Chapter 9: Priority Queues ADT

CS401

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Revised Nell Dale Presentation

9.1 Priority Queues

- A priority queue is an abstract data type with an interesting accessing protocol - only the *highest-priority* element can be accessed
- Priority queues are useful for any application that involves processing items by priority

The Interface

```
package ch09.priorityQueues;

public interface PriQueueInterface<T>
{
  void enqueue(T element);
  // Throws PriQOverflowException if this priority queue is full;
  // otherwise, adds element to this priority queue.

  T dequeue();
  // Throws PriQUnderflowException if this priority queue is empty;
  // otherwise, removes element with highest priority from this
  // priority queue and returns it.

  boolean isEmpty();
  // Returns true if this priority queue is empty; otherwise, returns false.

  boolean isFull();
  // Returns true if this priority queue is full; otherwise, returns false.

  int size();
  // Returns the number of elements in this priority queue. }
```

9.2 Priority Queue Implementations

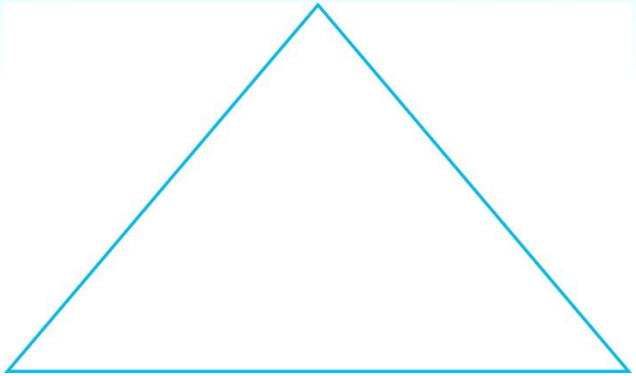
- There are many ways to implement a priority queue
  - **An Unsorted List -** dequeuing would require searching through the entire list
  - **An Array-Based Sorted List -** Enqueuing is expensive
  - **A Sorted Linked List -** Enqueuing again is 0(N)
  - **A Binary Search Tree -** On average, $0(\log_2 N)$ steps for both enqueue and dequeue
  - **A Heap -** (next section) guarantees $0(\log_2 N)$ steps, even in the worst case

9.3 The Heap

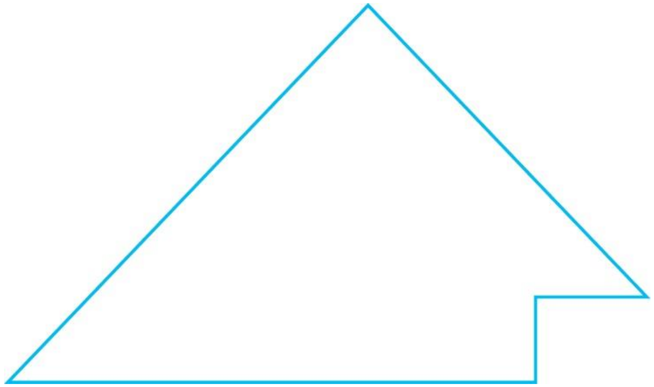
- **Heap** An implementation of a Priority Queue based on a complete binary tree, each of whose elements contains a value that is greater than or equal to the value of each of its children
- In other words, a heap is an implementation of a Priority Queue that uses a binary tree that satisfies two properties
  - the *shape property*: the tree must be a complete binary tree
  - the *order property:* for every node in the tree, the value stored in that node is greater than or equal to the value in each of its children.

