for the wave in Problem 28 traveling to the right, if its amplitude is 0.020 cm, and $D = -0.020\,\mathrm{cm}$, at $t = 0$ and $x = 0$.

**30.** (II) A sinusoidal wave traveling on a string in the negative $x$ direction has amplitude 1.00 cm, wavelength 3.00 cm, and frequency 245 Hz. At $t = 0$, the particle of string at $x = 0$ is displaced a distance $D = 0.80\,\mathrm{cm}$ above the origin and is moving upward. (*a*) Sketch the shape of the wave at $t = 0$ and (*b*) determine the function of $x$ and $t$ that describes the wave.

## *15–5 The Wave Equation

***31.** (II) Determine if the function $D = A\sin kx\cos\omega t$ is a solution of the wave equation.

***32.** (II) Show by direct substitution that the following functions satisfy the wave equation: (*a*) $D(x,t) = A\ln(x + vt)$; (*b*) $D(x,t) = (x - vt)^4$.

***33.** (II) Show that the wave forms of Eqs. 15–13 and 15–15 satisfy the wave equation, Eq. 15–16.

***34.** (II) Let two linear waves be represented by $D_1 = f_1(x,t)$ and $D_2 = f_2(x,t)$. If both these waves satisfy the wave equation (Eq. 15–16), show that any combination $D = C_1 D_1 + C_2 D_2$ does as well, where $C_1$ and $C_2$ are constants.

***35.** (II) Does the function $D(x,t) = e^{-(kx-\omega t)^2}$ satisfy the wave equation? Why or why not?

***36.** (II) In deriving Eq. 15–2, $v = \sqrt{F_T/\mu}$, for the speed of a transverse wave on a string, it was assumed that the wave's amplitude $A$ is much less than its wavelength $\lambda$. Assuming a sinusoidal wave shape $D = A\sin(kx - \omega t)$, show via the partial derivative $v' = \partial D/\partial t$ that the assumption $A \ll \lambda$ implies that the maximum transverse speed $v'_{max}$ of the string itself is much less than the wave velocity. If $A = \lambda/100$ determine the ratio $v'_{max}/v$.

## 15–7 Reflection and Transmission

**37.** (II) A cord has two sections with linear densities of 0.10 kg/m and 0.20 kg/m, Fig. 15–34. An incident wave, given by $D = (0.050\,\mathrm{m})\sin(7.5x - 12.0t)$, where $x$ is in meters and $t$ in seconds, travels along the lighter cord. (*a*) What is the wavelength on the lighter section of the cord? (*b*) What is the tension in the cord? (*c*) What is the wavelength when the wave travels on the heavier section?

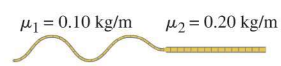



**FIGURE 15–34** Problem 37.

38. (II) Consider a sine wave traveling down the stretched two-part cord of Fig. 15–19. Determine a formula (*a*) for the ratio of the speeds of the wave in the two sections, $v_H/v_L$, and (*b*) for the ratio of the wavelengths in the two sections. (The frequency is the same in both sections. Why?) (*c*) Is the wavelength larger in the heavier cord or the lighter?

39. (II) Seismic reflection prospecting is commonly used to map deeply buried formations containing oil. In this technique, a seismic wave generated on the Earth's surface (for example, by an explosion or falling weight) reflects from the subsurface formation and is detected upon its return to ground level. By placing ground-level detectors at a variety of locations relative to the source, and observing the variation in the source-to-detector travel times, the depth of the subsurface formation can be determined. (*a*) Assume a ground-level detector is placed a distance $x$ away from a seismic-wave source and that a horizontal boundary between overlying rock and a subsurface formation exists at depth $D$ (Fig. 15–35a). Determine an expression for the time $t$ taken by the reflected wave to travel from source to detector, assuming the seismic wave propagates at constant speed $v$. (*b*) Suppose several detectors are placed along a line at different distances $x$ from the source as in Fig. 15–35b. Then, when a seismic wave is generated, the different travel times $t$ for each detector are measured. Starting with your result from part (*a*), explain how a graph of $t^2$ vs. $x^2$ can be used to determine $D$.

