"If music be the food of physics, play on." [See Shakespeare, *Twelfth Night*, line 1.]

Stringed instruments depend on transverse standing waves on strings to produce their harmonious sounds. The sound of wind instruments originates in longitudinal standing waves of an air column. Percussion instruments create more complicated standing waves.

Besides examining sources of sound, we also study the decibel scale of sound level, sound wave interference and beats, the Doppler effect, shock waves and sonic booms, and ultrasound imaging.

CHAPTER 16

Sound

CONTENTS

CHAPTER-OPENING QUESTION—Guess now!

A pianist plays the note "middle C." The sound is made by the vibration of the piano string and is propagated outward as a vibration of the air (which can reach your ear). Comparing the vibration on the string to the vibration in the air, which of the following is true?

(a) The vibration on the string and the vibration in the air have the same wavelength.
(b) They have the same frequency.
(c) They have the same speed.
(d) Neither wavelength, frequency, nor speed are the same in the air as on the string.

Sound is associated with our sense of hearing and, therefore, with the physiology of our ears and the psychology of our brain, which interprets the sensations that reach our ears. The term *sound* also refers to the physical sensation that stimulates our ears: namely, longitudinal waves.

We can distinguish three aspects of any sound. First, there must be a *source* for a sound; as with any mechanical wave, the source of a sound wave is a vibrating object. Second, the energy is transferred from the source in the form of longitudinal sound *waves*. And third, the sound is *detected* by an ear or by a microphone. We start this Chapter by looking at some aspects of sound waves themselves.

16–1 Characteristics of Sound

We saw in Chapter 15, Fig. 15–5, how a vibrating drumhead produces a sound wave in air. Indeed, we usually think of sound waves traveling in the air, for normally it is the vibrations of the air that force our eardrums to vibrate. But sound waves can also travel in other materials.

Two stones struck together under water can be heard by a swimmer beneath the surface, for the vibrations are carried to the ear by the water. When you put your ear flat against the ground, you can hear an approaching train or truck. In this case the ground does not actually touch your eardrum, but the longitudinal wave transmitted by the ground is called a sound wave just the same, for its vibrations cause the outer ear and the air within it to vibrate. Sound cannot travel in the absence of matter. For example, a bell ringing inside an evacuated jar cannot be heard, nor does sound travel through the empty reaches of outer space.

The **speed of sound** is different in different materials. In air at 0°C and 1 atm, sound travels at a speed of 331 m/s. We saw in Eq. 15–4 $\left(v = \sqrt{B/\rho}\right)$ that the speed depends on the elastic modulus, B, and the density, ρ, of the material. Thus for helium, whose density is much less than that of air but whose elastic modulus is not greatly different, the speed is about three times as great as in air. In liquids and solids, which are much less compressible and therefore have much greater elastic moduli, the speed is larger still. The speed of sound in various materials is given in Table 16–1. The values depend somewhat on temperature, but this is significant mainly for gases. For example, in air at normal (ambient) temperatures, the speed increases approximately 0.60 m/s for each Celsius degree increase in temperature:

TABLE 16–1 Speed of Sound in Various Materials
(20°C and 1 atm)

Material	Speed (m/s)
Air	343
Air (0°C)	331
Helium	1005
Hydrogen	1300
Water	1440
Sea water	1560
Iron and steel	≈ 5000
Glass	≈ 4500
Aluminum	≈ 5100
Hardwood	≈ 4000
Concrete	≈ 3000

$$v \approx (331 + 0.60\,T)\ \text{m/s}, \qquad \text{[speed of sound in air]}$$

where T is the temperature in °C. Unless stated otherwise, we will assume in this Chapter that $T = 20°\text{C}$, so that† $v = [331 + (0.60)(20)]\ \text{m/s} = 343\ \text{m/s}$.

CONCEPTUAL EXAMPLE 16–1 **Distance from a lightning strike.** A rule of thumb that tells how close lightning has struck is, "one mile for every five seconds before the thunder is heard." Explain why this works, noting that the speed of light is so high (3×10^8 m/s, almost a million times faster than sound) that the time for light to travel to us is negligible compared to the time for the sound.

PHYSICS APPLIED
How far away is the lightning?

RESPONSE The speed of sound in air is about 340 m/s, so to travel $1\ \text{km} = 1000\ \text{m}$ takes about 3 seconds. One mile is about 1.6 kilometers, so the time for the thunder to travel a mile is about $(1.6)(3) \approx 5$ seconds.

EXERCISE A What would be the rule used in Example 16–1 in terms of kilometers?

Two aspects of any sound are immediately evident to a human listener: "loudness" and "pitch." Each refers to a sensation in the consciousness of the listener. But to each of these subjective sensations there corresponds a physically measurable quantity. **Loudness** is related to the intensity (energy per unit time crossing unit area) in the sound wave, and we shall discuss it in Section 16–3.

The **pitch** of a sound refers to whether it is high, like the sound of a piccolo or violin, or low, like the sound of a bass drum or string bass. The physical quantity that determines pitch is the frequency, as was first noted by Galileo. The lower the frequency, the lower the pitch; the higher the frequency, the higher the pitch.‡ The best human ears can respond to frequencies from about 20 Hz to almost 20,000 Hz. (Recall that 1 Hz is 1 cycle per second.) This frequency range is called the **audible range**. These limits vary somewhat from one individual to another. One general trend is that as people age, they are less able to hear high frequencies, so the high-frequency limit may be 10,000 Hz or less.

†We treat the 20°C ("room temperature") as accurate to 2 significant figures.

‡Although pitch is determined mainly by frequency, it also depends to a slight extent on loudness. For example, a very loud sound may seem slightly lower in pitch than a quiet sound of the same frequency.

CAUTION

Do not confuse ultrasonic (high frequency) with supersonic (high speed)

Sound waves whose frequencies are outside the audible range may reach the ear, but we are not generally aware of them. Frequencies above 20,000 Hz are called **ultrasonic** (do not confuse with *supersonic*, which is used for an object moving with a speed faster than the speed of sound). Many animals can hear ultrasonic frequencies; dogs, for example, can hear sounds as high as 50,000 Hz, and bats can detect frequencies as high as 100,000 Hz. Ultrasonic waves have many useful applications in medicine and other fields, which we discuss later in this Chapter.

PHYSICS APPLIED

Autofocusing camera

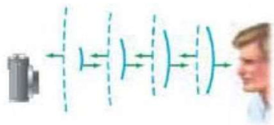


FIGURE 16–1 Example 16–2. Autofocusing camera emits an ultrasonic pulse. Solid lines represent the wave front of the outgoing wave pulse moving to the right; dashed lines represent the wave front of the pulse reflected off the person's face, returning to the camera. The time information allows the camera mechanism to adjust the lens to focus at the proper distance.

EXAMPLE 16–2 **Autofocusing with sound waves.** Older autofocusing cameras determine the distance by emitting a pulse of very high frequency (ultrasonic) sound that travels to the object being photographed, and include a sensor that detects the returning reflected sound, as shown in Fig. 16–1. To get an idea of the time sensitivity of the detector, calculate the travel time of the pulse for an object (*a*) 1.0 m away, and (*b*) 20 m away.

APPROACH If we assume the temperature is about 20°C, then the speed of sound is 343 m/s. Using this speed v and the total distance d back and forth in each case, we can obtain the time $(v = d/t)$.

SOLUTION (*a*) The pulse travels 1.0 m to the object and 1.0 m back, for a total of 2.0 m. We solve for t in $v = d/t$:

$$t = \frac{d}{v} = \frac{2.0\ \text{m}}{343\ \text{m/s}} = 0.0058\ \text{s} = 5.8\ \text{ms}.$$

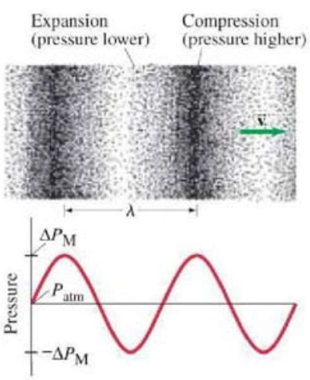
(*b*) The total distance now is $2 \times 20\ \text{m} = 40\ \text{m}$, so

$$t = \frac{40\ \text{m}}{343\ \text{m/s}} = 0.12\ \text{s} = 120\ \text{ms}.$$

NOTE Newer autofocus cameras use infrared light $(v = 3 \times 10^8\ \text{m/s})$ instead of ultrasound, and/or a digital sensor array that detects light intensity differences between adjacent receptors as the lens is automatically moved back and forth, choosing the lens position that provides maximum intensity differences (sharpest focus).

Sound waves whose frequencies are below the audible range (that is, less than 20 Hz) are called **infrasonic**. Sources of infrasonic waves include earthquakes, thunder, volcanoes, and waves produced by vibrating heavy machinery. This last source can be particularly troublesome to workers, for infrasonic waves—even though inaudible—can cause damage to the human body. These low-frequency waves act in a resonant fashion, causing motion and irritation of the body's organs.

FIGURE 16–2 Longitudinal sound wave traveling to the right, and its graphical representation in terms of pressure.



16–2 Mathematical Representation of Longitudinal Waves

In Section 15–4, we saw that a one-dimensional sinusoidal wave traveling along the x axis can be represented by the relation (Eq. 15–10c)

$$D = A\sin(kx - \omega t), \tag{16–1}$$

where D is the displacement of the wave at position x and time t, and A is its *amplitude* (maximum value). The wave number k is related to the wavelength λ by $k = 2\pi/\lambda$, and $\omega = 2\pi f$ where f is the frequency. For a transverse wave—such as a wave on a string—the displacement D is perpendicular to the direction of wave propagation along the x axis. But for a longitudinal wave the displacement D is *along the direction of wave propagation*. That is, D is parallel to x and represents the displacement of a tiny volume element of the medium from its equilibrium position.

Longitudinal (sound) waves can also be considered from the point of view of variations in pressure rather than displacement. Indeed, longitudinal waves are often called **pressure waves**. The pressure variation is usually easier to measure than the displacement (see Example 16–7). As can be seen in Fig. 16–2, in a wave "compression" (where molecules are closest together), the pressure is higher than normal, whereas in an expansion (or rarefaction) the pressure is less than normal.

Figure 16–3 shows a graphical representation of a sound wave in air in terms of (a) displacement and (b) pressure. Note that the displacement wave is a quarter wavelength, or 90° ($\pi/2$ rad), out of phase with the pressure wave: where the pressure is a maximum or minimum, the displacement from equilibrium is zero; and where the pressure variation is zero, the displacement is a maximum or minimum.

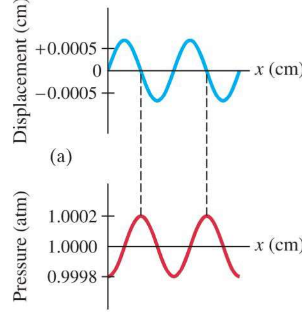


FIGURE 16–3 Representation of a sound wave in space at a given instant in terms of (a) displacement, and (b) pressure.

Pressure Wave Derivation

Let us now derive the mathematical representation of the pressure variation in a traveling longitudinal wave. From the definition of the bulk modulus, B (Eq. 12–7),

$$\Delta P = -B(\Delta V/V),$$

where ΔP represents the pressure difference from the normal pressure P_0 (no wave present) and $\Delta V/V$ is the fractional change in volume of the medium due to the pressure change ΔP. The negative sign reflects the fact that the volume decreases ($\Delta V < 0$) if the pressure is increased. Consider now a layer of fluid through which the longitudinal wave is passing (Fig. 16–4). If this layer has thickness Δx and area S, then its volume is $V = S\,\Delta x$. As a result of pressure variation in the wave, the volume will change by an amount $\Delta V = S\,\Delta D$, where ΔD is the change in thickness of this layer as it compresses or expands. (Remember that D represents the displacement of the medium.) Thus we have

$$\Delta P = -B\frac{S\,\Delta D}{S\,\Delta x}.$$

To be precise, we take the limit of $\Delta x \to 0$, so we obtain

$$\Delta P = -B\frac{\partial D}{\partial x}, \tag{16–2}$$

where we use the partial derivative notation since D is a function of both x and t. If the displacement D is sinusoidal as given by Eq. 16–1, then we have from Eq. 16–2 that

$$\Delta P = -(BAk)\cos(kx - \omega t). \tag{16–3}$$

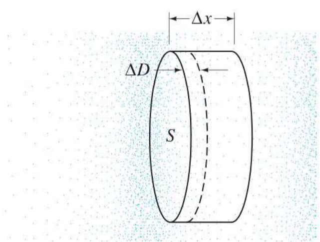
(Here A is the displacement amplitude, not area which is S.) Thus the pressure varies sinusoidally as well, but is out of phase from the displacement by 90° or a quarter wavelength, as in Fig. 16–3. The quantity BAk is called the **pressure amplitude**, ΔP_{M}. It represents the maximum and minimum amounts by which the pressure varies from the normal ambient pressure. We can thus write

$$\Delta P = -\Delta P_{\mathrm{M}}\cos(kx - \omega t), \tag{16–4}$$

where, using $v = \sqrt{B/\rho}$ (Eq. 15–4), and $k = \omega/v = 2\pi f/v$ (Eq. 15–12), then

$$\begin{aligned}\Delta P_{\mathrm{M}} &= BAk \\ &= \rho v^2 Ak \\ &= 2\pi\rho v\,Af.\end{aligned} \tag{16–5}$$

FIGURE 16–4 Longitudinal wave in a fluid moves to the right. A thin layer of fluid, in a thin cylinder of area S and thickness Δx, changes in volume as a result of pressure variation as the wave passes. At the moment shown, the pressure will increase as the wave moves to the right, so the thickness of our layer will decrease, by an amount ΔD.



16–3 Intensity of Sound: Decibels

Loudness is a sensation in the consciousness of a human being and is related to a physically measurable quantity, the **intensity** of the wave. Intensity is defined as the energy transported by a wave per unit time across a unit area perpendicular to the energy flow. As we saw in Chapter 15, intensity is proportional to the square of the wave amplitude. Intensity has units of power per unit area, or watts/meter^2 (W/m^2).

PHYSICS APPLIED
Wide range of human hearing

The human ear can detect sounds with an intensity as low as $10^{-12}\,\mathrm{W/m^2}$ and as high as $1\,\mathrm{W/m^2}$ (and even higher, although above this it is painful). This is an incredibly wide range of intensity, spanning a factor of 10^{12} from lowest to highest. Presumably because of this wide range, what we perceive as loudness is not directly proportional to the intensity. To produce a sound that sounds about twice as loud requires a sound wave that has about 10 times the intensity. This is roughly valid at any sound level for frequencies near the middle of the audible range. For example, a sound wave of intensity $10^{-2}\,\mathrm{W/m^2}$ sounds to an average human being like it is about twice as loud as one whose intensity is $10^{-3}\,\mathrm{W/m^2}$, and four times as loud as $10^{-4}\,\mathrm{W/m^2}$.

Sound Level

Because of this relationship between the subjective sensation of loudness and the physically measurable quantity "intensity," sound intensity levels are usually specified on a logarithmic scale. The unit on this scale is a **bel**, after the inventor Alexander Graham Bell, or much more commonly, the **decibel** (dB), which is $\frac{1}{10}$ bel (10 dB = 1 bel). The **sound level**, β, of any sound is defined in terms of its intensity, I, as

$$\beta\,(\text{in dB}) = 10 \log \frac{I}{I_0}, \tag{16–6}$$

where I_0 is the intensity of a chosen reference level, and the logarithm is to the base 10. I_0 is usually taken as the minimum intensity audible to a good ear—the "threshold of hearing," which is $I_0 = 1.0 \times 10^{-12}\,\mathrm{W/m^2}$. Thus, for example, the sound level of a sound whose intensity $I = 1.0 \times 10^{-10}\,\mathrm{W/m^2}$ will be

$$\beta = 10 \log\left(\frac{1.0 \times 10^{-10}\,\mathrm{W/m^2}}{1.0 \times 10^{-12}\,\mathrm{W/m^2}}\right) = 10 \log 100 = 20\,\mathrm{dB},$$

CAUTION

0 dB does not mean zero intensity

since log 100 is equal to 2.0. (Appendix A has a brief review of logarithms.) Notice that the sound level at the threshold of hearing is 0 dB. That is, $\beta = 10 \log 10^{-12}/10^{-12} = 10 \log 1 = 0$ since $\log 1 = 0$. Notice too that an increase in intensity by a factor of 10 corresponds to a sound level increase of 10 dB. An increase in intensity by a factor of 100 corresponds to a sound level increase of 20 dB. Thus a 50-dB sound is 100 times more intense than a 30-dB sound, and so on.