Tree Terminology

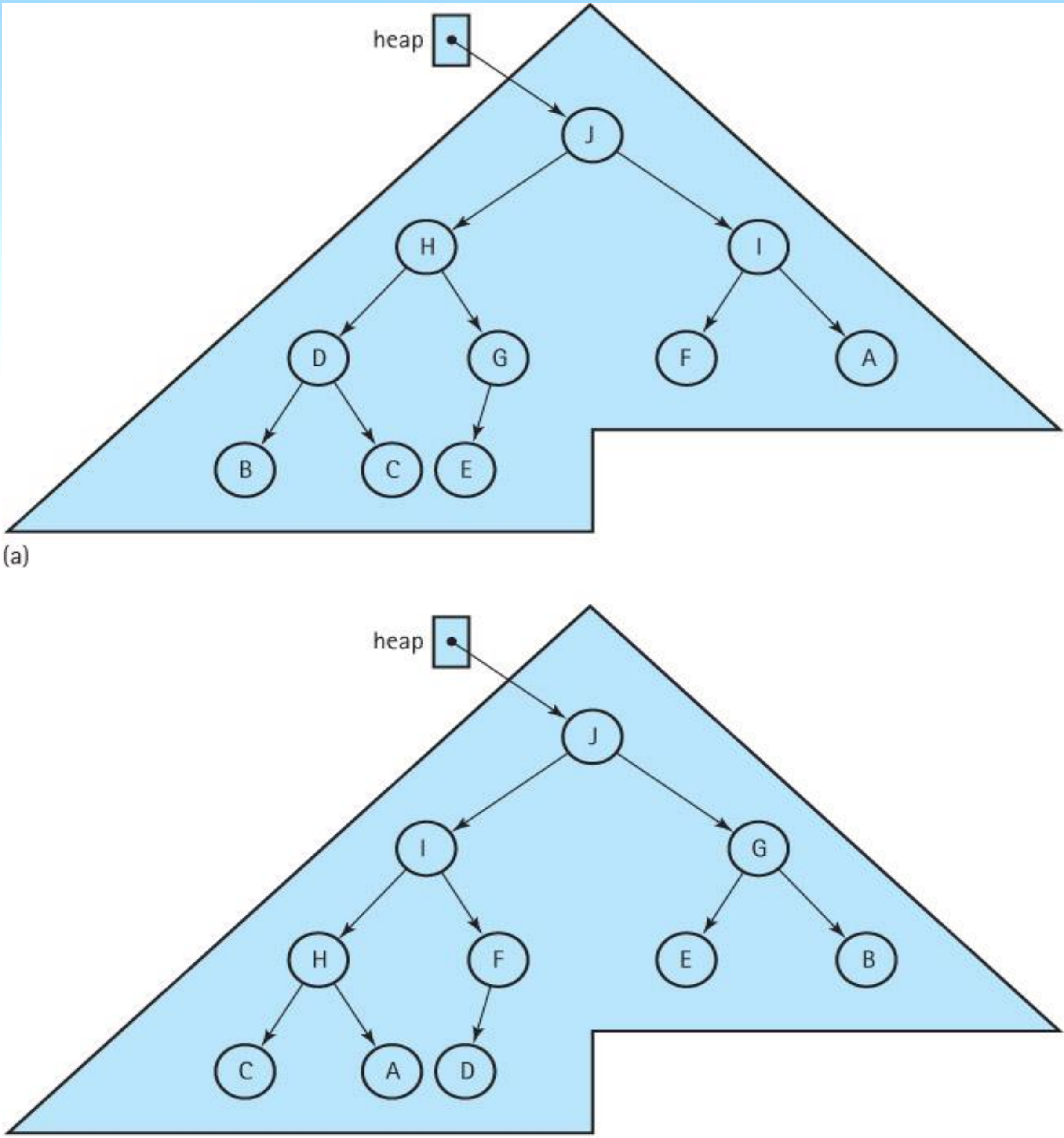
- A full binary tree



- A complete binary tree



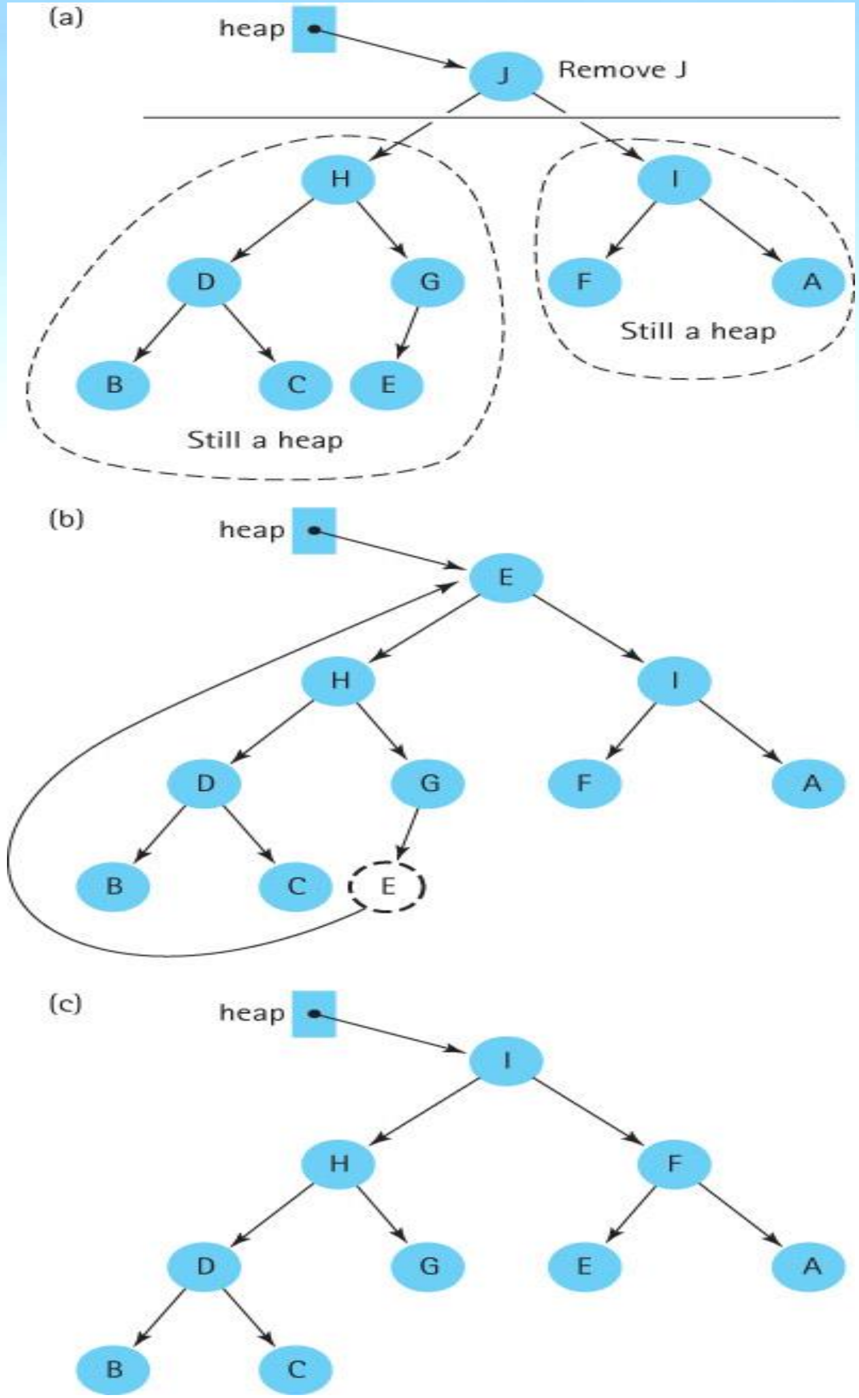
Two Heaps Containing the Letters ‘A’ through ‘J’



(a)

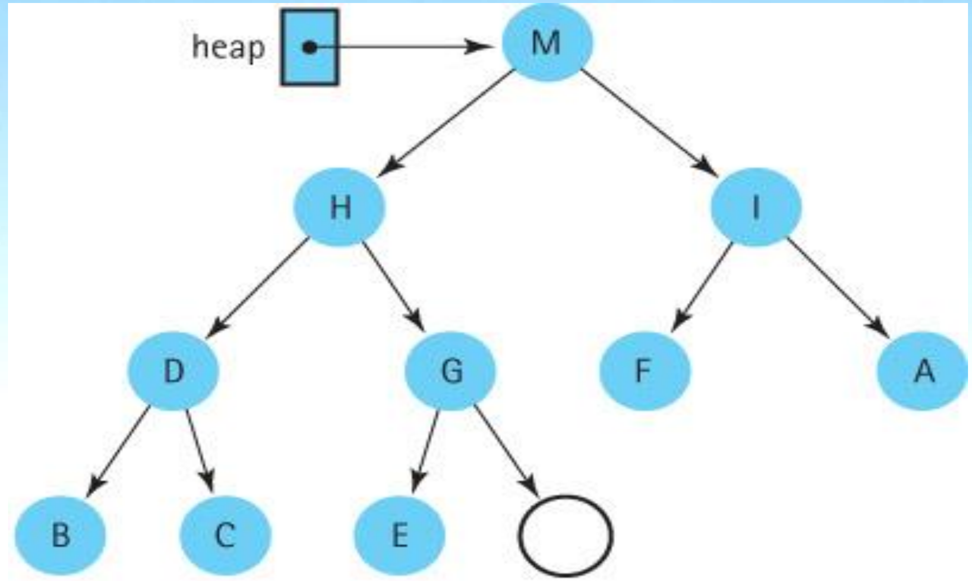
(b)

