Chapter 11: Sorting and Searching Algorithms

CS401

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Revised Nell Dale Presentation

Chapter 11: Sorting and Searching Algorithms

10.1 Sorting

- Putting an unsorted list of data elements into order – sorting - is a very common and useful operation
- We describe efficiency by relating the number of comparisons to the number of elements in the list (N)

A Test Harness

- To help us test our sorting algorithms we create an application class called `Sorts`:
- The class defines an array `values` that can hold 50 integers and static methods:
  - `initValues`: Initializes the `values` array with random numbers between 0 and 99
  - `isSorted`: Returns a boolean value indicating whether the `values` array is currently sorted
  - `swap`: swaps the integers between `values[index1]` and `values[index2]`, where `index1` and `index2` are parameters of the method
  - `printValues`: Prints the contents of the `values` array to the `System.out` stream; the output is arranged evenly in ten columns

Example of `Sorts` `main` method

```
public static void main(String[] args) throws IOException
 {
   initValues();
   printValues();
   System.out.println("values is sorted: " + isSorted());
   System.out.println();

   swap(0, 1);     // normally we put sorting algorithm here

   printValues();
   System.out.println("values is sorted: " + isSorted());
   System.out.println();
 }
```

Output from Example

```
the values array is:
20 49 07 50 45 69 20 07 88 02
89 87 35 98 23 98 61 03 75 48
25 81 97 79 40 78 47 56 24 07
63 39 52 80 11 63 51 45 25 78
35 62 72 05 98 83 05 14 30 23

values is sorted: false

the values array is:
49 20 07 50 45 69 20 07 88 02
89 87 35 98 23 98 61 03 75 48
25 81 97 79 40 78 47 56 24 07
63 39 52 80 11 63 51 45 25 78
35 62 72 05 98 83 05 14 30 23

values is sorted: false
```