Intensities and sound levels for a number of common sounds are listed in Table 16–2.

TABLE 16–2
Intensity of Various Sounds

Source of the Sound	Sound Level (dB)	Intensity ($\mathrm{W/m^2}$)
Jet plane at 30 m	140	100
Threshold of pain	120	1
Loud rock concert	120	1
Siren at 30 m	100	1×10^{-2}
Truck traffic	90	1×10^{-3}
Busy street traffic	80	1×10^{-4}
Noisy restaurant	70	1×10^{-5}
Talk, at 50 cm	65	3×10^{-6}
Quiet radio	40	1×10^{-8}
Whisper	30	1×10^{-9}
Rustle of leaves	10	1×10^{-11}
Threshold of hearing	0	1×10^{-12}

EXAMPLE 16–3 **Sound intensity on the street.** At a busy street corner, the sound level is 75 dB. What is the intensity of sound there?

APPROACH We have to solve Eq. 16–6 for intensity I, remembering that $I_0 = 1.0 \times 10^{-12}\,\mathrm{W/m^2}$.

SOLUTION From Eq. 16–6

$$\log \frac{I}{I_0} = \frac{\beta}{10},$$

so

$$\frac{I}{I_0} = 10^{\beta/10}.$$

With $\beta = 75$, then

$$I = I_0 10^{\beta/10} = \left(1.0 \times 10^{-12}\,\mathrm{W/m^2}\right)\left(10^{7.5}\right) = 3.2 \times 10^{-5}\,\mathrm{W/m^2}.$$

NOTE Recall that $x = \log y$ is the same as $y = 10^x$ (Appendix A).

PHYSICS APPLIED

Loudspeaker response (± 3 dB)

EXAMPLE 16–4 **Loudspeaker response.** A high-quality loudspeaker is advertised to reproduce, at full volume, frequencies from 30 Hz to 18,000 Hz with uniform sound level ± 3 dB. That is, over this frequency range, the sound level output does not vary by more than 3 dB for a given input level. By what factor does the intensity change for the maximum change of 3 dB in output sound level?

APPROACH Let us call the average intensity I_1 and the average sound level β_1. Then the maximum intensity, I_2, corresponds to a level $\beta_2 = \beta_1 + 3\,\mathrm{dB}$. We then use the relation between intensity and sound level, Eq. 16–6.

SOLUTION Equation 16–6 gives

$$\begin{aligned}\beta_2 - \beta_1 &= 10\log\frac{I_2}{I_0} - 10\log\frac{I_1}{I_0}\\ 3\text{ dB} &= 10\left(\log\frac{I_2}{I_0} - \log\frac{I_1}{I_0}\right)\\ &= 10\log\frac{I_2}{I_1}\end{aligned}$$

because $(\log a - \log b) = \log a/b$ (see Appendix A). This last equation gives

$$\log\frac{I_2}{I_1} = 0.30,$$

or

$$\frac{I_2}{I_1} = 10^{0.30} = 2.0.$$

So $\pm$ 3 dB corresponds to a doubling or halving of the intensity.

It is worth noting that a sound-level difference of 3 dB (which corresponds to a doubled intensity, as we just saw) corresponds to only a very small change in the subjective sensation of apparent loudness. Indeed, the average human can distinguish a difference in sound level of only about 1 or 2 dB.

EXERCISE B If an increase of 3 dB means "twice as intense," what does an increase of 6 dB mean?

CONCEPTUAL EXAMPLE 16–5 **Trumpet players.** A trumpeter plays at a sound level of 75 dB. Three equally loud trumpet players join in. What is the new sound level?

RESPONSE The intensity of four trumpets is four times the intensity of one trumpet $(= I_1)$ or $4I_1$. The sound level of the four trumpets would be

$$\begin{aligned}\beta = 10\log\frac{4I_1}{I_0} &= 10\log 4 + 10\log\frac{I_1}{I_0}\\ &= 6.0\text{ dB} + 75\text{ dB} = 81\text{ dB}.\end{aligned}$$

EXERCISE C From Table 16–2, we see that ordinary conversation corresponds to a sound level of about 65 dB. If two people are talking at once, the sound level is (*a*) 65 dB, (*b*) 68 dB, (*c*) 75 dB, (*d*) 130 dB, (*e*) 62 dB.

Normally, the loudness or intensity of a sound decreases as you get farther from the source of the sound. In interior rooms, this effect is altered because of reflections from the walls. However, if a source is in the open so that sound can radiate out freely in all directions, the intensity decreases as the inverse square of the distance,

$$I \propto \frac{1}{r^2},$$

as we saw in Section 15–3. Over large distances, the intensity decreases faster than $1/r^2$ because some of the energy is transferred into irregular motion of air molecules. This loss happens more for higher frequencies, so any sound of mixed frequencies will be less "bright" at a distance.

PHYSICS APPLIED
Jet plane noise

EXAMPLE 16–6 **Airplane roar.** The sound level measured 30 m from a jet plane is 140 dB. What is the sound level at 300 m? (Ignore reflections from the ground.)

APPROACH Given the sound level, we can determine the intensity at 30 m using Eq. 16–6. Because intensity decreases as the square of the distance, ignoring reflections, we can find I at 300 m and again apply Eq. 16–6 to obtain the sound level.

SOLUTION The intensity I at 30 m is

$$140\text{ dB} = 10\log\left(\frac{I}{10^{-12}\text{ W/m}^2}\right)$$

or

$$14 = \log\left(\frac{I}{10^{-12}\text{ W/m}^2}\right).$$

We raise both sides of this equation to the power 10 (recall $10^{\log x} = x$) and have

$$10^{14} = \frac{I}{10^{-12}\text{ W/m}^2},$$

so $I = (10^{14})(10^{-12}\text{ W/m}^2) = 10^2\text{ W/m}^2$. At 300 m, 10 times as far, the intensity will be $\left(\frac{1}{10}\right)^2 = 1/100$ as much, or 1 W/m^2. Hence, the sound level is

$$\beta = 10\log\left(\frac{1\text{ W/m}^2}{10^{-12}\text{ W/m}^2}\right) = 120\text{ dB}.$$

Even at 300 m, the sound is at the threshold of pain. This is why workers at airports wear ear covers to protect their ears from damage (Fig. 16–5).

FIGURE 16–5 Example 16–6. Airport worker with sound-intensity-reducing ear covers (headphones).

NOTE Here is a simpler approach that avoids Eq. 16–6. Because the intensity decreases as the square of the distance, at 10 times the distance the intensity decreases by $\left(\frac{1}{10}\right)^2 = \frac{1}{100}$. We can use the result that 10 dB corresponds to an intensity change by a factor of 10 (see text just before Example 16–3). Then an intensity change by a factor of 100 corresponds to a sound-level change of $(2)(10\text{ dB}) = 20\text{ dB}$. This confirms our result above: $140\text{ dB} - 20\text{ dB} = 120\text{ dB}$.

Intensity Related to Amplitude

The intensity I of a wave is proportional to the square of the wave amplitude, as we saw in Chapter 15. We can therefore relate the amplitude quantitatively to the intensity I or level β, as the following Example shows.

PHYSICS APPLIED
Incredible sensitivity of the ear

EXAMPLE 16–7 **How tiny the displacement is.** (*a*) Calculate the displacement of air molecules for a sound having a frequency of 1000 Hz at the threshold of hearing. (*b*) Determine the maximum pressure variation in such a sound wave.

APPROACH In Section 15–3 we found a relation between intensity I and displacement amplitude A of a wave, Eq. 15–7. The amplitude of oscillation of air molecules is what we want to solve for, given the intensity. The pressure is found from Eq. 16–5.

SOLUTION (*a*) At the threshold of hearing, $I = 1.0 \times 10^{-12}\text{ W/m}^2$ (Table 16–2). We solve for the amplitude A in Eq. 15–7:

$$\begin{aligned} A &= \frac{1}{\pi f}\sqrt{\frac{I}{2\rho v}} \\ &= \frac{1}{(3.14)(1.0 \times 10^3\text{ s}^{-1})}\sqrt{\frac{1.0 \times 10^{-12}\text{ W/m}^2}{(2)(1.29\text{ kg/m}^3)(343\text{ m/s})}} \\ &= 1.1 \times 10^{-11}\text{ m}, \end{aligned}$$

where we have taken the density of air to be 1.29 kg/m^3 and the speed of sound in air (assumed 20°C) as 343 m/s.

NOTE We see how incredibly sensitive the human ear is: it can detect displacements of air molecules which are actually less than the diameter of atoms (about 10^{-10} m).

(*b*) Now we are dealing with sound as a pressure wave (Section 16–2). From Eq. 16–5,

$$\begin{aligned} \Delta P_{\text{M}} &= 2\pi\rho v A f \\ &= 2\pi(1.29\text{ kg/m}^3)(343\text{ m/s})(1.1 \times 10^{-11}\text{ m})(1.0 \times 10^3\text{ s}^{-1}) = 3.1 \times 10^{-5}\text{ Pa} \end{aligned}$$

or 3.1×10^{-10} atm. Again we see that the human ear is incredibly sensitive.

By combining Eqs. 15–7 and 16–5, we can write the intensity in terms of the pressure amplitude, ΔP_{M}:

$$I = 2\pi^2 v\rho f^2 A^2 = 2\pi^2 v\rho f^2\left(\frac{\Delta P_{\mathrm{M}}}{2\pi\rho v f}\right)^2$$

$$I = \frac{(\Delta P_{\mathrm{M}})^2}{2v\rho}. \qquad \textbf{(16–7)}$$

The intensity, when given in terms of pressure amplitude, thus does not depend on frequency.

The Ear's Response

The ear is not equally sensitive to all frequencies. To hear the same loudness for sounds of different frequencies requires different intensities. Studies averaged over large numbers of people have produced the curves shown in Fig. 16–6. On this graph, each curve represents sounds that seemed to be equally loud. The number labeling each curve represents the **loudness level** (the units are called *phons*), which is numerically equal to the sound level in dB at 1000 Hz. For example, the curve labeled 40 represents sounds that are heard by an average person to have the same loudness as a 1000-Hz sound with a sound level of 40 dB. From this 40-phon curve, we see that a 100-Hz tone must be at a level of about 62 dB to be perceived as loud as a 1000-Hz tone of only 40 dB.

The lowest curve in Fig. 16–6 (labeled 0) represents the sound level, as a function of frequency, for the *threshold of hearing*, the softest sound that is just audible by a very good ear. Note that the ear is most sensitive to sounds of frequency between 2000 and 4000 Hz, which are common in speech and music. Note too that whereas a 1000-Hz sound is audible at a level of 0 dB, a 100-Hz sound must be nearly 40 dB to be heard. The top curve in Fig. 16–6, labeled 120 phons, represents the *threshold of pain*. Sounds above this level can actually be felt and cause pain.

Figure 16–6 shows that at lower sound levels, our ears are less sensitive to the high and low frequencies relative to middle frequencies. The "loudness" control on some stereo systems is intended to compensate for this low-volume insensitivity. As the volume is turned down, the loudness control boosts the high and low frequencies relative to the middle frequencies so that the sound will have a more "normal-sounding" frequency balance. Many listeners, however, find the sound more pleasing or natural without the loudness control.

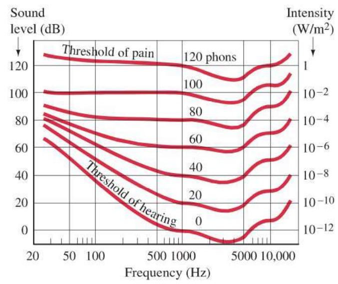


FIGURE 16–6 Sensitivity of the human ear as a function of frequency (see text). Note that the frequency scale is "logarithmic" in order to cover a wide range of frequencies.

16–4 Sources of Sound: Vibrating Strings and Air Columns

The source of any sound is a vibrating object. Almost any object can vibrate and hence be a source of sound. We now discuss some simple sources of sound, particularly musical instruments. In musical instruments, the source is set into vibration by striking, plucking, bowing, or blowing. Standing waves are produced and the source vibrates at its natural resonant frequencies. The vibrating source is in contact with the air (or other medium) and pushes on it to produce sound waves that travel outward. The frequencies of the waves are the same as those of the source, but the speed and wavelengths can be different. A drum has a stretched membrane that vibrates. Xylophones and marimbas have metal or wood bars that can be set into vibration. Bells, cymbals, and gongs also make use of a vibrating metal. Many instruments make use of vibrating strings, such as the violin, guitar, and piano, or make use of vibrating columns of air, such as the flute, trumpet, and pipe organ. We have already seen that the pitch of a pure sound is determined by the frequency. Typical frequencies for musical notes on the "equally tempered chromatic scale" are given in Table 16–3 for the octave beginning with middle C. Note that one octave corresponds to a doubling of frequency. For example, middle C has frequency of 262 Hz whereas C′ (C above middle C) has twice that frequency, 524 Hz. [Middle C is the C or "do" note at the middle of a piano keyboard.]

TABLE 16–3 Equally Tempered Chromatic Scale†

Note	Frequency (Hz)
C	262
C♯ or D♭	277
D	294
D♯ or E♭	311
E	330
F	349
F♯ or G♭	370
G	392
G♯ or A♭	415
A	440
A♯ or B♭	466
B	494
C′	524

†Only one octave is included.

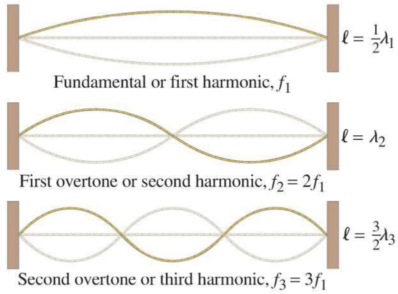


FIGURE 16–7 Standing waves on a string—only the lowest three frequencies are shown.

Stringed Instruments

PHYSICS APPLIED
Stringed instruments

We saw in Chapter 15, Fig. 15–26b, how standing waves are established on a string, and we show this again here in Fig. 16–7. Such standing waves are the basis for all stringed instruments. The pitch is normally determined by the lowest resonant frequency, the **fundamental**, which corresponds to nodes occurring only at the ends. The string vibrating up and down as a whole corresponds to a half wavelength as shown at the top of Fig. 16–7; so the wavelength of the fundamental on the string is equal to twice the length of the string. Therefore, the fundamental frequency is $f_1 = v/\lambda = v/2\ell$, where v is the velocity of the wave on the string (*not* in the air). The possible frequencies for standing waves on a stretched string are whole-number multiples of the fundamental frequency:

$$f_n = nf_1 = n\frac{v}{2\ell}, \qquad n = 1, 2, 3, \cdots$$

where $n = 1$ refers to the fundamental and $n = 2, 3, \cdots$ are the overtones. All of the standing waves, $n = 1, 2, 3, \cdots$, are called harmonics,† as we saw in Section 15–9.

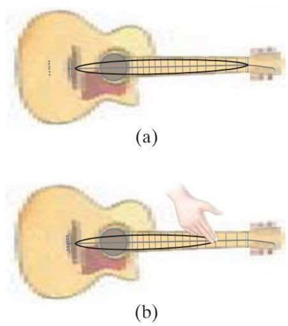


FIGURE 16–8 The wavelength of (a) an unfingered string is longer than that of (b) a fingered string. Hence, the frequency of the fingered string is higher. Only one string is shown on this guitar, and only the simplest standing wave, the fundamental, is shown.