The dequeue operation

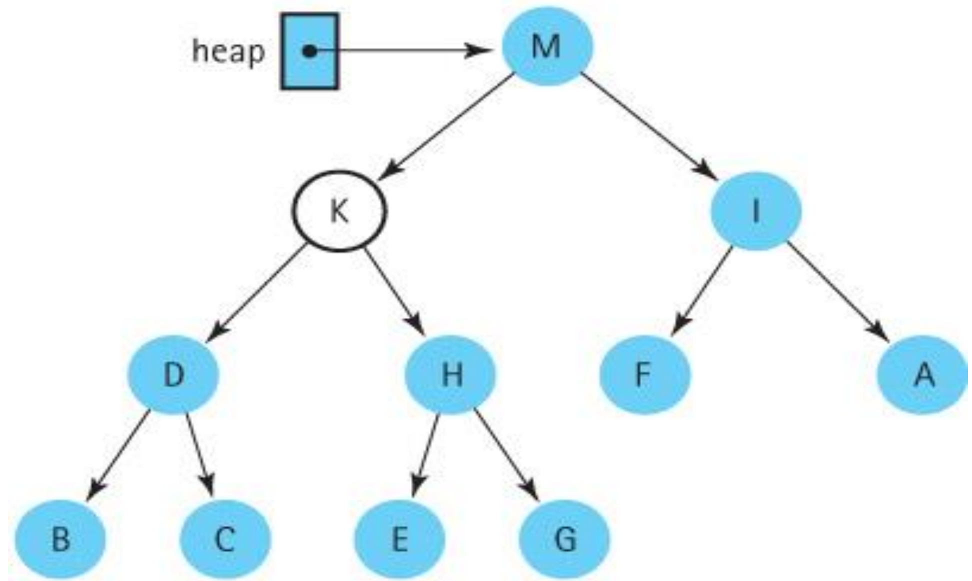


reheapDown (element)
Effect: Adds element to the heap.
Precondition: The root of the tree is empty.

