

Chapter 7: Binary Search Trees

CS401

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Revised Nell Dale Presentation

Chapter 7: The Binary Search Tree ADT

7.1 – Trees

7.2 – Binary Search Trees

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7.4 – The Implementation Level: Basics

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7.6 – The Implementation Level: Remaining Observers

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7.9 – Application: Word Frequency Counter

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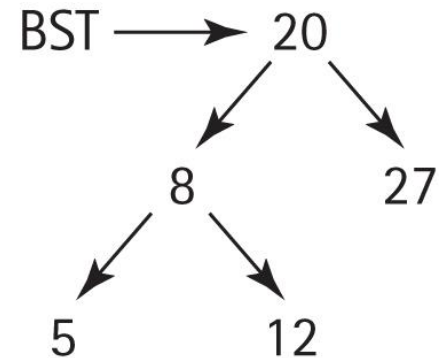
Binary Search Tree

- When maintaining a sorted list
 - Linked List requires $O(N)$ find location, $O(1)$ insertion
 - Array requires $O(\log_2 N)$ find location, $O(N)$ insertion
 - Binary Search Tree, in general allows $O(\log_2 N)$ find location, $O(1)$ insertion

Linked List \longrightarrow 5 \longrightarrow 8 \longrightarrow 12 \longrightarrow 20 \longrightarrow 27

Array \longrightarrow

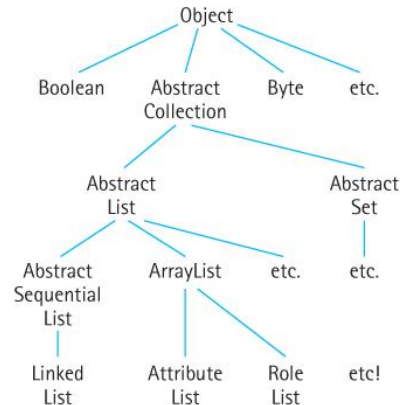
5	8	12	20	27
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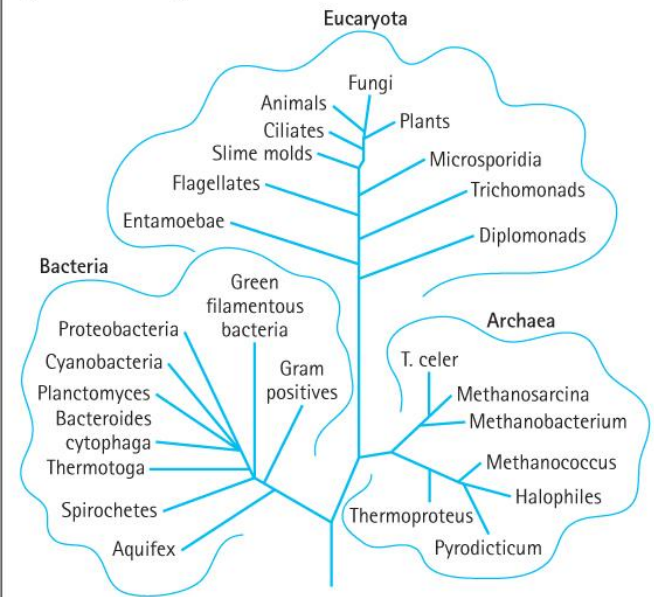
7.1 Trees

- A **tree** is a nonlinear structure in which each node is capable of having many successor nodes, called *children*.
- Trees are useful for representing lots of varied relationships among data items.

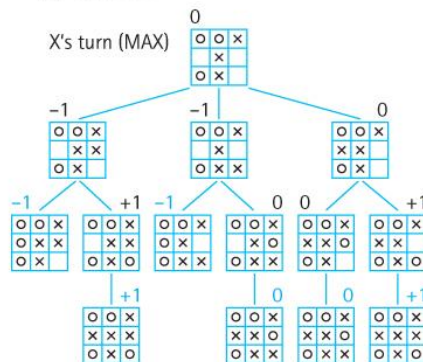
(a) Java classes



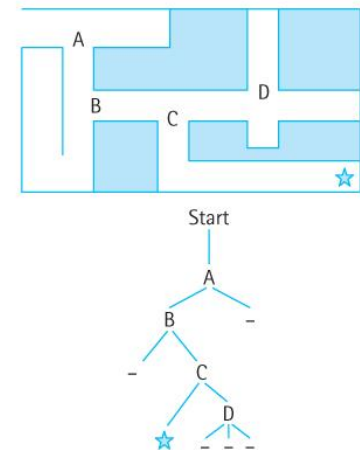
(b) Animal tree kingdom



(c) Game tree



(d) Maze tree

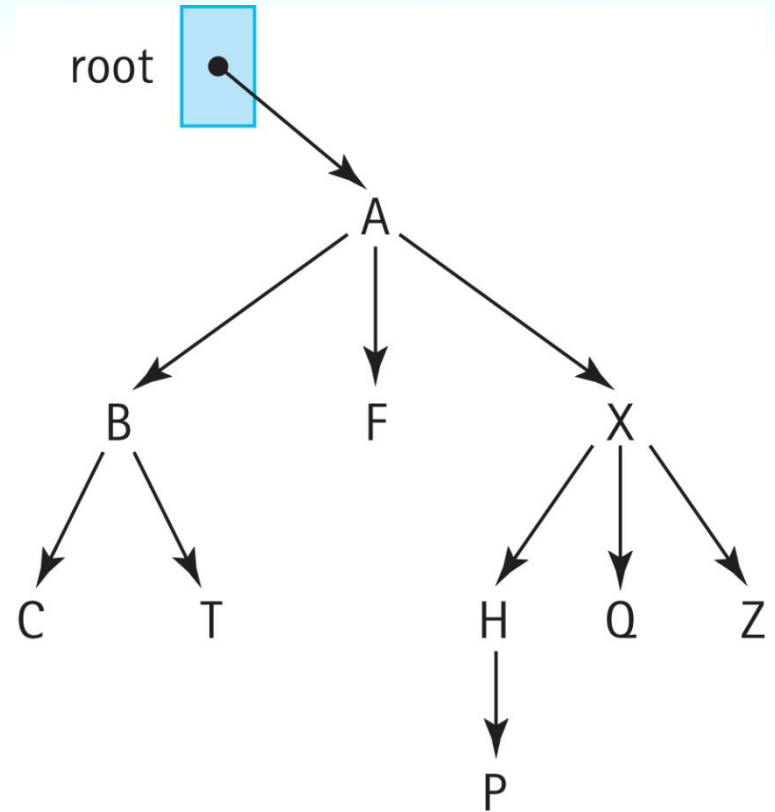


Definitions

- **Tree** A structure with a unique starting node (the root), in which each node is capable of having multiple successor nodes (its children), and in which a unique path exists from the root to every other node.
- **Root** The top node of a tree structure; a node with no parent
- **Parent node** The predecessor node of a node is its parent
- **Subtree** A node and all of its descendants form a subtree rooted at the node

Examples

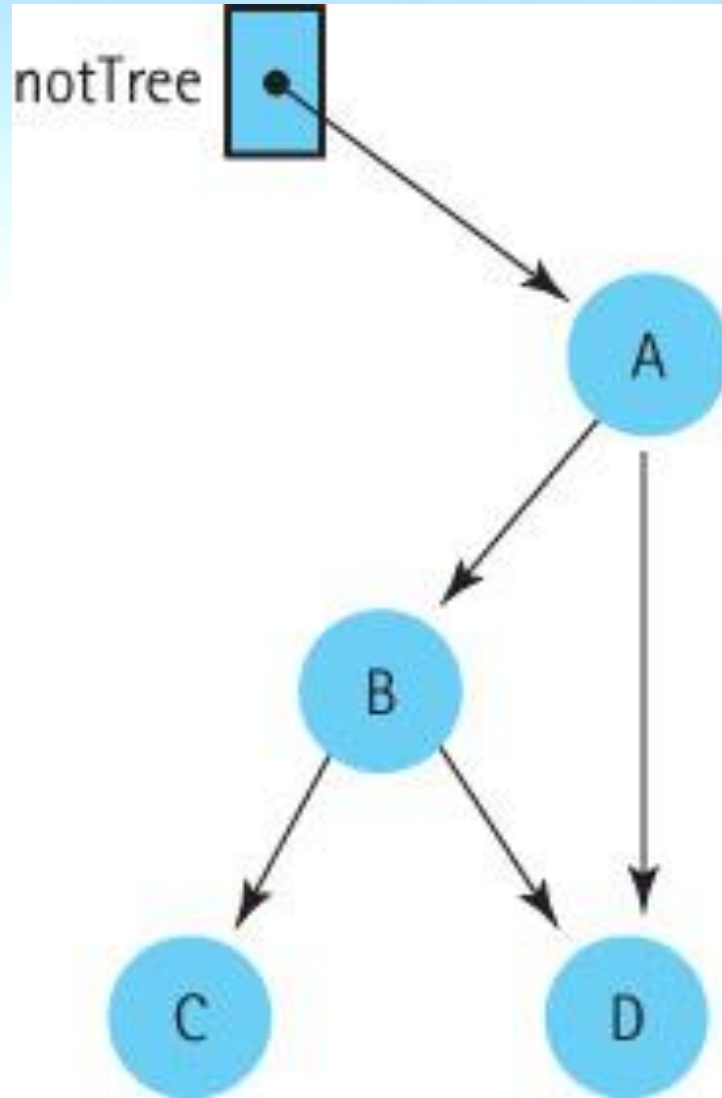
- The root is A
- The children of A are B, F, and X
- The parent of Q is X
- The leftmost subtree of X contains H and P



Required

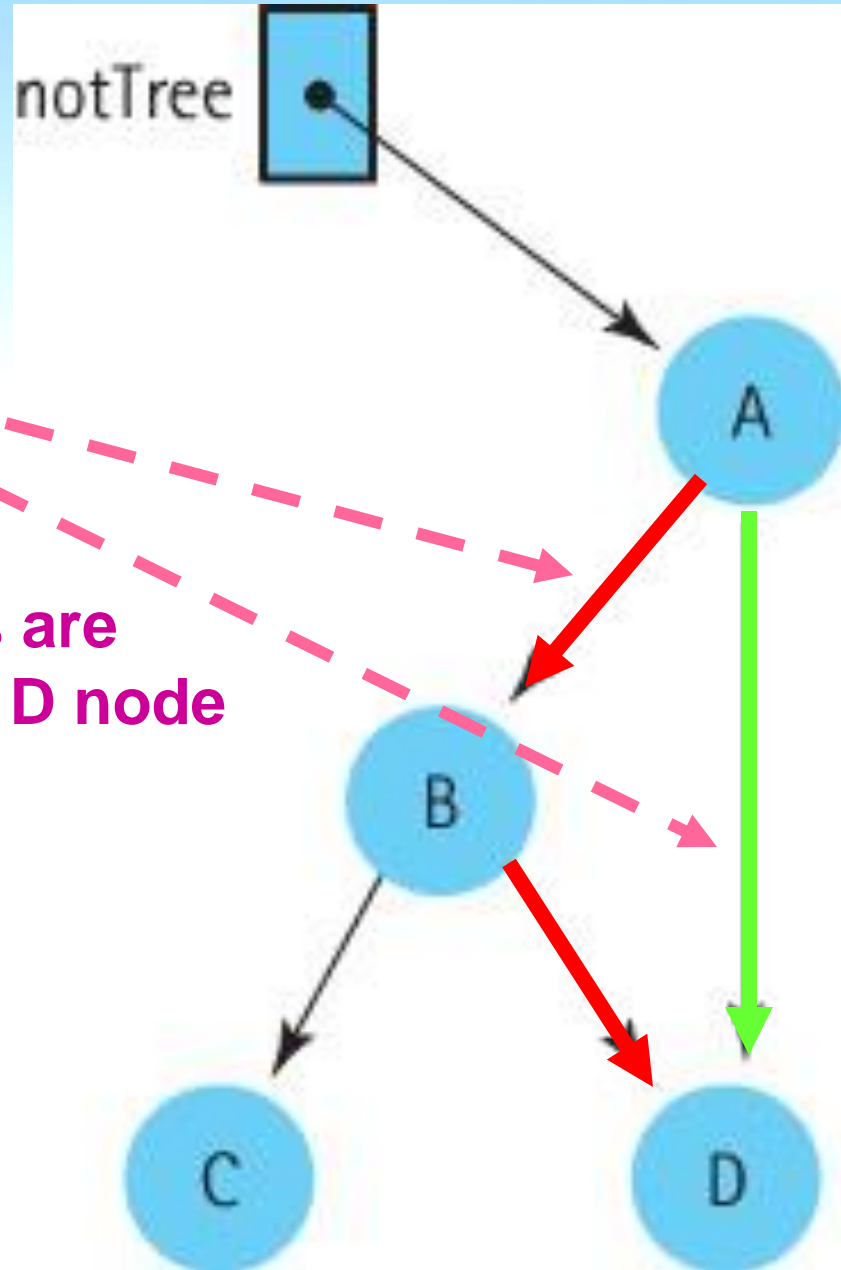
- A tree's subtrees must be disjoint
- There is a unique path from the root of a tree to any other node of the tree

Not a Tree →



Not a Tree

Because two paths are available from A to D node

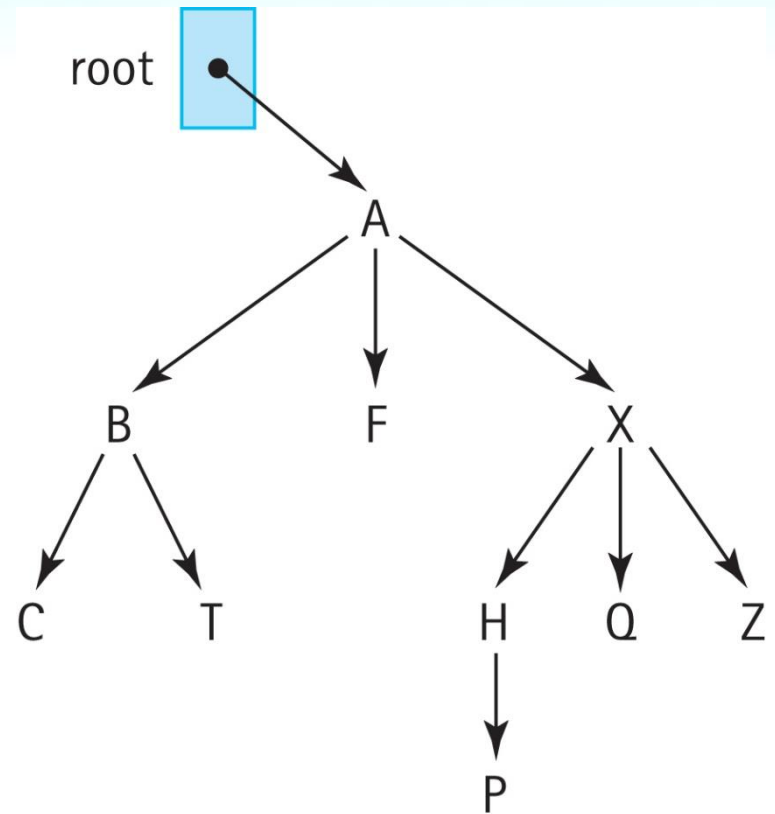


Definitions

- **Ancestor** A parent of a node, or a parent of an ancestor
- **Descendant** A child of a node, or a child of a descendant
- **Leaf** A node that has no children
- **Interior node** A node that is not a leaf
- **Siblings** Nodes with the same parent

Examples

- The ancestors of P are H, X, and A
- The descendants of X are H, Q, Z, and P
- The leaf nodes are C, T, F, P, Q, and Z
- The interior nodes are A, B, X
- The siblings of B are F and X

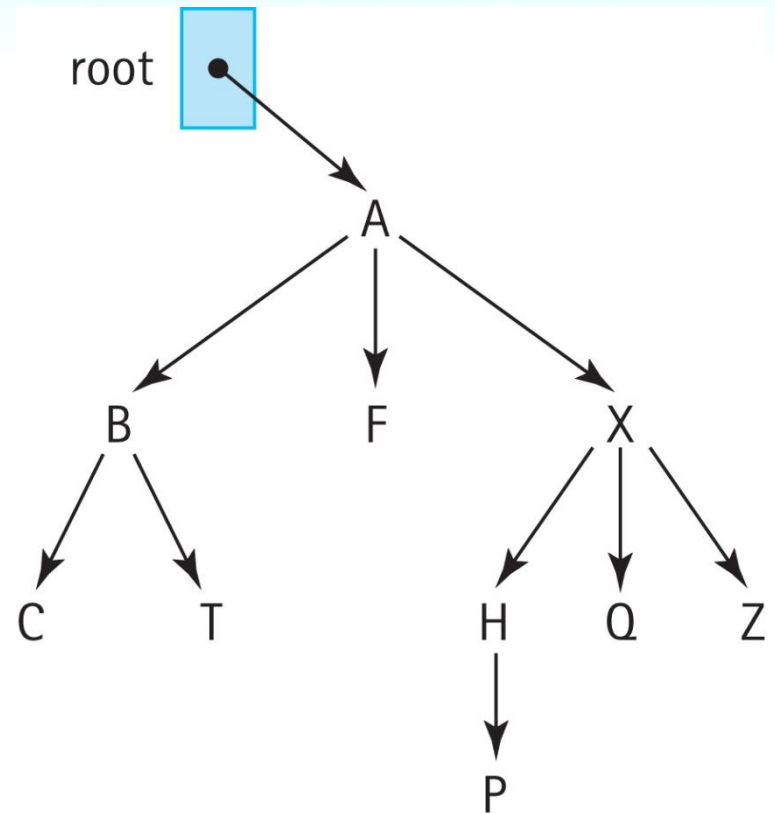


Definitions

- **Level** The level of a node is its distance from the root (the number of connections between itself and the root)
- **Height** The maximum level of the tree

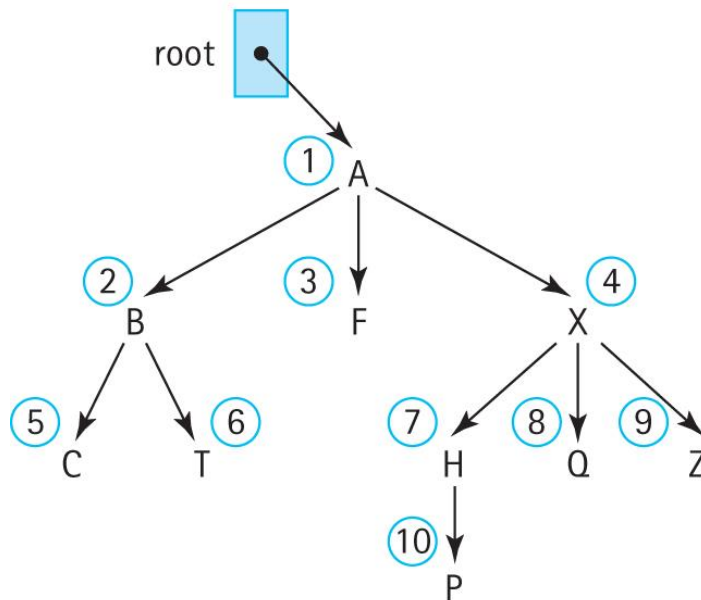
Examples

- The level of A is 0
- The level of P is 3
- The height of the tree is 3



Breadth-First Traversal

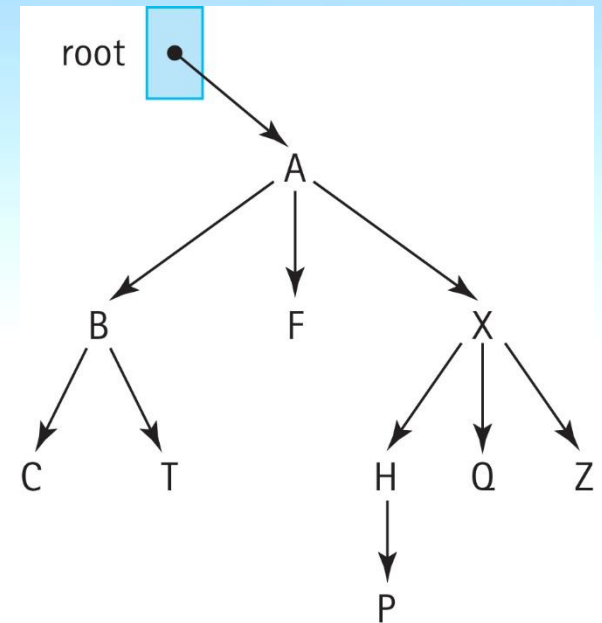
- Also called “level-order traversal”
- A B F X C T H Q Z P



Breadth-First Traversal(root)

Instantiate a queue of nodes
if (root is not null)

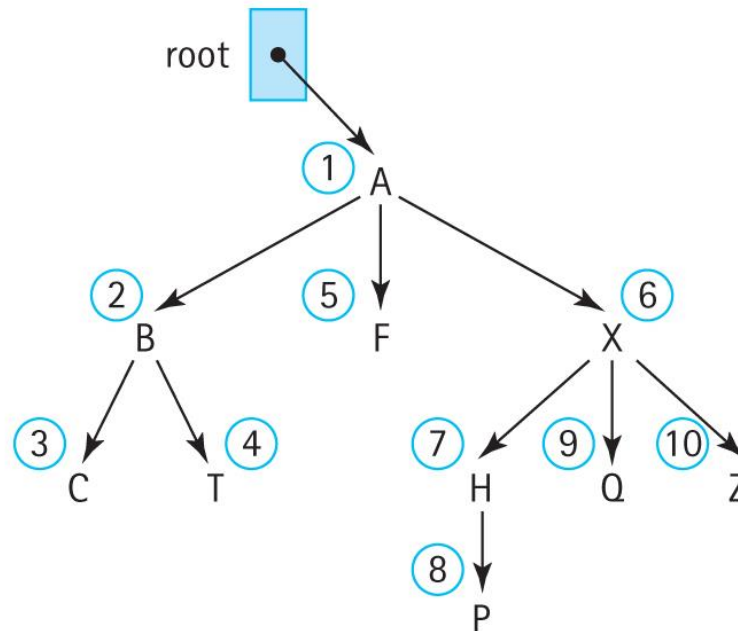
```
{  
    queue.enqueue(root)  
    while (!queue.isEmpty())  
    {  
        node = queue.dequeue()  
        Visit node  
        Enqueue the children of node  
            (from left to right) into queue  
    }  
}
```



Try It!