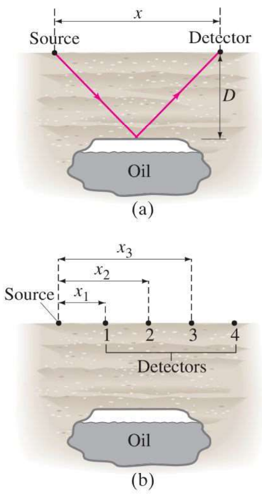



**FIGURE 15–35** Problem 39.

40. (III) A cord stretched to a tension $F_T$ consists of two sections (as in Fig. 15–19) whose linear densities are $\mu_1$ and $\mu_2$. Take $x = 0$ to be the point (a knot) where they are joined, with $\mu_1$ referring to that section of cord to the left and $\mu_2$ that to the right. A sinusoidal wave, $D = A\sin[k_1(x - v_1 t)]$, starts at the left end of the cord. When it reaches the knot, part of it is reflected and part is transmitted. Let the equation of the reflected wave be $D_R = A_R \sin[k_1(x + v_1 t)]$ and that for the transmitted wave be $D_T = A_T \sin[k_2(x - v_2 t)]$. Since the frequency must be the same in both sections, we have $\omega_1 = \omega_2$ or $k_1 v_1 = k_2 v_2$. (*a*) Because the cord is continuous, a point an infinitesimal distance to the left of the knot has the same displacement at any moment (due to incident plus reflected waves) as a point just to the right of the knot (due to the transmitted wave). Thus show that $A = A_T + A_R$. (*b*) Assuming that the slope $(\partial D/\partial x)$ of the cord just to the left of the knot is the same as the slope just to the right of the knot, show that the amplitude of the reflected wave is given by

$$A_R = \left(\frac{v_1 - v_2}{v_1 + v_2}\right)A = \left(\frac{k_2 - k_1}{k_2 + k_1}\right)A.$$

(*c*) What is $A_T$ in terms of $A$?

## 15–8 Interference

41. (I) The two pulses shown in Fig. 15–36 are moving toward each other. (*a*) Sketch the shape of the string at the moment they directly overlap. (*b*) Sketch the shape of the string a few moments later. (*c*) In Fig. 15–22a, at the moment the pulses pass each other, the string is straight. What has happened to the energy at this moment?

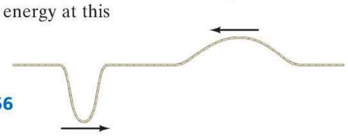


**FIGURE 15–36** Problem 41.

42. (II) Suppose two linear waves of equal amplitude and frequency have a phase difference $\phi$ as they travel in the same medium. They can be represented by

$$D_1 = A\sin(kx - \omega t)$$
$$D_2 = A\sin(kx - \omega t + \phi).$$

(*a*) Use the trigonometric identity $\sin\theta_1 + \sin\theta_2 = 2\sin\frac{1}{2}(\theta_1 + \theta_2)\cos\frac{1}{2}(\theta_1 - \theta_2)$ to show that the resultant wave is given by

$$D = \left(2A\cos\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right).$$

(*b*) What is the amplitude of this resultant wave? Is the wave purely sinusoidal, or not? (*c*) Show that constructive interference occurs if $\phi = 0, 2\pi, 4\pi$, and so on, and destructive interference occurs if $\phi = \pi, 3\pi, 5\pi$, etc. (*d*) Describe the resultant wave, by equation and in words, if $\phi = \pi/2$.

## 15–9 Standing Waves; Resonance

43. (I) A violin string vibrates at 441 Hz when unfingered. At what frequency will it vibrate if it is fingered one-third of the way down from the end? (That is, only two-thirds of the string vibrates as a standing wave.)

44. (I) If a violin string vibrates at 294 Hz as its fundamental frequency, what are the frequencies of the first four harmonics?

45. (I) In an earthquake, it is noted that a footbridge oscillated up and down in a one-loop (fundamental standing wave) pattern once every 1.5 s. What other possible resonant periods of motion are there for this bridge? What frequencies do they correspond to?

46. (I) A particular string resonates in four loops at a frequency of 280 Hz. Name at least three other frequencies at which it will resonate.

47. (II) A cord of length 1.0 m has two equal-length sections with linear densities of 0.50 kg/m and 1.00 kg/m. The tension in the entire cord is constant. The ends of the cord are oscillated so that a standing wave is set up in the cord with a single node where the two sections meet. What is the ratio of the oscillatory frequencies?