The enqueue operation



(a) Add K

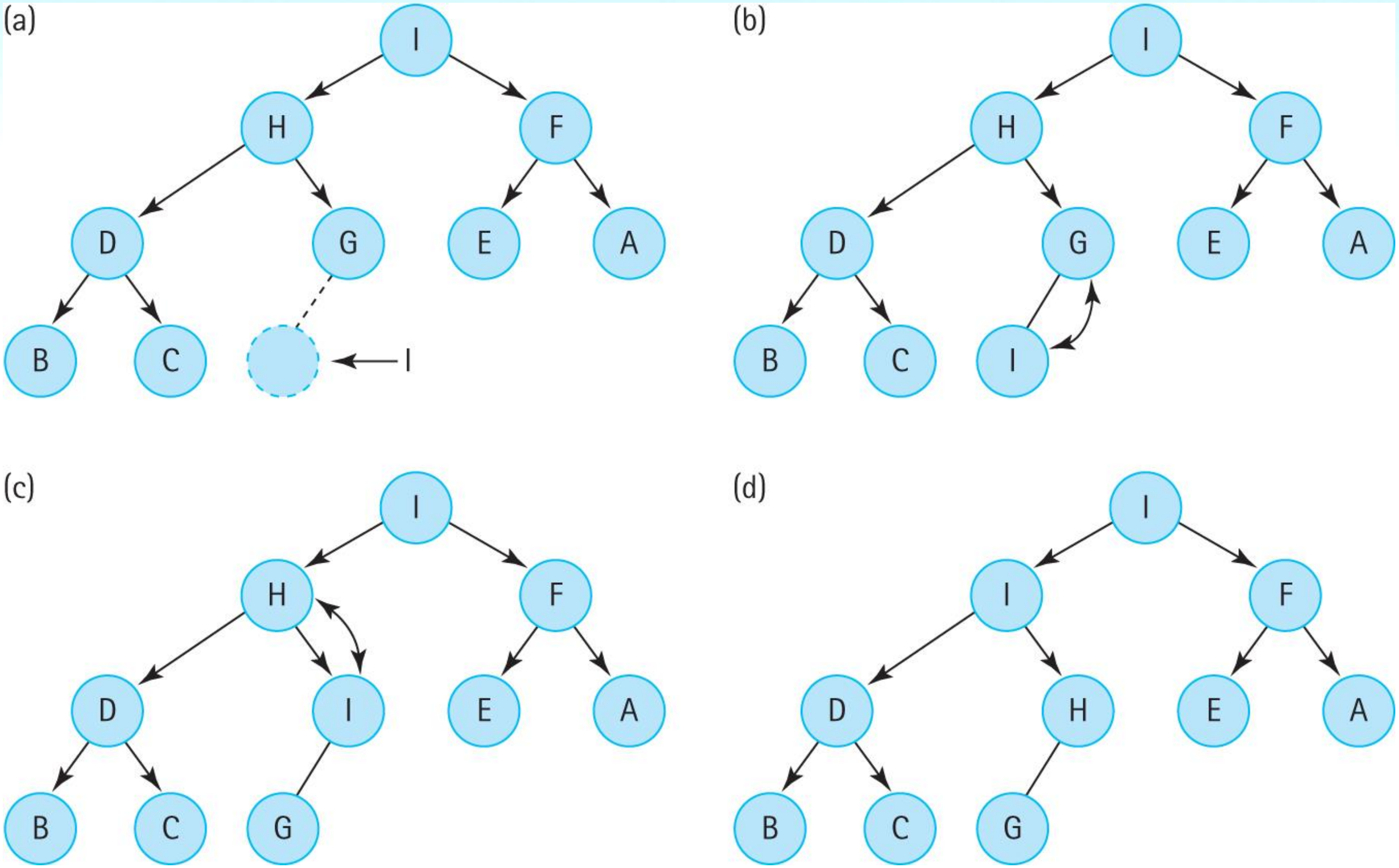


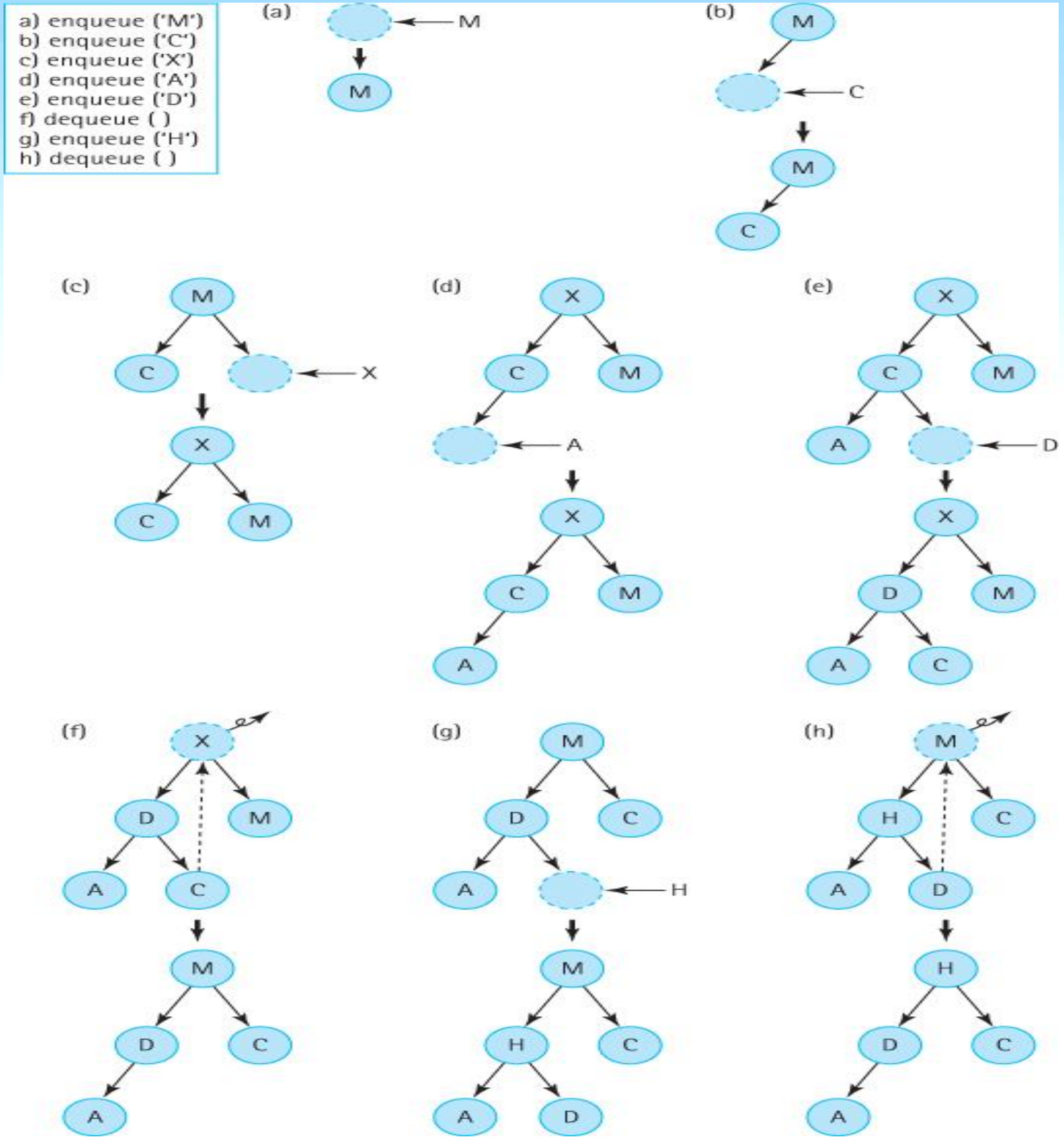
(b) reheapUp

reheapUp (element)
Effect: Adds element to the heap.
Precondition: The last index position of the tree is empty.