Depth-First Traversal

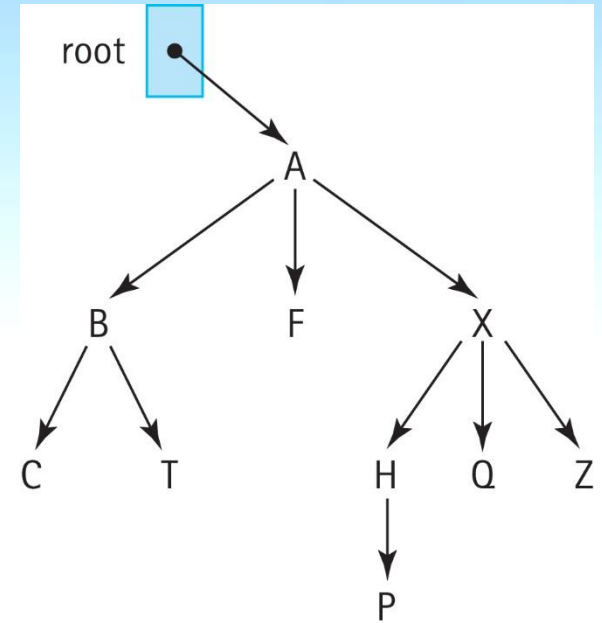
- A B C T F X H P Q Z



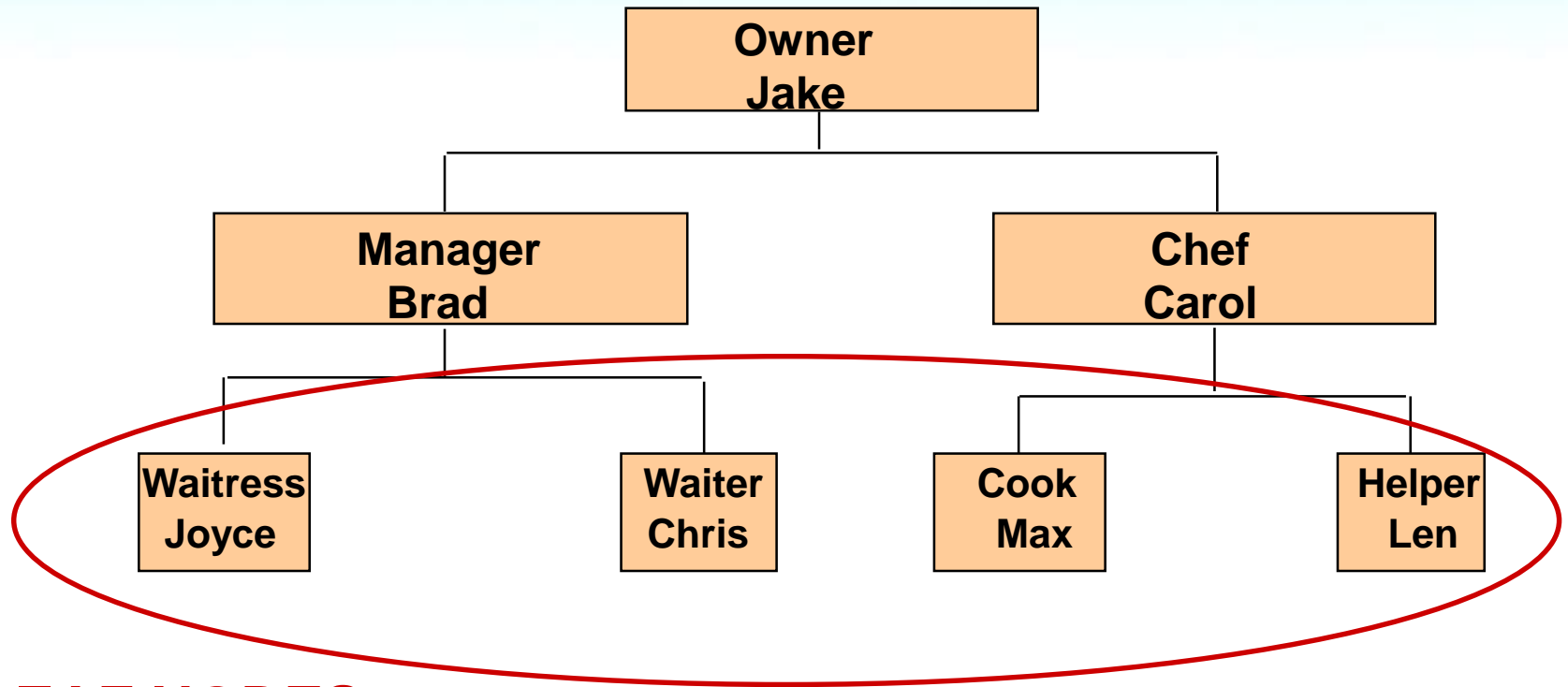
Depth-First Traversal(root)

Instantiate a stack of nodes
if (root is not null)

```
{  
    stack.push(root)  
    while (!stack.isEmpty())  
    {  
        node = stack.top()  
        stack.pop()  
        Visit node  
        Push the children of node  
            (from right to left) onto queue  
    }  
}
```

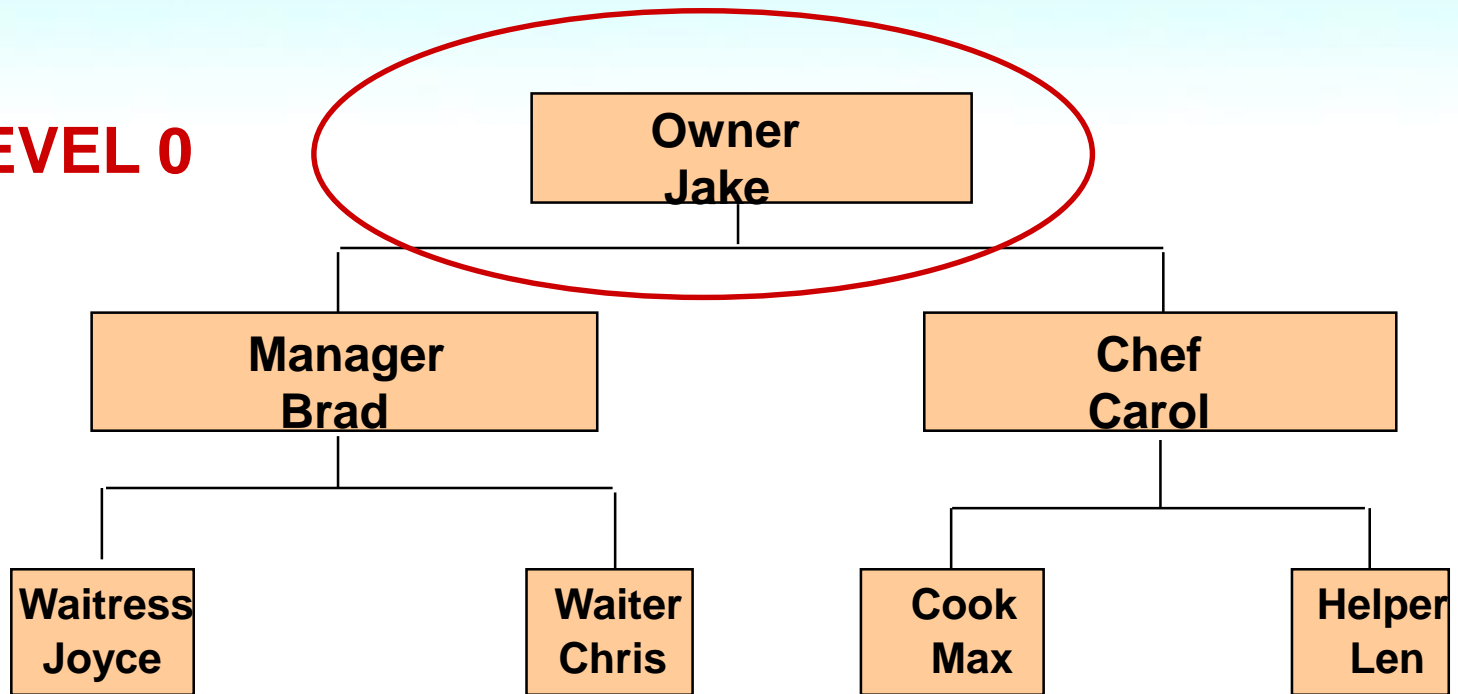


Try It!