This part varies
for each sample run

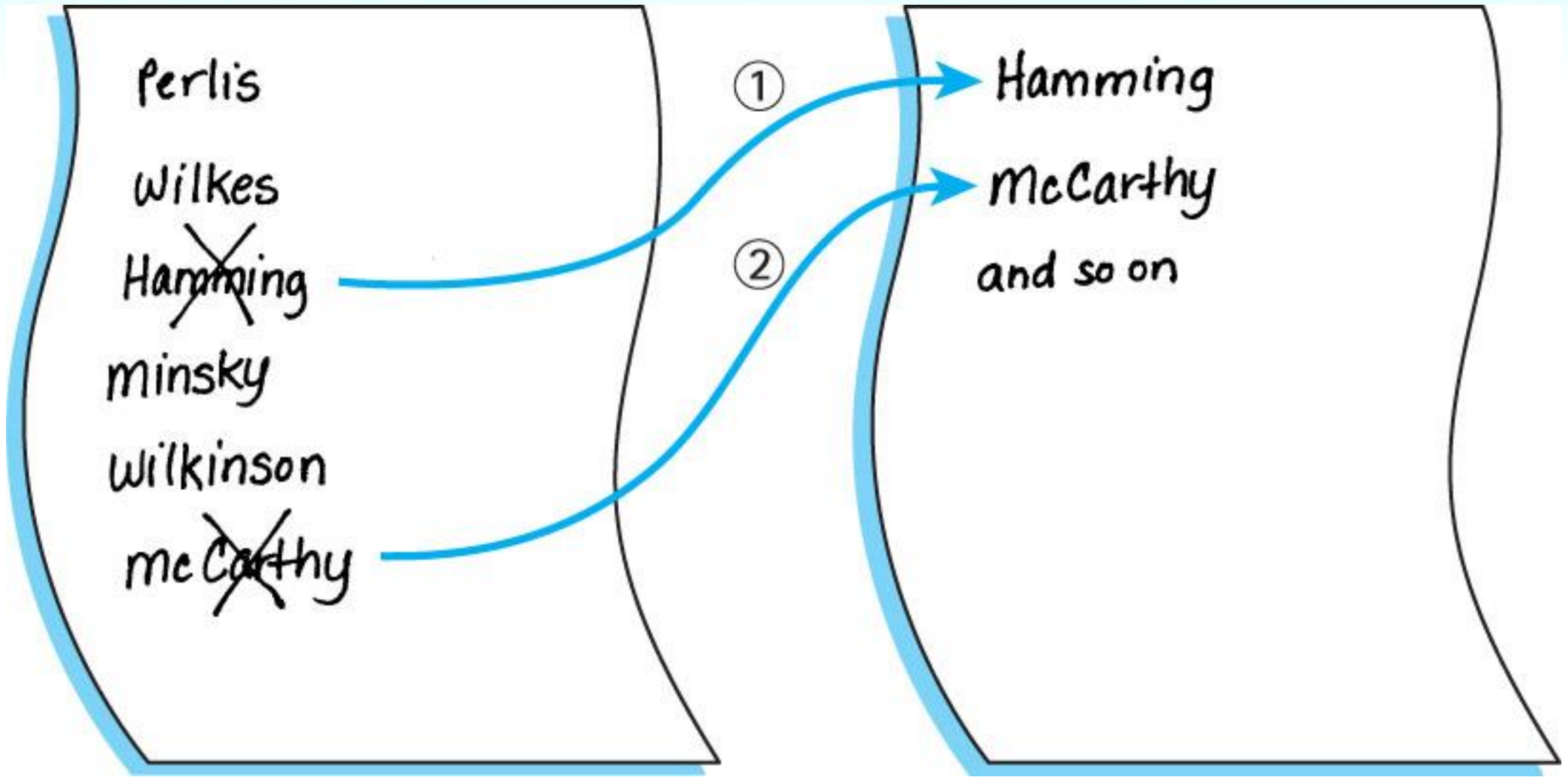
This does to, of course

11.2 Simple Sorts

- In this section we present three “simple” sorts
  - Selection Sort
  - Bubble Sort
  - Insertion Sort
- Properties of these sorts
  - use an unsophisticated brute force approach
  - are not very efficient
  - are easy to understand and to implement