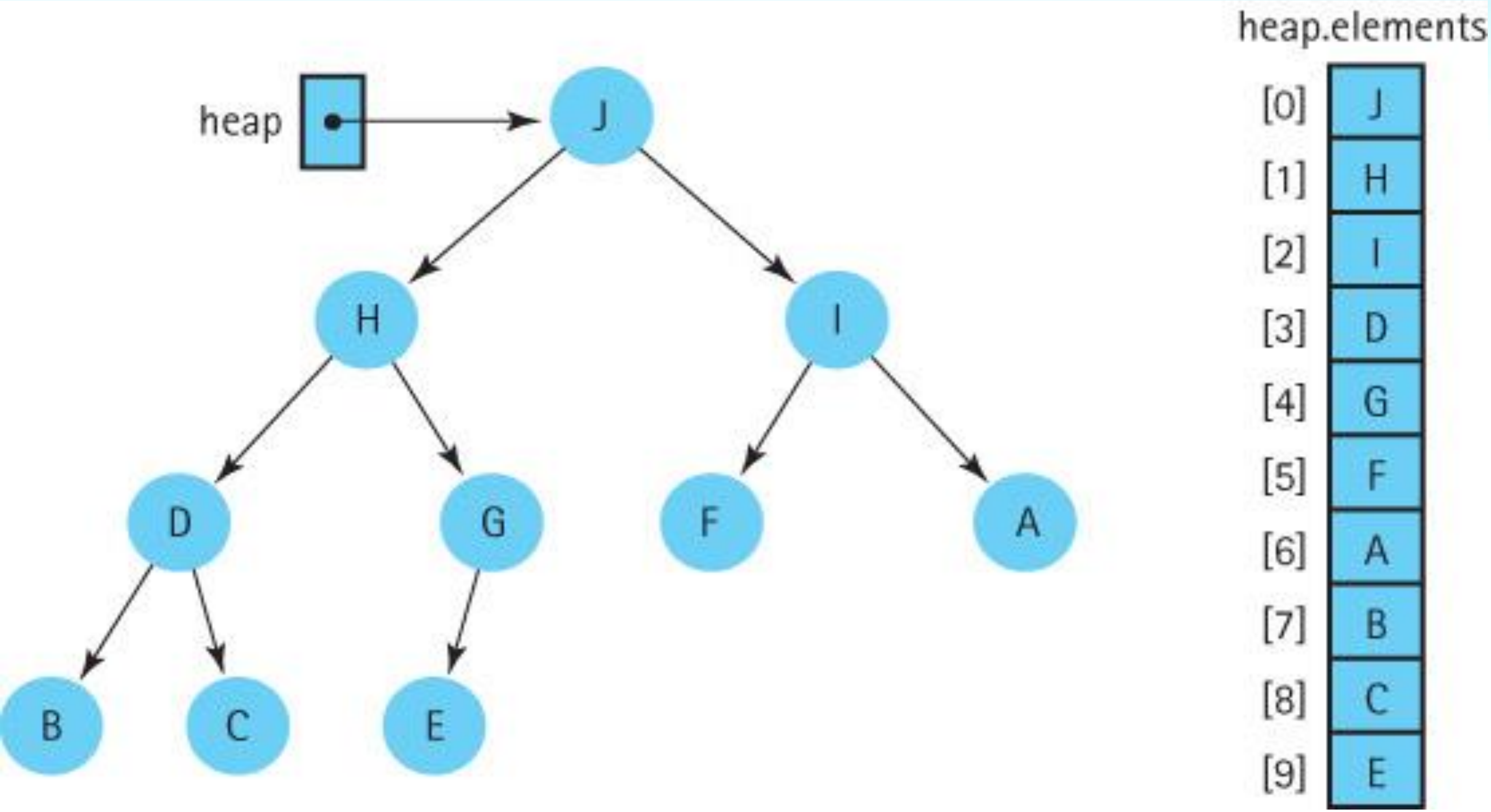
The enqueue operation

steps b, c represent the “reheap up” operation




a) enqueue ('M')
b) enqueue ('C')
c) enqueue ('X')
d) enqueue ('A')
e) enqueue ('D')
f) dequeue ( )
g) enqueue ('H')
h) dequeue ( )
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)