When a finger is placed on the string of a guitar or violin, the effective length of the string is shortened. So its fundamental frequency, and pitch, is higher since the wavelength of the fundamental is shorter (Fig. 16–8). The strings on a guitar or violin are all the same length. They sound at a different pitch because the strings have different mass per unit length, μ, which affects the velocity on the string, Eq. 15–2,

$$v = \sqrt{F_T/\mu}. \qquad \text{[stretched string]}$$

Thus the velocity on a heavier string is lower and the frequency will be lower for the same wavelength. The tension F_T may also be different. Adjusting the tension is the means for tuning the pitch of each string. In pianos and harps the strings are of different lengths. For the lower notes the strings are not only longer, but heavier as well, and the reason is illustrated in the following Example.

EXAMPLE 16–8 **Piano strings.** The highest key on a piano corresponds to a frequency about 150 times that of the lowest key. If the string for the highest note is 5.0 cm long, how long would the string for the lowest note have to be if it had the same mass per unit length and was under the same tension?

APPROACH Since $v = \sqrt{F_T/\mu}$, the velocity would be the same on each string. So the frequency is inversely proportional to the length ℓ of the string $(f = v/\lambda = v/2\ell)$.

SOLUTION We can write, for the fundamental frequencies of each string, the ratio

$$\frac{\ell_L}{\ell_H} = \frac{f_H}{f_L},$$

where the subscripts L and H refer to the lowest and highest notes, respectively. Thus $\ell_L = \ell_H(f_H/f_L) = (5.0\text{ cm})(150) = 750\text{ cm}$, or 7.5 m. This would be ridiculously long ($\approx$ 25 ft) for a piano.

NOTE The longer strings of lower frequency are made heavier, of higher mass per unit length, so even on grand pianos the strings are less than 3 m long.

†When the resonant frequencies above the fundamental (that is, the overtones) are integral multiples of the fundamental, as here, they are called harmonics. But if the overtones are not integral multiples of the fundamental, as is the case for a vibrating drumhead, for example, they are not harmonics.

EXERCISE D Two strings have the same length and tension, but one is more massive than the other. Which plays the higher note?

CAUTION
Speed of standing wave on string ≠ speed of sound wave in air

EXAMPLE 16–9 **Frequencies and wavelengths in the violin.** A 0.32-m-long violin string is tuned to play A above middle C at 440 Hz. (*a*) What is the wavelength of the fundamental string vibration, and (*b*) what are the frequency and wavelength of the sound wave produced? (*c*) Why is there a difference?

APPROACH The wavelength of the fundamental string vibration equals twice the length of the string (Fig. 16–7). As the string vibrates, it pushes on the air, which is thus forced to oscillate at the same frequency as the string.

SOLUTION (*a*) From Fig. 16–7 the wavelength of the fundamental is

$$\lambda = 2\ell = 2(0.32\text{ m}) = 0.64\text{ m} = 64\text{ cm}.$$

This is the wavelength of the standing wave on the string.

(*b*) The sound wave that travels outward in the air (to reach our ears) has the same frequency, 440 Hz. Its wavelength is

$$\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{440\text{ Hz}} = 0.78\text{ m} = 78\text{ cm},$$

where v is the speed of sound in air (assumed at 20°C), Section 16–1.

(*c*) The wavelength of the sound wave is different from that of the standing wave on the string because the speed of sound in air (343 m/s at 20°C) is different from the speed of the wave on the string $(= f\lambda = 440\text{ Hz} \times 0.64\text{ m} = 280\text{ m/s})$ which depends on the tension in the string and its mass per unit length.

NOTE The frequencies on the string and in the air are the same: the string and air are in contact, and the string "forces" the air to vibrate at the same frequency. But the wavelengths are different because the wave speed on the string is different than that in air.

(a)

(b)

FIGURE 16–9 (a) Piano, showing sounding board to which the strings are attached; (b) sounding box (guitar).

Stringed instruments would not be very loud if they relied on their vibrating strings to produce the sound waves since the strings are too thin to compress and expand much air. Stringed instruments therefore make use of a kind of mechanical amplifier known as a *sounding board* (piano) or *sounding box* (guitar, violin), which acts to amplify the sound by putting a greater surface area in contact with the air (Fig. 16–9). When the strings are set into vibration, the sounding board or box is set into vibration as well. Since it has much greater area in contact with the air, it can produce a more intense sound wave. On an electric guitar, the sounding box is not so important since the vibrations of the strings are amplified electronically.

Wind Instruments

Instruments such as woodwinds, the brasses, and the pipe organ produce sound from the vibrations of standing waves in a column of air within a tube (Fig. 16–10). Standing waves can occur in the air of any cavity, but the frequencies present are complicated for any but very simple shapes such as the uniform, narrow tube of a flute or an organ pipe. In some instruments, a vibrating reed or the vibrating lip of the player helps to set up vibrations of the air column. In others, a stream of air is directed against one edge of the opening or mouthpiece, leading to turbulence which sets up the vibrations. Because of the disturbance, whatever its source, the air within the tube vibrates with a variety of frequencies, but only frequencies that correspond to standing waves will persist.

FIGURE 16–10 Wind instruments: flute (left) and clarinet.

For a string fixed at both ends, Fig. 16–7, we saw that the standing waves have nodes (no movement) at the two ends, and one or more antinodes (large amplitude of vibration) in between. A node separates successive antinodes. The lowest-frequency standing wave, the *fundamental*, corresponds to a single antinode. The higher-frequency standing waves are called **overtones** or **harmonics**, as we saw in Section 15–9. Specifically, the first harmonic is the fundamental, the second harmonic (= first overtone) has twice the frequency of the fundamental, and so on.

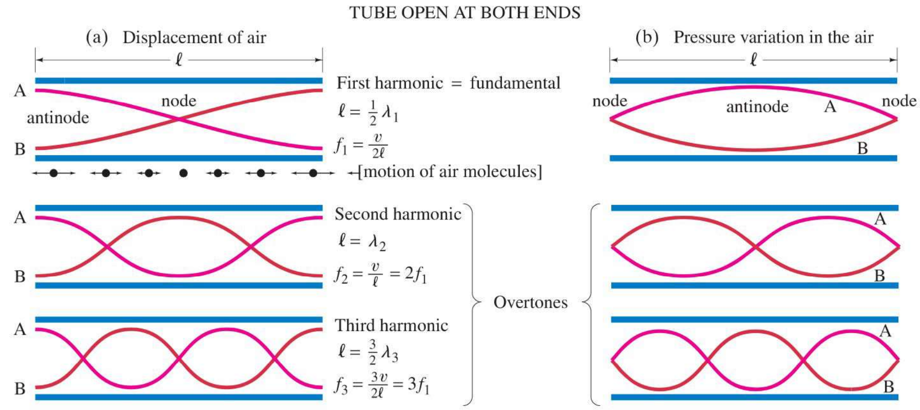


FIGURE 16–11 Graphs of the three simplest modes of vibration (standing waves) for a uniform tube open at both ends ("open tube"). These simplest modes of vibration are graphed in (a), on the left, in terms of the motion of the air (displacement), and in (b), on the right, in terms of air pressure. Each graph shows the wave format at two times, A and B, a half period apart. The actual motion of molecules for one case, the fundamental, is shown just below the tube at top left.

The situation is similar for a column of air in a tube of uniform diameter, but we must remember that it is now air itself that is vibrating. We can describe the waves either in terms of the flow of the air—that is, in terms of the *displacement* of air—or in terms of the *pressure* in the air (see Figs. 16–2 and 16–3). In terms of displacement, the air at the closed end of a tube is a displacement node since the air is not free to move there, whereas near the open end of a tube there will be an antinode because the air can move freely in and out. The air within the tube vibrates in the form of longitudinal standing waves. The possible modes of vibration for a tube open at both ends (called an **open tube**) are shown graphically in Fig. 16–11. They are shown for a tube that is open at one end but closed at the other (called a **closed tube**) in Fig. 16–12. [A tube closed at *both* ends, having no connection to the outside air, would be useless as an instrument.] The graphs in part (a) of each Figure (left-hand sides) represent the displacement amplitude of the vibrating air in the tube. Note that these are graphs, and that the air molecules themselves oscillate *horizontally*, parallel to the tube length, as shown by the small arrows in the top diagram of Fig. 16–11a (on the left). The exact position of the antinode near the open end of a tube depends on the diameter of the tube, but if the diameter is small compared to the length, which is the usual case, the antinode occurs very close to the end as shown. We assume this is the case in what follows. (The position of the antinode may also depend slightly on the wavelength and other factors.)

PHYSICS APPLIED
Wind instruments

Let us look in detail at the open tube, in Fig. 16–11a, which might be an organ pipe or a flute. An open tube has displacement antinodes at both ends since the air is free to move at open ends. There must be at least one node within an open tube if there is to be a standing wave at all. A single node corresponds to the *fundamental frequency* of the tube. Since the distance between two successive nodes, or between two successive antinodes, is $\frac{1}{2}\lambda$, there is one-half of a wavelength within the length of the tube for the simplest case of the fundamental (top diagram in Fig. 16–11a): $\ell = \frac{1}{2}\lambda$, or $\lambda = 2\ell$. So the fundamental frequency is $f_1 = v/\lambda = v/2\ell$, where v is the velocity of sound in air (the air in the tube). The standing wave with two nodes is the *first overtone* or *second harmonic* and has half the wavelength $(\ell = \lambda)$ and twice the frequency of the fundamental. Indeed, in a uniform tube open at both ends, the frequency of each overtone is an integral multiple of the fundamental frequency, as shown in Fig. 16–11a. This is just what is found for a string.

For a closed tube, shown in Fig. 16–12a, which could be an organ pipe, there is always a displacement node at the closed end (because the air is not free to move) and an antinode at the open end (where the air can move freely). Since the distance between a node and the nearest antinode is $\frac{1}{4}\lambda$, we see that the fundamental in a closed tube corresponds to only one-fourth of a wavelength within the length of the tube: $\ell = \lambda/4$, and $\lambda = 4\ell$. The fundamental frequency is thus $f_1 = v/4\ell$, or half that for an open pipe of the same length. There is another

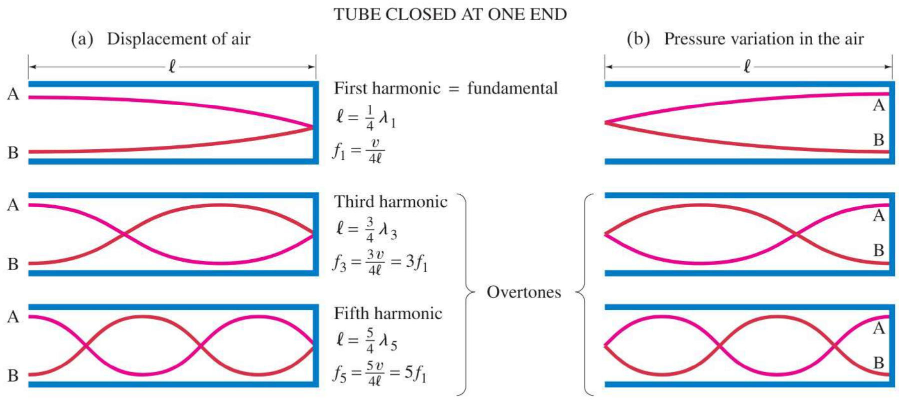


FIGURE 16–12 Modes of vibration (standing waves) for a tube closed at one end ("closed tube"). See caption for Fig. 16–11.

difference, for as we can see from Fig. 16–12a, only the odd harmonics are present in a closed tube: the overtones have frequencies equal to 3, 5, 7, … times the fundamental frequency. There is no way for waves with 2, 4, 6, … times the fundamental frequency to have a node at one end and an antinode at the other, and thus they cannot exist as standing waves in a closed tube.

Another way to analyze the vibrations in a uniform tube is to consider a description in terms of the *pressure* in the air, shown in part (b) of Figs. 16–11 and 16–12 (right-hand sides). Where the air in a wave is compressed, the pressure is higher, whereas in a wave expansion (or rarefaction), the pressure is less than normal. The open end of a tube is open to the atmosphere. Hence the pressure variation at an open end must be a *node*: the pressure does not alternate, but remains at the outside atmospheric pressure. If a tube has a closed end, the pressure at that closed end can readily alternate to be above or below atmospheric pressure. Hence there is a pressure *antinode* at a closed end of a tube. There can be pressure nodes and antinodes within the tube. Some of the possible vibrational modes in terms of pressure for an open tube are shown in Fig. 16–11b, and for a closed tube are shown in Fig. 16–12b.

EXAMPLE 16–10 **Organ pipes.** What will be the fundamental frequency and first three overtones for a 26-cm-long organ pipe at 20°C if it is (*a*) open and (*b*) closed?

APPROACH All our calculations can be based on Figs. 16–11a and 16–12a.

SOLUTION (*a*) For the open pipe, Fig. 16–11a, the fundamental frequency is

$$f_1 = \frac{v}{2\ell} = \frac{343\ \text{m/s}}{2(0.26\ \text{m})} = 660\ \text{Hz}.$$

The speed v is the speed of sound in air (the air vibrating in the pipe). The overtones include all harmonics: 1320 Hz, 1980 Hz, 2640 Hz, and so on.

(*b*) For a closed pipe, Fig. 16–12a, the fundamental frequency is

$$f_1 = \frac{v}{4\ell} = \frac{343\ \text{m/s}}{4(0.26\ \text{m})} = 330\ \text{Hz}.$$

Only odd harmonics are present: the first three overtones are 990 Hz, 1650 Hz, and 2310 Hz.

NOTE The closed pipe plays 330 Hz, which, from Table 16–3, is E above middle C, whereas the open pipe of the same length plays 660 Hz, an octave higher.

Pipe organs use both open and closed pipes, with lengths from a few centimeters to 5 m or more. A flute acts as an open tube, for it is open not only where you blow into it, but also at the opposite end. The different notes on a flute are obtained by shortening the length of the vibrating air column, by uncovering holes along the tube (so a displacement antinode can occur at the hole). The shorter the length of the vibrating air column, the higher the fundamental frequency.

EXAMPLE 16–11 **Flute.** A flute is designed to play middle C (262 Hz) as the fundamental frequency when all the holes are covered. Approximately how long should the distance be from the mouthpiece to the far end of the flute? (This is only approximate since the antinode does not occur precisely at the mouthpiece.) Assume the temperature is 20°C.

APPROACH When all holes are covered, the length of the vibrating air column is the full length. The speed of sound in air at 20°C is 343 m/s. Because a flute is open at both ends, we use Fig. 16–11: the fundamental frequency f_1 is related to the length ℓ of the vibrating air column by $f = v/2\ell$.

SOLUTION Solving for ℓ, we find

$$\ell = \frac{v}{2f} = \frac{343\ \text{m/s}}{2\left(262\ \text{s}^{-1}\right)} = 0.655\ \text{m} \approx 0.66\ \text{m}.$$

EXERCISE E To see why players of wind instruments "warm up" their instruments (so they will be in tune), determine the fundamental frequency of the flute of Example 16–11 when all holes are covered and the temperature is 10°C instead of 20°C.

EXERCISE F Return to the Chapter-Opening Question, page 424, and answer it again now. Try to explain why you may have answered differently the first time.

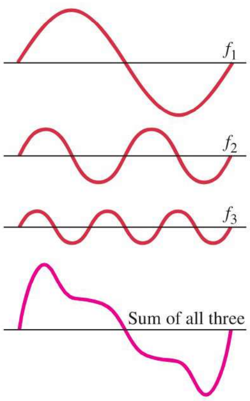
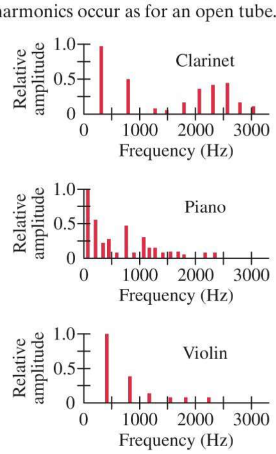


FIGURE 16–13 The amplitudes of the fundamental and first two overtones are added at each point to get the "sum," or composite waveform.