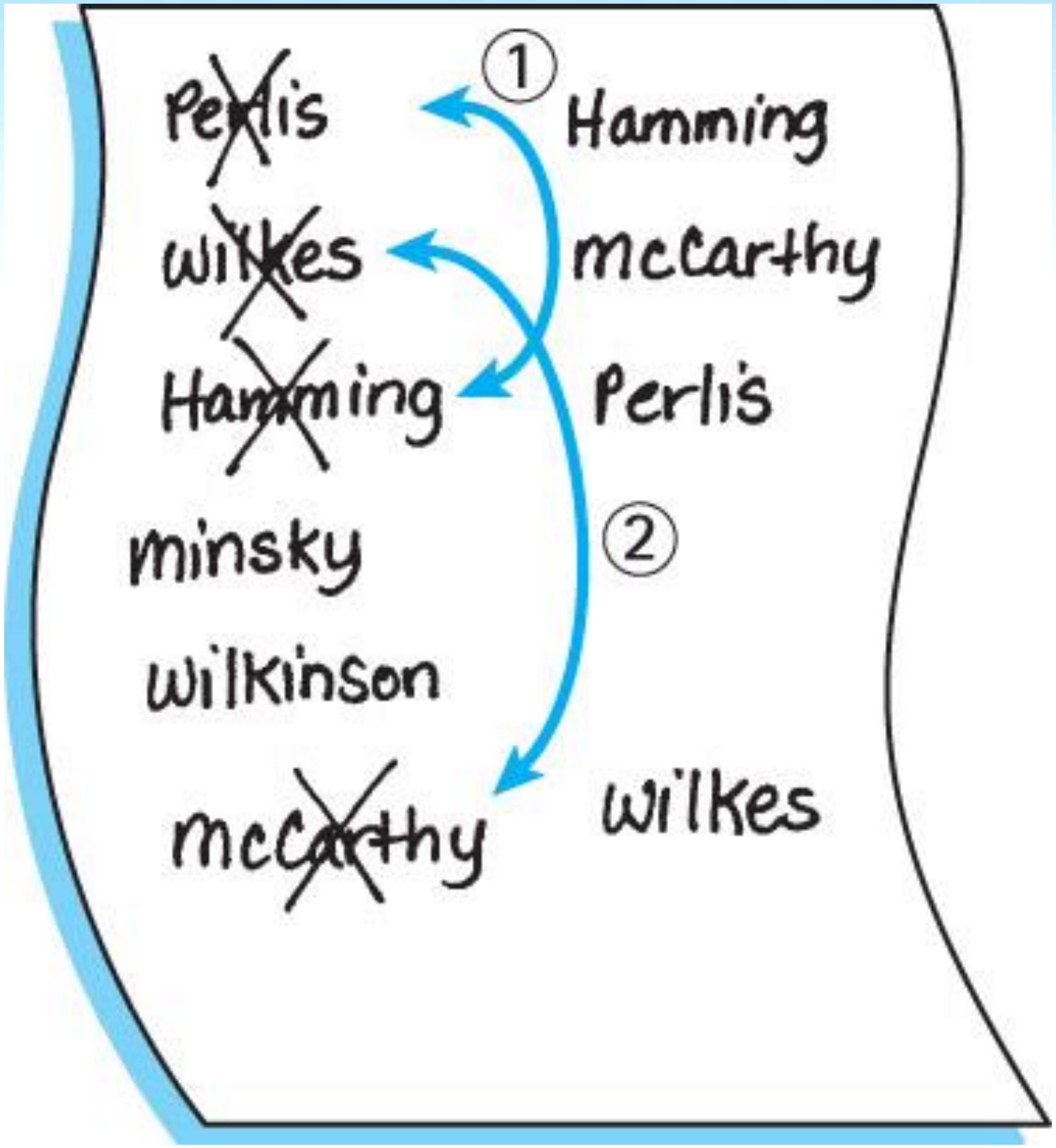
Selection Sort

- If we were handed a list of names on a sheet of paper and asked to put them in alphabetical order, we might use this general approach:
  - *Select* the name that comes first in alphabetical order, and write it on a second sheet of paper.
  - Cross the name out on the original sheet.
  - Repeat steps 1 and 2 for the second name, the third name, and so on until all the names on the original sheet have been crossed out and written onto the second sheet, at which point the list on the second sheet is sorted.


Perlis
wilkes
~~Hamming~~
minsky
wilkinson
~~mcCarthy~~
①
②
Hamming
McCarthy
and so on

An improvement

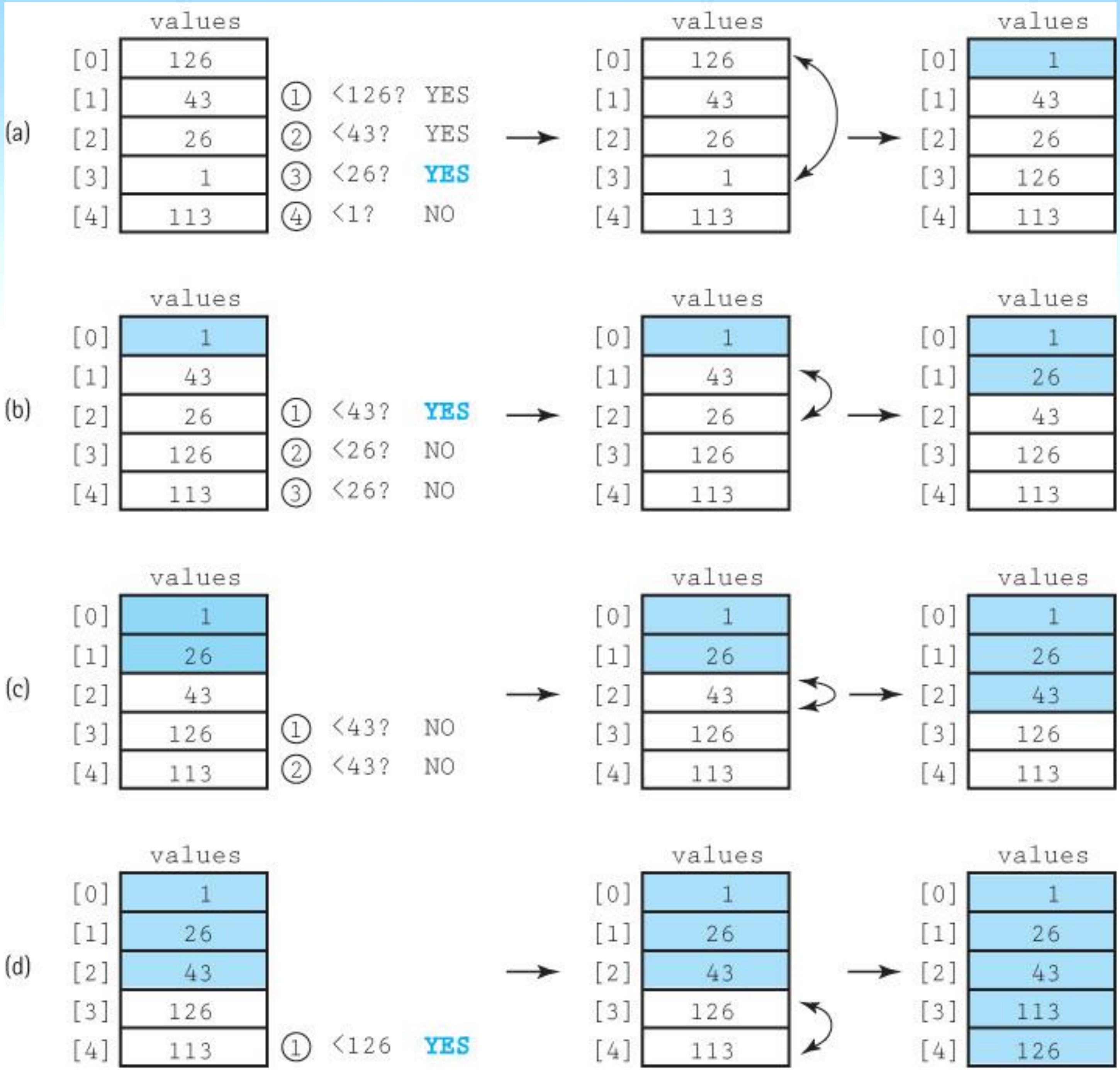
- Our algorithm is simple but it has one drawback: It requires $\boxed{\text{space to store two complete lists.}}$
- Instead of writing the “first” name onto a separate sheet of paper, exchange it with the name in the first location on the original sheet. And so on.