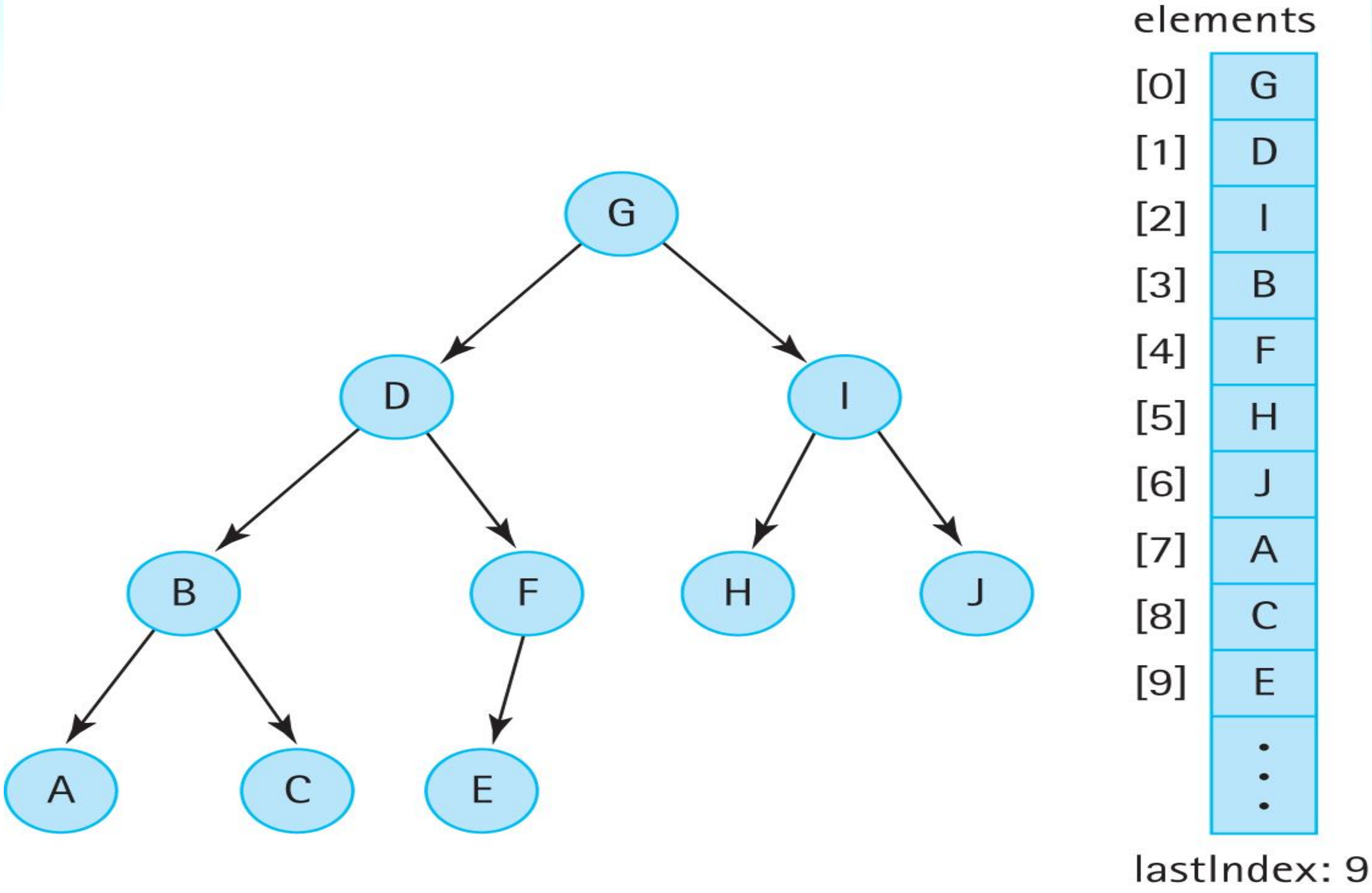
Heap Implementation



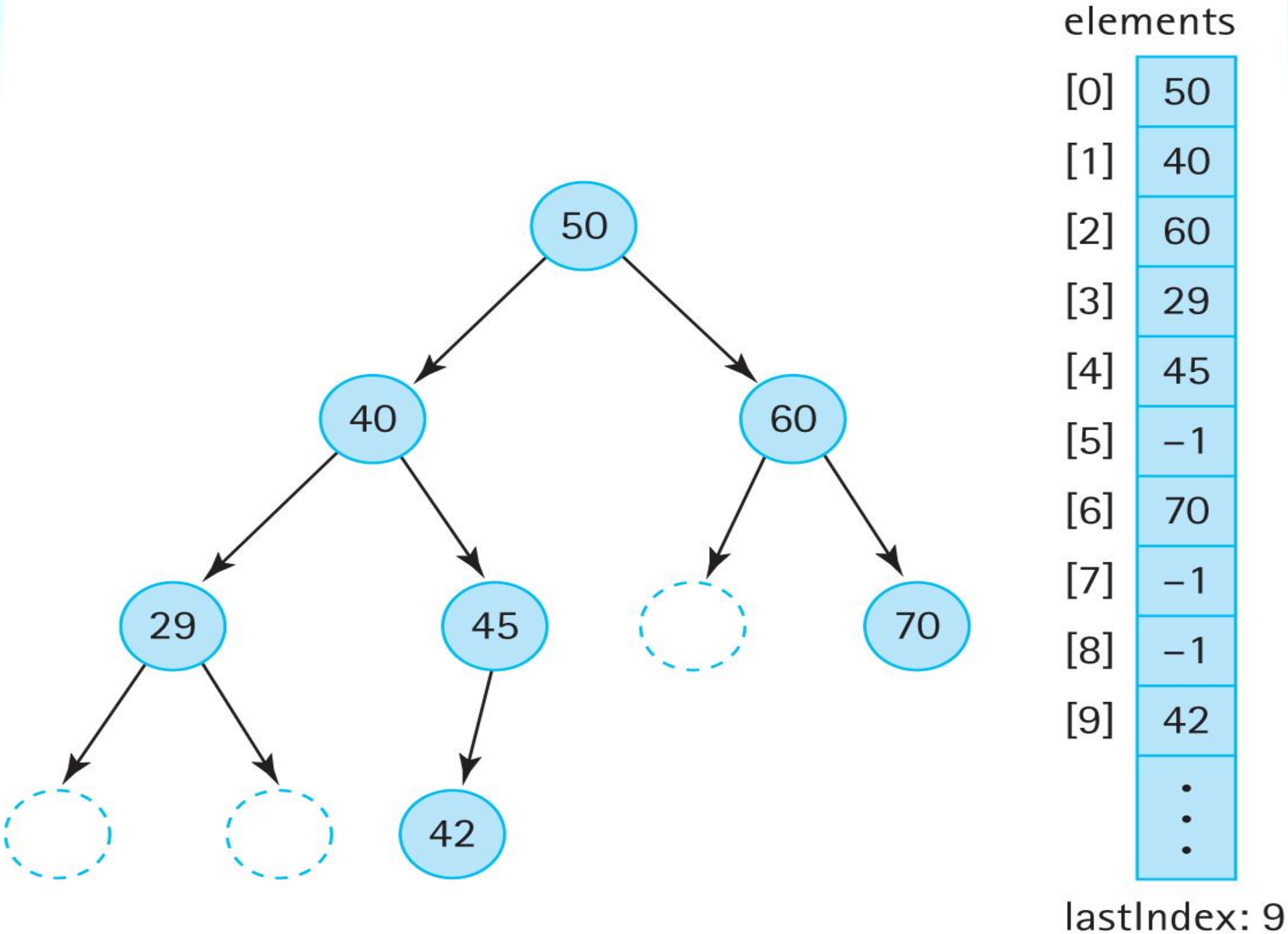
A Non-linked Representation of Binary Trees

- A binary tree can be stored in an array in such a way that the relationships in the tree are not physically represented by link members, but are *implicit* in the algorithms that manipulate the tree stored in the array.
- We store the tree elements in the array, level by level, left-to-right. We call the array `elements` and store the index of the last tree element in a variable `lastIndex`.
- The tree elements are stored with the root in `elements[0]` and the last node in `elements[lastIndex]`.

A Binary Tree and Its Array Representation



A Binary Search Tree Stored in an Array with Dummy Values



Array Representation continued

- To implement the algorithms that manipulate the tree, we must be able to find the left and right child of a node in the tree:
  - elements[index] left child is in elements[index*2 + 1]
  - elements[index] right child is in elements[index*2 + 2]
- We can also can determine the location of its parent node:
  - elements[index]’s parent is in elements[(index – 1)/2].
- This representation works best, space wise, if the tree is complete (which it is for a heap)

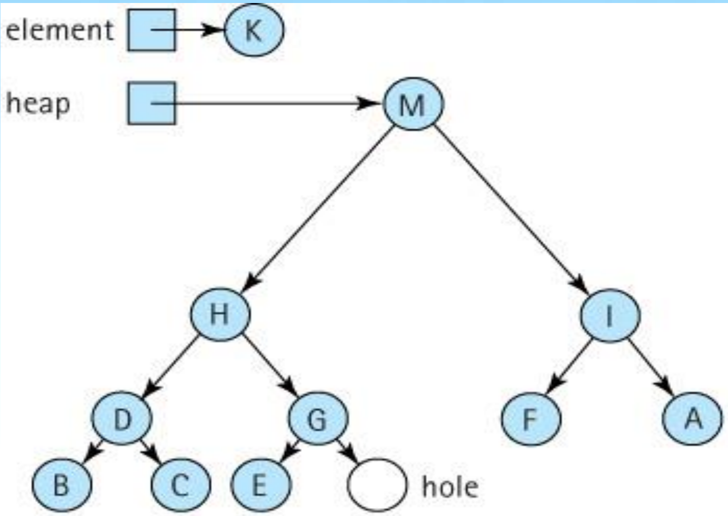
Beginning of `HeapPriQ.java`

```
//----------------------------------------------------------------------------
// HeapPriQ.java                by Dale/Joyce/Weems                    Chapter
9 // Priority Queue using Heap (implemented with an ArrayList)
//
// Two constructors are provided: one that use the natural order of the
// elements as defined by their compareTo method and one that uses an
// ordering based on a comparator argument.
//----------------------------------------------------------------------------
package ch09.priorityQueues;
import java.util.*;  // ArrayList, Comparator

public class HeapPriQ<T> implements PriQueueInterface<T>
{
  protected ArrayList<T> elements; // priority queue elements
  protected int lastIndex;         // index of last element in priority queue
  protected int maxIndex;          // index of last position in ArrayList

  protected Comparator<T> comp;

. . .
```