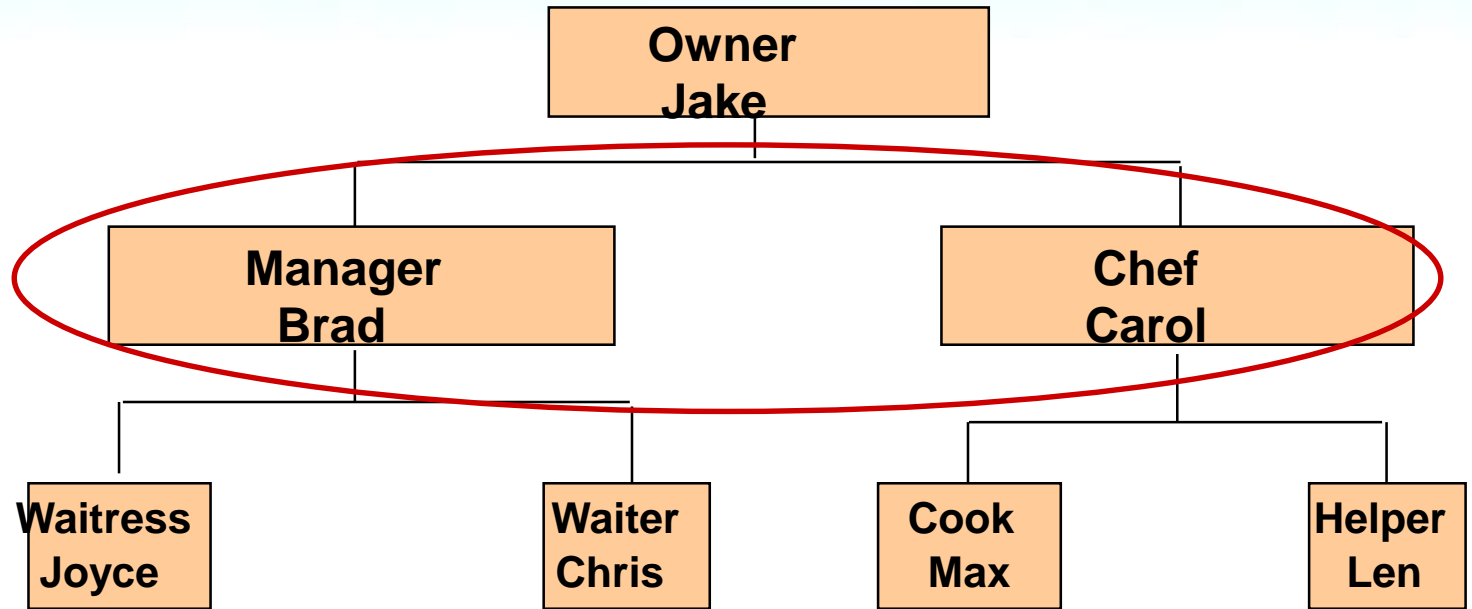


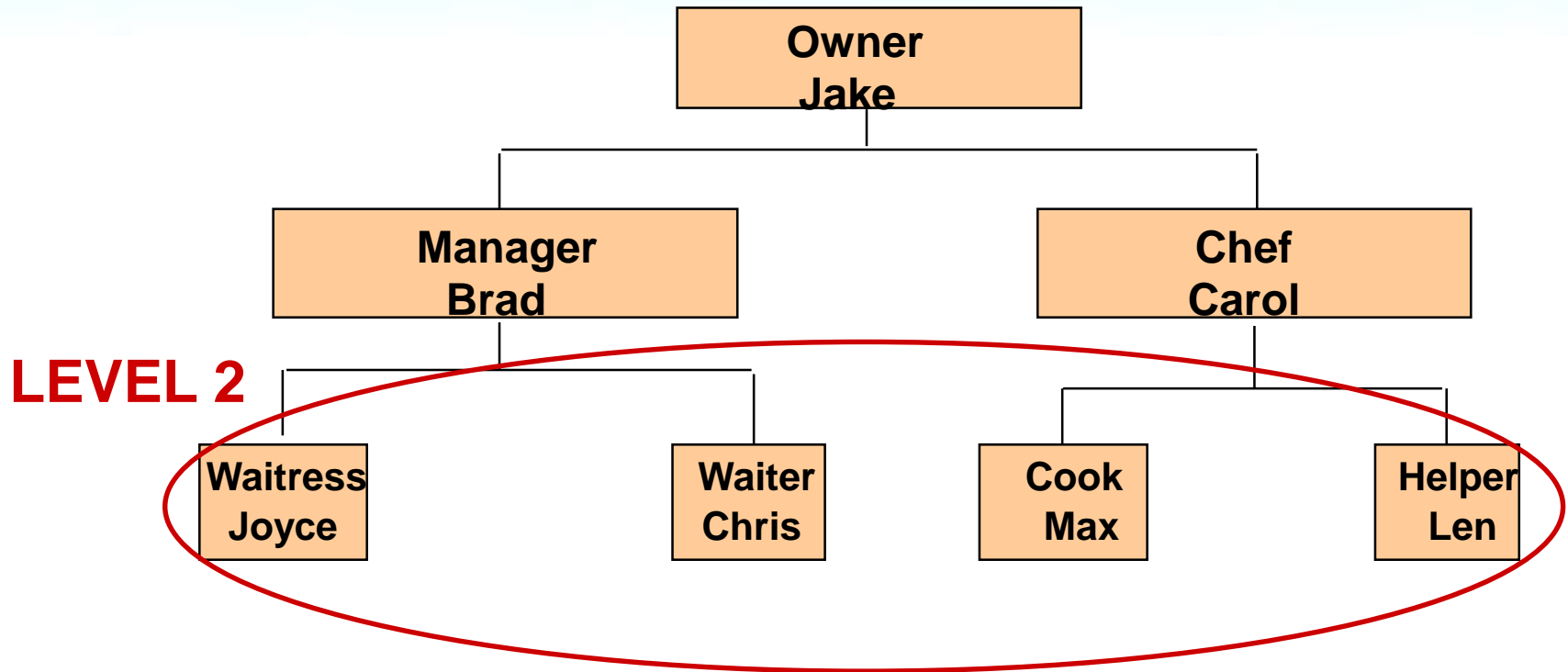
LEAF NODES

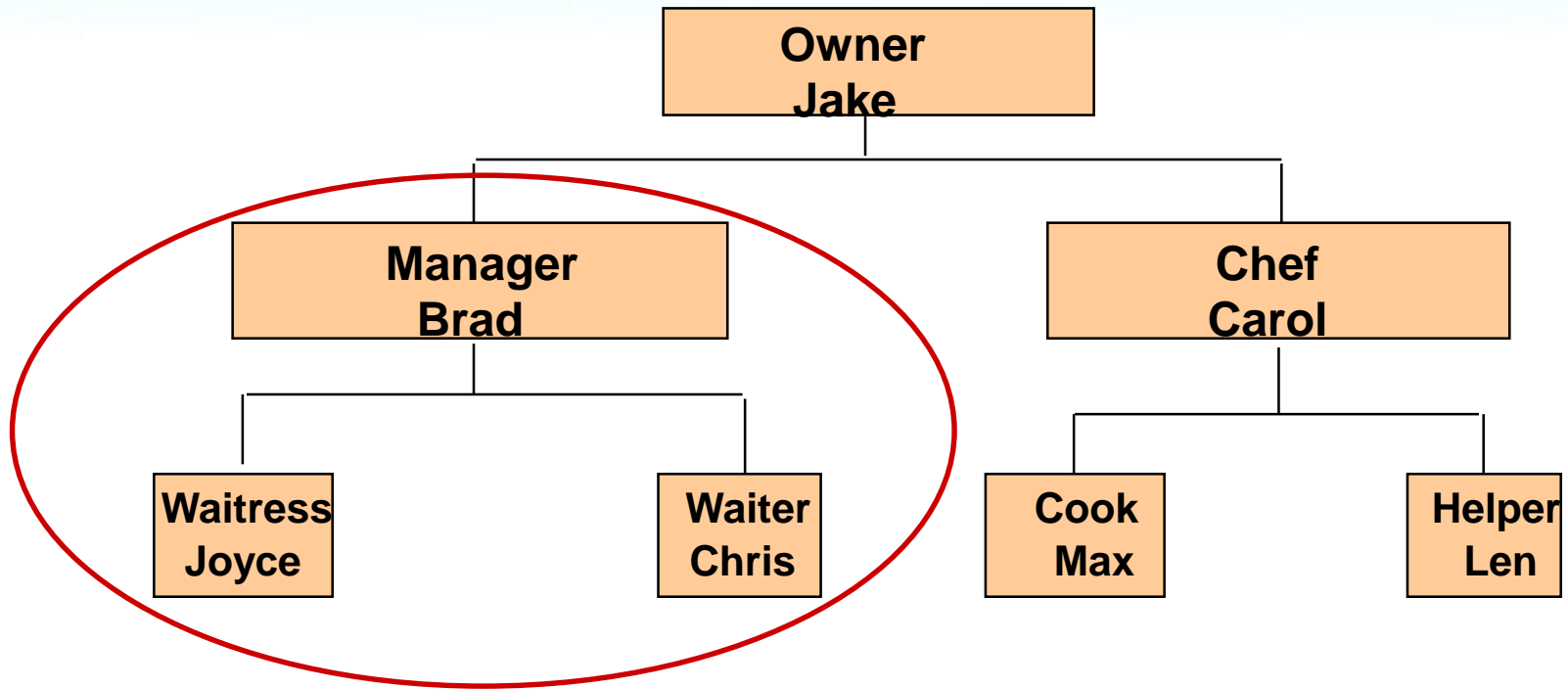
LEVEL 0



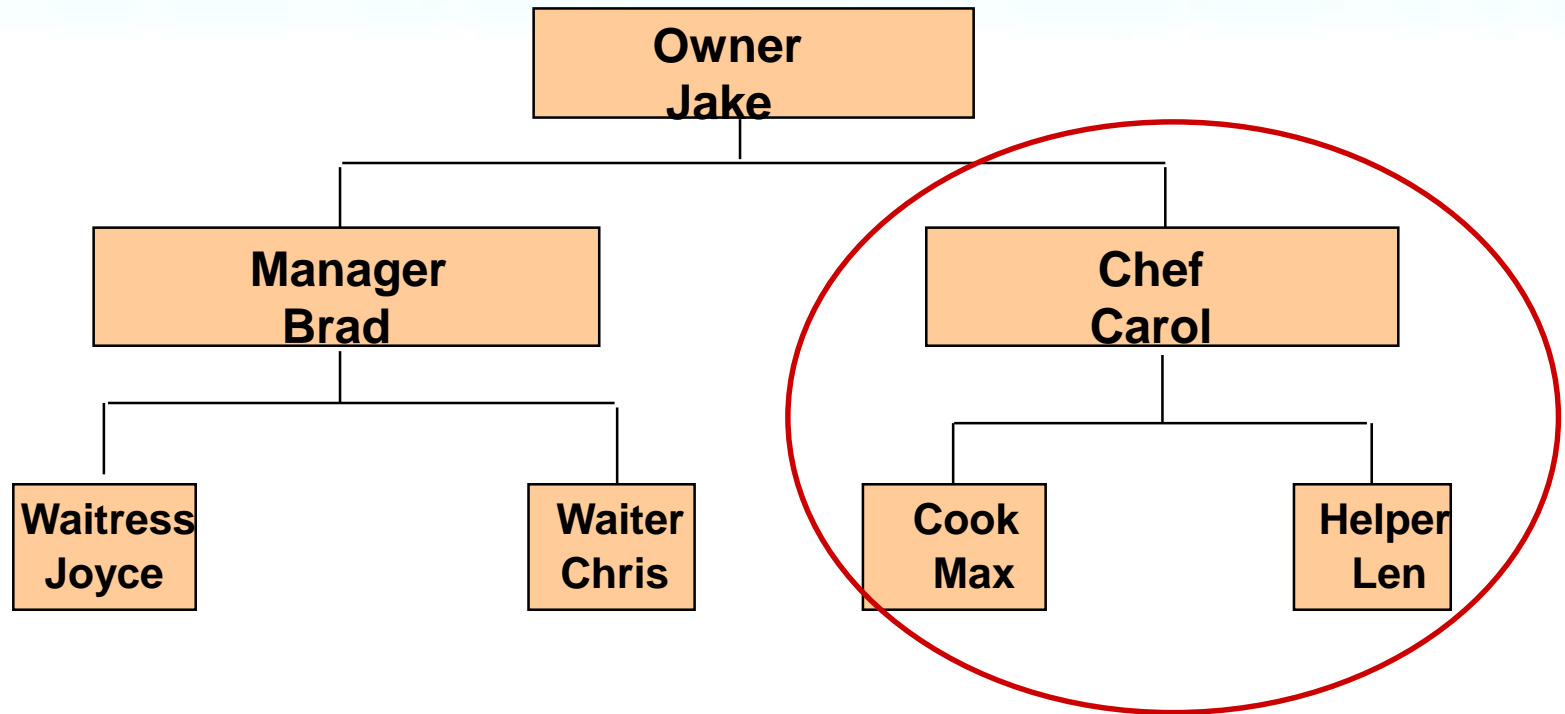
LEVEL 1





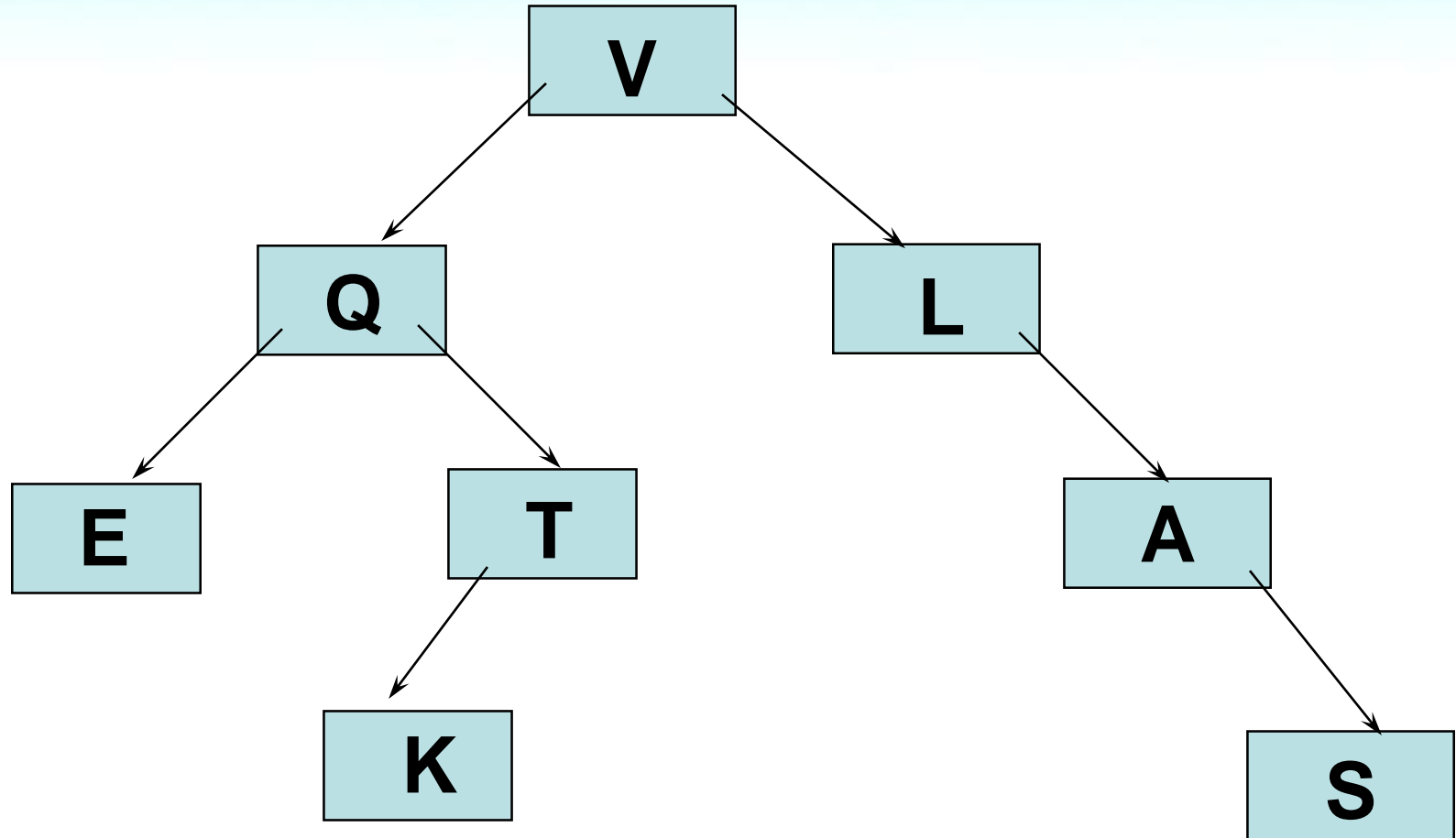


LEFT SUBTREE OF ROOT NODE

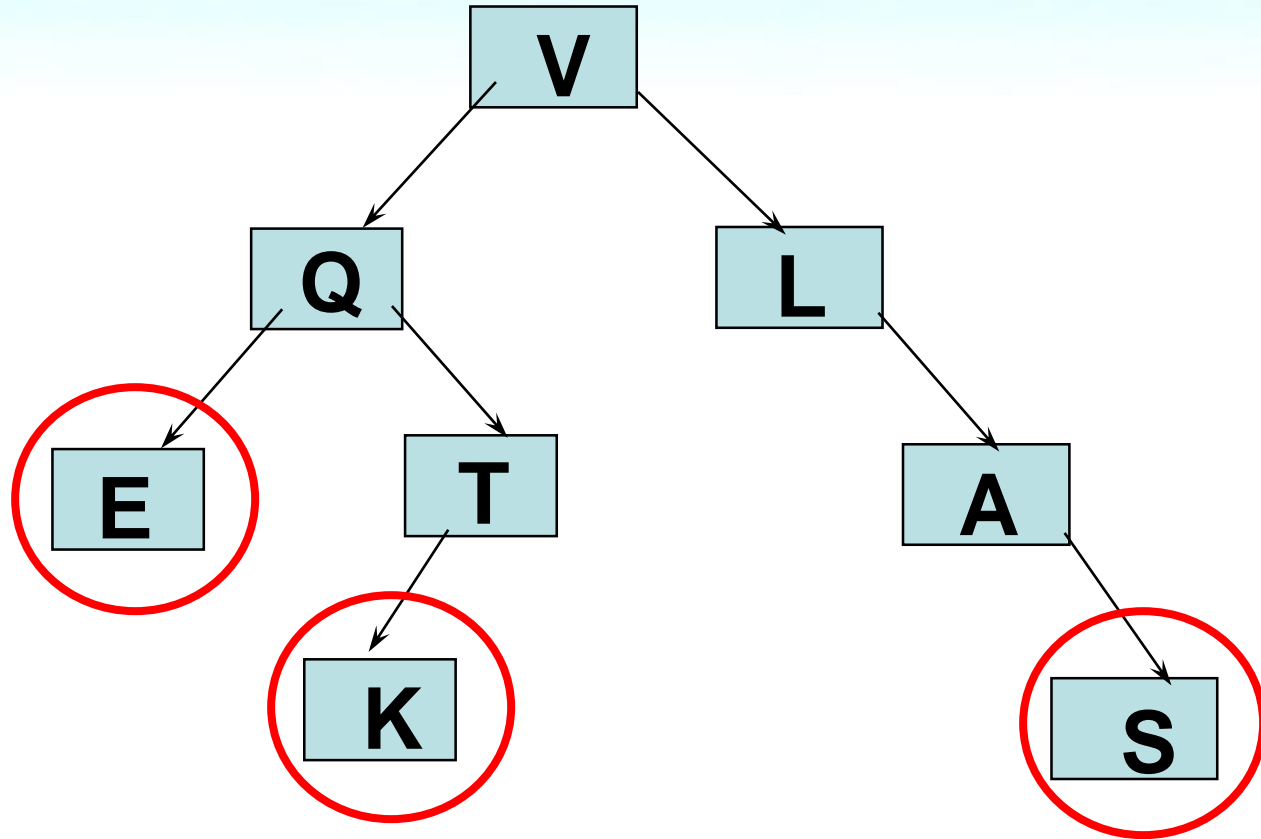


**RIGHT SUBTREE
OF ROOT NODE**

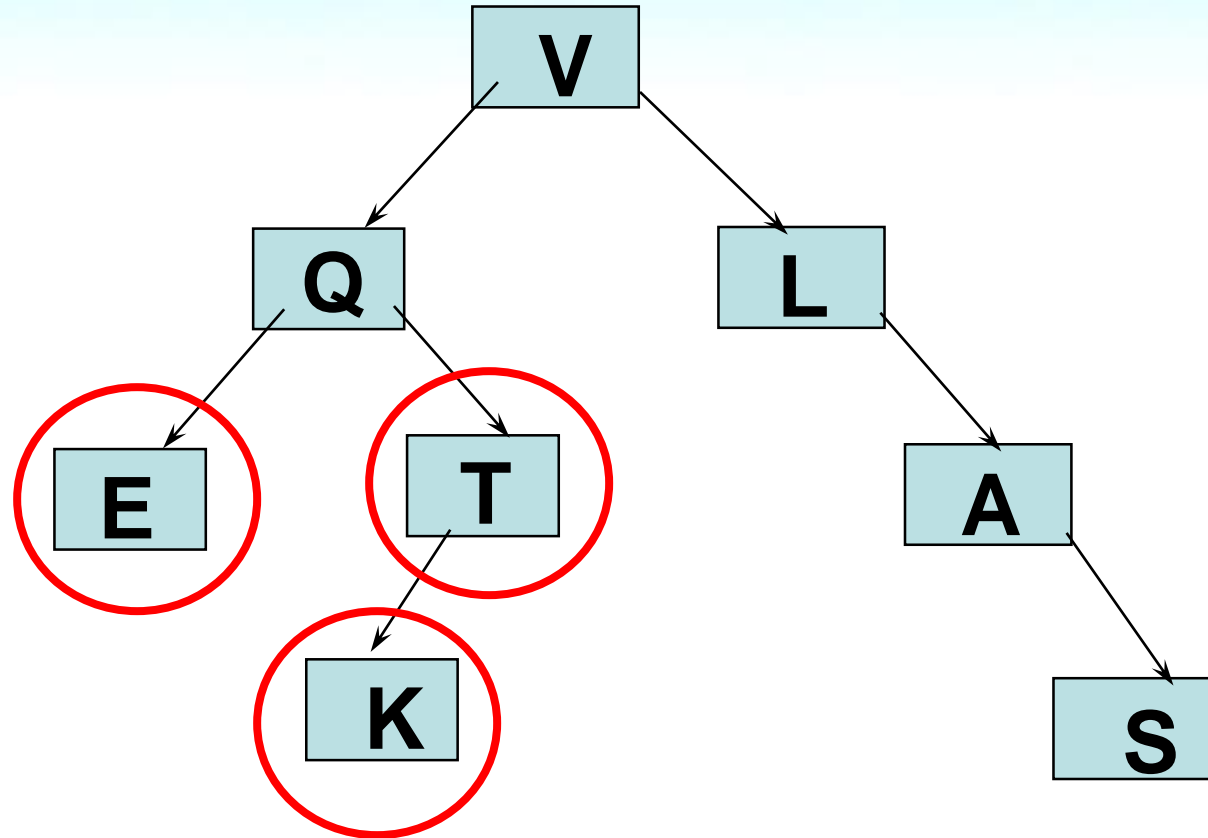
A Binary Tree



How many leaf nodes? 3

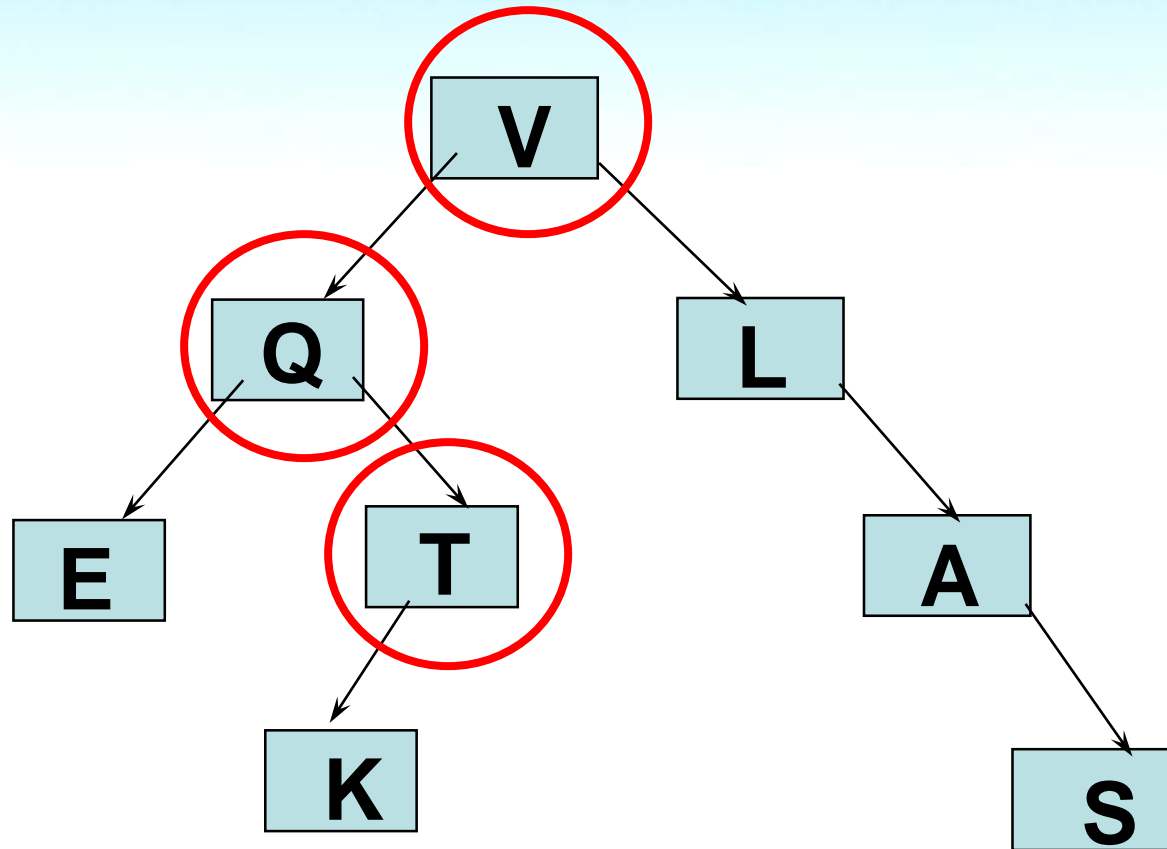


How many descendants of Q?



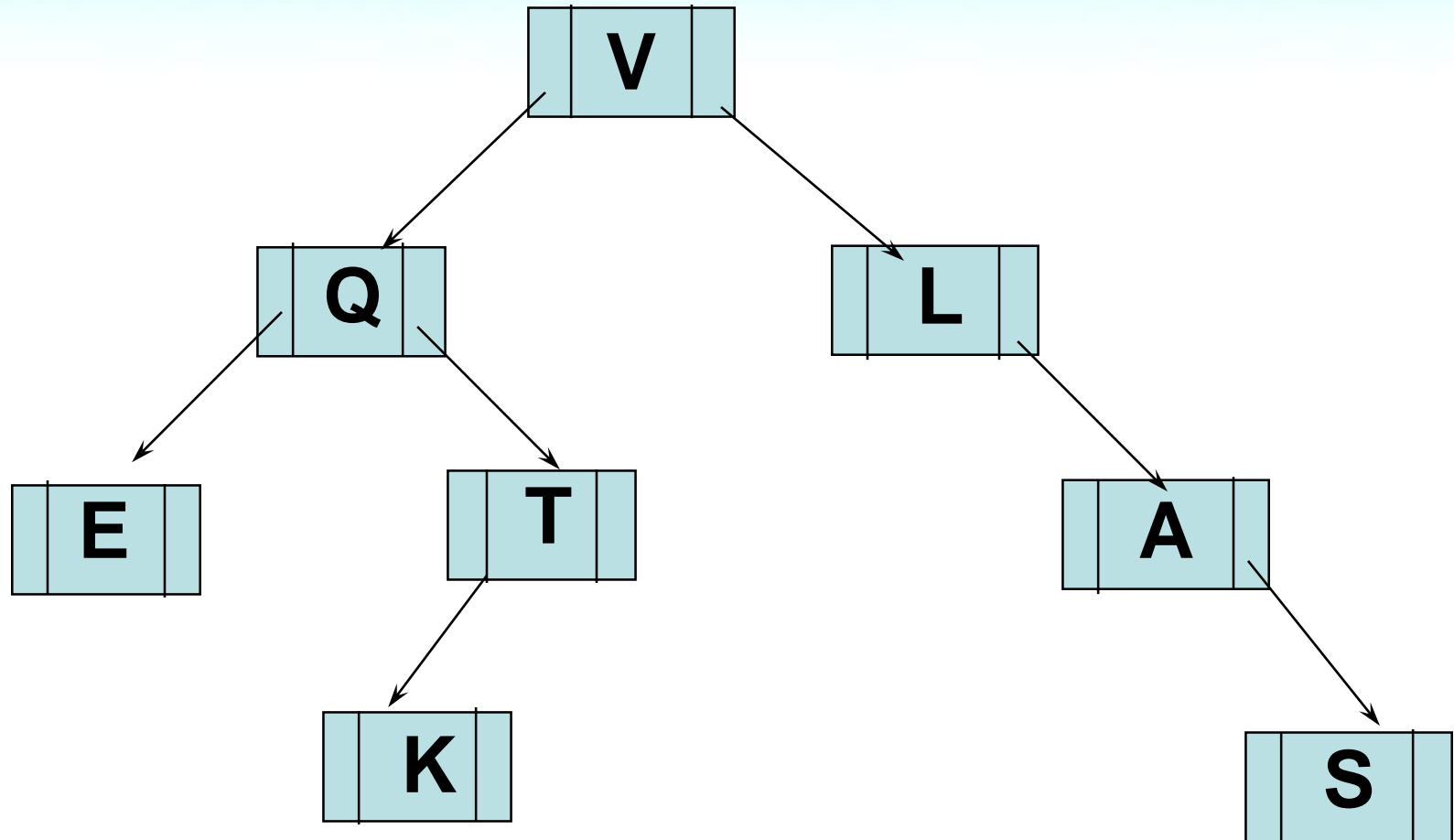
3

How many ancestors of K?



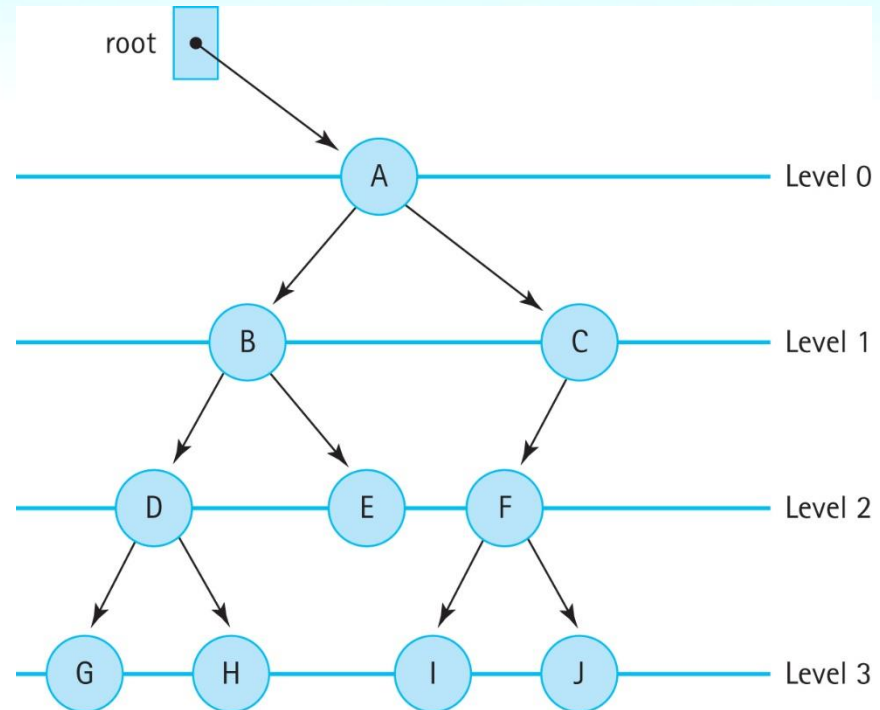
3

Implementing a Binary Tree with Reference links



7.2 Binary Search Trees

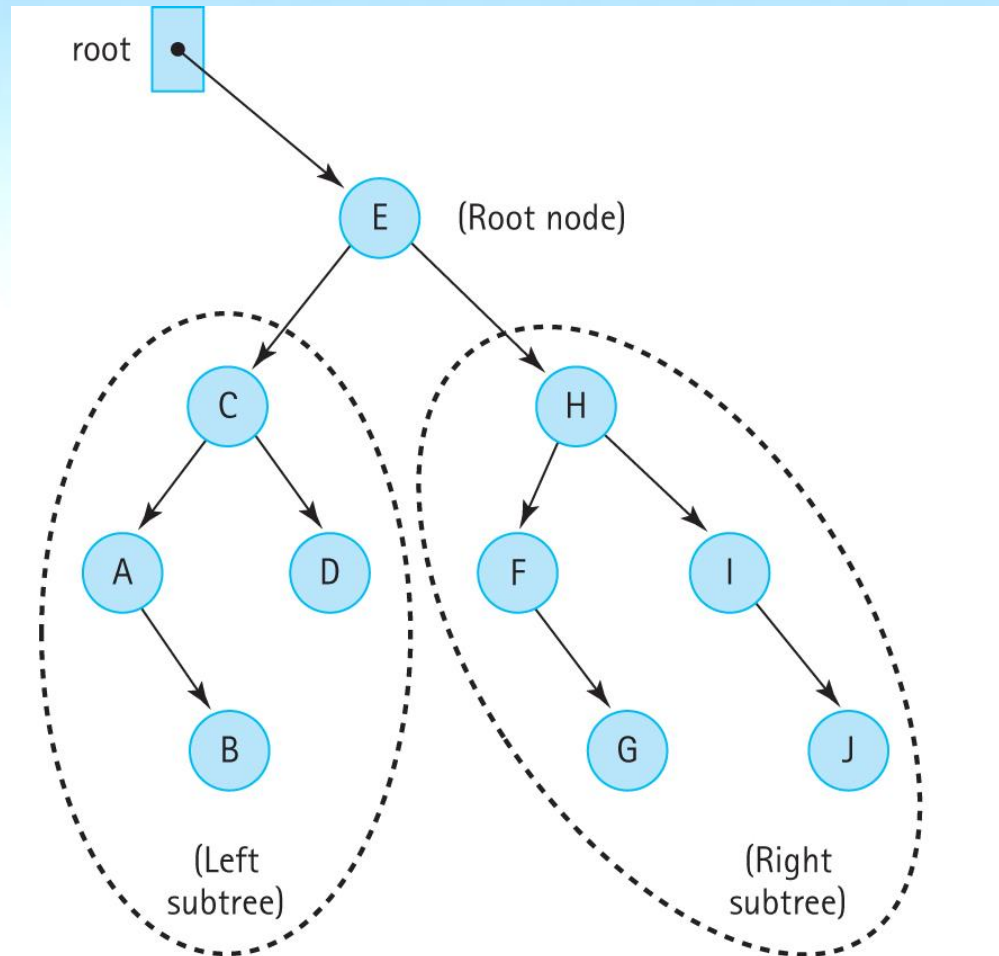
- **Binary tree** A tree in which each node is capable of having two child nodes, a left child node and a right child node



A Binary Tree

A Binary Search Tree

These trees facilitate searching for an element.