~~Perlis~~
~~wilkes~~
~~Hamming~~
minsky
wilkinson
~~mcCarthy~~
1
2
Hamming
mcCarthy
Perlis
wilkes

Selection Sort Algorithm

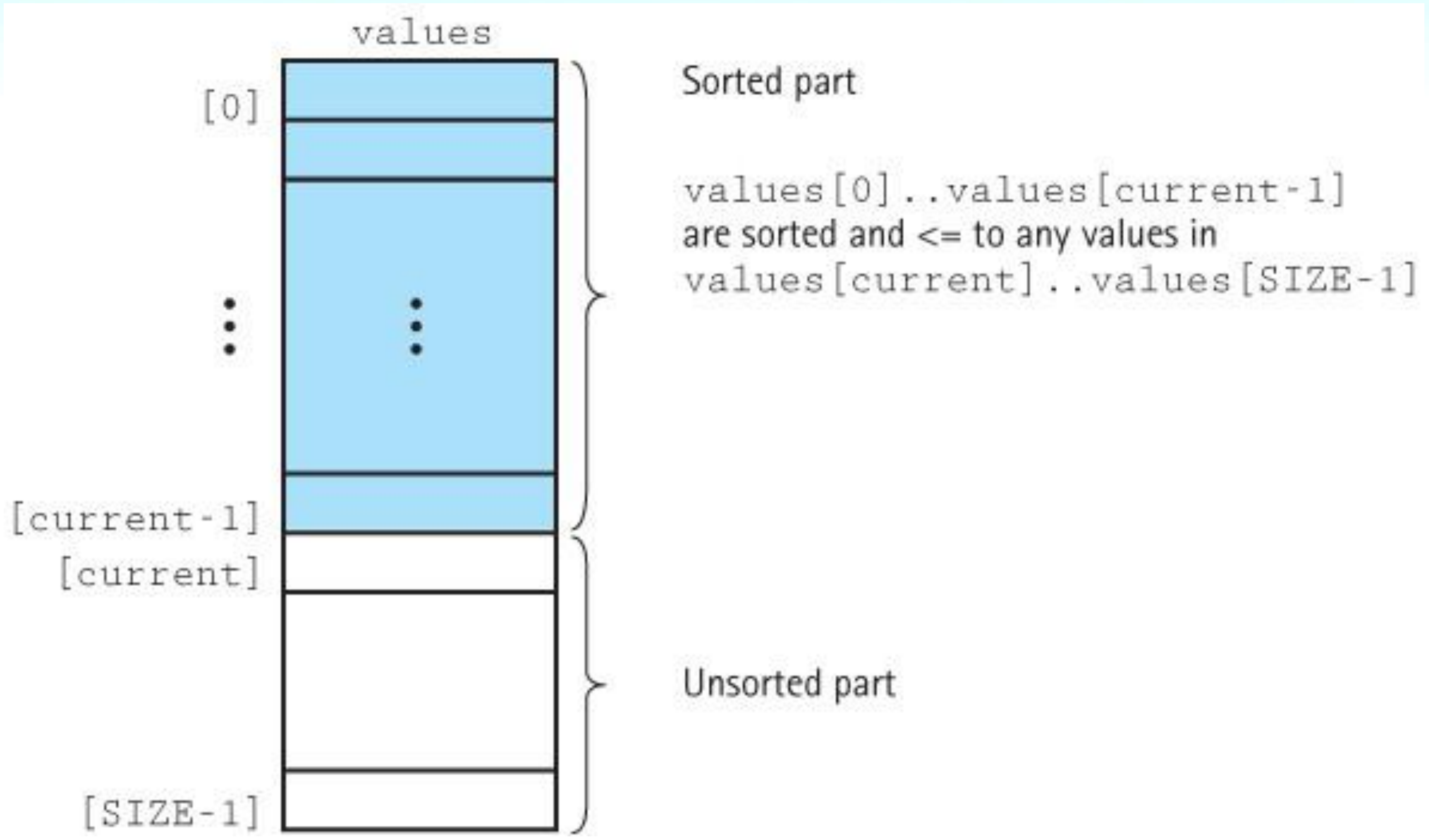
```
SelectionSort
for current going from 0 to SIZE - 2
    Find the index in the array of the smallest unsorted element
    Swap the current element with the smallest unsorted one
```