48. (II) The velocity of waves on a string is 96 m/s. If the frequency of standing waves is 445 Hz, how far apart are the two adjacent nodes?

49. (II) If two successive harmonics of a vibrating string are 240 Hz and 320 Hz, what is the frequency of the fundamental?

50. (II) A guitar string is 90.0 cm long and has a mass of 3.16 g. From the bridge to the support post ($=\ell$) is 60.0 cm and the string is under a tension of 520 N. What are the frequencies of the fundamental and first two overtones?

**51.** (II) Show that the frequency of standing waves on a cord of length $\ell$ and linear density $\mu$, which is stretched to a tension $F_T$, is given by

$$f = \frac{n}{2\ell}\sqrt{\frac{F_T}{\mu}}$$

where $n$ is an integer.

**52.** (II) One end of a horizontal string of linear density $6.6 \times 10^{-4}\,\text{kg/m}$ is attached to a small-amplitude mechanical 120-Hz oscillator. The string passes over a pulley, a distance $\ell = 1.50\,\text{m}$ away, and weights are hung from this end, Fig. 15–37. What mass $m$ must be hung from this end of the string to produce (*a*) one loop, (*b*) two loops, and (*c*) five loops of a standing wave? Assume the string at the oscillator is a node, which is nearly true.

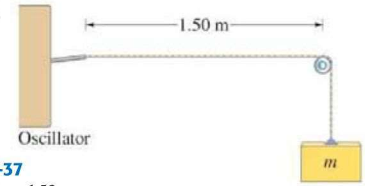



**FIGURE 15–37**
Problems 52 and 53.

**53.** (II) In Problem 52, Fig. 15–37, the length of the string may be adjusted by moving the pulley. If the hanging mass $m$ is fixed at 0.070 kg, how many different standing wave patterns may be achieved by varying $\ell$ between 10 cm and 1.5 m?

**54.** (II) The displacement of a standing wave on a string is given by $D = 2.4\sin(0.60x)\cos(42t)$, where $x$ and $D$ are in centimeters and $t$ is in seconds. (*a*) What is the distance (cm) between nodes? (*b*) Give the amplitude, frequency, and speed of each of the component waves. (*c*) Find the speed of a particle of the string at $x = 3.20\,\text{cm}$ when $t = 2.5\,\text{s}$.

**55.** (II) The displacement of a transverse wave traveling on a string is represented by $D_1 = 4.2\sin(0.84x - 47t + 2.1)$, where $D_1$ and $x$ are in cm and $t$ in s. (*a*) Find an equation that represents a wave which, when traveling in the opposite direction, will produce a standing wave when added to this one. (*b*) What is the equation describing the standing wave?

**56.** (II) When you slosh the water back and forth in a tub at just the right frequency, the water alternately rises and falls at each end, remaining relatively calm at the center. Suppose the frequency to produce such a standing wave in a 45-cm-wide tub is 0.85 Hz. What is the speed of the water wave?

**57.** (II) A particular violin string plays at a frequency of 294 Hz. If the tension is increased 15%, what will the new frequency be?

**58.** (II) Two traveling waves are described by the functions

$$D_1 = A\sin(kx - \omega t)$$
$$D_2 = A\sin(kx + \omega t),$$

where $A = 0.15\,\text{m}$, $k = 3.5\,\text{m}^{-1}$, and $\omega = 1.8\,\text{s}^{-1}$. (*a*) Plot these two waves, from $x = 0$ to a point $x(> 0)$ that includes one full wavelength. Choose $t = 1.0\,\text{s}$. (*b*) Plot the sum of the two waves and identify the nodes and antinodes in the plot, and compare to the analytic (mathematical) representation.

**59.** (II) Plot the two waves given in Problem 58 and their sum, as a function of time from $t = 0$ to $t = T$ (one period). Choose (*a*) $x = 0$ and (*b*) $x = \lambda/4$. Interpret your results.

**60.** (II) A standing wave on a 1.64-m-long horizontal string displays three loops when the string vibrates at 120 Hz. The maximum swing of the string (top to bottom) at the center of each loop is 8.00 cm. (*a*) What is the function describing the standing wave? (*b*) What are the functions describing the two equal-amplitude waves traveling in opposite directions that make up the standing wave?