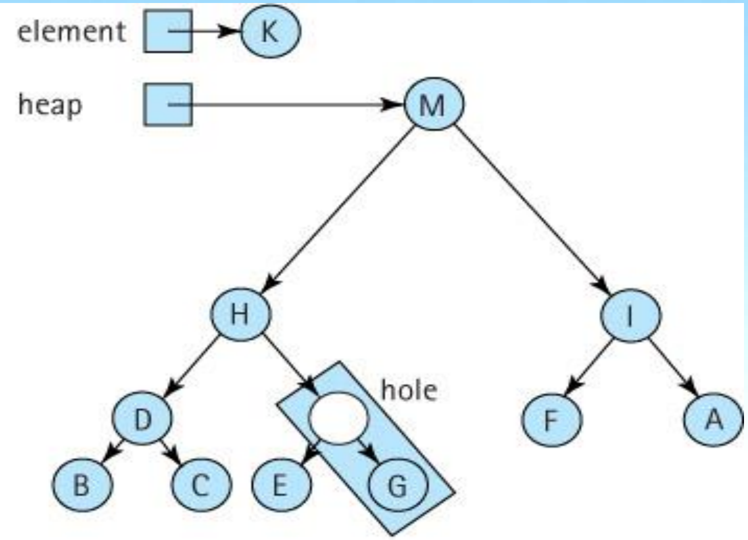
The enqueue method

```
public void enqueue(T element) throws PriQOverflowException
// Throws PriQOverflowException if this priority queue is full;
// otherwise, adds element to this priority queue.
{
  if (lastIndex == maxIndex)
    throw new PriQOverflowException("Priority queue is full");
  else
  {
    lastIndex++;
    elements.add(lastIndex, element);
    reheapUp(element);
  }
}
```

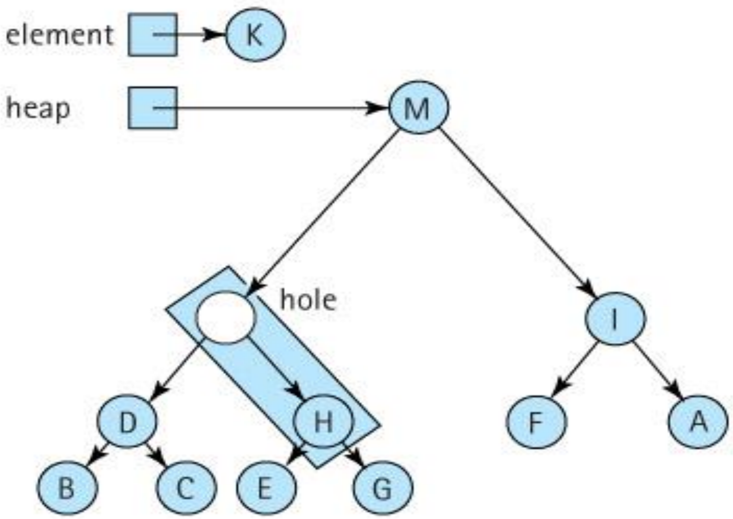
The `reheapUp` algorithm is pictured on the next slide



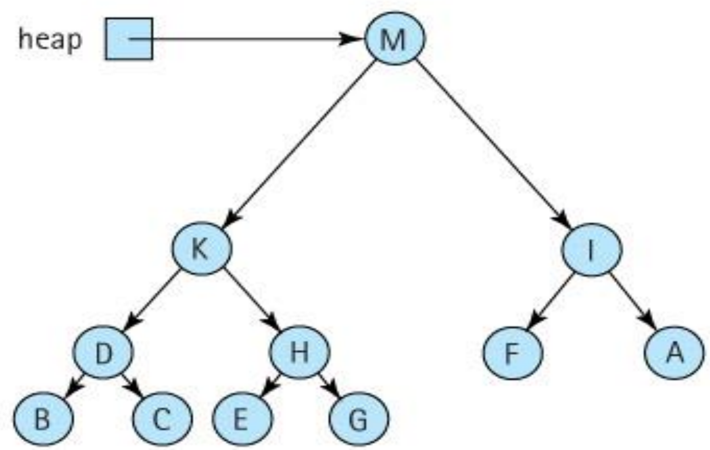
(a) Add K



(b) Move hole up



(c) Move hole up



(d) Place element into hole

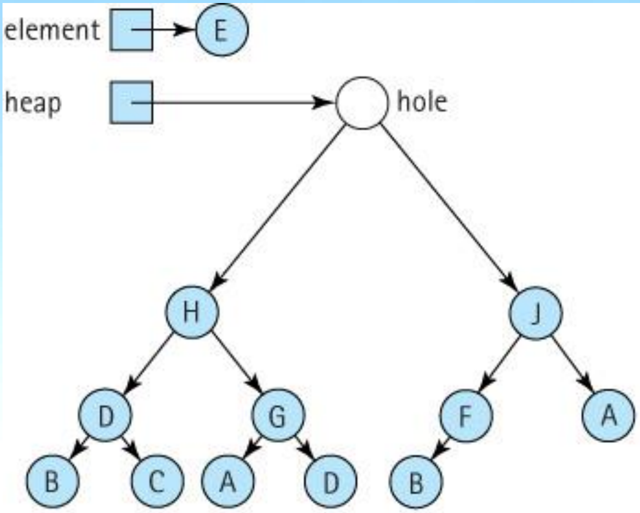
`reheapUp` operation

```
private void reheapUp(T element)
// Current lastIndex position is empty.
// Inserts element into the tree and ensures shape and order properties.
{
  int hole = lastIndex;
  while ((hole > 0)    // hole is not root and element > hole's parent
         &&
       (comp.compare(element, elements.get((hole - 1) / 2)) > 0))
  {
    elements.set(hole,elements.get((hole - 1) / 2));    // move hole's parent down
    hole = (hole - 1) / 2;                              // move hole up
  }
  elements.set(hole, element);                      // place element into final hole
}
```