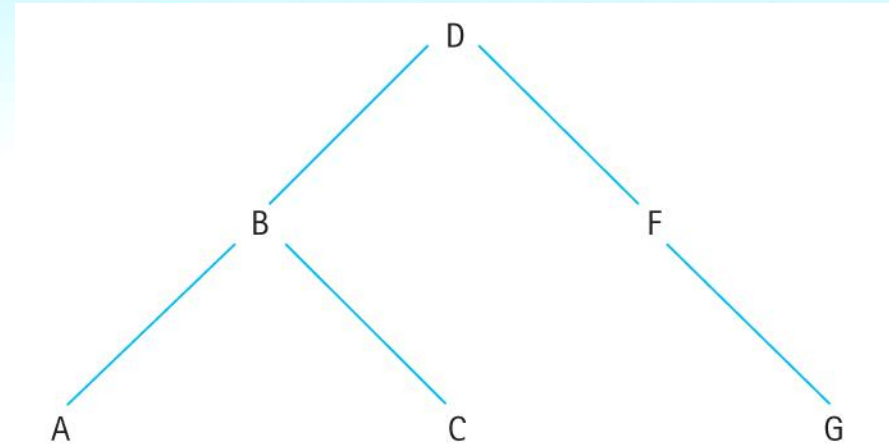


All values in the left subtree are less than or equal to the value in the root node.

All values in the right subtree are greater than the value in the root node.

Binary search tree

- A binary tree in which the key value in any node
 - is greater than or equal to the key value in its left child and any of its descendants (the nodes in the left subtree)
 - is less than the key value in its right child and any of its descendants (the nodes in the right subtree)
- We call this the “binary search tree” property



BST

A special kind of binary tree in which:

- 1. Each node contains a distinct data value,**
- 2. The key values in the tree can be compared using “greater than” and “less than”, and**
- 3. The key value of each node in the tree is less than every key value in its right subtree, and greater than every key value in its left subtree.**

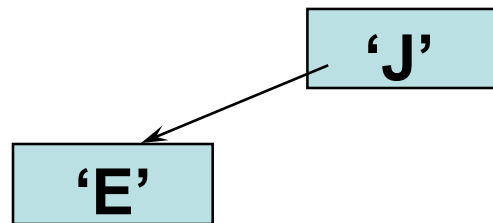
Shape of a binary search tree . . .

- **Depends on its key values and their order of insertion.**
- **Insert the elements 'J' 'E' 'F' 'T' 'A' in that order.**
- **The first value to be inserted is put into the root node.**

'E' < 'J'

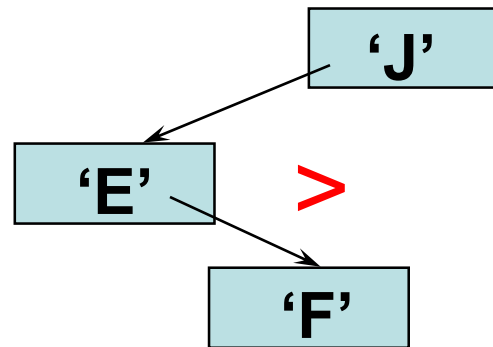
Inserting 'E' into the BST

- Thereafter, each value to be inserted begins by comparing itself to the value in the root node, moving left if it is less, or moving right if it is greater. This continues at each level until it can be inserted as a new leaf.



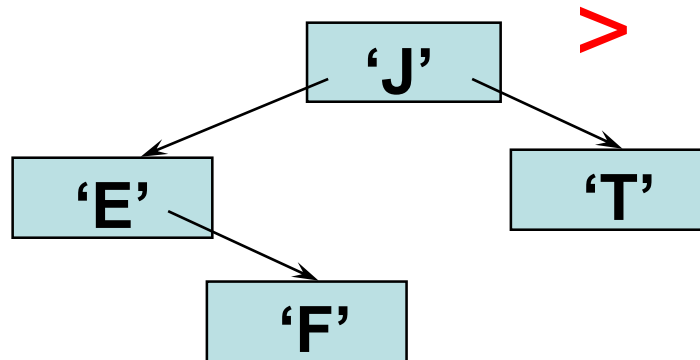
Inserting 'F' into the BST

- **Begin by comparing 'F' to the value in the root node, moving left if it is less, or moving right if it is greater. This continues until it can be inserted as a leaf.**



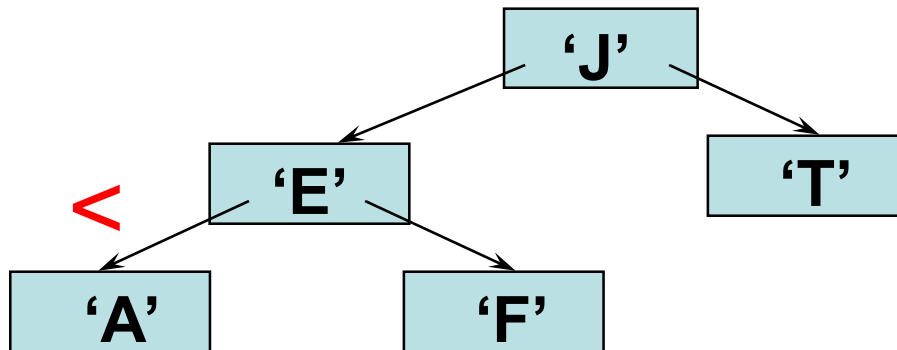
Inserting 'T' into the BST

Begin by comparing 'T' to the value in the root node, moving left if it is less, or moving right if it is greater. This continues until it can be inserted as a leaf.



Inserting 'A' into the BST

Begin by comparing 'A' to the value in the root node, moving left if it is less, or moving right if it is greater. This continues until it can be inserted as a leaf.



What binary search tree . . .

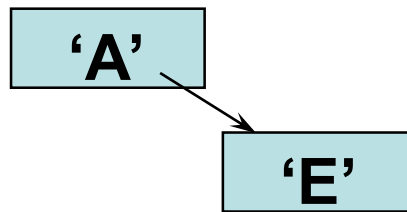
is obtained by inserting the elements 'A' 'E' 'F' 'J'
'T' in that order?



'A'

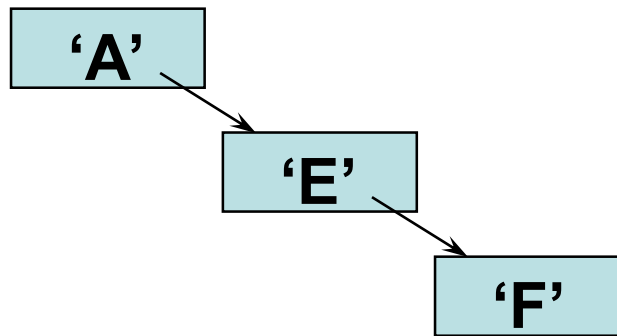
Binary search tree . . .

**obtained by inserting the elements 'A'
'E' 'F' 'J' 'T' in that order.**



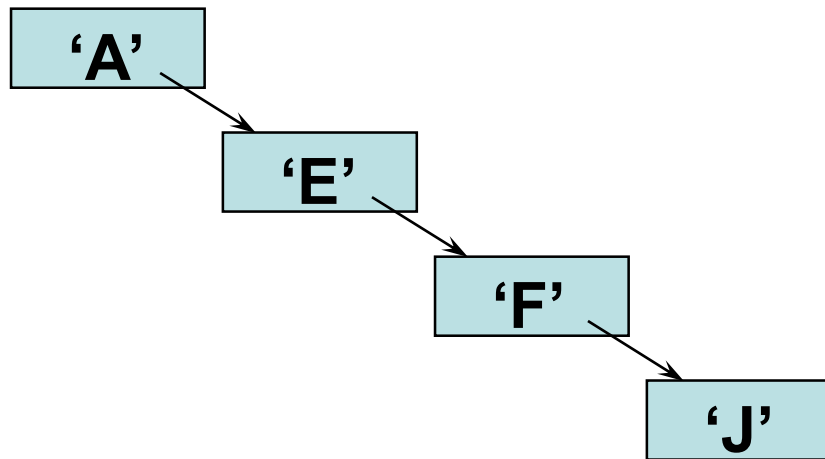
Binary search tree . . .

obtained by inserting the elements 'A'
'E' 'F' 'J' 'T' in that order.



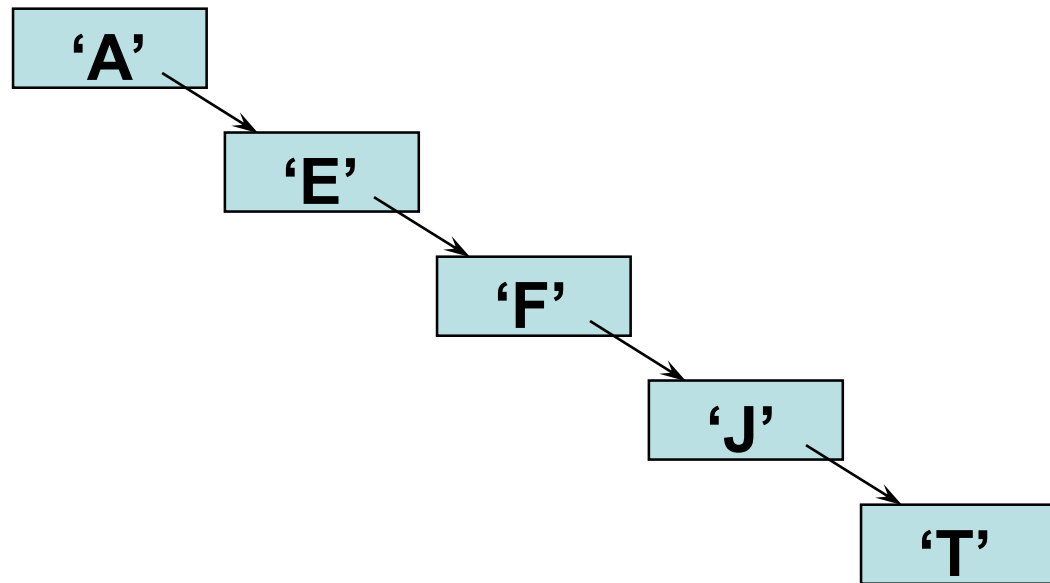
Binary search tree . . .

obtained by inserting the elements 'A'
'E' 'F' 'J' 'T' in that order.

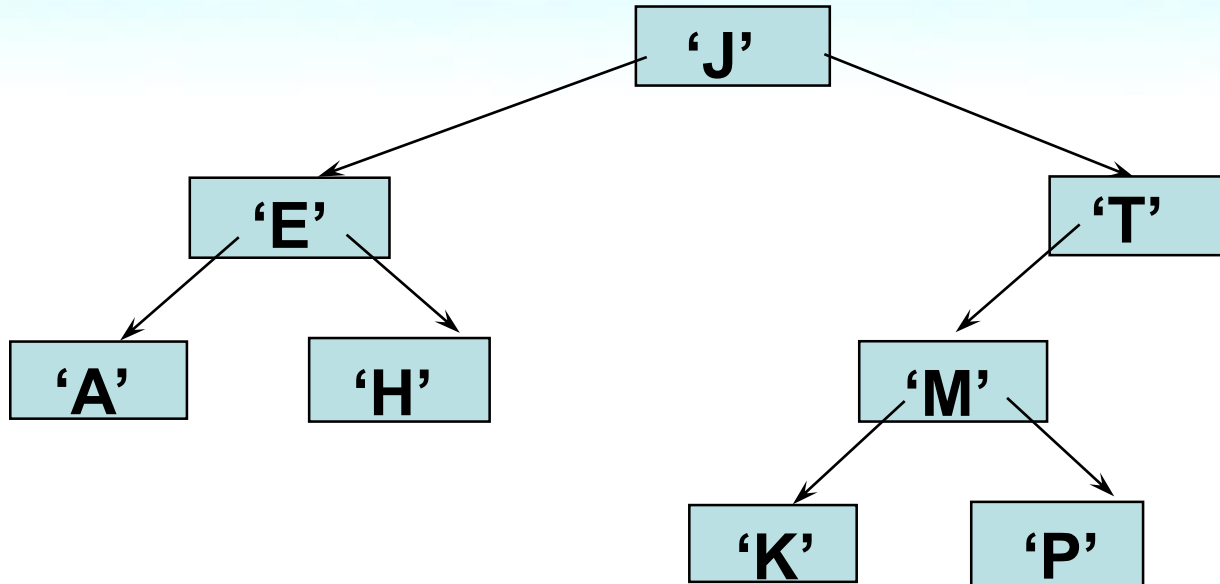


Binary search tree . . .

obtained by inserting the elements 'A'
'E' 'F' 'J' 'T' in that order.



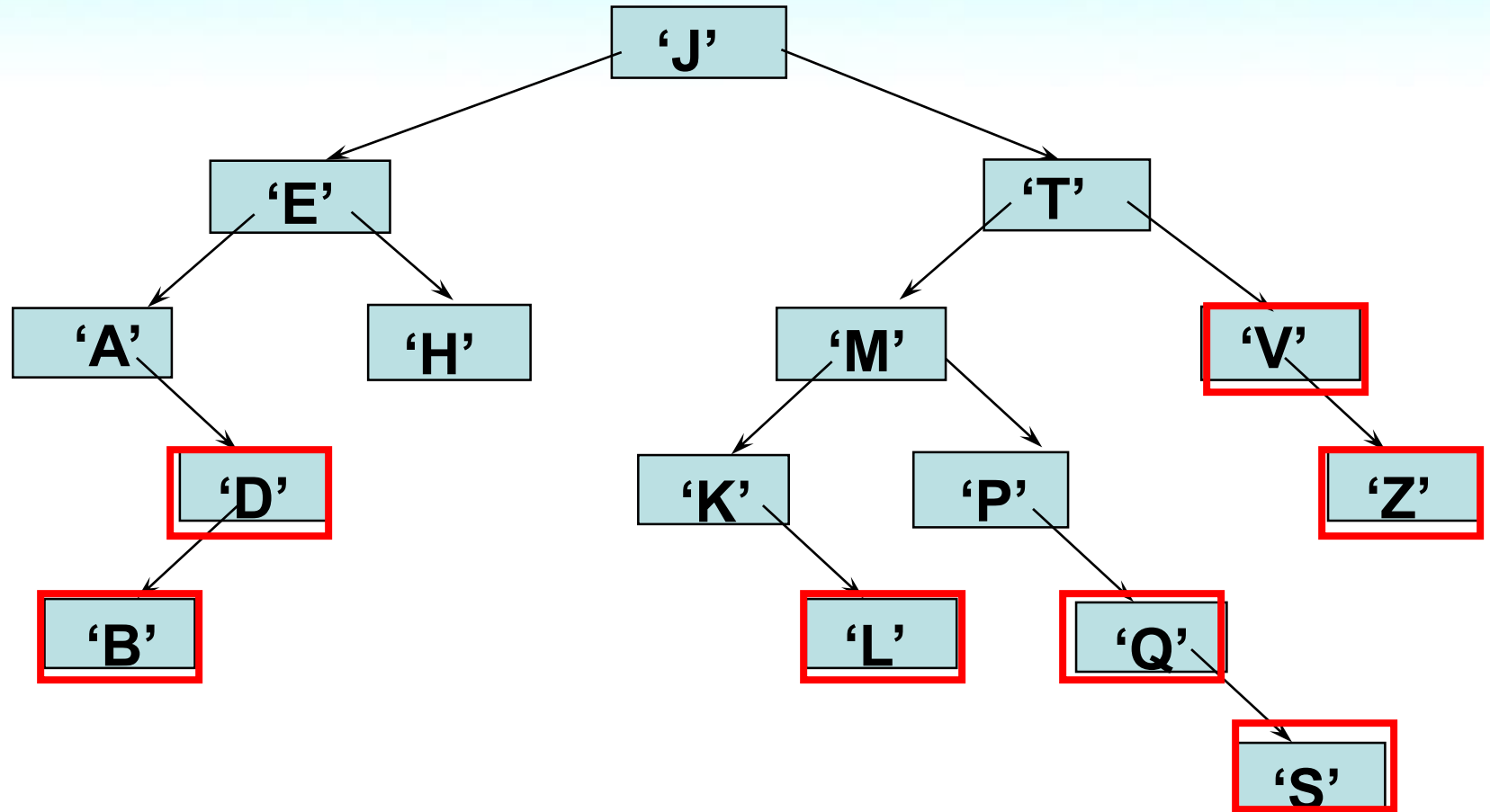
Another binary search tree



Add nodes containing these values in this order:

'D' 'B' 'L' 'Q' 'S' 'V' 'Z'

Is 'F' in the binary search tree?

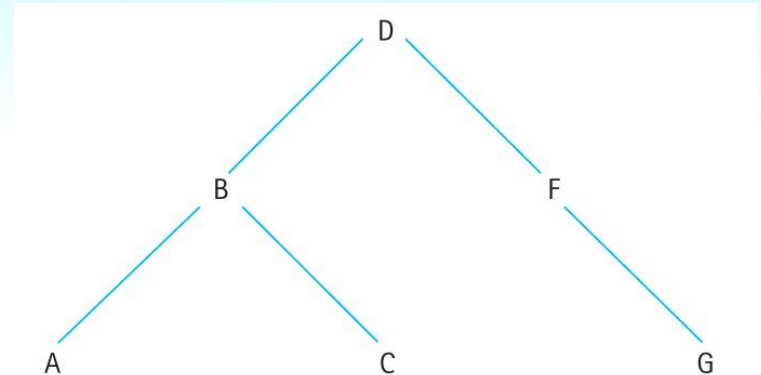


Traverse manner

- **In linear linked list**, set a temporary reference to the first element and follow **forward** and **backward**.
- Then how about tree? Nodes (elements) in a tree do not have linear relationship.
- Find good ways from next slides.

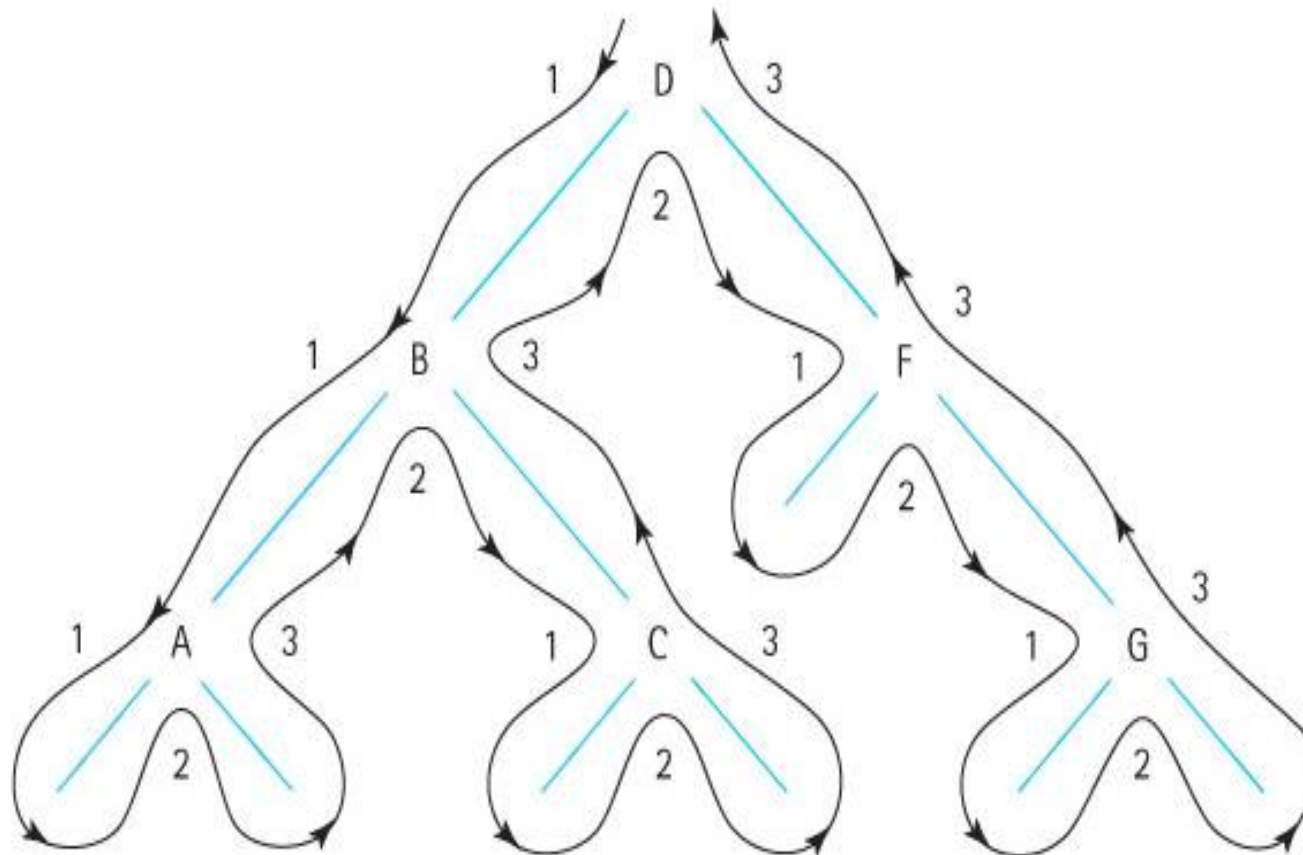
Binary Tree Traversals

- **Preorder traversal:** Visit the root, visit the left subtree, visit the right subtree:
 - D B A C F G
- **Inorder traversal:** Visit the left subtree, visit the root, visit the right subtree:
 - A B C D F G
- **Postorder traversal:** Visit the left subtree, visit the right subtree, visit the root:
 - A C B G F D