**61.** (II) On an electric guitar, a "pickup" under each string transforms the string's vibrations directly into an electrical signal. If a pickup is placed 16.25 cm from one of the fixed ends of a 65.00-cm-long string, which of the harmonics from $n = 1$ to $n = 12$ will not be "picked up" by this pickup?

**62.** (II) A 65-cm guitar string is fixed at both ends. In the frequency range between 1.0 and 2.0 kHz, the string is found to resonate only at frequencies 1.2, 1.5, and 1.8 kHz. What is the speed of traveling waves on this string?

**63.** (II) Two oppositely directed traveling waves given by $D_1 = (5.0\,\text{mm})\cos[(2.0\,\text{m}^{-1})x - (3.0\,\text{rad/s})t]$ and $D_2 = (5.0\,\text{mm})\cos[(2.0\,\text{m}^{-1})x + (3.0\,\text{rad/s})t]$ form a standing wave. Determine the position of nodes along the $x$ axis.

**64.** (II) A wire is composed of aluminum with length $\ell_1 = 0.600\,\text{m}$ and mass per unit length $\mu_1 = 2.70\,\text{g/m}$ joined to a steel section with length $\ell_2 = 0.882\,\text{m}$ and mass per unit length $\mu_2 = 7.80\,\text{g/m}$. This composite wire is fixed at both ends and held at a uniform tension of 135 N. Find the lowest frequency standing wave that can exist on this wire, assuming there is a node at the joint between aluminum and steel. How many nodes (including the two at the ends) does this standing wave possess?

## *15–10 Refraction

***65.** (I) An earthquake P wave traveling 8.0 km/s strikes a boundary within the Earth between two kinds of material. If it approaches the boundary at an incident angle of 52° and the angle of refraction is 31°, what is the speed in the second medium?

***66.** (I) Water waves approach an underwater "shelf" where the velocity changes from 2.8 m/s to 2.5 m/s. If the incident wave crests make a 35° angle with the shelf, what will be the angle of refraction?

***67.** (II) A sound wave is traveling in warm air (25°C) when it hits a layer of cold (−15°C) denser air. If the sound wave hits the cold air interface at an angle of 33°, what is the angle of refraction? The speed of sound as a function of temperature can be approximated by $v = (331 + 0.60\,T)\,\text{m/s}$, where $T$ is in °C.

***68.** (II) Any type of wave that reaches a boundary beyond which its speed is increased, there is a maximum incident angle if there is to be a transmitted refracted wave. This maximum incident angle $\theta_{iM}$ corresponds to an angle of refraction equal to 90°. If $\theta_i > \theta_{iM}$, all the wave is reflected at the boundary and none is refracted, because this would correspond to $\sin\theta_r > 1$ (where $\theta_r$ is the angle of refraction), which is impossible. This phenomenon is referred to as *total internal reflection*. (*a*) Find a formula for $\theta_{iM}$ using the law of refraction, Eq. 15–19. (*b*) How far from the bank should a trout fisherman stand (Fig. 15–38) so trout won't be frightened by his voice (1.8 m above the ground)? The speed of sound is about 343 m/s in air and 1440 m/s in water.

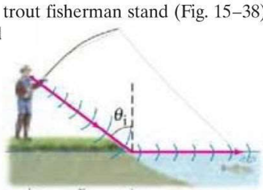



**FIGURE 15–38**
Problem 68b.

*69. (II) A longitudinal earthquake wave strikes a boundary between two types of rock at a 38° angle. As the wave crosses the boundary, the specific gravity of the rock changes from 3.6 to 2.8. Assuming that the elastic modulus is the same for both types of rock, determine the angle of refraction.

### *15–11 Diffraction

*70. (II) A satellite dish is about 0.5 m in diameter. According to the user's manual, the dish has to be pointed in the direction of the satellite, but an error of about 2° to either side is allowed without loss of reception. Estimate the wavelength of the electromagnetic waves (speed $= 3 \times 10^8\,\mathrm{m/s}$) received by the dish.

## General Problems

71. A sinusoidal traveling wave has frequency 880 Hz and phase velocity 440 m/s. (*a*) At a given time, find the distance between any two locations that correspond to a difference in phase of $\pi/6$ rad. (*b*) At a fixed location, by how much does the phase change during a time interval of $1.0 \times 10^{-4}$ s?