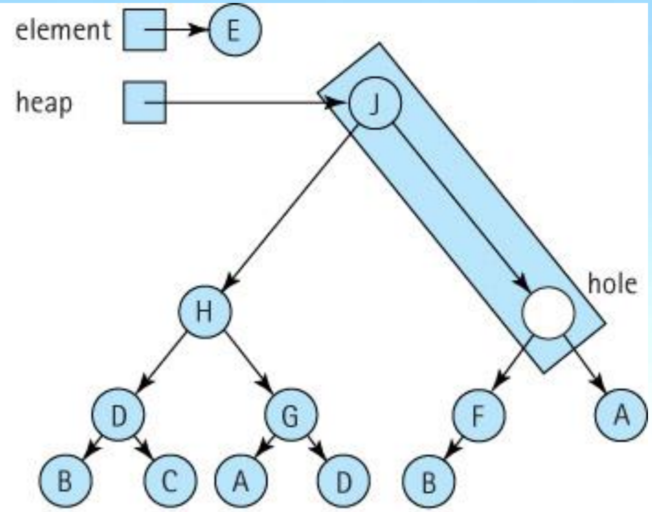
The `dequeue` method

```
public T dequeue() throws PriQUnderflowException
// Throws PriQUnderflowException if this priority queue is empty;
// otherwise, removes element with highest priority from this
// priority queue and returns it.
{
  T hold;      // element to be dequeued and returned
  T toMove;    // element to move down heap

  if (lastIndex == -1)
    throw new PriQUnderflowException("Priority queue is empty");
  else
  {
    hold = elements.get(0);                  // remember element to be returned
    toMove = elements.remove(lastIndex); // element to reheap down
    lastIndex--;                             // decrease priority queue size
    if (lastIndex != -1)
       reheapDown(toMove);                   // restore heap properties
    return hold;                             // return largest element
  }
}
```

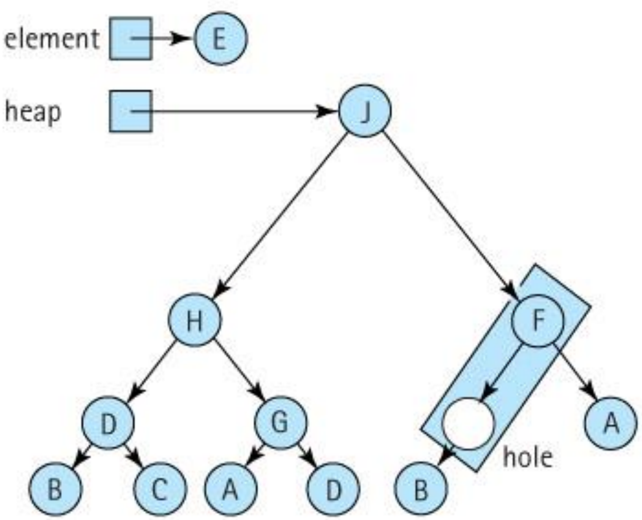
The `reheapDown` algorithm is pictured on the next slide



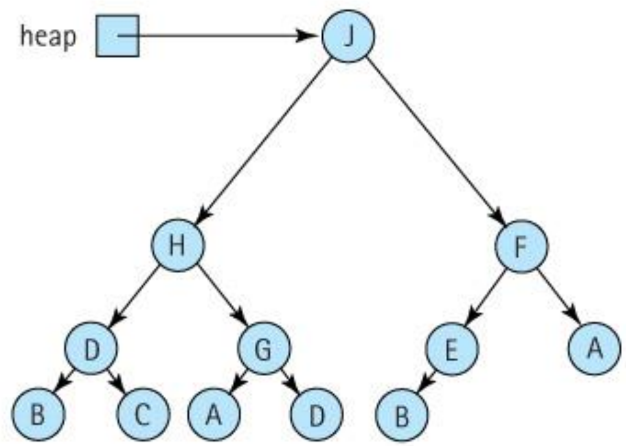
(a) reheapDown (E);



(b) Move hole down



(c) Move hole down



(d) Fill in final hole

`reheapDown` operation

```
private void reheapDown(T element)
// Current root position is "empty";
// Inserts element into the tree and ensures shape and order properties.
{
  int hole = 0;         // current index of hole
  int next;             // next index where hole should move to

  next = newHole(hole, element);    // find next hole
  while (next != hole)
  {
    elements.set(hole,elements.get(next));  // move element up
    hole = next;                            // move hole down
    next = newHole(hole, element);          // find next hole
  }
  elements.set(hole, element);              // fill in the final hole
}
```

The `newHole` method

```
private int newHole(int hole, T element)
// If either child of hole is larger than element return the index
// of the larger child; otherwise return the index of hole.
{
  int left = (hole * 2) + 1;
  int right = (hole * 2) + 2;
  if (left > lastIndex)
    // hole has no children
    return hole;
  else
  if (left == lastIndex)
    // hole has left child only
    if (comp.compare(element, elements.get(left)) < 0)
      // element < left child
      return left;
    else
      // element >= left child
      return hole;
  else
  // hole has two children
  if (comp.compare(elements.get(left), elements.get(right)) < 0)
    // left child < right child
    if (comp.compare(elements.get(right), element) <= 0)
      // right child <= element
      return hole;
    else
      // element < right child
      return right;
  else
  // left child >= right child
  if (comp.compare(elements.get(left), element) <= 0)
    // left child <= element
    return hole;
  else
    // element < left child
    return left;
}
```

Heaps Versus Other Representations of Priority Queues

	enqueue	dequeue
Heap	$O(\log_2 N)$	$O(\log_2 N)$
Linked List	$O(N)$	$O(1)$
Binary Search Tree		
Balanced	$O(\log_2 N)$	$O(\log_2 N)$
Skewed	$O(N)$	$O(N)$