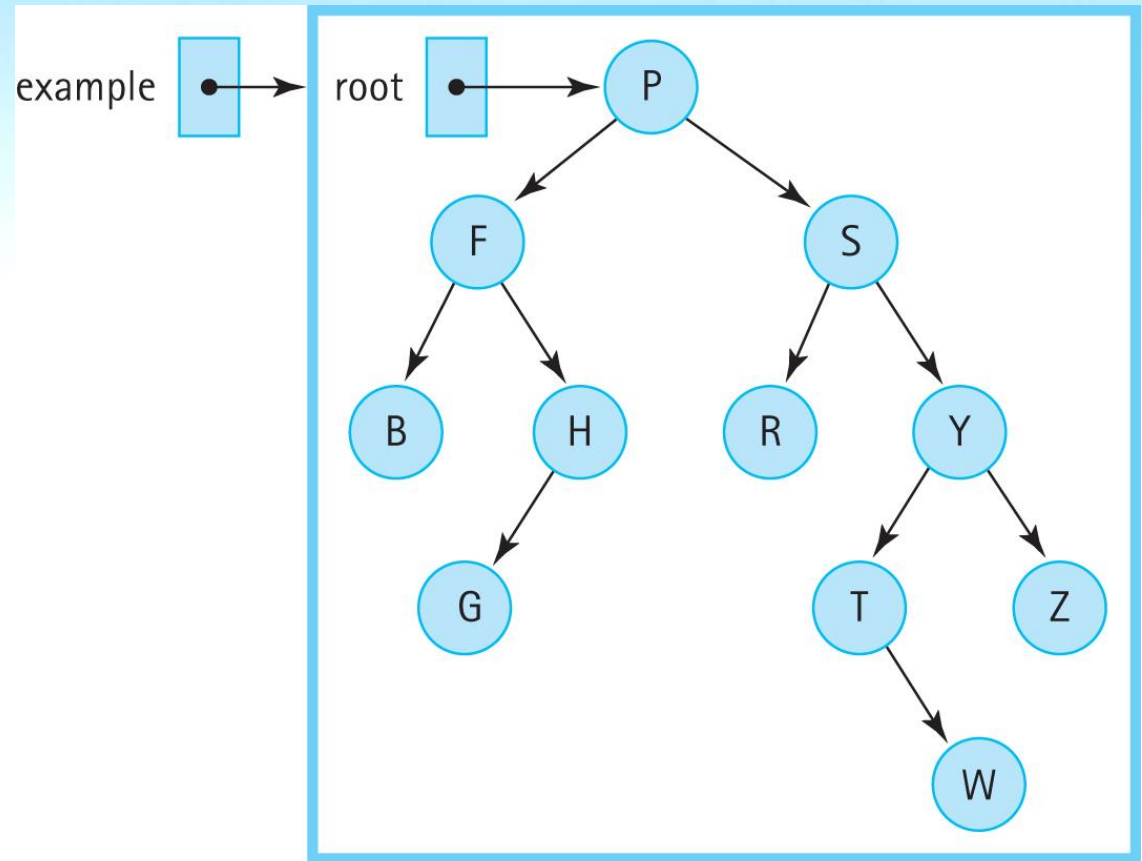
Visualizing Binary Tree Traversals

The extended tree



Preorder: DBACFG
Inorder: ABCDFG
Postorder: ACBGFD

Three Binary Tree Traversals



Inorder: B F G H P R S T W Y Z
Preorder: P F B H G S R Y T W Z
Postorder: B G H F R W T Z Y S P

7.3 – The Binary Search Tree Interface

- Our binary search trees
 - are similar to the sorted lists of Chapter 6
 - implement this text's `CollectionInterface`
 - implement the Java Library's `Iterable` interface
 - are unbounded, allow duplicate elements, and disallow `null` elements
 - support max and min operations
 - support preorder, inorder, and postorder traversals


```
package ch07.trees;

import ch05.collections.CollectionInterface;
import java.util.Iterator;

public interface BSTInterface<T> extends CollectionInterface<T>,
                                         Iterable<T>
{
    // Used to specify traversal order.
    public enum Traversal {Inorder, Preorder, Postorder};

    T min();
    // If this BST is empty, returns null;
    // otherwise returns the smallest element of the tree.

    T max();
    // If this BST is empty, returns null;
    // otherwise returns the largest element of the tree.

    public Iterator<T> getIterator(Traversal orderType);
    // Creates and returns an Iterator providing a traversal of a "snapshot"
    // of the current tree in the order indicated by the argument.
}
```

Iteration

- In addition to the `getIterator` method, a class that implements the `BSTInterface` must provide a separate `iterator` method, because `BSTInterface` **extends** `Iterable`.
- This method should return an `Iterator` that provides iteration in the “natural” order of the tree elements.
- For most applications this would be an inorder traversal, and we make that assumption in our implementation.

Iteration

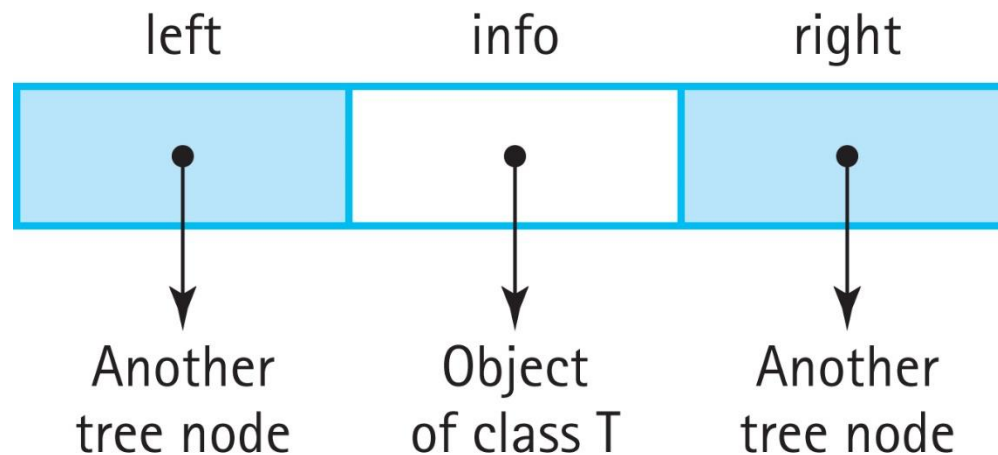
- We intend the iterators created and returned by `getIterator` and `iterator` to provide a **snapshot** of the tree as it exists at the time the iterator is requested.
- They represent the state of the tree at that time and subsequent changes to the tree should not affect the results returned by the iterator's `hasNext` and `next` methods.
- Our iterators will throw an `UnsupportedOperationException` if `remove` is invoked.

Example Application

- Instructors can review and demonstrate the application `BSTExample` found in the `ch07.apps` package
- This application demonstrates the use of iterators with our binary search tree

7.4 The Implementation Level: Basics

- We define `BSTNode.java` in our `support` package to provide nodes for our binary search trees
- Visually, a `BSTNode` object is:



BSTNode.java

instance variables:

```
private T info;           // The info in a BST node
private BSTNode<T> left;  // A link to the left child node
private BSTNode<T> right; // A link to the right child node
```

Constructor:

```
public BSTNode(T info)
{
    this.info = info;
    left = null;
    right = null;
}
```

Plus it includes the standard setters and getters

Beginning of BinarySearchTree.java

```
package ch07.trees;

import java.util.*;    // Iterator, Comparator

import ch04.queues.*;
import ch02.stacks.*;
import support.BSTNode;

public class BinarySearchTree<T> implements BSTInterface<T>
{
    protected BSTNode<T> root;        // reference to the root of this BST
    protected Comparator<T> comp;     // used for all comparisons

    protected boolean found;          // used by remove
    . . .
```

The Constructors

```
public BinarySearchTree()  
// Precondition: T implements Comparable  
// Creates an empty BST object - uses the natural order of elements.  
{  
    root = null;  
    comp = new Comparator<T>()  
    {  
        public int compare(T element1, T element2)  
        {  
            return ((Comparable)element1).compareTo(element2);  
        }  
    };  
}  
  
public BinarySearchTree(Comparator<T> comp)  
// Creates an empty BST object - uses Comparator comp for order of elements.  
{  
    root = null;  
    this.comp = comp;  
}  
. . .
```


Example Observer methods

```
public boolean isFull()  
// Returns false; this link-based BST is never full.  
{  
    return false;  
}  
  
public boolean isEmpty()  
// Returns true if this BST is empty; otherwise, returns false.  
{  
    return (root == null);  
}  
  
public T min()  
// If this BST is empty, returns null;  
// otherwise returns the smallest element of the tree.  
{  
    if (isEmpty())  
        return null;  
    else  
    {  
        BSTNode<T> node = root;  
        while (node.getLeft() != null)  
            node = node.getLeft();  
        return node.getInfo();  
    }  
}
```

7.5 Iterative versus Recursive Method Invocations

- Trees are inherently recursive; a tree consists of subtrees
- In this section we look at recursive and iterative approaches to the `size` method
- We then discuss the benefits of recursion versus iteration for this problem

Recursive Approach

- We create a public method, `size`, that calls a private recursive method, `recSize` and passes it a reference to the root of the tree.

```
public int size()  
// Returns the number of elements in this BST.  
{  
    return recSize(root);  
}
```

- We design the `recSize` method to return the number of nodes in the subtree referenced by the argument passed to it.
- Note that the number of nodes in a tree is:
 $1 + \text{number of nodes in left subtree} + \text{number of nodes in right subtree}$

recSize Algorithm

Version 1

recSize(node): returns int

if (node.getLeft() is null) AND (node.getRight() is null)

 return 1

else

 return 1 + recSize(node.getLeft()) + recSize(node.getRight())

- The corresponding method would crash when we try to access `node.getLeft` when `node` is `null`.

recSize Algorithm Version 2

recSize(node): returns int Version 2

if (node is null)

 return 0

else

if (node.getLeft() is null) AND (node.getRight() is null)

 return 1

else

 return 1 + recSize(node.getLeft()) + recSize(node.getRight())

- Works, but can be simplified. There is no need to make the leaf node a special case – it would also be handled properly by the final else clause

recSize Algorithm

Version 3

recSize(node): returns int Version 3

if node is null

 return 0

else

 return 1 + recSize(node.getLeft()) + recSize(node.getRight())

- Works and is “simple”.
- This example illustrates two important points about recursion with trees:
 - always check for the empty tree first
 - leaf nodes do not need to be treated as separate cases.

The recSize Code

```
private int recSize(BSTNode<T> node)
// Returns the number of elements in subtree rooted at node.
{
    if (node == null)
        return 0;
    else
        return 1 + recSize(node.getLeft()) + recSize(node.getRight());
}
```

Iterative Version

- We use a stack to hold nodes we have encountered but not yet processed
- We must be careful that we process each node in the tree exactly once. We follow these rules:
 - Process a node immediately after removing it from the stack.
 - Do not process nodes at any other time.
 - Once a node is removed from the stack, do not push it back onto the stack.

Code for the iterative approach

```
public int size()
// Returns the number of elements in this BST.
{
    int count = 0;
    if (root != null)
    {
        LinkedStack<BSTNode<T>> nodeStack = new LinkedStack<BSTNode<T>> ();
        BSTNode<T> currNode;
        nodeStack.push(root);
        while (!nodeStack.isEmpty())
        {
            currNode = nodeStack.top();
            nodeStack.pop();
            count++;
            if (currNode.getLeft() != null)
                nodeStack.push(currNode.getLeft());
            if (currNode.getRight() != null)
                nodeStack.push(currNode.getRight());
        }
    }
    return count;
}
```

Recursion or Iteration?

- *Is the depth of recursion relatively shallow?* **Yes**
- *Is the recursive solution shorter or clearer than the nonrecursive version?* **Yes**
- *Is the recursive version much less efficient than the nonrecursive version?* **No**
- This is a good use of recursion.

7.6 The Implementation Level: Remaining Operations

- In this section, we use recursion to implement the remaining Binary Search Tree operations
 - `contains`
 - `get`
 - `add`
 - `remove`
 - `iterator`
 - `getIterator`

The contains operation

- We implement `contains` using a private recursive method called `recContains` which
 - is passed the `target` we are searching for and a `node` representing the root of the subtree in which to search
 - has two base cases
 - if `node` is `null` returns `false`
 - if `node` contains `target` returns `true`
 - has two recursive cases
 - one searches the left subtree of `node`
 - one searches the right subtree of `node`

The contains method

```
public boolean contains (T target)
// Returns true if this BST contains a node with info i such that
// comp.compare(target, i) == 0; otherwise, returns false.
{
    return recContains(target, root);
}

private boolean recContains(T target, BSTNode<T> node)
// Returns true if the subtree rooted at node contains info i such that
// comp.compare(target, i) == 0; otherwise, returns false.
{
    if (node == null)
        return false;          // target is not found
    else if (comp.compare(target, node.getInfo()) < 0)
        return recContains(target, node.getLeft());    // Search left subtree
    else if (comp.compare(target, node.getInfo()) > 0)
        return recContains(target, node.getRight());   // Search right subtree
    else
        return true;          // target is found
}
```

The get method is similar

```
public T get(T target)
// Returns info i from node of this BST where comp.compare(target, i) == 0;
// if no such node exists, returns null.
{
    return recGet(target, root);
}

private T recGet(T target, BSTNode<T> node)
// Returns info i from the subtree rooted at node such that
// comp.compare(target, i) == 0; if no such info exists, returns null.
{
    if (node == null)
        return null;                // target is not found
    else if (comp.compare(target, node.getInfo()) < 0)
        return recGet(target, node.getLeft());        // get from left subtree
    else
        if (comp.compare(target, node.getInfo()) > 0)
            return recGet(target, node.getRight());    // get from right subtree
        else
            return node.getInfo();    // target is found
}
```

Iteration: Review of Traversal Definitions

- **Preorder traversal:** Visit the root, visit the left subtree, visit the right subtree
- **Inorder traversal:** Visit the left subtree, visit the root, visit the right subtree
- **Postorder traversal:** Visit the left subtree, visit the right subtree, visit the root

the `getIterator` method

- client passes `getIterator` an argument indicating one of the three traversal orders
- `getIterator` creates the appropriate `iterator` and returns it by traversing the tree in the desired order and building a queue of T
- It then creates an `iterator` using the anonymous inner class approach
- returned `iterator` represents a snapshot of the tree at the time `getIterator` is invoked and does not support `remove`


```
public Iterator<T> getIterator(BSTInterface.Traversal orderType)
{
    final LinkedList<T> infoQueue = new LinkedList<T>();
    if (orderType == BSTInterface.Traversal.Preorder)
        preOrder(root, infoQueue);
    else if (orderType == BSTInterface.Traversal.Inorder)
        inOrder(root, infoQueue);
    else if (orderType == BSTInterface.Traversal.Postorder)
        postOrder(root, infoQueue);

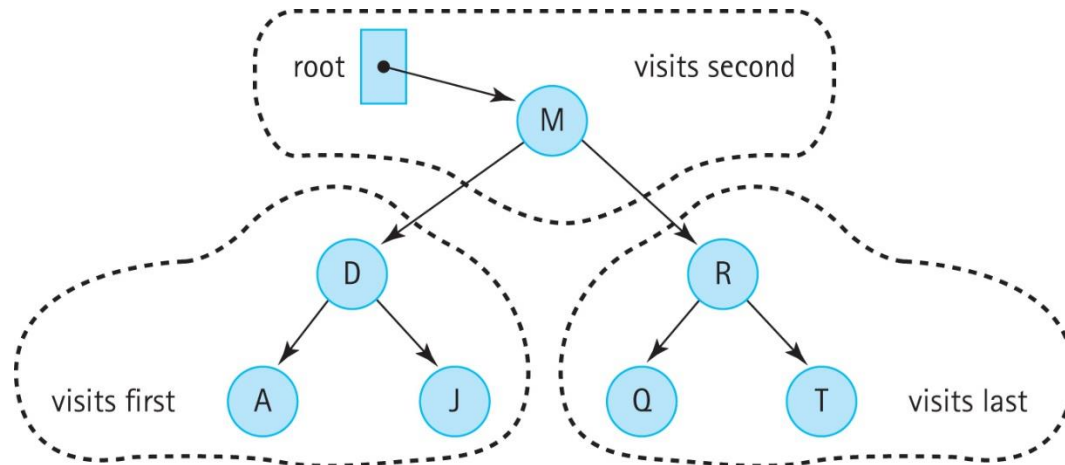
    return new Iterator<T>()
    {
        public boolean hasNext()
        {
            return !infoQueue.isEmpty();
        }

        public T next()
        {
            if (!hasNext())
                throw new IndexOutOfBoundsException("illegal invocation of next ...");
            return infoQueue.dequeue();
        }

        public void remove()
        {
            throw new UnsupportedOperationException("Unsupported remove attempted ...");
        }
    };
}
```

InOrder traversal

```
private void inOrder(BSTNode<T> node, LinkedQueue<T> q)
// Enqueues the elements from the subtree rooted at node into q in inOrder.
{  if (node != null)
    {
        inOrder(node.getLeft(), q);
        q.enqueue(node.getInfo());
        inOrder(node.getRight(), q);
    }
}
```



preOrder and postorder traversals

```
private void preOrder(BSTNode<T> node, LinkedQueue<T> q)
{
    if (node != null)
    {
        q.enqueue(node.getInfo());
        preOrder(node.getLeft(), q);
        preOrder(node.getRight(), q);
    }
}
```

```
private void postOrder(BSTNode<T> node, LinkedQueue<T> q)
{
    if (node != null)
    {
        postOrder(node.getLeft(), q);
        postOrder(node.getRight(), q);
        q.enqueue(node.getInfo());
    }
}
```


7.7 The Implementation Level: Transformers


- To complete our implementation of the Binary Search Tree ADT we need to create the transformer methods add and remove.
- These are the most complex operations.
- We use a similar approach as used in the subsection “Transforming a Linked List” in Section 3.4, “Recursive Processing of Linked Lists”


The add operation


- A new node is always inserted into its appropriate position in the tree as a leaf


(a) root 


(b) add 5
root  → 5


(c) add 9
root  → 5
 ↓
 9


(d) add 7
root  → 5
 ↓
 9
 ↓
 7


(e) add 3
root  → 5
 ↓
 3 9
 ↓
 7

(f) add 8
root  → 5
 ↓
 3 9
 ↓
 7
 ↓
 8

(g) add 12
root  → 5
 ↓
 3 9
 ↓
 7 12
 ↓
 8

(h) add 6
root  → 5
 ↓
 3 9
 ↓
 7 12
 ↓
 6 8

(i) add 5
root  → 5
 ↓
 3 9
 ↓
 5 7 12
 ↓
 6 8

(j) add 20
root  → 5
 ↓
 3 9
 ↓
 5 7 12
 ↓
 6 8 20

The add operation

- The `add` method invokes the recursive method, `recAdd`, and passes it the element to be added plus a reference to the root of the tree.

```
public boolean add (T element)
// Adds element to this BST. The tree retains its BST property.
{
    root = recAdd(element, root);
    return true;
}
```

- The call to `recAdd` returns a `BSTNode`. It returns a reference to the new tree, that is, to the tree that includes element. The statement

```
root = recAdd(element, root);
```

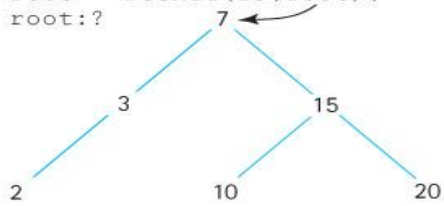
can be interpreted as “Set the reference of the root of this tree to the root of the tree that is generated when element is added to this tree.”

The add method

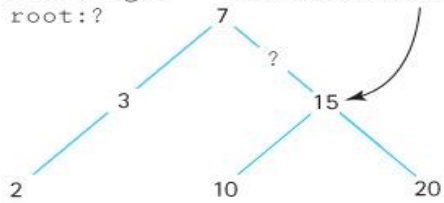
```
private BSTNode<T> recAdd(T element, BSTNode<T> node)
// Adds element to tree rooted at node; tree retains its BST property.
{
    if (node == null)
        // Addition place found
        node = new BSTNode<T>(element);
    else if (element.compareTo(node.getInfo()) <= 0)
        node.setLeft(recAdd(element, node.getLeft()));    // Add in left subtree
    else
        node.setRight(recAdd(element, node.getRight()));    // Add in right subtree
    return tree;
}

public boolean add (T element)
// Adds element to this BST. The tree retains its BST property.
{
    root = recAdd(element, root);
    return true;
}
```

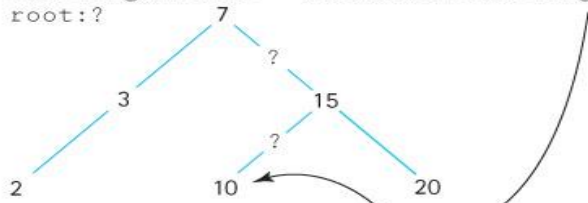
(a) `root = recAdd(13, root);`
`root: ?`



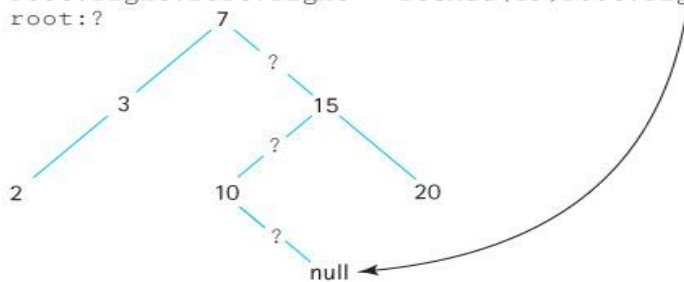
(b) `root.right = recAdd(13, root.right);`
`root: ?`



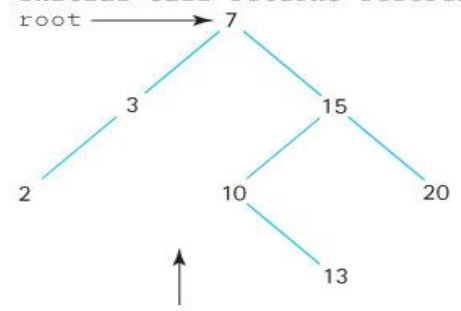
(c) `root.right.left = recAdd(13, root.right.left);`
`root: ?`



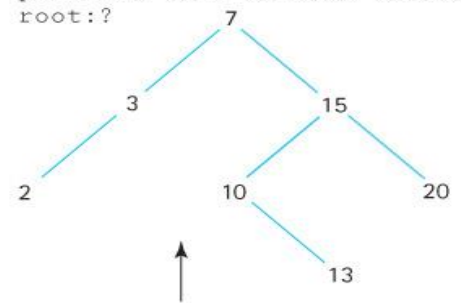
(d) `root.right.left.right = recAdd(13, root.right.left.right);`
`root: ?`



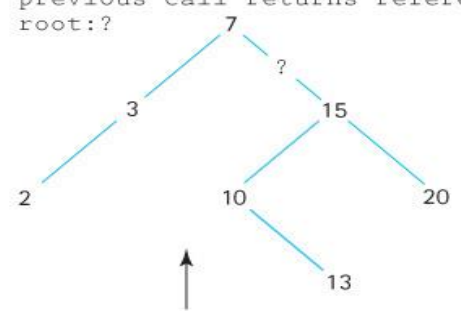
(h) Initial call returns reference
`root` →



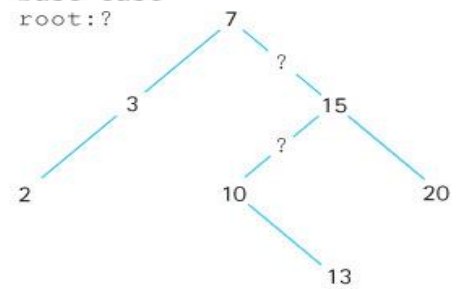
(g) previous call returns reference
`root: ?`



(f) previous call returns reference
`root: ?`



(e) Base case
`root: ?`



The `remove` Operation

- The most complicated of the binary search tree operations.
- We must ensure when we remove an element we maintain the binary search tree property.