72. When you walk with a cup of coffee (diameter 8 cm) at just the right pace of about one step per second, the coffee sloshes higher and higher in your cup until eventually it starts to spill over the top, Fig 15–39. Estimate the speed of the waves in the coffee.

**FIGURE 15–39** Problem 72.

73. Two solid rods have the same bulk modulus but one is 2.5 times as dense as the other. In which rod will the speed of longitudinal waves be greater, and by what factor?

74. Two waves traveling along a stretched string have the same frequency, but one transports 2.5 times the power of the other. What is the ratio of the amplitudes of the two waves?

75. A bug on the surface of a pond is observed to move up and down a total vertical distance of 0.10 m, lowest to highest point, as a wave passes. (*a*) What is the amplitude of the wave? (*b*) If the amplitude increases to 0.15 m, by what factor does the bug's maximum kinetic energy change?

76. A guitar string is supposed to vibrate at 247 Hz, but is measured to actually vibrate at 255 Hz. By what percentage should the tension in the string be changed to get the frequency to the correct value?

77. An earthquake-produced surface wave can be approximated by a sinusoidal transverse wave. Assuming a frequency of 0.60 Hz (typical of earthquakes, which actually include a mixture of frequencies), what amplitude is needed so that objects begin to leave contact with the ground? [*Hint*: Set the acceleration $a > g$.]

78. A uniform cord of length $\ell$ and mass $m$ is hung vertically from a support. (*a*) Show that the speed of transverse waves in this cord is $\sqrt{gh}$, where $h$ is the height above the lower end. (*b*) How long does it take for a pulse to travel upward from one end to the other?

79. A transverse wave pulse travels to the right along a string with a speed $v = 2.4\,\mathrm{m/s}$. At $t = 0$ the shape of the pulse is given by the function

$$D = \frac{4.0\,\mathrm{m}^3}{x^2 + 2.0\,\mathrm{m}^2},$$

where $D$ and $x$ are in meters. (*a*) Plot $D$ vs. $x$ at $t = 0$ from $x = -10$ m to $x = +10$ m. (*b*) Determine a formula for the wave pulse at any time $t$ assuming there are no frictional losses. (*c*) Plot $D(x, t)$ vs. $x$ at $t = 1.00$ s. (*d*) Repeat parts (*b*) and (*c*) assuming the pulse is traveling to the left.

80. (*a*) Show that if the tension in a stretched string is changed by a small amount $\Delta F_T$, the frequency of the fundamental is changed by an amount $\Delta f = \frac{1}{2}(\Delta F_T/F_T)f$. (*b*) By what percent must the tension in a piano string be increased or decreased to raise the frequency from 436 Hz to 442 Hz. (*c*) Does the formula in part (*a*) apply to the overtones as well?

81. Two strings on a musical instrument are tuned to play at 392 Hz (G) and 494 Hz (B). (*a*) What are the frequencies of the first two overtones for each string? (*b*) If the two strings have the same length and are under the same tension, what must be the ratio of their masses $(m_G/m_A)$? (*c*) If the strings, instead, have the same mass per unit length and are under the same tension, what is the ratio of their lengths $(\ell_G/\ell_A)$? (*d*) If their masses and lengths are the same, what must be the ratio of the tensions in the two strings?

82. The ripples in a certain groove 10.8 cm from the center of a 33-rpm phonograph record have a wavelength of 1.55 mm. What will be the frequency of the sound emitted?

83. A 10.0-m-long wire of mass 152 g is stretched under a tension of 255 N. A pulse is generated at one end, and 20.0 ms later a second pulse is generated at the opposite end. Where will the two pulses first meet?

84. A wave with a frequency of 220 Hz and a wavelength of 10.0 cm is traveling along a cord. The maximum speed of particles on the cord is the same as the wave speed. What is the amplitude of the wave?

85. A string can have a "free" end if that end is attached to a ring that can slide without friction on a vertical pole (Fig. 15–40). Determine the wavelengths of the resonant vibrations of such a string with one end fixed and the other free.

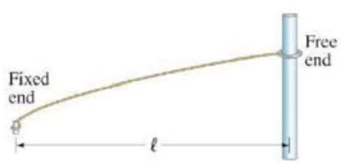



**FIGURE 15–40** Problem 85.

**86.** A highway overpass was observed to resonate as one full loop $(\frac{1}{2}\lambda)$ when a small earthquake shook the ground vertically at 3.0 Hz. The highway department put a support at the center of the overpass, anchoring it to the ground as shown in Fig. 15–41. What resonant frequency would you now expect for the overpass? It is noted that earthquakes rarely do significant shaking above 5 or 6 Hz. Did the modifications do any good? Explain.