FIGURE 16–14 Sound spectra for different instruments. The spectra change when the instruments play different notes. The clarinet is a bit complicated: it acts like a closed tube at lower frequencies, having only odd harmonics, but at higher frequencies all harmonics occur as for an open tube.



*16–5 Quality of Sound, and Noise; Superposition

Whenever we hear a sound, particularly a musical sound, we are aware of its loudness, its pitch, and also of a third aspect called its *timbre* or "quality." For example, when a piano and then a flute play a note of the same loudness and pitch (say, middle C), there is a clear difference in the overall sound. We would never mistake a piano for a flute. This is what is meant by the timbre or *quality* of a sound. For musical instruments, the term *tone color* is also used.

Just as loudness and pitch can be related to physically measurable quantities, so too can quality. The quality of a sound depends on the presence of overtones—their number and their relative amplitudes. Generally, when a note is played on a musical instrument, the fundamental as well as overtones are present simultaneously. Figure 16–13 illustrates how the *principle of superposition* (Section 15–6) applies to three wave forms, in this case the fundamental and first two overtones (with particular amplitudes): they add together at each point to give a composite *waveform*. Normally, more than two overtones are present. [Any complex wave can be analyzed into a superposition of sinusoidal waves of appropriate amplitudes, wavelengths, and frequencies—see Section 15–6. Such an analysis is called a *Fourier analysis*.]

The relative amplitudes of the overtones for a given note are different for different musical instruments, which is what gives each instrument its characteristic quality or timbre. A bar graph showing the relative amplitudes of the harmonics for a given note produced by an instrument is called a *sound spectrum*. Several typical examples for different musical instruments are shown in Fig. 16–14. The fundamental usually has the greatest amplitude, and its frequency is what is heard as the pitch.

The manner in which an instrument is played strongly influences the sound quality. Plucking a violin string, for example, makes a very different sound than pulling a bow across it. The sound spectrum at the very start (or end) of a note (as when a hammer strikes a piano string) can be very different from the subsequent sustained tone. This too affects the subjective tone quality of an instrument.

An ordinary sound, like that made by striking two stones together, is a noise that has a certain quality, but a clear pitch is not discernible. Such a noise is a mixture of many frequencies which bear little relation to one another. A sound spectrum made of that noise would not show discrete lines like those of Fig. 16–14. Instead it would show a continuous, or nearly continuous, spectrum of frequencies. Such a sound we call "noise" in comparison with the more harmonious sounds which contain frequencies that are simple multiples of the fundamental.

16–6 Interference of Sound Waves; Beats

Interference in Space

We saw in Section 15–8 that when two waves simultaneously pass through the same region of space, they interfere with one another. Interference also occurs with sound waves.

Consider two large loudspeakers, A and B, a distance d apart on the stage of an auditorium as shown in Fig. 16–15. Let us assume the two speakers are emitting sound waves of the same single frequency and that they are in phase: that is, when one speaker is forming a compression, so is the other. (We ignore reflections from walls, floor, etc.) The curved lines in the diagram represent the crests of sound waves from each speaker at one instant in time. We must remember that for a sound wave, a crest is a compression in the air whereas a trough—which falls between two crests—is a rarefaction. A human ear or detector at a point such as C, which is the same distance from each speaker, will experience a loud sound because the interference will be constructive—two crests reach it at one moment, two troughs reach it a moment later. On the other hand, at a point such as D in the diagram, little if any sound will be heard because destructive interference occurs—compressions of one wave meet rarefactions of the other and vice versa (see Fig. 15–24 and the related discussion on water waves in Section 15–8).

An analysis of this situation is perhaps clearer if we graphically represent the waveforms as in Fig. 16–16. In Fig. 16–16a, it can be seen that at point C, constructive interference occurs since both waves simultaneously have crests or simultaneously have troughs when they arrive at C. In Fig. 16–16b we see that, to reach point D, the wave from speaker B must travel a greater distance than the wave from A. Thus the wave from B lags behind that from A. In this diagram, point E is chosen so that the distance ED is equal to AD. Thus we see that if the distance BE is equal to precisely one-half the wavelength of the sound, the two waves will be exactly out of phase when they reach D, and destructive interference occurs. This then is the criterion for determining at what points destructive interference occurs: destructive interference occurs at any point whose distance from one speaker is one-half wavelength greater than its distance from the other speaker. Notice that if this extra distance (BE in Fig. 16–16b) is equal to a whole wavelength (or 2, 3, … wavelengths), then the two waves will be in phase and *constructive interference* occurs. If the distance BE equals $\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, \ldots$ wavelengths, *destructive interference* occurs.

It is important to realize that a person at point D in Fig. 16–15 or 16–16 hears nothing at all (or nearly so), yet sound is coming from both speakers. Indeed, if one of the speakers is turned off, the sound from the other speaker will be clearly heard.

If a loudspeaker emits a whole range of frequencies, only specific wavelengths will destructively interfere completely at a given point.

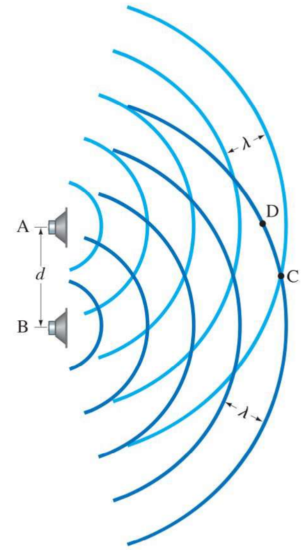
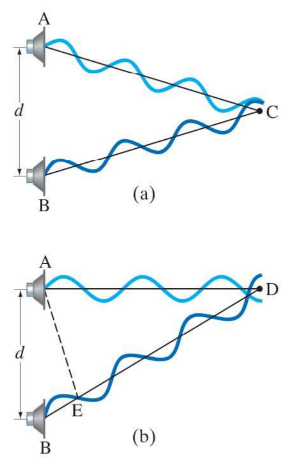


FIGURE 16–15 Sound waves from two loudspeakers interfere.

FIGURE 16–16 Sound waves of a single frequency from loudspeakers A and B (see Fig. 16–15) constructively interfere at C and destructively interfere at D. [Shown here are graphical representations, not the actual longitudinal sound waves.]



EXAMPLE 16–12 **Loudspeakers' interference.** Two loudspeakers are 1.00 m apart. A person stands 4.00 m from one speaker. How far must this person be from the second speaker to detect destructive interference when the speakers emit an 1150-Hz sound? Assume the temperature is 20°C.

APPROACH To sense destructive interference, the person must be one-half wavelength closer to or farther from one speaker than from the other—that is, at a distance $= 4.00\,\mathrm{m} \pm \lambda/2$. We can determine λ since we know f and v.

SOLUTION The speed of sound at 20°C is 343 m/s, so the wavelength of this sound is (Eq. 15–1)

$$\lambda = \frac{v}{f} = \frac{343\,\mathrm{m/s}}{1150\,\mathrm{Hz}} = 0.30\,\mathrm{m}.$$

For destructive interference to occur, the person must be one-half wavelength farther from one loudspeaker than from the other, or 0.15 m. Thus the person must be 3.85 m or 4.15 m from the second speaker.

NOTE If the speakers are less than 0.15 m apart, there will be no point that is 0.15 m farther from one speaker than the other, and there will be no point where destructive interference could occur.

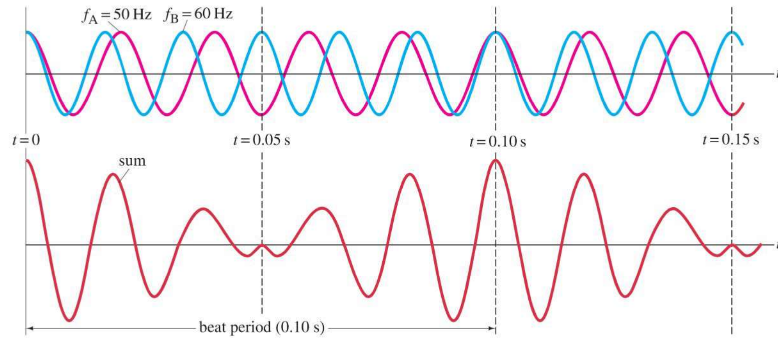


FIGURE 16–17 Beats occur as a result of the superposition of two sound waves of slightly different frequency.

Beats—Interference in Time

We have been discussing interference of sound waves that takes place in space. An interesting and important example of interference that occurs in time is the phenomenon known as **beats**: If two sources of sound—say, two tuning forks—are close in frequency but not exactly the same, sound waves from the two sources interfere with each other. The sound level at a given position alternately rises and falls in time, because the two waves are sometimes in phase and sometimes out of phase due to their different wavelengths. The regularly spaced intensity changes are called beats.

To see how beats arise, consider two equal-amplitude sound waves of frequency $f_A = 50\,\text{Hz}$ and $f_B = 60\,\text{Hz}$, respectively. In 1.00 s, the first source makes 50 vibrations whereas the second makes 60. We now examine the waves at one point in space equidistant from the two sources. The waveforms for each wave as a function of time, at a fixed position, are shown on the top graph of Fig. 16–17; the red line represents the 50-Hz wave, and the blue line represents the 60-Hz wave. The lower graph in Fig. 16–17 shows the sum of the two waves as a function of time. At time $t = 0$ the two waves are shown to be in phase and interfere constructively. Because the two waves vibrate at different rates, at time $t = 0.05\,\text{s}$ they are completely out of phase and interfere destructively. At $t = 0.10\,\text{s}$, they are again in phase and the resultant amplitude again is large. Thus the resultant amplitude is large every 0.10 s and drops drastically in between. This rising and falling of the intensity is what is heard as beats.† In this case the beats are 0.10 s apart. That is, the **beat frequency** is ten per second, or 10 Hz. This result, that the beat frequency equals the difference in frequency of the two waves, is valid in general, as we now show.

Let the two waves, of frequencies f_1 and f_2, be represented at a fixed point in space by

$$D_1 = A \sin 2\pi f_1 t$$

and

$$D_2 = A \sin 2\pi f_2 t.$$

The resultant displacement, by the principle of superposition, is

$$D = D_1 + D_2 = A(\sin 2\pi f_1 t + \sin 2\pi f_2 t).$$

Using the trigonometric identity $\sin\theta_1 + \sin\theta_2 = 2\sin\frac{1}{2}(\theta_1 + \theta_2)\cos\frac{1}{2}(\theta_1 - \theta_2)$, we have

$$D = \left[2A\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t\right]\sin 2\pi\left(\frac{f_1 + f_2}{2}\right)t. \tag{16–8}$$

We can interpret Eq. 16–8 as follows. The superposition of the two waves results in a wave that vibrates at the average frequency of the two components, $(f_1 + f_2)/2$. This vibration has an amplitude given by the expression in brackets, and this amplitude varies in time, from zero to a maximum of $2A$ (the sum of the separate amplitudes), with a frequency of $(f_1 - f_2)/2$. A beat occurs whenever $\cos 2\pi[(f_1 - f_2)/2]t$ equals $+1$ or -1 (see Fig. 16–17); that is, two beats occur per cycle, so the beat frequency is twice $(f_1 - f_2)/2$ which is just $f_1 - f_2$, the difference in frequency of the component waves.

†Beats will be heard even if the amplitudes are not equal, as long as the difference in amplitude is not great.

The phenomenon of beats can occur with any kind of wave and is a very sensitive method for comparing frequencies. For example, to tune a piano, a piano tuner listens for beats produced between his standard tuning fork and that of a particular string on the piano, and knows it is in tune when the beats disappear. The members of an orchestra tune up by listening for beats between their instruments and that of a standard tone (usually A above middle C at 440 Hz) produced by a piano or an oboe. A beat frequency is perceived as an intensity modulation (a wavering between loud and soft) for beat frequencies below 20 Hz or so, and as a separate low tone for higher beat frequencies (audible if the tones are strong enough).

PHYSICS APPLIED
Tuning a piano

EXAMPLE 16–13 **Beats.** A tuning fork produces a steady 400-Hz tone. When this tuning fork is struck and held near a vibrating guitar string, twenty beats are counted in five seconds. What are the possible frequencies produced by the guitar string?

APPROACH For beats to occur, the string must vibrate at a frequency different from 400 Hz by whatever the beat frequency is.

SOLUTION The beat frequency is

$$f_{\text{beat}} = 20\text{ vibrations}/5\text{ s} = 4\text{ Hz}.$$

This is the difference of the frequencies of the two waves. Because one wave is known to be 400 Hz, the other must be either 404 Hz or 396 Hz.

16–7 Doppler Effect

You may have noticed that you hear the pitch of the siren on a speeding fire truck drop abruptly as it passes you. Or you may have noticed the change in pitch of a blaring horn on a fast-moving car as it passes by you. The pitch of the engine noise of a racecar changes as the car passes an observer. When a source of sound is moving toward an observer, the pitch the observer hears is higher than when the source is at rest; and when the source is traveling away from the observer, the pitch is lower. This phenomenon is known as the **Doppler effect**† and occurs for all types of waves. Let us now see why it occurs, and calculate the difference between the perceived and source frequencies when there is relative motion between source and observer.

Consider the siren of a fire truck at rest, which is emitting sound of a particular frequency in all directions as shown in Fig. 16–18a. The sound waves are moving at the speed of sound in air, v_{snd}, which is independent of the velocity of the source or observer. If our source, the fire truck, is moving, the siren emits sound at the same frequency as it does at rest. But the sound wavefronts it emits forward, in front of it, are closer together than when the fire truck is at rest, as shown in Fig. 16–18b. This is because the fire truck, as it moves, is "chasing" the previously emitted wavefronts, and emits each crest closer to the previous one. Thus an observer on the sidewalk in front of the truck will detect more wave crests passing per second, so the frequency heard is higher. The wavefronts emitted behind the truck, on the other hand, are farther apart than when the truck is at rest because the truck is speeding away from them. Hence, fewer wave crests per second pass by an observer behind the moving truck (Fig. 16–18b) and the perceived pitch is lower.

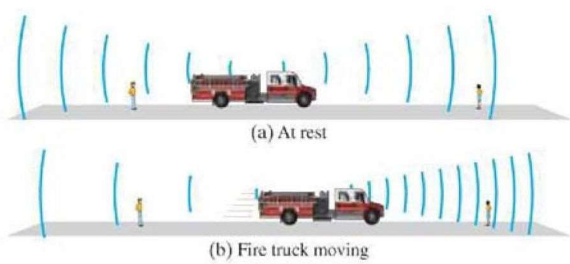


FIGURE 16–18 (a) Both observers on the sidewalk hear the same frequency from a fire truck at rest. (b) Doppler effect: observer toward whom the fire truck moves hears a higher-frequency sound, and observer behind the fire truck hears a lower-frequency sound.

†After J. C. Doppler (1803–1853).

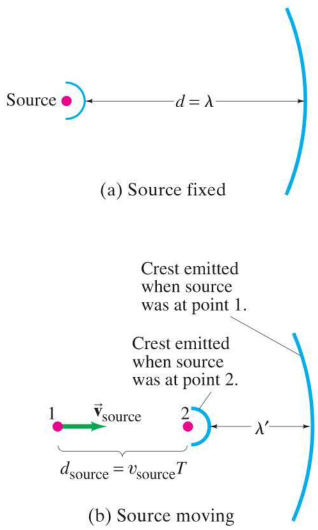


FIGURE 16–19 Determination of the frequency shift in the Doppler effect (see text). The red dot is the source.