The code for `remove`:

- The set up for the `remove` operation is the same as that for the `add` operation.
- The private `recRemove` method is invoked from the public `remove` method with arguments equal to the target element to be removed and the root of the tree to remove it from.
- The recursive method returns a reference to the revised tree
- The `remove` method returns the boolean value stored in `found`, indicating the result of the `remove` operation.

```
public boolean remove (T target)
// Removes a node with info i from tree such that
// comp.compare(target,i) == 0 and returns true;
// if no such node exists, returns false.
{
    root = recRemove(target, root);
    return found;
}
```

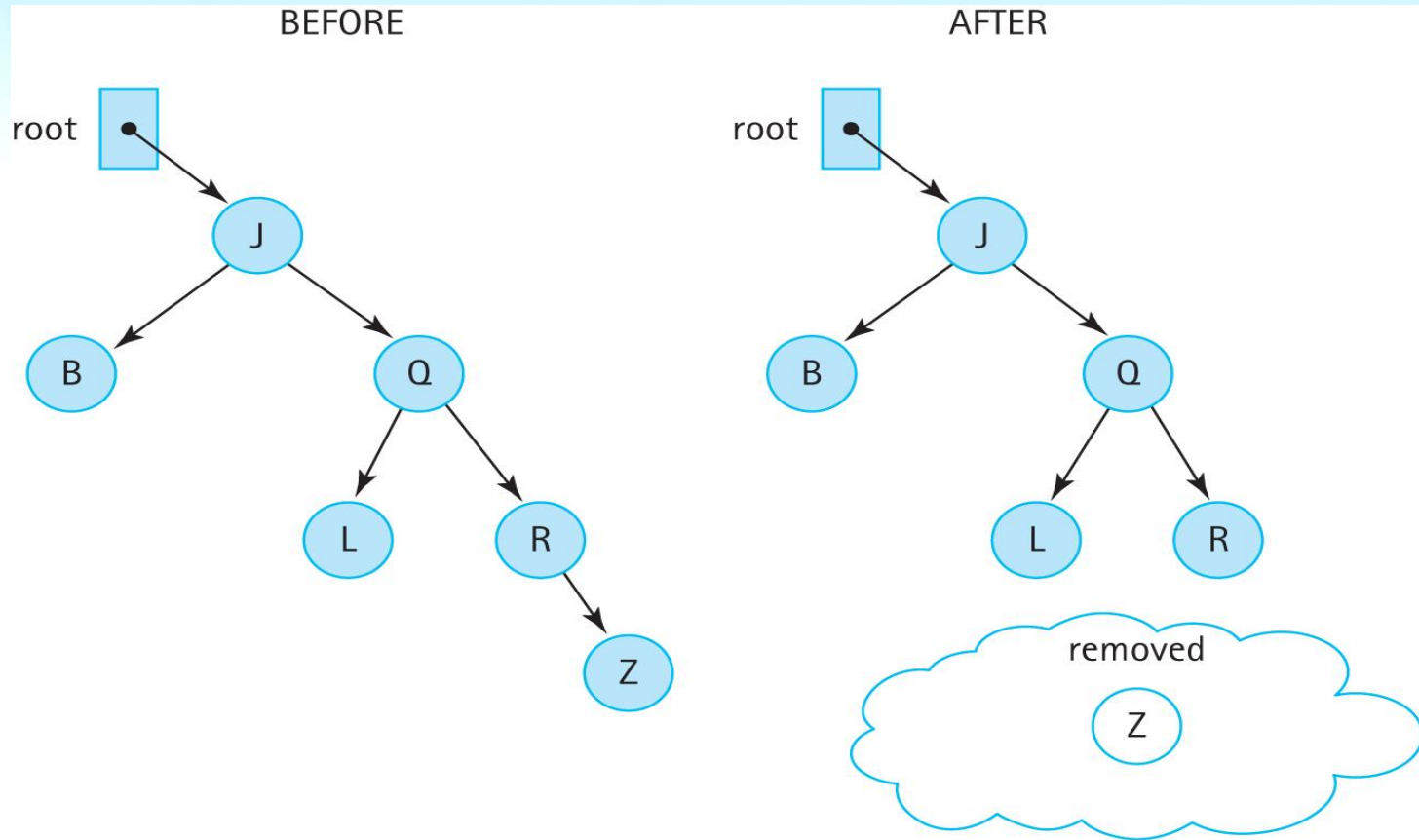
The recRemove method

```
private BSTNode<T> recRemove(T target, BSTNode<T> node)
// Removes element with info i from tree rooted at node such that
// comp.compare(target, i) == 0 and returns true;
// if no such node exists, returns false.
{
    if (node == null)
        found = false;
    else if (comp.compare(target, node.getInfo()) < 0)
        node.setLeft(recRemove(target, node.getLeft()));
    else if (comp.compare(target, node.getInfo()) > 0)
        node.setRight(recRemove(target, node.getRight()));
    else
    {
        node = removeNode(node);
        found = true;
    }
    return node;
}
```

Three cases for the `removeNode` operation

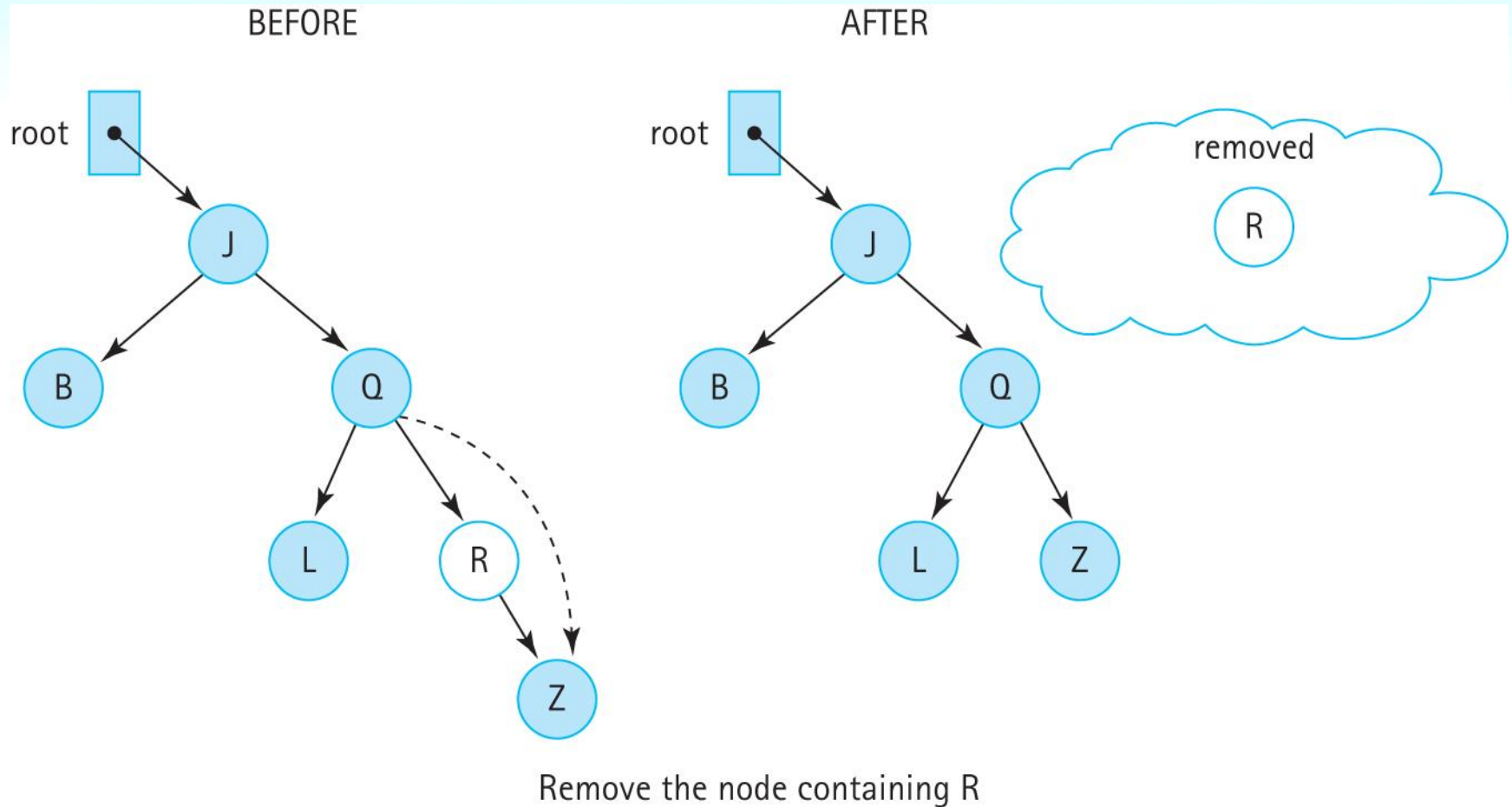
- Removing a leaf (no children): removing a leaf is simply a matter of setting the appropriate link of its parent to null.
- Removing a node with only one child: make the reference from the parent skip over the removed node and point instead to the child of the node we intend to remove
- Removing a node with two children: replaces the node's info with the info from another node in the tree so that the search property is retained - then remove this other node

Removing a Leaf Node

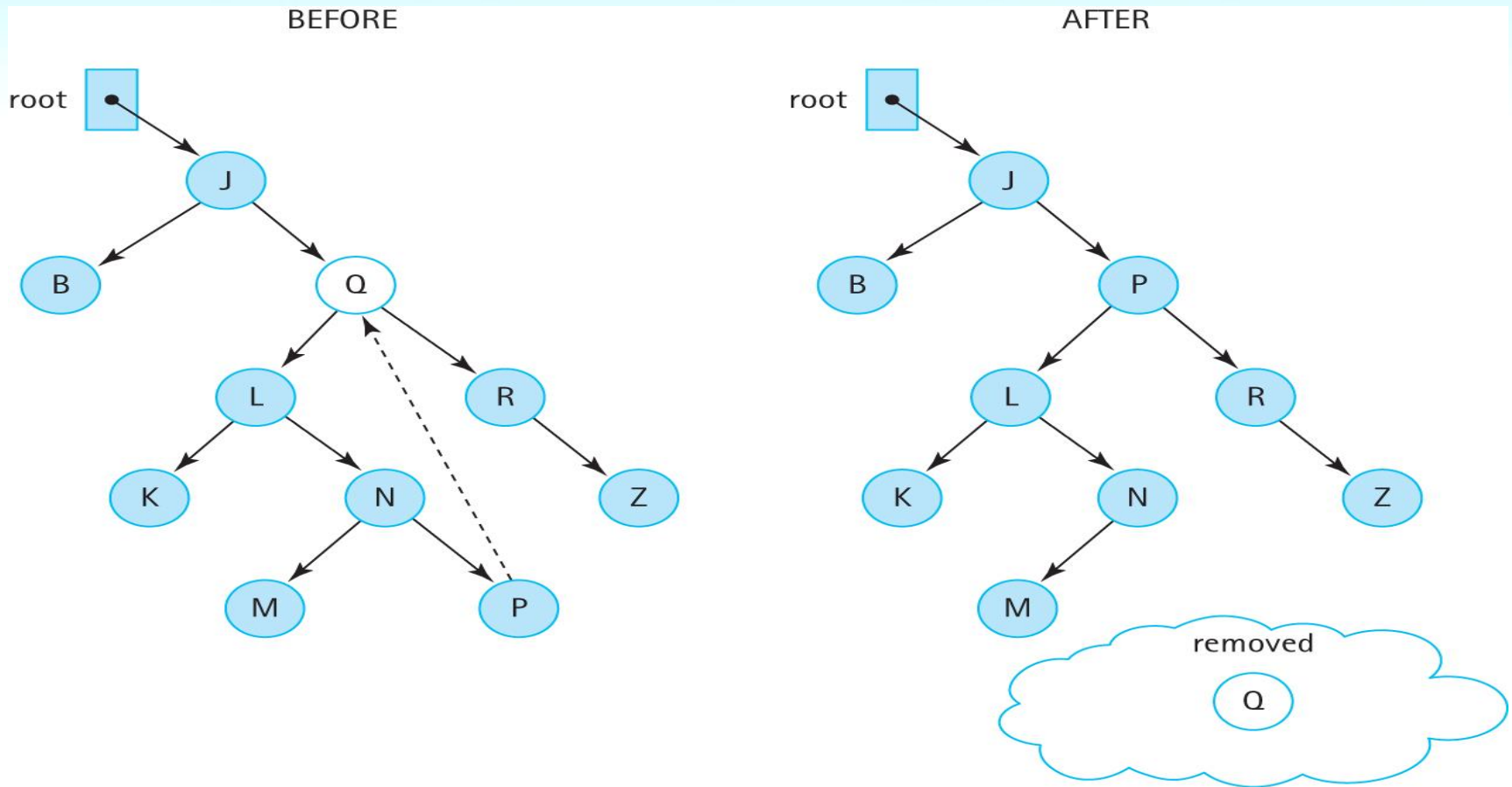


Remove the node containing Z

Removing a node with one child



Removing a Node With Two Children



Remove the node containing Q

The Remove Node Algorithm

removeNode (node): returns BSTNode

```
if (node.getLeft( ) is null) AND (node.getRight( ) is null)
    return null
else if node.getLeft( ) is null
    return node.getRight( )
else if node.getRight( ) is null
    return node.getLeft( )
else
    Find predecessor
    node.setInfo(predecessor.getInfo( ))
    node.setLeft(recRemove(predecessor.getInfo( ), node.getLeft( )))
return node
```

Note: We can remove one of the tests if we notice that the action taken when the left child reference is null also takes care of the case in which both child references are null. When the left child reference is null, the right child reference is returned. If the right child reference is also null, then null is returned, which is what we want if they are both null.

The removeNode method

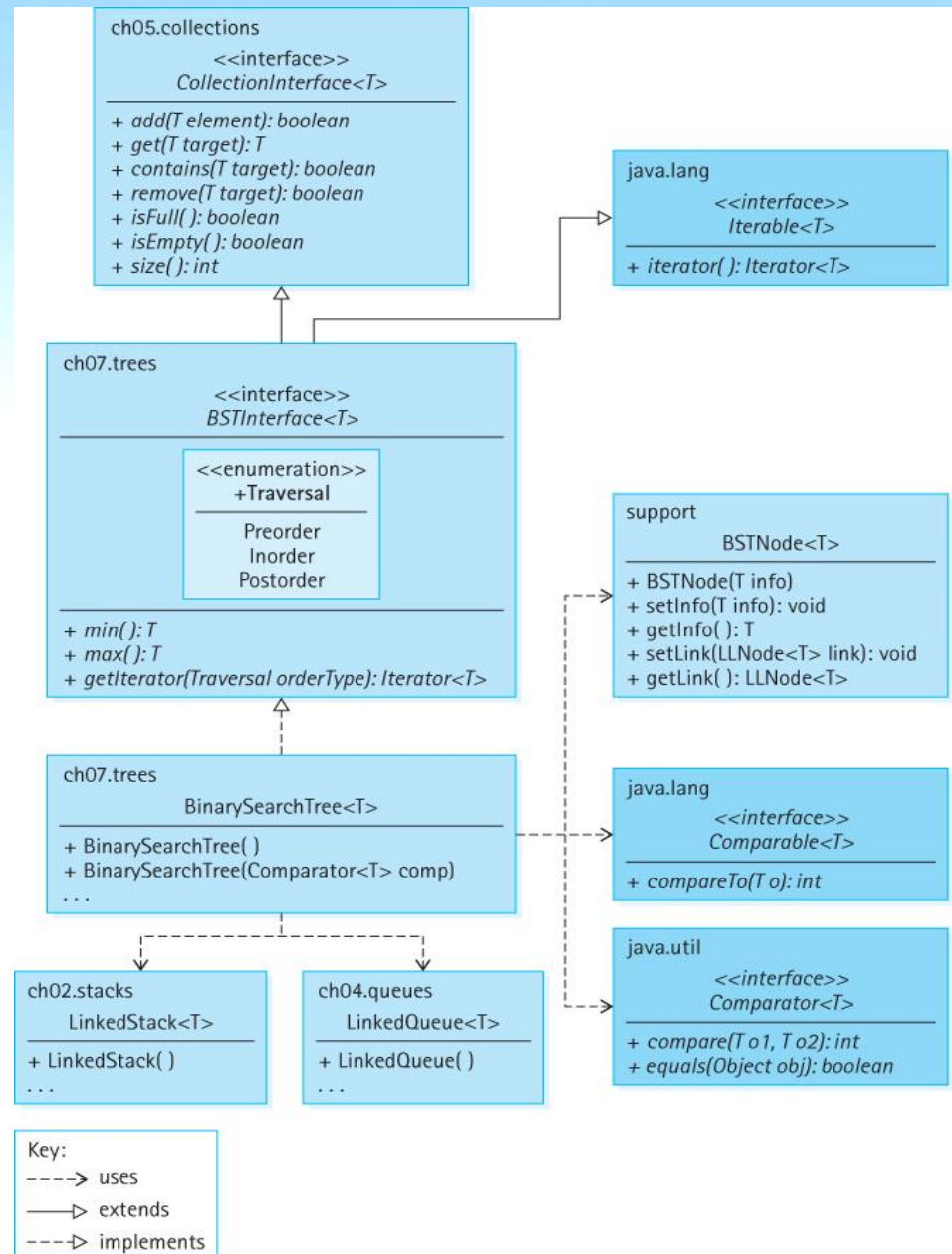
```
private BSTNode<T> removeNode(BSTNode<T> node)
// Removes the information at node from the tree.
{
    T data;
    if (node.getLeft() == null)
        return node.getRight();
    else if (node.getRight() == null)
        return node.getLeft();
    else
    {
        data = getPredecessor(node.getLeft());
        node.setInfo(data);
        node.setLeft(recRemove(data, node.getLeft()));
        return node;
    }
}
```

The getPredecessor method

- The logical predecessor is the maximum value in node's left subtree.
- The maximum value in a binary search tree is in its rightmost node.
- Therefore, given node's left subtree, we just keep moving right until the right child is null.
- When this occurs, we return the info reference of the node.

```
private T getPredecessor(BSTNode<T> subtree)
// Returns the information held in the rightmost node of subtree
{
    BSTNode temp = subtree;
    while (temp.getRight() != null)
        temp = temp.getRight();
    return temp.getInfo();
}
```

Our Binary Search Tree Architecture



7.8 Binary Search Tree Performance

- A binary search tree is an appropriate structure for many of the same applications discussed previously in conjunction with sorted lists.
- Similar to a sorted array-based list, it can be searched quickly, using a binary search.
- Similar to a linked list, it allows insertions and removals without having to move large amounts of data.
- There is a space cost - the binary search tree, with its extra reference in each node, takes up more memory space than a singly linked list.