An example is depicted on the following slide …



(a)
(b)
(c)
(d)
values
[0] [1] [2] [3] [4]
126 43 26 1 113
① <126? YES
② <43? YES
③ <26? YES
④ <1? NO
1 43 26 126 113
① <43? YES
② <26? NO
③ <26? NO
1 26 43 126 113
① <43? NO
② <43? NO
① <126 YES
1 26 43 113 126

Selection Sort Snapshot


values
[0]
[current-1]
[current]
[SIZE-1]
Sorted part
values[0]..values[current-1]
are sorted and <= to any values in
values[current]..values[SIZE-1]
Unsorted part

Selection Sort Code

```
static int minIndex(int startIndex, int endIndex)
// Returns the index of the smallest value in
// values[startIndex]..values[endIndex].
{
  int indexOfMin = startIndex;
  for (int index = startIndex + 1; index <= endIndex; index++)
    if (values[index] < values[indexOfMin])
      indexOfMin = index;
  return indexOfMin;
}

static void selectionSort()
// Sorts the values array using the selection sort algorithm.
{
  int endIndex = SIZE – 1;
  for (int current = 0; current < endIndex; current++)
    swap(current, minIndex(current, endIndex));
}
```

Testing Selection Sort

The test harness:

```
initValues();
printValues();
System.out.println("values is sorted: "
                    + isSorted());
System.out.println();

selectionSort();

System.out.println("Selection Sort called\n");
printValues();
System.out.println("values is sorted: "
                    + isSorted());
System.out.println();
```

The resultant output:

```
the values array is:
92 66 38 17 21 78 10 43 69 19
17 96 29 19 77 24 47 01 97 91
13 33 84 93 49 85 09 54 13 06
21 21 93 49 67 42 25 29 05 74
96 82 26 25 11 74 03 76 29 10

values is sorted: false

Selection Sort called
the values array is:
01 03 05 06 09 10 10 11 13 13
17 17 19 19 21 21 21 24 25 25
26 29 29 29 33 38 42 43 47 49
49 54 66 67 69 74 74 76 77 78
82 84 85 91 92 93 93 96 96 97

values is sorted: true
```

Selection Sort Analysis

- We describe the number of comparisons as a function of the number of elements in the array, i.e., `SIZE`. To be concise, in this discussion we refer to SIZE as N
- The `minIndex` method is called N - 1 times
- Within `minIndex`, the number of comparisons varies:
  - in the first call there are N - 1 comparisons
  - in the next call there are N - 2 comparisons
  - and so on, until in the last call, when there is only 1 comparison
- The total number of comparisons is

  $(N – 1) + (N – 2) + (N – 3) + \ldots + 1$

  $= N(N – 1)/2 = 1/2N^2 – 1/2N$
- The Selection Sort algorithm is $\boxed{O(N^2)}$

Number of Comparisons Required to Sort Arrays of Different Sizes Using Selection Sort

Number of Elements	**Number of Comparisons**
10	45
20	190
100	4,950
1,000	499,500
10,000	49,995,000