We can calculate the frequency shift perceived by making use of Fig. 16–19, and we assume the air (or other medium) is at rest in our reference frame. (The stationary observer is off to the right.) In Fig. 16–19a, the source of the sound is shown as a red dot, and is at rest. Two successive wave crests are shown, the second of which has just been emitted and so is still near the source. The distance between these crests is λ, the wavelength. If the frequency of the source is f, then the time between emissions of wave crests is

$$T = \frac{1}{f} = \frac{\lambda}{v_{\text{snd}}}.$$

In Fig. 16–19b, the source is moving with a velocity v_{source} toward the observer. In a time T (as just defined), the first wave crest has moved a distance $d = v_{\text{snd}}T = \lambda$, where v_{snd} is the velocity of the sound wave in air (which is the same whether the source is moving or not). In this same time, the source has moved a distance $d_{\text{source}} = v_{\text{source}}T$. Then the distance between successive wave crests, which is the wavelength λ' the observer will perceive, is

$$\begin{aligned} \lambda' &= d - d_{\text{source}} \\ &= \lambda - v_{\text{source}}T \\ &= \lambda - v_{\text{source}}\frac{\lambda}{v_{\text{snd}}} \\ &= \lambda\left(1 - \frac{v_{\text{source}}}{v_{\text{snd}}}\right). \end{aligned}$$

We subtract λ from both sides of this equation and find that the shift in wavelength, $\Delta\lambda$, is

$$\Delta\lambda = \lambda' - \lambda = -\lambda\frac{v_{\text{source}}}{v_{\text{snd}}}.$$

So the shift in wavelength is directly proportional to the source speed v_{source}. The frequency f' that will be perceived by our stationary observer on the ground is given by

$$f' = \frac{v_{\text{snd}}}{\lambda'} = \frac{v_{\text{snd}}}{\lambda\left(1 - \frac{v_{\text{source}}}{v_{\text{snd}}}\right)}.$$

Since $v_{\text{snd}}/\lambda = f$, then

$$f' = \frac{f}{\left(1 - \frac{v_{\text{source}}}{v_{\text{snd}}}\right)}. \qquad \left[\begin{array}{l}\text{source moving toward}\\ \text{stationary observer}\end{array}\right] \qquad \textbf{(16–9a)}$$

Because the denominator is less than 1, the observed frequency f' is greater than the source frequency f. That is, $f' > f$. For example, if a source emits a sound of frequency 400 Hz when at rest, then when the source moves toward a fixed observer with a speed of 30 m/s, the observer hears a frequency (at 20°C) of

$$f' = \frac{400\ \text{Hz}}{1 - \frac{30\ \text{m/s}}{343\ \text{m/s}}} = 438\ \text{Hz}.$$

Now consider a source moving *away* from the stationary observer at a speed v_{source}. Using the same arguments as above, the wavelength λ' perceived by our observer will have the minus sign on d_{source} (second equation on this page) changed to plus:

$$\begin{aligned} \lambda' &= d + d_{\text{source}} \\ &= \lambda\left(1 + \frac{v_{\text{source}}}{v_{\text{snd}}}\right). \end{aligned}$$

The difference between the observed and emitted wavelengths will be $\Delta\lambda = \lambda' - \lambda = +\lambda(v_{\text{source}}/v_{\text{snd}})$. The observed frequency of the wave, $f' = v_{\text{snd}}/\lambda'$, will be

$$f' = \frac{f}{\left(1 + \frac{v_{\text{source}}}{v_{\text{snd}}}\right)}. \qquad \left[\begin{array}{l}\text{source moving away from}\\ \text{stationary observer}\end{array}\right] \qquad \textbf{(16–9b)}$$

If a source emitting at 400 Hz is moving away from a fixed observer at 30 m/s, the observer hears a frequency $f' = (400\ \text{Hz})/[1 + (30\ \text{m/s})/(343\ \text{m/s})] = 368\ \text{Hz}$.

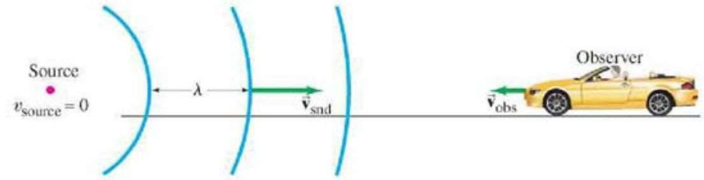


FIGURE 16–20 Observer moving with speed v_{obs} toward a stationary source detects wave crests passing at speed $v' = v_{snd} + v_{obs}$ where v_{snd} is the speed of the sound waves in air.

The Doppler effect also occurs when the source is at rest and the observer is in motion. If the observer is traveling *toward* the source, the pitch heard is higher than that of the emitted source frequency. If the observer is traveling *away* from the source, the pitch heard is lower. Quantitatively the change in frequency is different than for the case of a moving source. With a fixed source and a moving observer, the distance between wave crests, the wavelength λ, is not changed. But the velocity of the crests with respect to the observer *is* changed. If the observer is moving toward the source, Fig. 16–20, the speed v' of the waves relative to the observer is a simple addition of velocities: $v' = v_{snd} + v_{obs}$, where v_{snd} is the velocity of sound in air (we assume the air is still) and v_{obs} is the velocity of the observer. Hence, the frequency heard is

$$f' = \frac{v'}{\lambda} = \frac{v_{snd} + v_{obs}}{\lambda}.$$

Because $\lambda = v_{snd}/f$, then

$$f' = \frac{(v_{snd} + v_{obs})f}{v_{snd}},$$

or

$$f' = \left(1 + \frac{v_{obs}}{v_{snd}}\right)f. \qquad \begin{bmatrix}\text{observer moving toward} \\ \text{stationary source}\end{bmatrix} \qquad \textbf{(16–10a)}$$

If the observer is moving away from the source, the relative velocity is $v' = v_{snd} - v_{obs}$, so

$$f' = \left(1 - \frac{v_{obs}}{v_{snd}}\right)f. \qquad \begin{bmatrix}\text{observer moving away} \\ \text{from stationary source}\end{bmatrix} \qquad \textbf{(16–10b)}$$

EXAMPLE 16–14 **A moving siren.** The siren of a police car at rest emits at a predominant frequency of 1600 Hz. What frequency will you hear if you are at rest and the police car moves at 25.0 m/s (*a*) toward you, and (*b*) away from you?

APPROACH The observer is fixed, and the source moves, so we use Eqs. 16–9. The frequency you (the observer) hear is the emitted frequency f divided by the factor $(1 \pm v_{source}/v_{snd})$ where v_{source} is the speed of the police car. Use the minus sign when the car moves toward you (giving a higher frequency); use the plus sign when the car moves away from you (lower frequency).

SOLUTION (*a*) The car is moving toward you, so (Eq. 16–9a)

$$f' = \frac{f}{\left(1 - \dfrac{v_{source}}{v_{snd}}\right)} = \frac{1600\text{ Hz}}{\left(1 - \dfrac{25.0\text{ m/s}}{343\text{ m/s}}\right)} = 1726\text{ Hz} \approx 1730\text{ Hz}.$$

(*b*) The car is moving away from you, so (Eq. 16–9b)

$$f' = \frac{f}{\left(1 + \dfrac{v_{source}}{v_{snd}}\right)} = \frac{1600\text{ Hz}}{\left(1 + \dfrac{25.0\text{ m/s}}{343\text{ m/s}}\right)} = 1491\text{ Hz} \approx 1490\text{ Hz}.$$

EXERCISE G Suppose the police car of Example 16–14 is at rest and emits at 1600 Hz. What frequency would you hear if you were moving at 25.0 m/s (*a*) toward it, and (*b*) away from it?

When a sound wave is reflected from a moving obstacle, the frequency of the reflected wave will, because of the Doppler effect, be different from that of the incident wave. This is illustrated in the following Example.

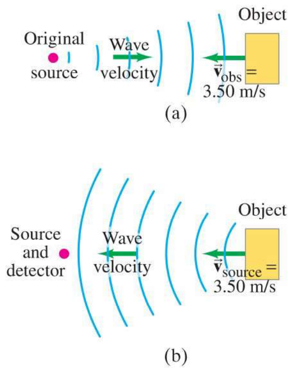


FIGURE 16–21 Example 16–15.

EXAMPLE 16–15 **Two Doppler shifts.** A 5000-Hz sound wave is emitted by a stationary source. This sound wave reflects from an object moving 3.50 m/s toward the source (Fig. 16–21). What is the frequency of the wave reflected by the moving object as detected by a detector at rest near the source?

APPROACH There are actually two Doppler shifts in this situation. First, the moving object acts like an observer moving toward the source with speed $v_{obs} = 3.50\,\mathrm{m/s}$ (Fig. 16–21a) and so "detects" a sound wave of frequency (Eq. 16–10a) $f' = f\left[1 + \left(v_{obs}/v_{snd}\right)\right]$. Second, reflection of the wave from the moving object is equivalent to the object reemitting the wave, acting effectively as a moving source with speed $v_{source} = 3.50\,\mathrm{m/s}$ (Fig. 16–21b). The final frequency detected, f'', is given by $f'' = f'/\left[1 - v_{source}/v_{snd}\right]$, Eq. 16–9a.

SOLUTION The frequency f' that is "detected" by the moving object is (Eq. 16–10a):

$$f' = \left(1 + \frac{v_{obs}}{v_{snd}}\right)f = \left(1 + \frac{3.50\,\mathrm{m/s}}{343\,\mathrm{m/s}}\right)(5000\,\mathrm{Hz}) = 5051\,\mathrm{Hz}.$$

The moving object now "emits" (reflects) a sound of frequency (Eq. 16–9a)

$$f'' = \frac{f'}{\left(1 - \dfrac{v_{source}}{v_{snd}}\right)} = \frac{5051\,\mathrm{Hz}}{\left(1 - \dfrac{3.50\,\mathrm{m/s}}{343\,\mathrm{m/s}}\right)} = 5103\,\mathrm{Hz}.$$

Thus the frequency shifts by 103 Hz.

NOTE Bats use this technique to be aware of their surroundings. This is also the principle behind Doppler radar as speed-measuring devices for vehicles and other objects.

PHYSICS APPLIED
Doppler blood-flow meter and other medical uses

The incident wave and the reflected wave in Example 16–15, when mixed together (say, electronically), interfere with one another and beats are produced. The beat frequency is equal to the difference in the two frequencies, 103 Hz. This Doppler technique is used in a variety of medical applications, usually with ultrasonic waves in the megahertz frequency range. For example, ultrasonic waves reflected from red blood cells can be used to determine the velocity of blood flow. Similarly, the technique can be used to detect the movement of the chest of a young fetus and to monitor its heartbeat.

For convenience, we can write Eqs. 16–9 and 16–10 as a single equation that covers all cases of both source and observer in motion:

$$f' = f\left(\frac{v_{snd} \pm v_{obs}}{v_{snd} \mp v_{source}}\right). \qquad \left[\begin{array}{l}\text{source and}\\ \text{observer moving}\end{array}\right] \qquad \textbf{(16–11)}$$

PROBLEM SOLVING
Getting the signs right

To get the signs right, recall from your own experience that the frequency is higher when observer and source approach each other, and lower when they move apart. Thus the upper signs in numerator and denominator apply if source and/or observer move toward each other; the lower signs apply if they are moving apart.

EXERCISE H How fast would a source have to approach an observer for the observed frequency to be one octave above (twice) the produced frequency? (*a*) $\frac{1}{2}v_{snd}$, (*b*) v_{snd}, (*c*) $2v_{snd}$, (*d*) $4v_{snd}$.

Doppler Effect for Light

The Doppler effect occurs for other types of waves as well. Light and other types of electromagnetic waves (such as radar) exhibit the Doppler effect: although the formulas for the frequency shift are not identical to Eqs. 16–9 and 16–10, as discussed in Chapter 44, the effect is similar. One important application is for weather forecasting using radar. The time delay between the emission of radar pulses and their reception after being reflected off raindrops gives the position of precipitation. Measuring the Doppler shift in frequency (as in Example 16–15) tells how fast the storm is moving and in which direction.

PHYSICS APPLIED
Doppler effect for EM waves and weather forecasting

Another important application is to astronomy, where the velocities of distant galaxies can be determined from the Doppler shift. Light from distant galaxies is shifted toward lower frequencies, indicating that the galaxies are moving away from us. This is called the **redshift** since red has the lowest frequency of visible light. The greater the frequency shift, the greater the velocity of recession. It is found that the farther the galaxies are from us, the faster they move away. This observation is the basis for the idea that the universe is expanding, and is one basis for the idea that the universe began as a great explosion, affectionately called the "Big Bang" (Chapter 44).

PHYSICS APPLIED
Redshift in cosmology

*16–8 Shock Waves and the Sonic Boom

An object such as an airplane traveling faster than the speed of sound is said to have a **supersonic speed**. Such a speed is often given as a **Mach**† number, which is defined as the ratio of the speed of the object to the speed of sound in the surrounding medium. For example, a plane traveling 600 m/s high in the atmosphere, where the speed of sound is only 300 m/s, has a speed of Mach 2.

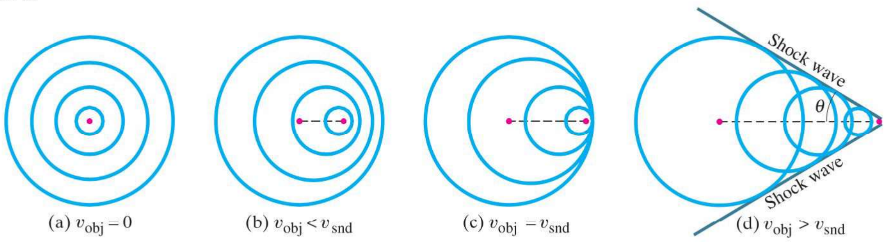


FIGURE 16–22 Sound waves emitted by an object (a) at rest or (b, c, and d) moving. (b) If the object's velocity is less than the velocity of sound, the Doppler effect occurs; (d) if its velocity is greater than the velocity of sound, a shock wave is produced.

When a source of sound moves at subsonic speeds (less than the speed of sound), the pitch of the sound is altered as we have seen (the Doppler effect); see also Fig. 16–22a and b. But if a source of sound moves faster than the speed of sound, a more dramatic effect known as a **shock wave** occurs. In this case, the source is actually "outrunning" the waves it produces. As shown in Fig. 16–22c, when the source is traveling *at* the speed of sound, the wave fronts it emits in the forward direction "pile up" directly in front of it. When the object moves faster, at a supersonic speed, the wave fronts pile up on one another along the sides, as shown in Fig. 16–22d. The different wave crests overlap one another and form a single very large crest which is the shock wave. Behind this very large crest there is usually a very large trough. A shock wave is essentially the result of constructive interference of a large number of wave fronts. A shock wave in air is analogous to the bow wave of a boat traveling faster than the speed of the water waves it produces, Fig. 16–23.

FIGURE 16–23 Bow waves produced by a boat.

†After the Austrian physicist Ernst Mach (1838–1916).

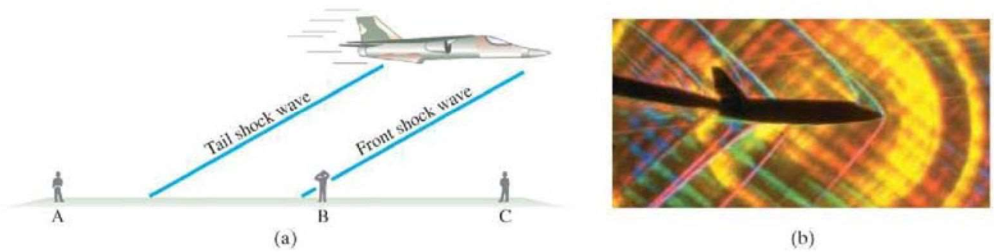


FIGURE 16–24 (a) The (double) sonic boom has already been heard by person A on the left. The front shock wave is just being heard by person B in the center. And it will shortly be heard by person C on the right. (b) Special photo of supersonic aircraft showing shock waves produced in the air. (Several closely spaced shock waves are produced by different parts of the aircraft.)

PHYSICS APPLIED
Sonic boom

When an airplane travels at supersonic speeds, the noise it makes and its disturbance of the air form into a shock wave containing a tremendous amount of sound energy. When the shock wave passes a listener, it is heard as a loud *sonic boom*. A sonic boom lasts only a fraction of a second, but the energy it contains is often sufficient to break windows and cause other damage. Actually, a sonic boom is made up of two or more booms since major shock waves can form at the front and the rear of the aircraft, as well as at the wings, etc. (Fig. 16–24). Bow waves of a boat are also multiple, as can be seen in Fig. 16–23.