Text Analysis Experiment Revisited

Table 7.2 Results of Vocabulary Density Experiment

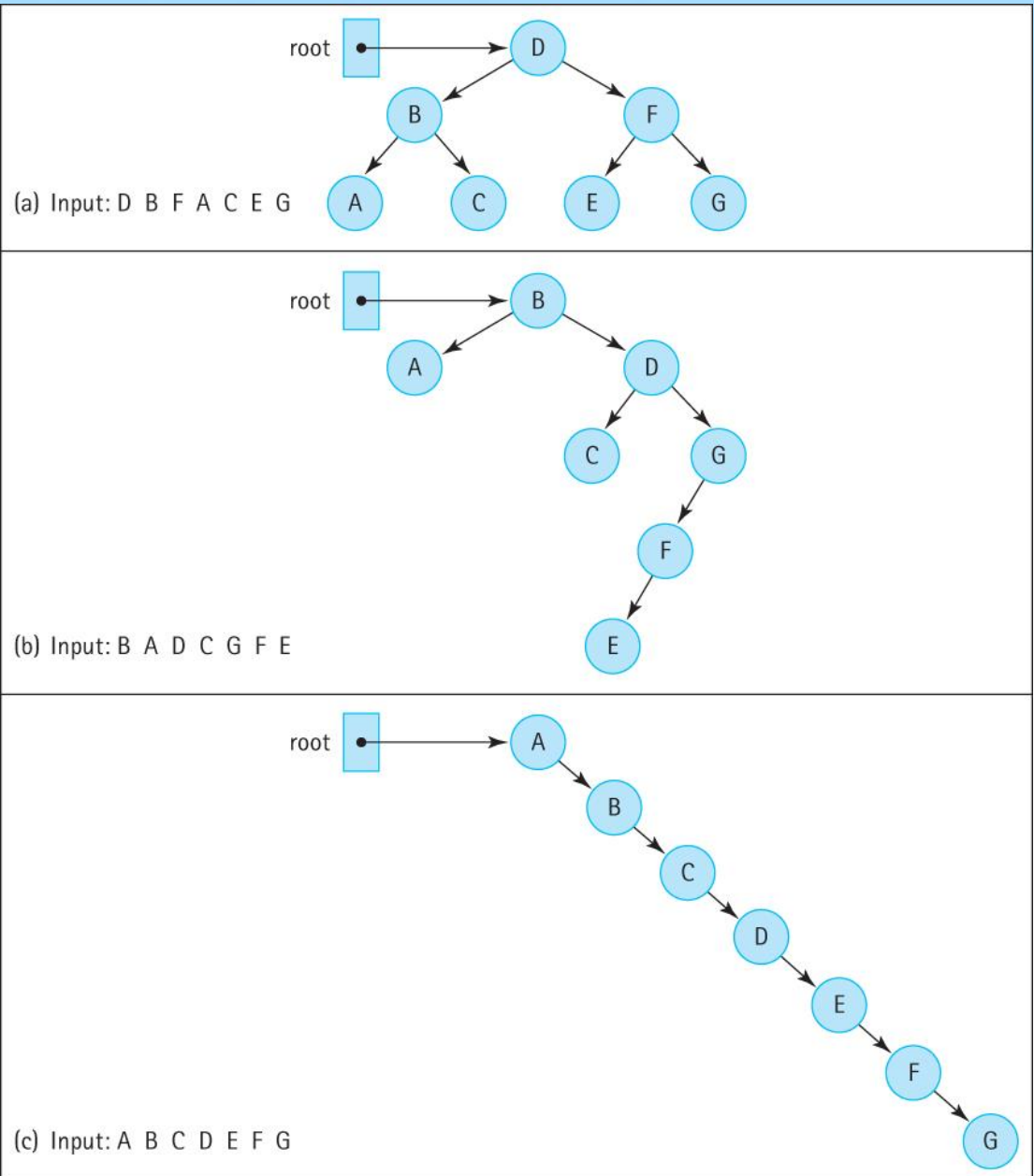
Source	File Size	Array	Sorted Array	Binary Search Tree
Shakespeare's 18th Sonnet	1 KB	20 msec	23 msec	22 msec
Shakespeare's <i>Hamlet</i>	177 KB	236 msec	128 msec	127 msec
Linux Word File	400 KB	9,100 msec	182 msec	Stack overflow error, 33,760 msec with revision
Melville's <i>Moby-Dick</i>	1,227 KB	2,278 msec or 2.3 seconds	382 msec	334 msec
<i>Complete Works of William Shakespeare</i>	5,542 KB	9.7 seconds	1.2 seconds	0.9 seconds
<i>Webster's Unabridged Dictionary</i>	28,278 KB	4.7 minutes	9.5 seconds	4.2 seconds
11th Edition of the <i>Encyclopaedia Britannica</i>	291,644 KB	56.4 minutes	2.5 minutes	41.8 seconds
Mashup	608,274 KB	10 hours	7.2 minutes	1.7 minutes

Text Analysis Experiment Revisited

- Due to operation complexity, performance gains from the binary search tree are only clearly evident as the size of the file increases.
- The table also reveals a serious issue when the binary search tree structure is used for the Linux Word file. The application bombs—it stops executing and reports a “Stack overflow error.”
- The problem is that the underlying tree is completely skewed.

Insertion Order and Tree Shape

**Linux Word
File is like this →**



Testing the Binary Search Tree operations

- The code for the entire `BinarySearchTree` class is included with the rest of the book files.
- It provides both the recursive `size` method, and the iterative version (`size2`).
- We have also included an interactive test driver for the ADT called `ITDBinarySearchTree`.
 - it allows you to create, manipulate, and observe trees containing strings.
 - it also supports a print operation that allows you to indicate one of the traversal orders and “prints” the contents of the tree, in that order.
- You are invited to use the test driver to test the various tree operations.

8.7 Comparing Binary Search Trees to Linear Lists

	Binary Search Tree	Array-Based Linear List	Linked List
Class constructor	$O(1)$	$O(N)$	$O(1)$
isEmpty	$O(1)$	$O(1)$	$O(1)$
reset	$O(N)$	$O(1)$	$O(1)$
getNext	$O(1)$	$O(1)$	$O(1)$
contains	$O(\log_2 N)$	$O(\log_2 N)$	$O(N)$
get			
Find	$O(\log_2 N)$	$O(\log_2 N)$	$O(N)$
Process	$O(1)$	$O(1)$	$O(1)$
Total	$O(\log_2 N)$	$O(\log_2 N)$	$O(N)$
add			
Find	$O(\log_2 N)$	$O(\log_2 N)$	$O(N)$
Process	$O(1)$	$O(N)$	$O(N)$
Total	$O(\log_2 N)$	$O(N)$	$O(N)$
remove			
Find	$O(\log_2 N)$	$O(\log_2 N)$	$O(N)$
Process	$O(1)$	$O(N)$	$O(1)$
Total	$O(\log_2 N)$	$O(N)$	$O(N)$

Balancing a Binary Search Tree

- A beneficial addition to our Binary Search Tree ADT operations is a `balance` operation
- The specification of the operation is:

```
public balance();  
// Restructures this BST to be optimally balanced
```

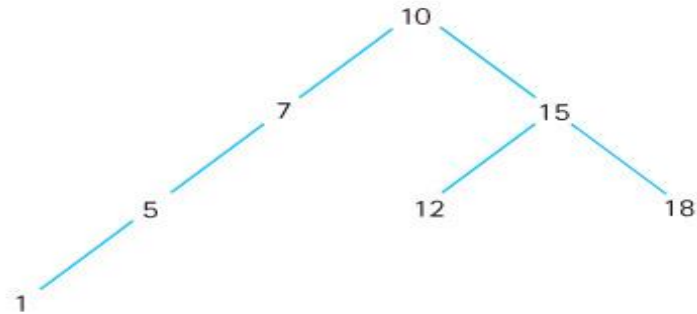
- It is up to the client program to use the `balance` method appropriately

Our Approach

- Basic algorithm:
 - Save the tree information in an array
 - Insert the information from the array back into the tree
- The structure of the new tree depends on the order that we save the information into the array, or the order in which we insert the information back into the tree, or both
- We save the information “in order” so that the array is sorted

Using inOrder traversal

(a) The original tree

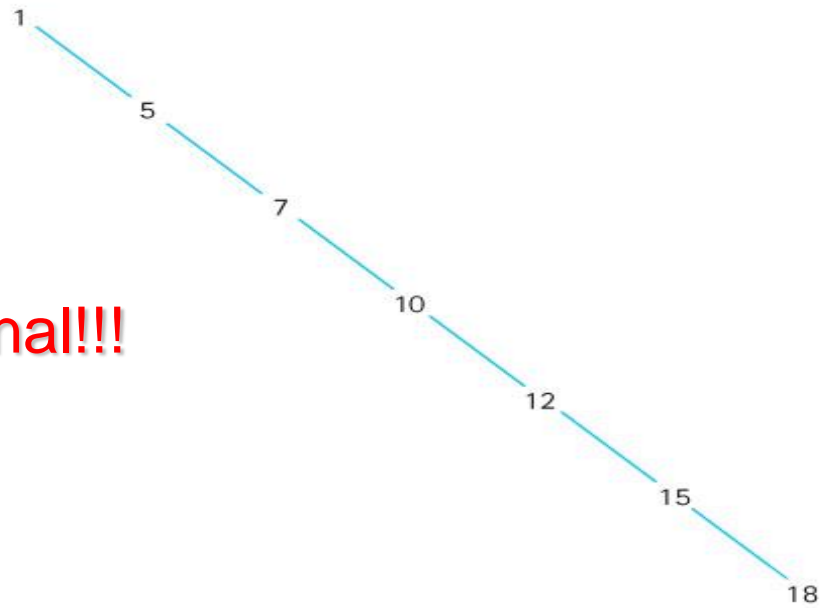


(b) The inorder traversal

array:

0	1	2	3	4	5	6
1	5	7	10	12	15	18

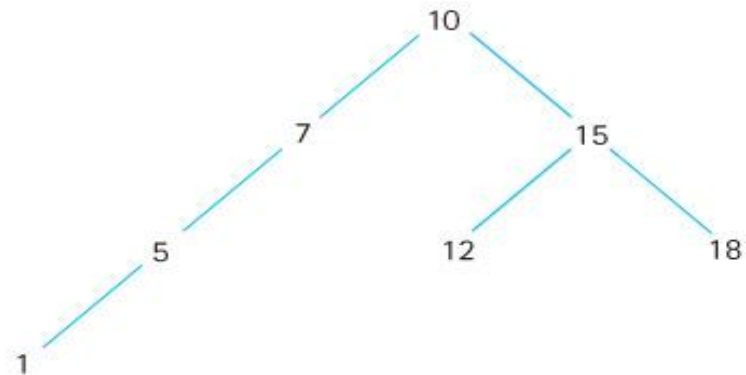
(c) The resultant tree if linear traversal of array is used



Worse than original!!!

Using preOrder traversal

(a) The original tree

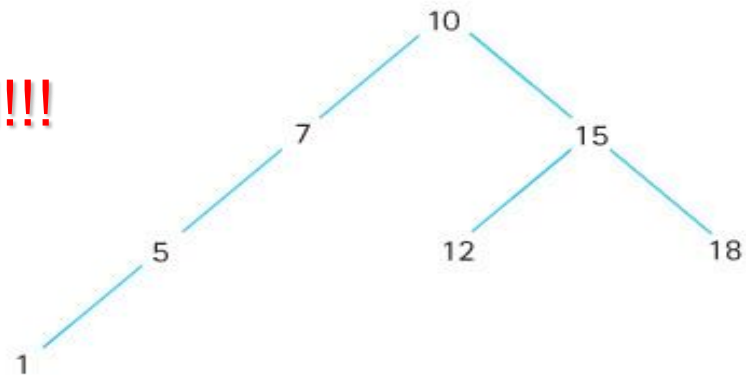


(b) The preorder traversal

array:

0	1	2	3	4	5	6
10	7	5	1	15	12	18

(c) The resultant tree if linear traversal of array is used



Same as original!!!

To Ensure a Balanced Tree

- Even out as much as possible, the number of descendants in each node's left and right subtrees
- First insert the “middle” item of the inOrder array
 - Then insert the left half of the array using the same approach
 - Then insert the right half of the array using the same approach

Our Balance Tree Algorithm

Balance

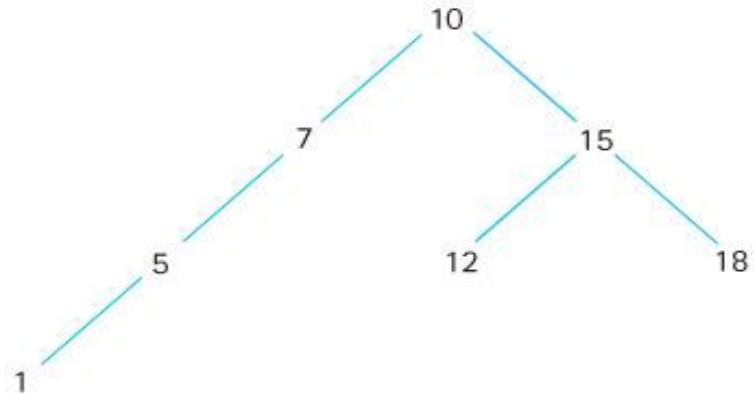
```
Iterator iter = tree.getIterator(Inorder )
int index = 0
while (iter.hasNext())
    array[index] = iter.next( )
    index++
tree = new BinarySearchTree()
tree.InsertTree(0, index - 1)
```

InsertTree(low, high)

```
if (low == high)                // Base case 1
    tree.add(array[low])
else if ((low + 1) == high)      // Base case 2
    tree.add(array[low])
    tree.add(array[high])
else
    mid = (low + high) / 2
    tree.add(array[mid])
    tree.InsertTree(low, mid - 1)
    tree.InsertTree(mid + 1, high)
```

Using recursive insertTree

(a) The original tree

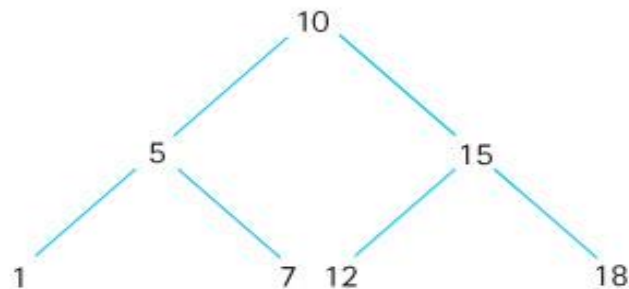


(b) The inorder traversal

array:

0	1	2	3	4	5	6
1	5	7	10	12	15	18

(c) The resultant tree if InsertTree (0,6) is used



7.9 Application: Word Frequency Counter

- In this section we develop a word frequency counter that uses our binary search tree
- Read a text file and generate an alphabetical listing of the unique words, along with a count of how many times each word occurs
- We allow users to specify a minimum word size and a minimum frequency count
- Present a few summary statistics

```
package support;
import java.text.DecimalFormat;

public class WordFreq implements Comparable<WordFreq>
{
    private String word;  private int freq;
    DecimalFormat fmt = new DecimalFormat("00000");

    public WordFreq(String newWord)
    {
        word = newWord;    freq = 0;
    }

    public String getWordIs(){return word;}

    public int getFreq(){return freq;}

    public void inc()  {    freq++;  }

    public int compareTo(WordFreq other)
    {
        return this.word.compareTo(other.word);
    }

    public String toString()
    {
        return(fmt.format(freq) + " " + word);
    }
}
```

The WordFreq Class

The Application

- Scans the input file for words, and after reading a word, it checks to see if a match is already in the tree, and if not it inserts a WordFreq object that holds the word and a frequency count of 1
- If the word is already in the tree, increment the frequency associated with the word

Algorithm to process a word

```
wordToTry = new WordFreq(word)
wordInTree = tree.get(wordToTry)
if (wordInTree != null)
    wordInTree.inc()
else
    wordToTry.inc() // to set its frequency to 1
    tree.add(wordToTry);
```

Processing a word

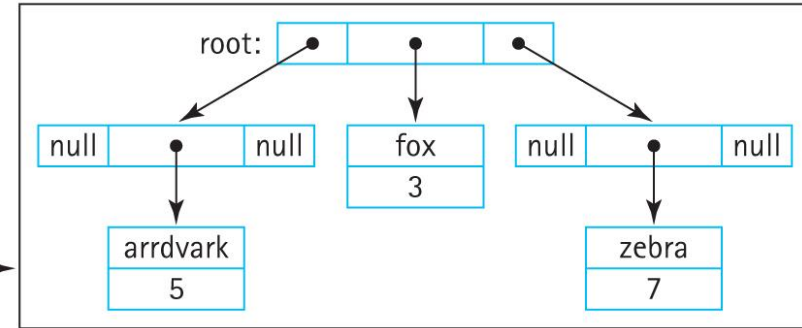
(a) Status: Before processing

wordToTry: →

arrdvark
0

wordInTree: null

tree: →



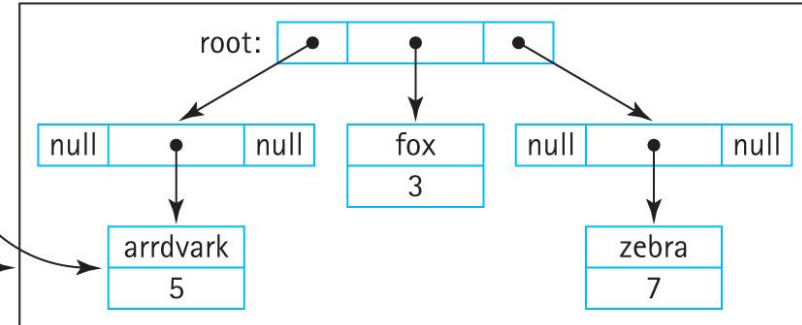
(b) Status: After 'wordInTree = tree.get (wordToTry)'

wordToTry: →

arrdvark
0

wordInTree: →

tree: →



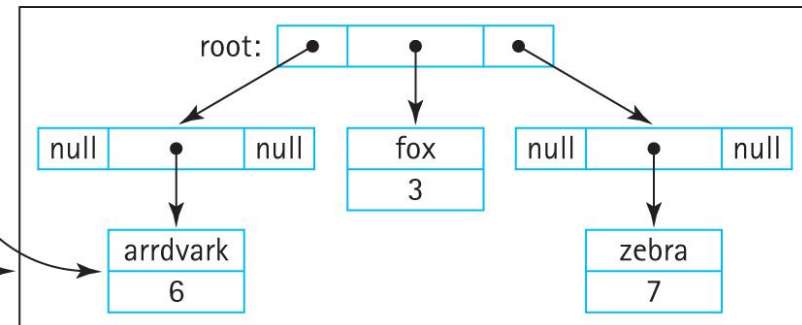
(c) Status: After 'wordInTree.inc()'

wordToTry: →

arrdvark
0

wordInTree: →

tree: →



The Application

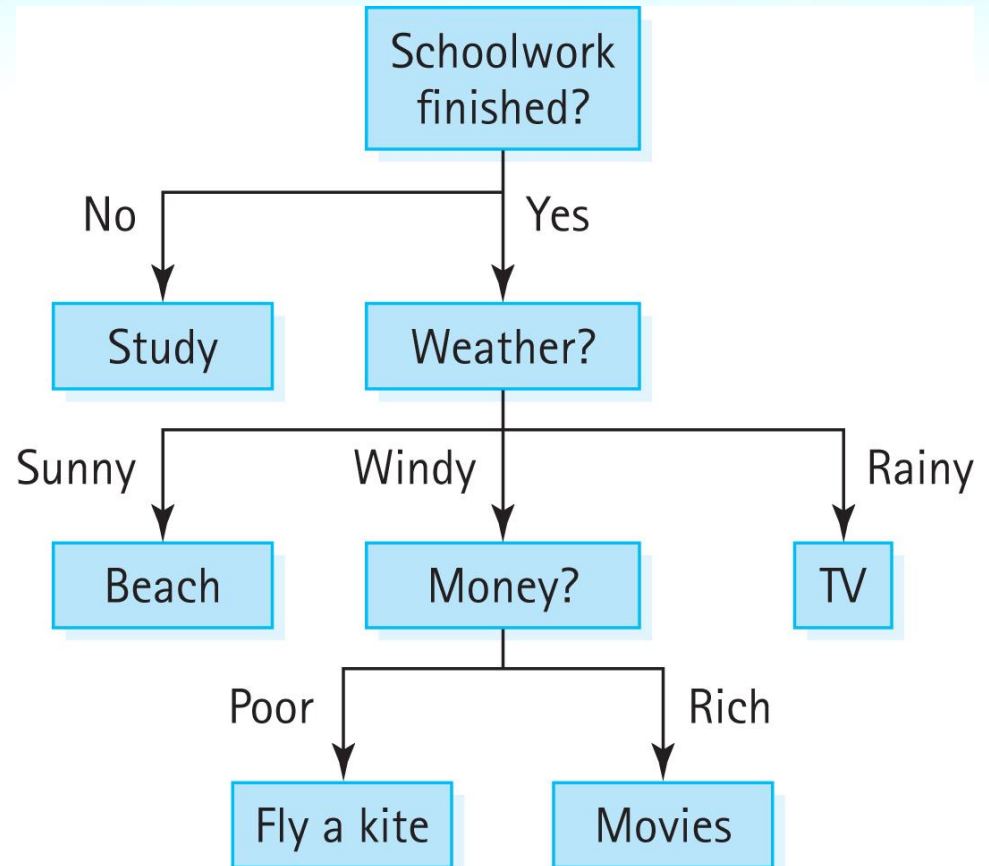
- Instructors can now review the code and demonstrate/discuss the application.

7.10 Tree Variations

- Trees can be binary, trinary (up to three children per node), n-ary (up to n children per node), alternating ary, balanced, not balanced, partially balanced, self-adjusting, and store information in their nodes or their edges or both.
- The Java Library does not include a general tree structure and neither do most textbooks.
- Often with trees we let the specific target application dictate the definition, assumptions, rules, and operations associated with a specific implementation.