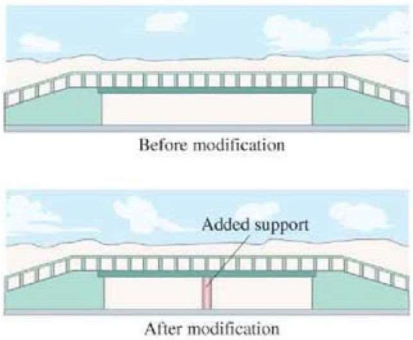



**FIGURE 15–41** Problem 86.

**87.** Figure 15–42 shows the wave shape at two instants of time for a sinusoidal wave traveling to the right. What is the mathematical representation of this wave?

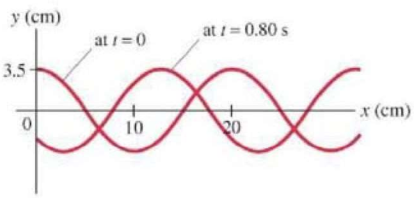



**FIGURE 15–42** Problem 87.

**88.** Estimate the average power of a water wave when it hits the chest of an adult standing in the water at the seashore. Assume that the amplitude of the wave is 0.50 m, the wavelength is 2.5 m, and the period is 4.0 s.

**89.** A tsunami of wavelength 215 km and velocity 550 km/h travels across the Pacific Ocean. As it approaches Hawaii, people observe an unusual decrease of sea level in the harbors. Approximately how much time do they have to run to safety? (In the absence of knowledge and warning, people have died during tsunamis, some of them attracted to the shore to see stranded fishes and boats.)

**90.** Two wave pulses are traveling in opposite directions with the same speed of 7.0 cm/s as shown in Fig. 15–43. At $t = 0$, the leading edges of the two pulses are 15 cm apart. Sketch the wave pulses at $t = 1.0, 2.0$ and 3.0 s.

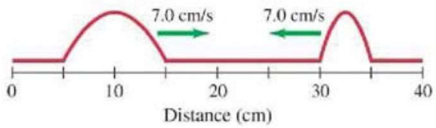



**FIGURE 15–43** Problem 90.

**91.** For a spherical wave traveling uniformly away from a point source, show that the displacement can be represented by

$$D = \left(\frac{A}{r}\right)\sin(kr - \omega t),$$

where $r$ is the radial distance from the source and $A$ is a constant.

**92.** What frequency of sound would have a wavelength the same size as a 1.0-m-wide window? (The speed of sound is 344 m/s at 20°C.) What frequencies would diffract through the window?

## *Numerical/Computer

***93.** (II) Consider a wave generated by the periodic vibration of a source and given by the expression $D(x,t) = A\sin^2 k(x - ct)$, where $x$ represents position (in meters), $t$ represents time (in seconds), and $c$ is a positive constant. We choose $A = 5.0$ m and $c = 0.50$ m/s. Use a spreadsheet to make a graph with three curves of $D(x,t)$ from $x = -5.0$ m to $+5.0$ m in steps of 0.050 m at times $t = 0.0$, 1.0, and 2.0 s. Determine the speed, direction of motion, period, and wavelength of the wave.

***94.** (II) The displacement of a bell-shaped wave pulse is described by a relation that involves the exponential function:

$$D(x,t) = Ae^{-\alpha(x-vt)^2}$$

where the constants $A = 10.0$ m, $\alpha = 2.0\ \text{m}^{-2}$, and $v = 3.0$ m/s. (*a*) Over the range $-10.0\ \text{m} \leq x \leq +10.0$ m, use a graphing calculator or computer program to plot $D(x,t)$ at each of the three times $t = 0$, $t = 1.0$, and $t = 2.0$ s. Do these three plots demonstrate the wave-pulse shape shifting along the $x$ axis by the expected amount over the span of each 1.0-s interval? (*b*) Repeat part (*a*) but assume $D(x,t) = Ae^{-\alpha(x+vt)^2}$.

## Answers to Exercises

**A:** (*c*).

**B:** (*d*).

**C:** (*c*).

**D:** (*a*).