When an aircraft approaches the speed of sound, it encounters a barrier of sound waves in front of it (see Fig. 16–22c). To exceed the speed of sound, the aircraft needs extra thrust to pass through this "sound barrier." This is called "breaking the sound barrier." Once a supersonic speed is attained, this barrier no longer impedes the motion. It is sometimes erroneously thought that a sonic boom is produced only at the moment an aircraft is breaking through the sound barrier. Actually, a shock wave follows the aircraft at all times it is traveling at supersonic speeds. A series of observers on the ground will each hear a loud "boom" as the shock wave passes, Fig. 16–24. The shock wave consists of a cone whose apex is at the aircraft. The angle of this cone, θ (see Fig. 16–22d), is given by

$$\sin\theta = \frac{v_{\text{snd}}}{v_{\text{obj}}}, \tag{16–12}$$

where v_{obj} is the velocity of the object (the aircraft) and v_{snd} is the velocity of sound in the medium. (The proof is left as Problem 75.)

*16–9 Applications: Sonar, Ultrasound, and Medical Imaging

*Sonar

PHYSICS APPLIED
Sonar: depth finding, Earth soundings

The reflection of sound is used in many applications to determine distance. The **sonar**† or pulse-echo technique is used to locate underwater objects. A transmitter sends out a sound pulse through the water, and a detector receives its reflection, or echo, a short time later. This time interval is carefully measured, and from it the distance to the reflecting object can be determined since the speed of sound in water is known. The depth of the sea and the location of reefs, sunken ships, submarines, or schools of fish can be determined in this way. The interior structure of the Earth is studied in a similar way by detecting reflections of waves traveling through the Earth whose source was a deliberate explosion (called "soundings"). An analysis of waves reflected from various structures and boundaries within the Earth reveals characteristic patterns that are also useful in the exploration for oil and minerals.

†Sonar stands for "*so*und *na*vigation *r*anging."

Sonar generally makes use of **ultrasonic** frequencies: that is, waves whose frequencies are above 20 kHz, beyond the range of human detection. For sonar, the frequencies are typically in the range 20 kHz to 100 kHz. One reason for using ultrasound waves, other than the fact that they are inaudible, is that for shorter wavelengths there is less diffraction (Section 15–11) so the beam spreads less and smaller objects can be detected.

* Ultrasound Medical Imaging

The diagnostic use of ultrasound in medicine, in the form of images (sometimes called *sonograms*), is an important and interesting application of physical principles. A **pulse-echo technique** is used, much like sonar, except that the frequencies used are in the range of 1 to 10 MHz $(1\text{ MHz} = 10^6\text{ Hz})$. A high-frequency sound pulse is directed into the body, and its reflections from boundaries or interfaces between organs and other structures and lesions in the body are then detected. Tumors and other abnormal growths, or pockets of fluid, can be distinguished; the action of heart valves and the development of a fetus can be examined; and information about various organs of the body, such as the brain, heart, liver, and kidneys, can be obtained. Although ultrasound does not replace X-rays, for certain kinds of diagnosis it is more helpful. Some kinds of tissue or fluid are not detected in X-ray photographs, but ultrasound waves are reflected from their boundaries. "Real-time" ultrasound images are like a movie of a section of the interior of the body.

PHYSICS APPLIED
Ultrasound medical imaging

The pulse-echo technique for medical imaging works as follows. A brief pulse of ultrasound is emitted by a transducer that transforms an electrical pulse into a sound-wave pulse. Part of the pulse is reflected as echoes at each interface in the body, and most of the pulse (usually) continues on, Fig. 16–25a. The detection of reflected pulses by the same transducer can then be displayed on the screen of a display terminal or monitor. The time elapsed from when the pulse is emitted to when each reflection (echo) is received is proportional to the distance to the reflecting surface. For example, if the distance from transducer to the vertebra is 25 cm, the pulse travels a round-trip distance of $2 \times 25\text{ cm} = 0.50\text{ m}$. The speed of sound in human tissue is about 1540 m/s (close to that of sea water), so the time taken is

$$t = \frac{d}{v} = \frac{(0.50\text{ m})}{(1540\text{ m/s})} = 320\ \mu\text{s}.$$

The *strength* of a reflected pulse depends mainly on the difference in density of the two materials on either side of the interface and can be displayed as a pulse or as a dot (Figs. 16–25b and c). Each echo dot (Fig. 16–25c) can be represented as a point whose position is given by the time delay and whose brightness depends on the strength of the echo.

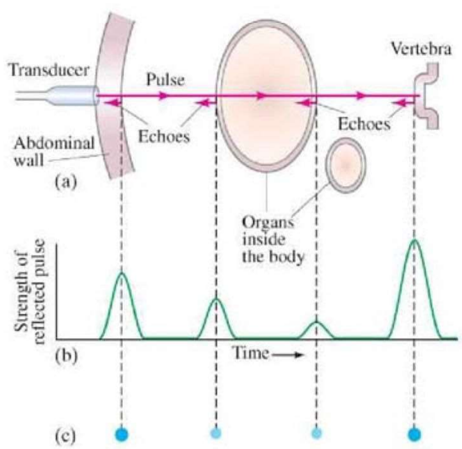


FIGURE 16–25 (a) Ultrasound pulse passes through the abdomen, reflecting from surfaces in its path. (b) Reflected pulses plotted as a function of time when received by transducer. The vertical dashed lines point out which reflected pulse goes with which surface. (c) Dot display for the same echoes: brightness of each dot is related to signal strength.

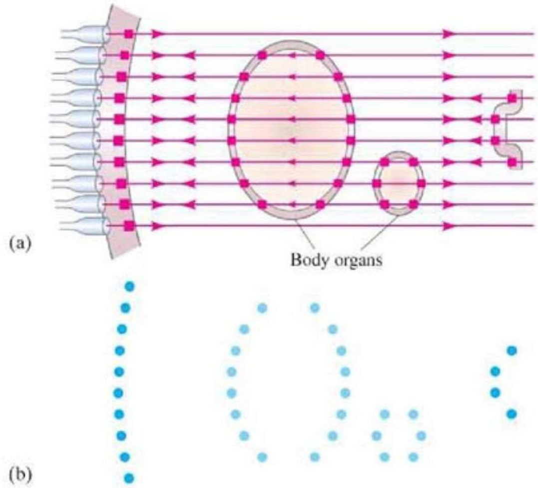


FIGURE 16–26 (a) Ten traces are made across the abdomen by moving the transducer, or by using an array of transducers. (b) The echoes are plotted as dots to produce the image. More closely spaced traces would give a more detailed image.

FIGURE 16–27 Ultrasound image of a human fetus within the uterus.

A two-dimensional image can then be formed out of these dots from a series of scans. The transducer is moved, or an array of transducers is used, each of which sends out a pulse at each position and receives echoes as shown in Fig. 16–26a. Each trace can be plotted, spaced appropriately one below the other, to form an image on a monitor screen as shown in Fig. 16–26b. Only 10 lines are shown in Fig. 16–26, so the image is crude. More lines give a more precise image.† An ultrasound image is shown in Fig. 16–27.

†*Radar* used for aircraft involves a similar pulse-echo technique except that it uses electromagnetic (EM) waves, which, like light, travel with a speed of 3×10^8 m/s.

Summary

Sound travels as a longitudinal wave in air and other materials. In air, the speed of sound increases with temperature; at 20°C, it is about 343 m/s.

The **pitch** of a sound is determined by the frequency; the higher the frequency, the higher the pitch.

The **audible range** of frequencies for humans is roughly 20 Hz to 20,000 Hz (1 Hz = 1 cycle per second).

The **loudness** or **intensity** of a sound is related to the amplitude squared of the wave. Because the human ear can detect sound intensities from 10^{-12} W/m^2 to over 1 W/m^2, sound levels are specified on a logarithmic scale. The **sound level** β, specified in decibels, is defined in terms of intensity I as

$$\beta(\text{in dB}) = 10 \log\left(\frac{I}{I_0}\right), \qquad \textbf{(16–6)}$$

where the reference intensity I_0 is usually taken to be 10^{-12} W/m^2.

Musical instruments are simple sources of sound in which *standing waves* are produced.

The strings of a stringed instrument may vibrate as a whole with nodes only at the ends; the frequency at which this standing wave occurs is called the **fundamental**. The fundamental frequency corresponds to a wavelength equal to twice the length of the string, $\lambda_1 = 2\ell$. The string can also vibrate at higher frequencies, called **overtones** or **harmonics**, in which there are one or more additional nodes. The frequency of each harmonic is a whole-number multiple of the fundamental.

In wind instruments, standing waves are set up in the column of air within the tube.

The vibrating air in an **open tube** (open at both ends) has displacement antinodes at both ends. The fundamental frequency corresponds to a wavelength equal to twice the tube length: $\lambda_1 = 2\ell$. The harmonics have frequencies that are 1, 2, 3, 4, … times the fundamental frequency, just as for strings.

For a **closed tube** (closed at one end), the fundamental corresponds to a wavelength four times the length of the tube: $\lambda_1 = 4\ell$. Only the odd harmonics are present, equal to 1, 3, 5, 7, … times the fundamental frequency.

Sound waves from different sources can interfere with each other. If two sounds are at slightly different frequencies, **beats** can be heard at a frequency equal to the difference in frequency of the two sources.

The **Doppler effect** refers to the change in pitch of a sound due to the motion either of the source or of the listener. If source and listener are approaching each other, the perceived pitch is higher; if they are moving apart, the perceived pitch is lower.

[*Shock waves and a sonic boom occur when an object moves at a supersonic speed—faster than the speed of sound. Ultrasonic-frequency (higher than 20 kHz) sound waves are used in many applications, including sonar and medical imaging.]

Questions

1. What is the evidence that sound travels as a wave?
2. What is the evidence that sound is a form of energy?
3. Children sometimes play with a homemade "telephone" by attaching a string to the bottoms of two paper cups. When the string is stretched and a child speaks into one cup, the sound can be heard at the other cup (Fig. 16–28). Explain clearly how the sound wave travels from one cup to the other.

FIGURE 16–28 Question 3.

4. When a sound wave passes from air into water, do you expect the frequency or wavelength to change?
5. What evidence can you give that the speed of sound in air does not depend significantly on frequency?
6. The voice of a person who has inhaled helium sounds very high-pitched. Why?
7. What is the main reason the speed of sound in hydrogen is greater than the speed of sound in air?
8. Two tuning forks oscillate with the same amplitude, but one has twice the frequency. Which (if either) produces the more intense sound?
9. How will the air temperature in a room affect the pitch of organ pipes?
10. Explain how a tube might be used as a filter to reduce the amplitude of sounds in various frequency ranges. (An example is a car muffler.)
11. Why are the frets on a guitar (Fig. 16–29) spaced closer together as you move up the fingerboard toward the bridge?

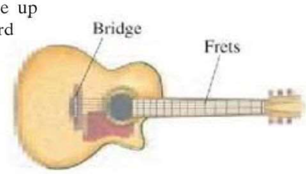


FIGURE 16–29 Question 11.

12. A noisy truck approaches you from behind a building. Initially you hear it but cannot see it. When it emerges and you do see it, its sound is suddenly "brighter"—you hear more of the high-frequency noise. Explain. [*Hint*: See Section 15–11 on diffraction.]
13. Standing waves can be said to be due to "interference in space," whereas beats can be said to be due to "interference in time." Explain.
14. In Fig. 16–15, if the frequency of the speakers is lowered, would the points D and C (where destructive and constructive interference occur) move farther apart or closer together?
15. Traditional methods of protecting the hearing of people who work in areas with very high noise levels have consisted mainly of efforts to block or reduce noise levels. With a relatively new technology, headphones are worn that do not block the ambient noise. Instead, a device is used which detects the noise, inverts it electronically, then feeds it to the headphones *in addition to* the ambient noise. How could adding *more* noise reduce the sound levels reaching the ears?
16. Consider the two waves shown in Fig. 16–30. Each wave can be thought of as a superposition of two sound waves with slightly different frequencies, as in Fig. 16–17. In which of the waves, (a) or (b), are the two component frequencies farther apart? Explain.

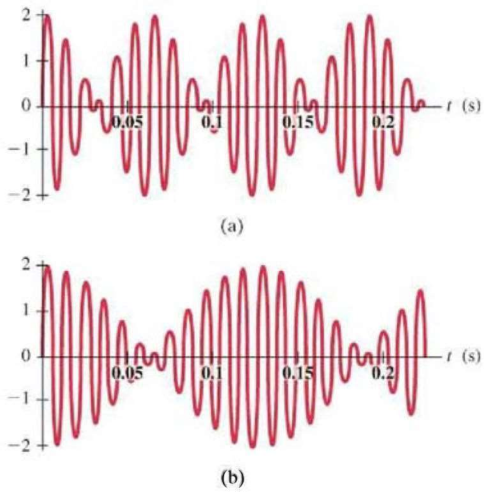


FIGURE 16–30 Question 16.

17. Is there a Doppler shift if the source and observer move in the same direction, with the same velocity? Explain.
18. If a wind is blowing, will this alter the frequency of the sound heard by a person at rest with respect to the source? Is the wavelength or velocity changed?
19. Figure 16–31 shows various positions of a child on a swing moving toward a person on the ground who is blowing a whistle. At which position, A through E, will the child hear the highest frequency for the sound of the whistle? Explain your reasoning.

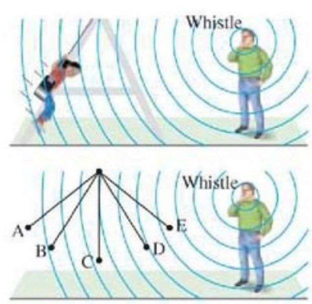


FIGURE 16–31 Question 19.

20. Approximately how many octaves are there in the human audible range?
21. At a race track, you can estimate the speed of cars just by listening to the difference in pitch of the engine noise between approaching and receding cars. Suppose the sound of a certain car drops by a full octave (frequency halved) as it goes by on the straightaway. How fast is it going?

Problems

[Unless stated otherwise, assume $T = 20°C$ and $v_{sound} = 343$ m/s in air.]

16–1 Characteristics of Sound

1. (I) A hiker determines the length of a lake by listening for the echo of her shout reflected by a cliff at the far end of the lake. She hears the echo 2.0 s after shouting. Estimate the length of the lake.
2. (I) A sailor strikes the side of his ship just below the waterline. He hears the echo of the sound reflected from the ocean floor directly below 2.5 s later. How deep is the ocean at this point? Assume the speed of sound in sea water is 1560 m/s (Table 16–1) and does not vary significantly with depth.
3. (I) (*a*) Calculate the wavelengths in air at 20°C for sounds in the maximum range of human hearing, 20 Hz to 20,000 Hz. (*b*) What is the wavelength of a 15-MHz ultrasonic wave?
4. (I) On a warm summer day (27°C), it takes 4.70 s for an echo to return from a cliff across a lake. On a winter day, it takes 5.20 s. What is the temperature on the winter day?
5. (II) A *motion sensor* can accurately measure the distance d to an object repeatedly via the sonar technique used in Example 16–2. A short ultrasonic pulse is emitted and reflects from any objects it encounters, creating echo pulses upon their arrival back at the sensor. The sensor measures the time interval t between the emission of the original pulse and the arrival of the first echo. (*a*) The smallest time interval t that can be measured with high precision is 1.0 ms. What is the smallest distance (at 20°C) that can be measured with the motion sensor? (*b*) If the motion sensor makes 15 distance measurements every second (that is, it emits 15 sound pulses per second at evenly spaced time intervals), the measurement of t must be completed within the time interval between the emissions of successive pulses. What is the largest distance (at 20°C) that can be measured with the motion sensor? (*c*) Assume that during a lab period the room's temperature increases from 20°C to 23°C. What percent error will this introduce into the motion sensor's distance measurements?
6. (II) An ocean fishing boat is drifting just above a school of tuna on a foggy day. Without warning, an engine backfire occurs on another boat 1.35 km away (Fig. 16–32). How much time elapses before the backfire is heard (*a*) by the fish, and (*b*) by the fishermen?

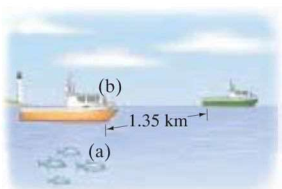


FIGURE 16–32 Problem 6.