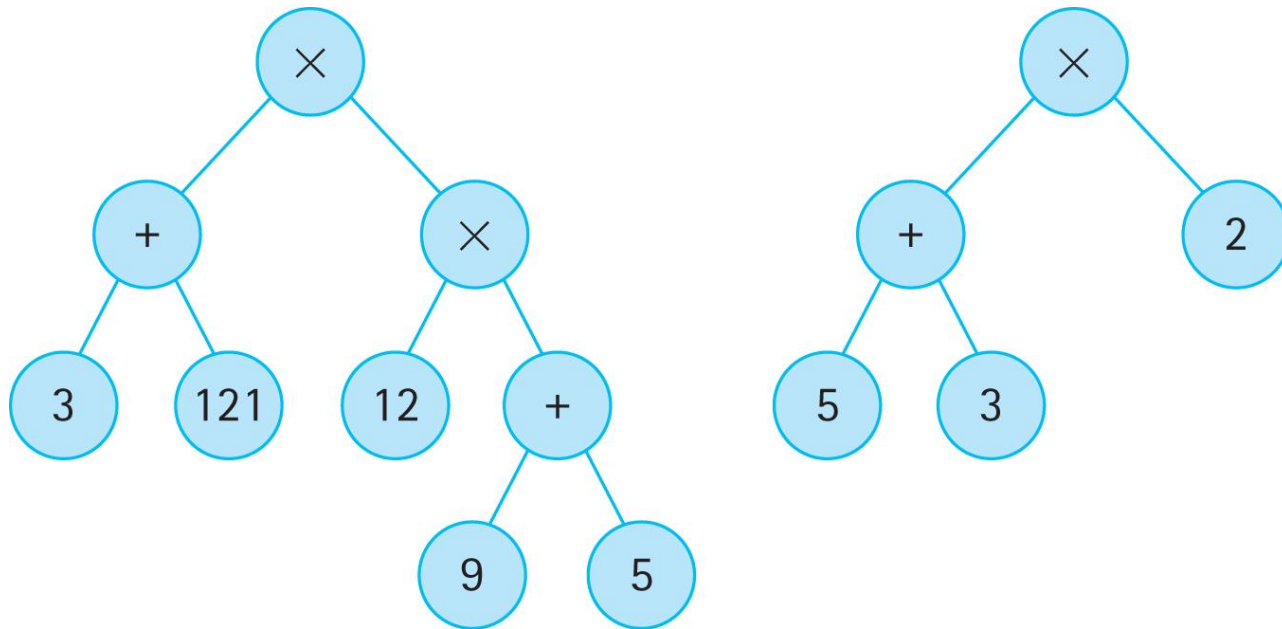
Application-Specific Variations

- Decision Trees:



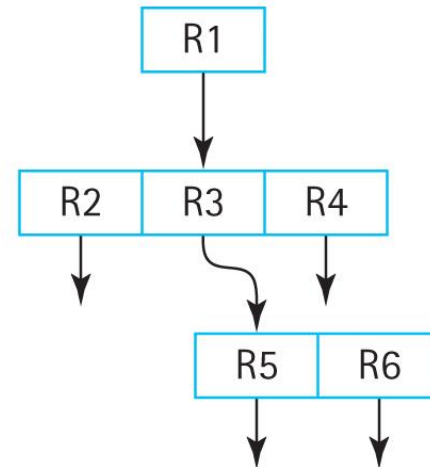
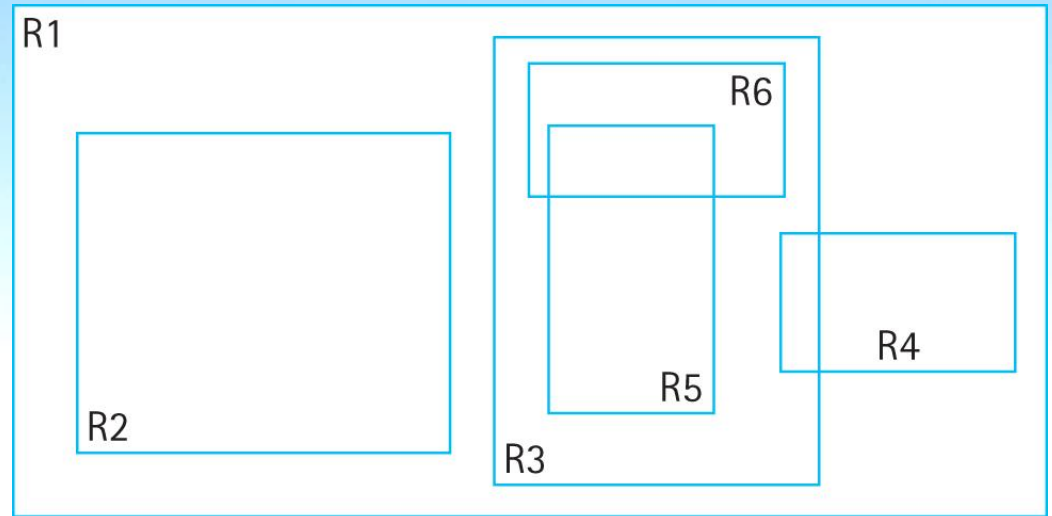
Application-Specific Variations

- Expression/Parse Trees:



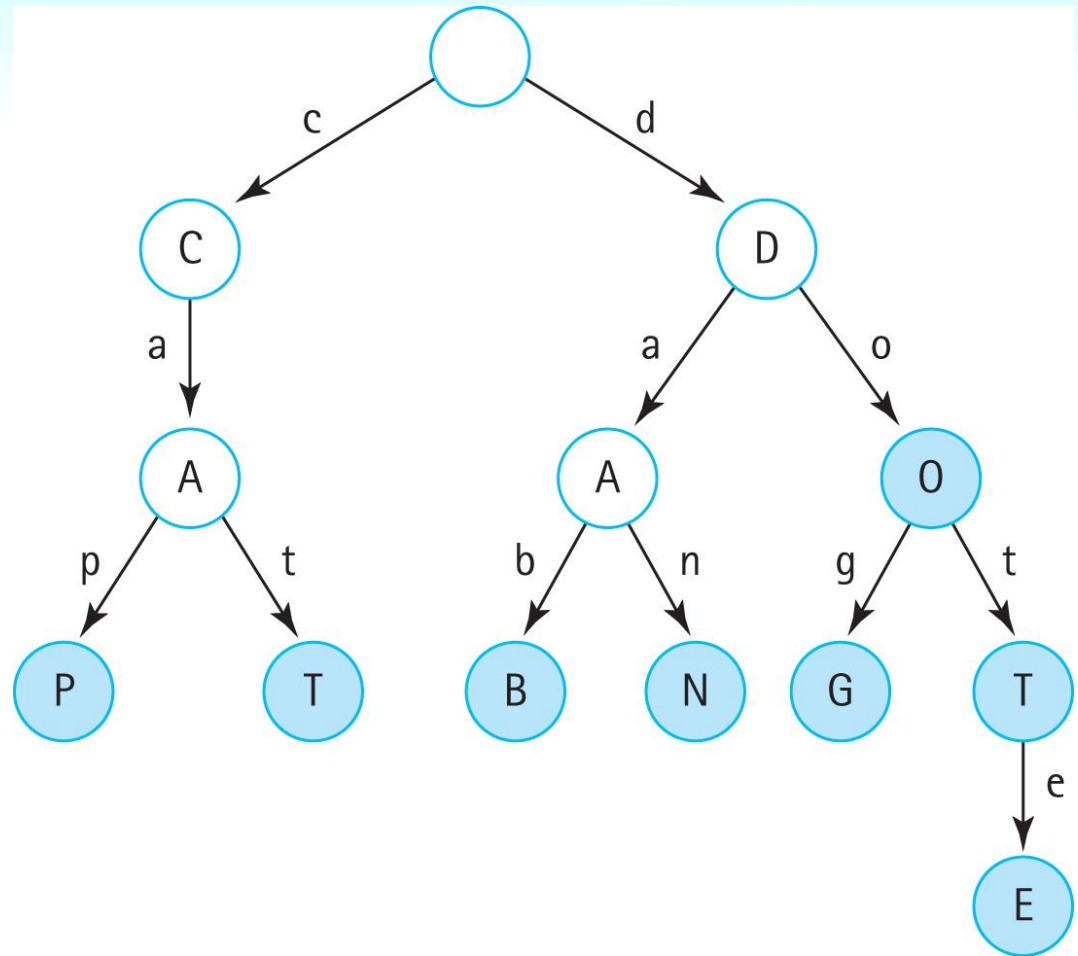
Application-Specific Variations

- R Trees:



Application-Specific Variations

- Tries/Prefix Trees:

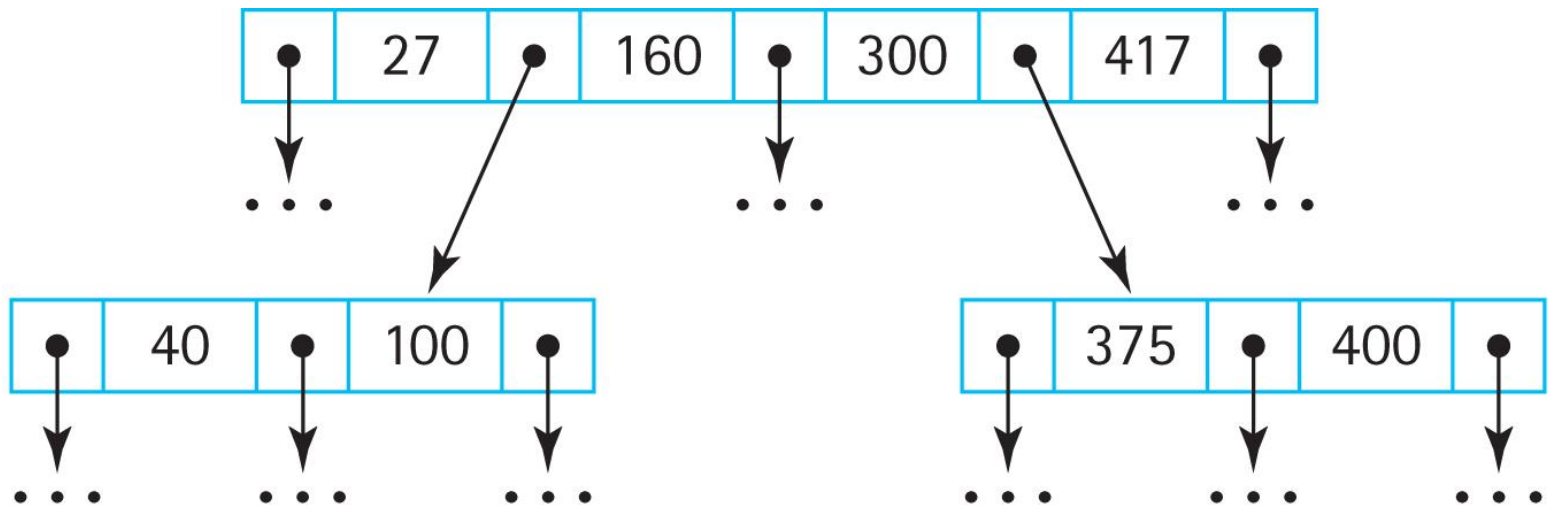


Balanced Search Trees

- The Binary Search Tree ADT presented in this chapter is an excellent collection structure for holding “random” data, but suffers from one major weakness—it can become unbalanced.
- Many search tree variations have been invented to address this weakness, each in turn with their own strengths and weaknesses.

Balanced Search Trees

- B-Trees: a search tree that allows internal nodes to hold multiple values and have multiple children



Balanced Search Trees

- B-Tree Variants: A popular variant of the B-Tree is the 2-3-4 tree, where nodes are constrained to hold 1, 2, or 3 values and therefore have 2, 3, or 4 subtrees (thus the name of the tree)
- Searching, insertion, and removal of information is always $O(\log_2 N)$ and the amount of work required to process the information within any node is constant.

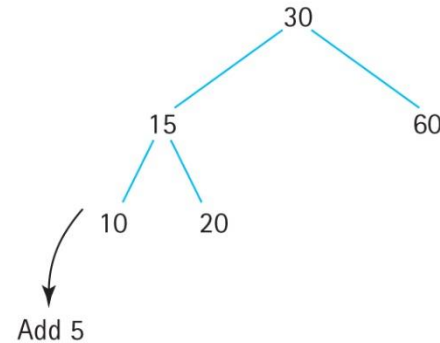
Balanced Search Trees

- A 2-3-4 tree can be implemented using a binary tree structure called a red-black tree. In this structure nodes are “colored” either red or black (each node must hold an extra boolean value that indicates this information). The color of a node indicates how its value fits into the corresponding 2-3-4 tree.
- The Java Library includes two important classes that use tree implementations. The Java TreeMap class supports maps (see Chapter 8) and the Java TreeSet class supports sets (see Section 5.7, “Collection Variations”). In both cases the internal representation is a red-black tree.

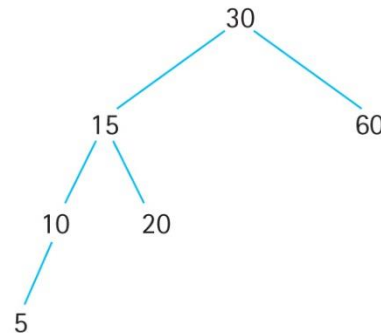
Balanced Search Trees

- AVL Trees: the difference in height between a node's two subtrees can be at most 1.
- During addition and removal of nodes extra work is required to ensure the tree remains balanced.

(a) An AVL tree



(b) Not an AVL tree



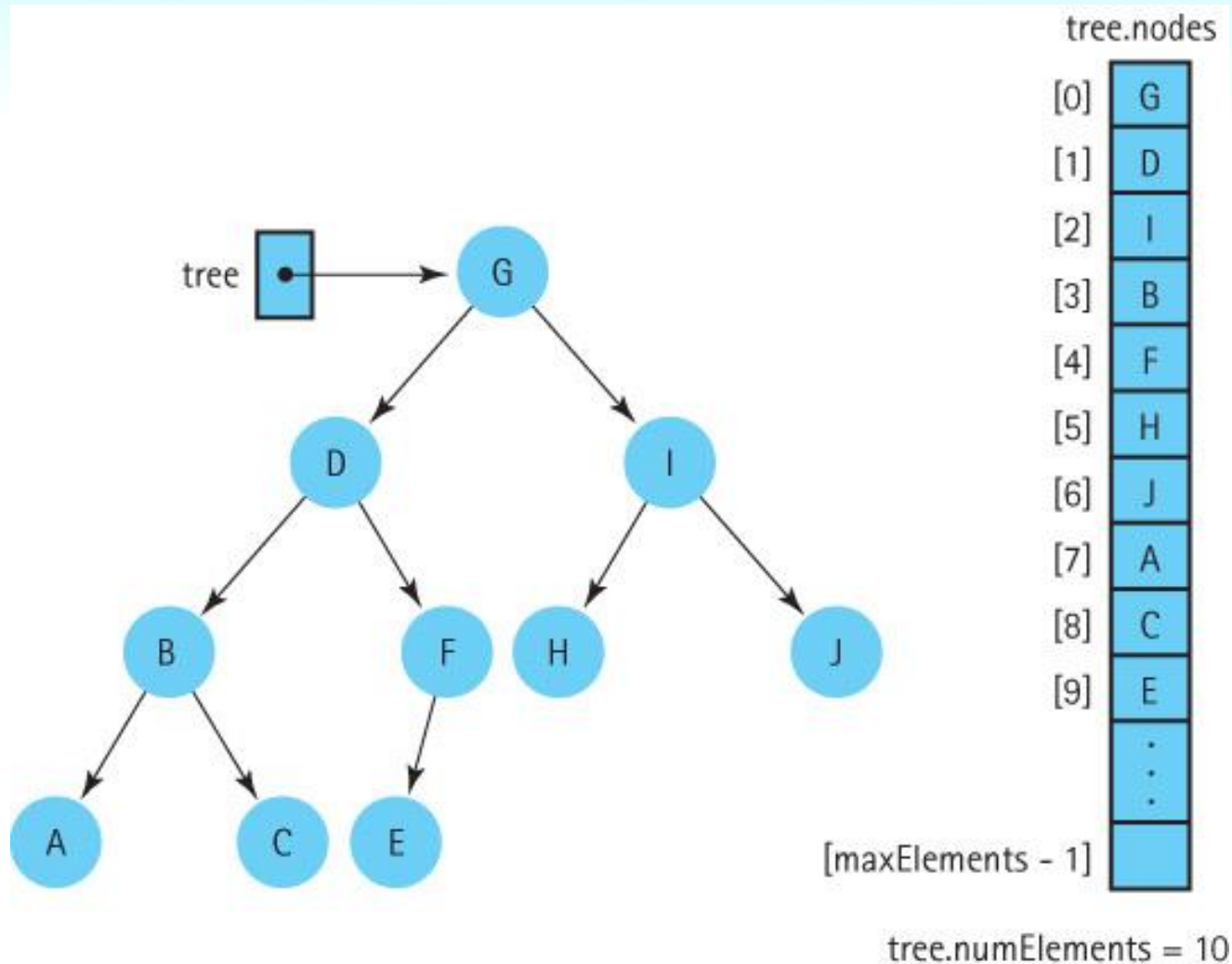
(c) An AVL tree again



Extra: A Nonlinked Representation of Binary Trees

- A binary tree can be stored in an array in such a way that the relationships in the tree are not physically represented by link members, but are *implicit* in the algorithms that manipulate the tree stored in the array.
- We store the tree elements in the array, level by level, left-to-right. If the number of nodes in the tree is `numElements`, we can package the array and `numElements` into an object
- The tree elements are stored with the root in `tree.nodes[0]` and the last node in `tree.nodes[numElements - 1]`.

A Binary Tree and Its Array Representation



Array Representation continued

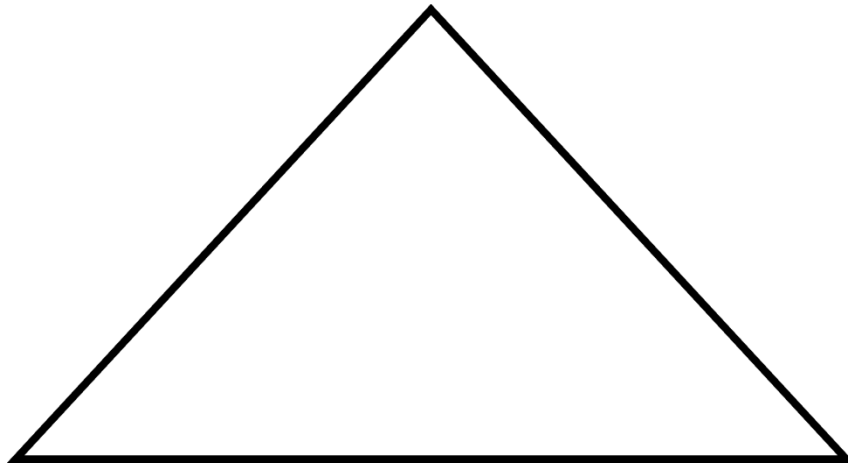
- To implement the algorithms that manipulate the tree, we must be able to find the left and right child of a node in the tree:
 - `tree.nodes[index]` left child is in `tree.nodes[index*2 + 1]`
 - `tree.nodes[index]` right child is in `tree.nodes[index*2 + 2]`
- We can also determine the location of its parent node:
 - `tree.nodes[index]`'s parent is in `tree.nodes[(index - 1)/2]`.
- This representation works best, space wise, if the tree is complete

Array Representation continued

- For any node `tree.nodes[index]`
 - its left child is in `tree.nodes[index*2 + 1]`
 - right child is in `tree.nodes[index*2 + 2]`
 - its parent is in `tree.nodes[(index - 1)/2]`.

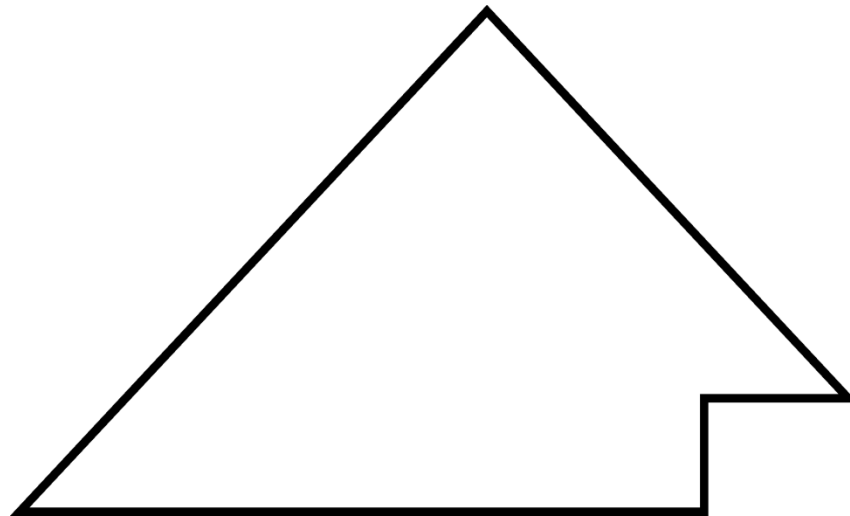
Definitions

- **Full Binary Tree:** A binary tree in which all of the leaves are on the same level and every nonleaf node has two children



Definitions *(Cont'd)*

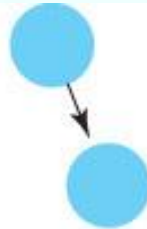
- **Complete Binary Tree:** A binary tree that is either full or full through the next-to-last level, with the leaves on the last level as far to the left as possible



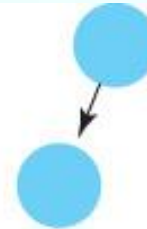
Examples of Different Types of Binary Trees



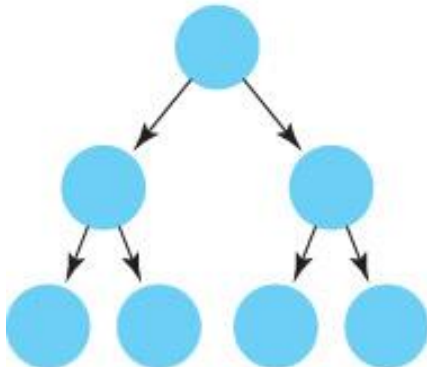
(a) Full and complete



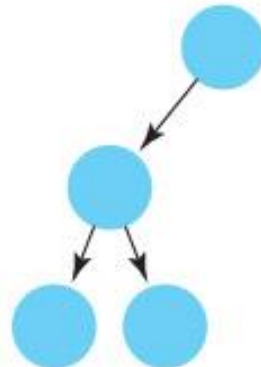
(b) Neither full nor complete



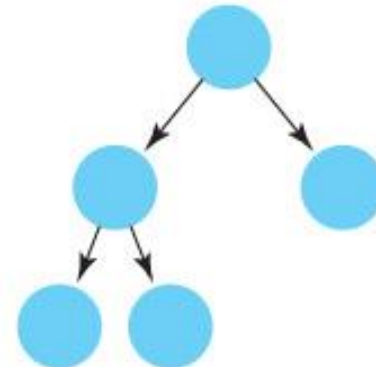
(c) Complete



(d) Full and complete



(e) Neither full nor complete



(f) Complete

A Binary Search Tree Stored in an Array with Dummy Values