7. (II) A stone is dropped from the top of a cliff. The splash it makes when striking the water below is heard 3.0 s later. How high is the cliff?
8. (II) A person, with his ear to the ground, sees a huge stone strike the concrete pavement. A moment later two sounds are heard from the impact: one travels in the air and the other in the concrete, and they are 0.75 s apart. How far away did the impact occur? See Table 16–1.
9. (II) Calculate the percent error made over one mile of distance by the "5-second rule" for estimating the distance from a lightning strike if the temperature is (*a*) 30°C, and (*b*) 10°C.

16–2 Mathematical Representation of Waves

10. (I) The pressure amplitude of a sound wave in air ($\rho = 1.29\ \mathrm{kg/m^3}$) at 0°C is 3.0×10^{-3} Pa. What is the displacement amplitude if the frequency is (*a*) 150 Hz and (*b*) 15 kHz?
11. (I) What must be the pressure amplitude in a sound wave in air (0°C) if the air molecules undergo a maximum displacement equal to the diameter of an oxygen molecule, about 3.0×10^{-10} m? Assume a sound-wave frequency of (*a*) 55 Hz and (*b*) 5.5 kHz.
12. (II) Write an expression that describes the pressure variation as a function of x and t for the waves described in Problem 11.
13. (II) The pressure variation in a sound wave is given by
$$\Delta P = 0.0035 \sin(0.38\pi x - 1350\pi t),$$
where ΔP is in pascals, x in meters, and t in seconds. Determine (*a*) the wavelength, (*b*) the frequency, (*c*) the speed, and (*d*) the displacement amplitude of the wave. Assume the density of the medium to be $\rho = 2.3 \times 10^3\ \mathrm{kg/m^3}$.

16–3 Intensity of Sound; Decibels

14. (I) What is the intensity of a sound at the pain level of 120 dB? Compare it to that of a whisper at 20 dB.
15. (I) What is the sound level of a sound whose intensity is $2.0 \times 10^{-6}\ \mathrm{W/m^2}$?
16. (I) What are the lowest and highest frequencies that an ear can detect when the sound level is 40 dB? (See Fig. 16–6.)
17. (II) Your auditory system can accommodate a huge range of sound levels. What is the ratio of highest to lowest intensity at (*a*) 100 Hz, (*b*) 5000 Hz? (See Fig. 16–6.)
18. (II) You are trying to decide between two new stereo amplifiers. One is rated at 100 W per channel and the other is rated at 150 W per channel. In terms of dB, how much louder will the more powerful amplifier be when both are producing sound at their maximum levels?
19. (II) At a painfully loud concert, a 120-dB sound wave travels away from a loudspeaker at 343 m/s. How much sound wave energy is contained in each 1.0-cm^3 volume of air in the region near this loudspeaker?
20. (II) If two firecrackers produce a sound level of 95 dB when fired simultaneously at a certain place, what will be the sound level if only one is exploded?
21. (II) A person standing a certain distance from an airplane with four equally noisy jet engines is experiencing a sound level of 130 dB. What sound level would this person experience if the captain shut down all but one engine?
22. (II) A cassette player is said to have a signal-to-noise ratio of 62 dB, whereas for a CD player it is 98 dB. What is the ratio of intensities of the signal and the background noise for each device?
23. (II) (*a*) Estimate the power output of sound from a person speaking in normal conversation. Use Table 16–2. Assume the sound spreads roughly uniformly over a sphere centered on the mouth. (*b*) How many people would it take to produce a total sound output of 75 W of ordinary conversation? [*Hint*: Add intensities, not dBs.]
24. (II) A 50-dB sound wave strikes an eardrum whose area is $5.0 \times 10^{-5}\ \mathrm{m^2}$. (*a*) How much energy is received by the eardrum per second? (*b*) At this rate, how long would it take your eardrum to receive a total energy of 1.0 J?

25. (II) Expensive amplifier A is rated at 250 W, while the more modest amplifier B is rated at 45 W. (*a*) Estimate the sound level in decibels you would expect at a point 3.5 m from a loudspeaker connected in turn to each amp. (*b*) Will the expensive amp sound twice as loud as the cheaper one?

26. (II) At a rock concert, a dB meter registered 130 dB when placed 2.2 m in front of a loudspeaker on the stage. (*a*) What was the power output of the speaker, assuming uniform spherical spreading of the sound and neglecting absorption in the air? (*b*) How far away would the sound level be a somewhat reasonable 85 dB?

27. (II) A fireworks shell explodes 100 m above the ground, creating a colorful display of sparks. How much greater is the sound level of the explosion for a person standing at a point directly below the explosion than for a person a horizontal distance of 200 m away (Fig. 16–33)?

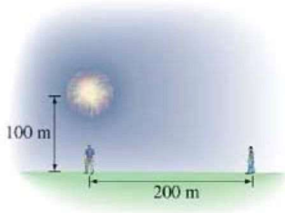


FIGURE 16–33
Problem 27.

28. (II) If the amplitude of a sound wave is made 2.5 times greater, (*a*) by what factor will the intensity increase? (*b*) By how many dB will the sound level increase?

29. (II) Two sound waves have equal displacement amplitudes, but one has 2.6 times the frequency of the other. (*a*) Which has the greater pressure amplitude and by what factor is it greater? (*b*) What is the ratio of their intensities?

30. (II) What would be the sound level (in dB) of a sound wave in air that corresponds to a displacement amplitude of vibrating air molecules of 0.13 mm at 380 Hz?

31. (II) (*a*) Calculate the maximum displacement of air molecules when a 330-Hz sound wave passes whose intensity is at the threshold of pain (120 dB). (*b*) What is the pressure amplitude in this wave?

32. (II) A jet plane emits 5.0×10^5 J of sound energy per second. (*a*) What is the sound level 25 m away? Air absorbs sound at a rate of about 7.0 dB/km; calculate what the sound level will be (*b*) 1.00 km and (*c*) 7.50 km away from this jet plane, taking into account air absorption.

16–4 Sources of Sound: Strings and Air Columns

33. (I) What would you estimate for the length of a bass clarinet, assuming that it is modeled as a closed tube and that the lowest note that it can play is a D$\flat$ whose frequency is 69.3 Hz?

34. (I) The A string on a violin has a fundamental frequency of 440 Hz. The length of the vibrating portion is 32 cm, and it has a mass of 0.35 g. Under what tension must the string be placed?

35. (I) An organ pipe is 124 cm long. Determine the fundamental and first three audible overtones if the pipe is (*a*) closed at one end, and (*b*) open at both ends.

36. (I) (*a*) What resonant frequency would you expect from blowing across the top of an empty soda bottle that is 21 cm deep, if you assumed it was a closed tube? (*b*) How would that change if it was one-third full of soda?

37. (I) If you were to build a pipe organ with open-tube pipes spanning the range of human hearing (20 Hz to 20 kHz), what would be the range of the lengths of pipes required?

38. (II) Estimate the frequency of the "sound of the ocean" when you put your ear very near a 20-cm-diameter seashell (Fig. 16–34).

FIGURE 16–34
Problem 38.

39. (II) An unfingered guitar string is 0.73 m long and is tuned to play E above middle C (330 Hz). (*a*) How far from the end of this string must a fret (and your finger) be placed to play A above middle C (440 Hz)? (*b*) What is the wavelength on the string of this 440-Hz wave? (*c*) What are the frequency and wavelength of the sound wave produced in air at 25°C by this fingered string?

40. (II) (*a*) Determine the length of an open organ pipe that emits middle C (262 Hz) when the temperature is 15°C. (*b*) What are the wavelength and frequency of the fundamental standing wave in the tube? (*c*) What are λ and f in the traveling sound wave produced in the outside air?

41. (II) An organ is in tune at 22.0°C. By what percent will the frequency be off at 5.0°C?

42. (II) How far from the mouthpiece of the flute in Example 16–11 should the hole be that must be uncovered to play F above middle C at 349 Hz?

43. (II) A bugle is simply a tube of fixed length that behaves as if it is open at both ends. A bugler, by adjusting his lips correctly and blowing with proper air pressure, can cause a harmonic (usually other than the fundamental) of the air column within the tube to sound loudly. Standard military tunes like *Taps* and *Reveille* require only four musical notes: G4 (392 Hz), C5 (523 Hz), E5 (659 Hz), and G5 (784 Hz). (*a*) For a certain length ℓ, a bugle will have a sequence of four consecutive harmonics whose frequencies very nearly equal those associated with the notes G4, C5, E5, and G5. Determine this ℓ. (*b*) Which harmonic is each of the (approximate) notes G4, C5, E5, and G5 for the bugle?

44. (II) A particular organ pipe can resonate at 264 Hz, 440 Hz, and 616 Hz, but not at any other frequencies in between. (*a*) Show why this is an open or a closed pipe. (*b*) What is the fundamental frequency of this pipe?

45. (II) When a player's finger presses a guitar string down onto a fret, the length of the vibrating portion of the string is shortened, thereby increasing the string's fundamental frequency (see Fig. 16–35). The string's tension and mass per unit length remain unchanged. If the unfingered length of the string is $\ell = 65.0$ cm, determine the positions x of the first six frets, if each fret raises the pitch of the fundamental by one musical note in comparison to the neighboring fret. On the equally tempered chromatic scale, the ratio of frequencies of neighboring notes is $2^{1/12}$.

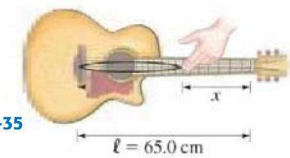


FIGURE 16–35
Problem 45.

46. (II) A uniform narrow tube 1.80 m long is open at both ends. It resonates at two successive harmonics of frequencies 275 Hz and 330 Hz. What is (*a*) the fundamental frequency, and (*b*) the speed of sound in the gas in the tube?

47. (II) A pipe in air at 23.0°C is to be designed to produce two successive harmonics at 240 Hz and 280 Hz. How long must the pipe be, and is it open or closed?

48. (II) How many overtones are present within the audible range for a 2.48-m-long organ pipe at 20°C (*a*) if it is open, and (*b*) if it is closed?

49. (II) Determine the fundamental and first overtone frequencies for an 8.0-m-long hallway with all doors closed. Model the hallway as a tube closed at both ends.

50. (II) In a *quartz oscillator*, used as a stable clock in electronic devices, a transverse (shear) standing sound wave is excited across the thickness d of a quartz disk and its frequency f is detected electronically. The parallel faces of the disk are unsupported and so behave as "free ends" when the sound wave reflects from them (see Fig. 16–36). If the oscillator is designed to operate with the first harmonic, determine the required disk thickness if $f = 12.0$ MHz. The density and shear modulus of quartz are $\rho = 2650\ \mathrm{kg/m^3}$ and $G = 2.95 \times 10^{10}\ \mathrm{N/m^2}$.

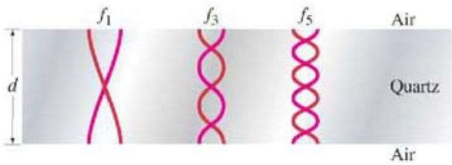


FIGURE 16–36 Problem 50.

51. (III) The human ear canal is approximately 2.5 cm long. It is open to the outside and is closed at the other end by the eardrum. Estimate the frequencies (in the audible range) of the standing waves in the ear canal. What is the relationship of your answer to the information in the graph of Fig. 16–6?

*16–5 Quality of Sound, Superposition

***52.** (II) Approximately what are the intensities of the first two overtones of a violin compared to the fundamental? How many decibels softer than the fundamental are the first and second overtones? (See Fig. 16–14.)

16–6 Interference; Beats

53. (I) A piano tuner hears one beat every 2.0 s when trying to adjust two strings, one of which is sounding 370 Hz. How far off in frequency is the other string?

54. (I) What is the beat frequency if middle C (262 Hz) and C$^\#$ (277 Hz) are played together? What if each is played two octaves lower (each frequency reduced by a factor of 4)?

55. (II) A guitar string produces 4 beats/s when sounded with a 350-Hz tuning fork and 9 beats/s when sounded with a 355-Hz tuning fork. What is the vibrational frequency of the string? Explain your reasoning.

56. (II) The two sources of sound in Fig. 16–15 face each other and emit sounds of equal amplitude and equal frequency (294 Hz) but 180° out of phase. For what minimum separation of the two speakers will there be some point at which (*a*) complete constructive interference occurs and (*b*) complete destructive interference occurs. (Assume $T = 20$°C.)

57. (II) How many beats will be heard if two identical flutes, each 0.66 m long, try to play middle C (262 Hz), but one is at 5.0°C and the other at 28°C?

58. (II) Two loudspeakers are placed 3.00 m apart, as shown in Fig. 16–37. They emit 494-Hz sounds, in phase. A microphone is placed 3.20 m distant from a point midway between the two speakers, where an intensity maximum is recorded. (*a*) How far must the microphone be moved to the right to find the first intensity minimum? (*b*) Suppose the speakers are reconnected so that the 494-Hz sounds they emit are exactly out of phase. At what positions are the intensity maximum and minimum now?

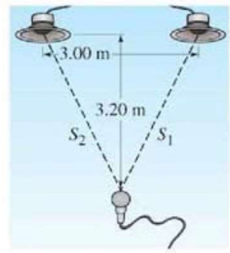


FIGURE 16–37 Problem 58.

59. (II) Two piano strings are supposed to be vibrating at 220 Hz, but a piano tuner hears three beats every 2.0 s when they are played together. (*a*) If one is vibrating at 220.0 Hz, what must be the frequency of the other (is there only one answer)? (*b*) By how much (in percent) must the tension be increased or decreased to bring them in tune?

60. (II) A source emits sound of wavelengths 2.64 m and 2.72 m in air. (*a*) How many beats per second will be heard? (Assume $T = 20$°C.) (*b*) How far apart in space are the regions of maximum intensity?

16–7 Doppler Effect

61. (I) The predominant frequency of a certain fire truck's siren is 1350 Hz when at rest. What frequency do you detect if you move with a speed of 30.0 m/s (*a*) toward the fire truck, and (*b*) away from it?

62. (I) A bat at rest sends out ultrasonic sound waves at 50.0 kHz and receives them returned from an object moving directly away from it at 30.0 m/s. What is the received sound frequency?

63. (II) (*a*) Compare the shift in frequency if a 2300-Hz source is moving toward you at 18 m/s, versus you moving toward it at 18 m/s. Are the two frequencies exactly the same? Are they close? (*b*) Repeat the calculation for 160 m/s and then again (*c*) for 320 m/s. What can you conclude about the asymmetry of the Doppler formulas? (*d*) Show that at low speeds (relative to the speed of sound), the two formulas—source approaching and detector approaching—yield the same result.

64. (II) Two automobiles are equipped with the same single-frequency horn. When one is at rest and the other is moving toward the first at 15 m/s, the driver at rest hears a beat frequency of 4.5 Hz. What is the frequency the horns emit? Assume $T = 20$°C.

65. (II) A police car sounding a siren with a frequency of 1280 Hz is traveling at 120.0 km/h. (*a*) What frequencies does an observer standing next to the road hear as the car approaches and as it recedes? (*b*) What frequencies are heard in a car traveling at 90.0 km/h in the opposite direction before and after passing the police car? (*c*) The police car passes a car traveling in the same direction at 80.0 km/h. What two frequencies are heard in this car?

66. (II) A bat flies toward a wall at a speed of 7.0 m/s. As it flies, the bat emits an ultrasonic sound wave with frequency 30.0 kHz. What frequency does the bat hear in the reflected wave?

67. (II) In one of the original Doppler experiments, a tuba was played on a moving flat train car at a frequency of 75 Hz, and a second identical tuba played the same tone while at rest in the railway station. What beat frequency was heard in the station if the train car approached the station at a speed of 12.0 m/s?

68. (II) If a speaker mounted on an automobile broadcasts a song, with what speed (km/h) does the automobile have to move toward a stationary listener so that the listener hears the song with each musical note shifted up by one note in comparison to the song heard by the automobile's driver? On the equally tempered chromatic scale, the ratio of frequencies of neighboring notes is $2^{1/12}$.

69. (II) A wave on the surface of the ocean with wavelength 44 m is traveling east at a speed of 18 m/s relative to the ocean floor. If, on this stretch of ocean surface, a powerboat is moving at 15 m/s (relative to the ocean floor), how often does the boat encounter a wave crest, if the boat is traveling (*a*) west, and (*b*) east?

70. (III) A factory whistle emits sound of frequency 720 Hz. When the wind velocity is 15.0 m/s from the north, what frequency will observers hear who are located, at rest, (*a*) due north, (*b*) due south, (*c*) due east, and (*d*) due west, of the whistle? What frequency is heard by a cyclist heading (*e*) north or (*f*) west, toward the whistle at 12.0 m/s? Assume $T = 20°\text{C}$.

71. (III) The Doppler effect using ultrasonic waves of frequency 2.25×10^6 Hz is used to monitor the heartbeat of a fetus. A (maximum) beat frequency of 260 Hz is observed. Assuming that the speed of sound in tissue is 1.54×10^3 m/s, calculate the maximum velocity of the surface of the beating heart.

*16–8 Shock Waves; Sonic Boom

***72.** (II) An airplane travels at Mach 2.0 where the speed of sound is 310 m/s. (*a*) What is the angle the shock wave makes with the direction of the airplane's motion? (*b*) If the plane is flying at a height of 6500 m, how long after it is directly overhead will a person on the ground hear the shock wave?

***73.** (II) A space probe enters the thin atmosphere of a planet where the speed of sound is only about 45 m/s. (*a*) What is the probe's Mach number if its initial speed is 15,000 km/h? (*b*) What is the angle of the shock wave relative to the direction of motion?

***74.** (II) A meteorite traveling 8800 m/s strikes the ocean. Determine the shock wave angle it produces (*a*) in the air just before entering the ocean, and (*b*) in the water just after entering. Assume $T = 20°\text{C}$.

***75.** (II) Show that the angle θ a sonic boom makes with the path of a supersonic object is given by Eq. 16–12.

***76.** (II) You look directly overhead and see a plane exactly 1.25 km above the ground flying faster than the speed of sound. By the time you hear the sonic boom, the plane has traveled a horizontal distance of 2.0 km. See Fig. 16–38. Determine (*a*) the angle of the shock cone, θ, and (*b*) the speed of the plane (the Mach number). Assume the speed of sound is 330 m/s.

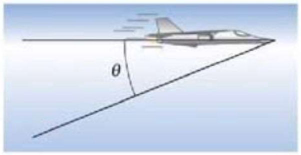


FIGURE 16–38 Problem 76.

***77.** (II) A supersonic jet traveling at Mach 2.2 at an altitude of 9500 m passes directly over an observer on the ground. Where will the plane be relative to the observer when the latter hears the sonic boom? (See Fig. 16–39.)

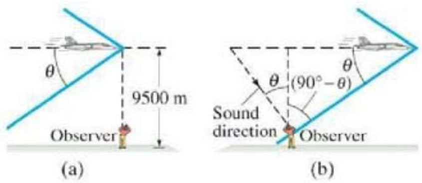


FIGURE 16–39 Problem 77.

General Problems

78. A fish finder uses a sonar device that sends 20,000-Hz sound pulses downward from the bottom of the boat, and then detects echoes. If the maximum depth for which it is designed to work is 75 m, what is the minimum time between pulses (in fresh water)?

79. A science museum has a display called a sewer pipe symphony. It consists of many plastic pipes of various lengths, which are open on both ends. (*a*) If the pipes have lengths of 3.0 m, 2.5 m, 2.0 m, 1.5 m and 1.0 m, what frequencies will be heard by a visitor's ear placed near the ends of the pipes? (*b*) Why does this display work better on a noisy day than on a quiet day?

80. A single mosquito 5.0 m from a person makes a sound close to the threshold of human hearing (0 dB). What will be the sound level of 100 such mosquitoes?

81. What is the resultant sound level when an 82-dB sound and an 89-dB sound are heard simultaneously?

82. The sound level 9.00 m from a loudspeaker, placed in the open, is 115 dB. What is the acoustic power output (W) of the speaker, assuming it radiates equally in all directions?

83. A stereo amplifier is rated at 175 W output at 1000 Hz. The power output drops by 12 dB at 15 kHz. What is the power output in watts at 15 kHz?

84. Workers around jet aircraft typically wear protective devices over their ears. Assume that the sound level of a jet airplane engine, at a distance of 30 m, is 130 dB, and that the average human ear has an effective radius of 2.0 cm. What would be the power intercepted by an unprotected ear at a distance of 30 m from a jet airplane engine?

85. In audio and communications systems, the *gain*, β, in decibels is defined as

$$\beta = 10 \log\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right),$$

where P_{in} is the power input to the system and P_{out} is the power output. A particular stereo amplifier puts out 125 W of power for an input of 1.0 mW. What is its gain in dB?

86. For large concerts, loudspeakers are sometimes used to amplify a singer's sound. The human brain interprets sounds that arrive within 50 ms of the original sound as if they came from the same source. Thus if the sound from a loudspeaker reaches a listener first, it would sound as if the loudspeaker is the source of the sound. Conversely, if the singer is heard first and the loudspeaker adds to the sound within 50 ms, the sound would seem to come from the singer, who would now seem to be singing louder. The second situation is desired. Because the signal to the loudspeaker travels at the speed of light $(3 \times 10^8\,\text{m/s})$, which is much faster than the speed of sound, a delay is added to the signal sent to the loudspeaker. How much delay must be added if the loudspeaker is 3.0 m behind the singer and we want its sound to arrive 30 ms after the singer's?

87. Manufacturers typically offer a particular guitar string in a choice of diameters so that players can tune their instruments with a preferred string tension. For example, a nylon high-E string is available in a low- and high-tension model with diameter 0.699 mm and 0.724 mm, respectively. Assuming the density ρ of nylon is the same for each model, compare (as a ratio) the tension in a tuned high- and low-tension string.

88. The high-E string on a guitar is fixed at both ends with length $\ell = 65.0\,\text{cm}$ and fundamental frequency $f_1 = 329.6\,\text{Hz}$. On an acoustic guitar, this string typically has a diameter of 0.33 mm and is commonly made of brass $(7760\,\text{kg/m}^3)$, while on an electric guitar it has a diameter of 0.25 mm and is made of nickel-coated steel $(7990\,\text{kg/m}^3)$. Compare (as a ratio) the high-E string tension on an acoustic versus an electric guitar.

89. The A string of a violin is 32 cm long between fixed points with a fundamental frequency of 440 Hz and a mass per unit length of $7.2 \times 10^{-4}\,\text{kg/m}$. (*a*) What are the wave speed and tension in the string? (*b*) What is the length of the tube of a simple wind instrument (say, an organ pipe) closed at one end whose fundamental is also 440 Hz if the speed of sound is 343 m/s in air? (*c*) What is the frequency of the first overtone of each instrument?

90. A tuning fork is set into vibration above a vertical open tube filled with water (Fig. 16–40). The water level is allowed to drop slowly. As it does so, the air in the tube above the water level is heard to resonate with the tuning fork when the distance from the tube opening to the water level is 0.125 m and again at 0.395 m. What is the frequency of the tuning fork?

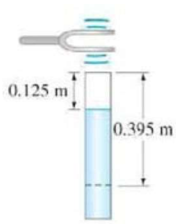


FIGURE 16–40 Problem 90.

91. Two identical tubes, each closed at one end, have a fundamental frequency of 349 Hz at 25.0°C. The air temperature is increased to 30.0°C in one tube. If the two pipes are sounded together now, what beat frequency results?

92. Each string on a violin is tuned to a frequency $1\frac{1}{2}$ times that of its neighbor. The four equal-length strings are to be placed under the same tension; what must be the mass per unit length of each string relative to that of the lowest string?

93. The diameter D of a tube does affect the node at the open end of a tube. The end correction can be roughly approximated as adding $D/3$ to the effective length of the tube. For a closed tube of length 0.60 m and diameter 3.0 cm, what are the first four harmonics, taking the end correction into consideration?

94. A person hears a pure tone in the 500 to 1000-Hz range coming from two sources. The sound is loudest at points equidistant from the two sources. To determine exactly what the frequency is, the person moves about and finds that the sound level is minimal at a point 0.28 m farther from one source than the other. What is the frequency of the sound?

95. The frequency of a steam train whistle as it approaches you is 552 Hz. After it passes you, its frequency is measured as 486 Hz. How fast was the train moving (assume constant velocity)?

96. Two trains emit 516-Hz whistles. One train is stationary. The conductor on the stationary train hears a 3.5-Hz beat frequency when the other train approaches. What is the speed of the moving train?

97. Two loudspeakers are at opposite ends of a railroad car as it moves past a stationary observer at 10.0 m/s, as shown in Fig. 16–41. If the speakers have identical sound frequencies of 348 Hz, what is the beat frequency heard by the observer when (*a*) he listens from the position A, in front of the car, (*b*) he is between the speakers, at B, and (*c*) he hears the speakers after they have passed him, at C?

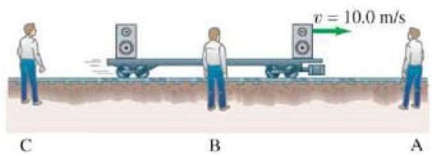


FIGURE 16–41 Problem 97.

98. Two open organ pipes, sounding together, produce a beat frequency of 8.0 Hz. The shorter one is 2.40 m long. How long is the other?

99. A bat flies toward a moth at speed 7.5 m/s while the moth is flying toward the bat at speed 5.0 m/s. The bat emits a sound wave of 51.35 kHz. What is the frequency of the wave detected by the bat after that wave reflects off the moth?

100. If the velocity of blood flow in the aorta is normally about 0.32 m/s, what beat frequency would you expect if 3.80-MHz ultrasound waves were directed along the flow and reflected from the red blood cells? Assume that the waves travel with a speed of $1.54 \times 10^3\,\text{m/s}$.

101. A bat emits a series of high-frequency sound pulses as it approaches a moth. The pulses are approximately 70.0 ms apart, and each is about 3.0 ms long. How far away can the moth be detected by the bat so that the echo from one pulse returns before the next pulse is emitted?

102. (*a*) Use the binomial expansion to show that Eqs. 16–9a and 16–10a become essentially the same for small relative velocity between source and observer. (*b*) What percent error would result if Eq. 16–10a were used instead of Eq. 16–9a for a relative velocity of 18.0 m/s?

103. Two loudspeakers face each other at opposite ends of a long corridor. They are connected to the same source which produces a pure tone of 282 Hz. A person walks from one speaker toward the other at a speed of 1.4 m/s. What "beat" frequency does the person hear?

104. A *Doppler flow meter* is used to measure the speed of blood flow. Transmitting and receiving elements are placed on the skin, as shown in Fig. 16–42. Typical sound-wave frequencies of about 5.0 MHz are used, which have a reasonable chance of being reflected from red blood cells. By measuring the frequency of the reflected waves, which are Doppler-shifted because the red blood cells are moving, the speed of the blood flow can be deduced. "Normal" blood flow speed is about 0.1 m/s. Suppose that an artery is partly constricted, so that the speed of the blood flow is increased, and the flow meter measures a Doppler shift of 780 Hz. What is the speed of blood flow in the constricted region? The effective angle between the sound waves (both transmitted and reflected) and the direction of blood flow is 45°. Assume the velocity of sound in tissue is 1540 m/s.

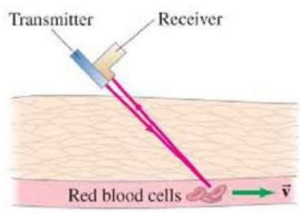


FIGURE 16–42 Problem 104.

105. The wake of a speedboat is 15° in a lake where the speed of the water wave is 2.2 km/h. What is the speed of the boat?

106. A source of sound waves (wavelength λ) is a distance ℓ from a detector. Sound reaches the detector directly, and also by reflecting off an obstacle, as shown in Fig. 16–43. The obstacle is equidistant from source and detector. When the obstacle is a distance d to the right of the line of sight between source and detector, as shown, the two waves arrive in phase. How much farther to the right must the obstacle be moved if the two waves are to be out of phase by $\frac{1}{2}$ wavelength, so destructive interference occurs? (Assume $\lambda \ll \ell, d$.)

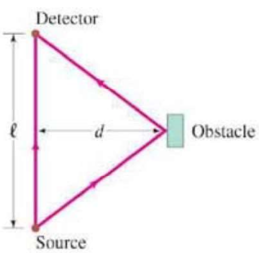


FIGURE 16–43 Problem 106.

107. A dramatic demonstration, called "singing rods," involves a long, slender aluminum rod held in the hand near the rod's midpoint. The rod is stroked with the other hand. With a little practice, the rod can be made to "sing," or emit a clear, loud, ringing sound. For a 75-cm-long rod, (*a*) what is the fundamental frequency of the sound? (*b*) What is its wavelength in the rod, and (*c*) what is the wavelength of the sound in air at 20°C?

108. Assuming that the maximum displacement of the air molecules in a sound wave is about the same as that of the speaker cone that produces the sound (Fig. 16–44), estimate by how much a loudspeaker cone moves for a fairly loud (105 dB) sound of (*a*) 8.0 kHz, and (*b*) 35 Hz.

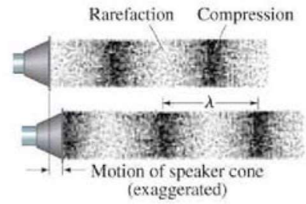


FIGURE 16–44 Problem 108.

*Numerical/Computer

*__109.__ (III) The manner in which a string is plucked determines the mixture of harmonic amplitudes in the resulting wave. Consider a string exactly $\frac{1}{2}$-m long that is fixed at both its ends located at $x = 0.0$ and $x = \frac{1}{2}$ m. The first five harmonics of this string have wavelengths $\lambda_1 = 1.0$ m, $\lambda_2 = \frac{1}{2}$ m, $\lambda_3 = \frac{1}{3}$ m, $\lambda_4 = \frac{1}{4}$ m, and $\lambda_5 = \frac{1}{5}$ m. According to Fourier's theorem, any shape of this string can be formed by a sum of its harmonics, with each harmonic having its own unique amplitude A. We limit the sum to the first five harmonics in the expression

$$D(x) = A_1 \sin\left(\frac{2\pi}{\lambda_1}x\right) + A_2 \sin\left(\frac{2\pi}{\lambda_2}x\right) + A_3 \sin\left(\frac{2\pi}{\lambda_3}x\right) + A_4 \sin\left(\frac{2\pi}{\lambda_4}x\right) + A_5 \sin\left(\frac{2\pi}{\lambda_5}x\right),$$

and D is the displacement of the string at a time $t = 0$. Imagine plucking this string at its midpoint (Fig. 16–45a) or at a point two-thirds from the left end (Fig. 16–45b). Using a graphing calculator or computer program, show that the above expression can fairly accurately represent the shape in: (*a*) Fig. 16–45a, if $A_1 = 1.00$, $A_2 = 0.00$, $A_3 = -0.11$, $A_4 = 0.00$, and $A_5 = 0.040$; and in (*b*) Fig. 16–45b, if $A_1 = 0.87$, $A_2 = -0.22$, $A_3 = 0.00$, $A_4 = 0.054$, and $A_5 = -0.035$.

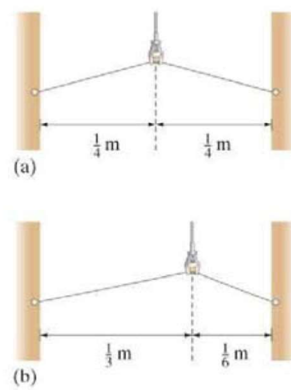


FIGURE 16–45 Problem 109.

Answers to Exercises

A: 1 km for every 3 s before the thunder is heard.

B: 4 times as intense.

C: (*b*).

D: The less-massive one.

E: 257 Hz.

F: (*b*).

G: (*a*) 1717 Hz, (*b*) 1483 Hz.

H: (*a*).