Bubble Sort

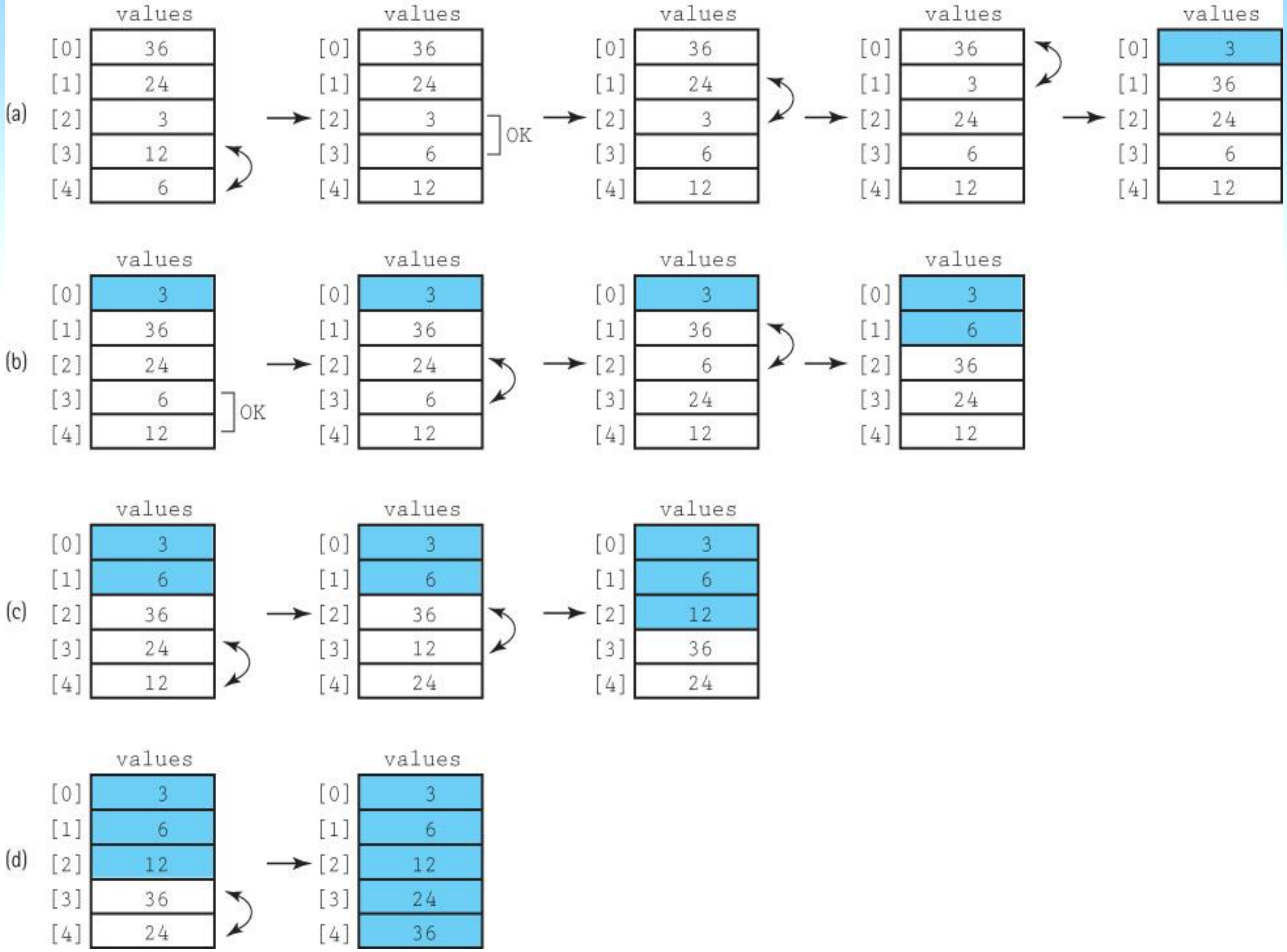
- With this approach the smaller data values “bubble up” to the front of the array …
- Each iteration puts the smallest unsorted element into its correct place, but it also makes changes in the locations of the other elements in the array.
- The first iteration puts the smallest element in the array into the first array position:
  - starting with the last array element, we compare successive pairs of elements, swapping whenever the bottom element of the pair is smaller than the one above it
  - in this way the smallest element “bubbles up” to the top of the array.
- The next iteration puts the smallest element in the unsorted part of the array into the second array position, using the same technique
- The rest of the sorting process continues in the same way

Bubble Sort Algorithm

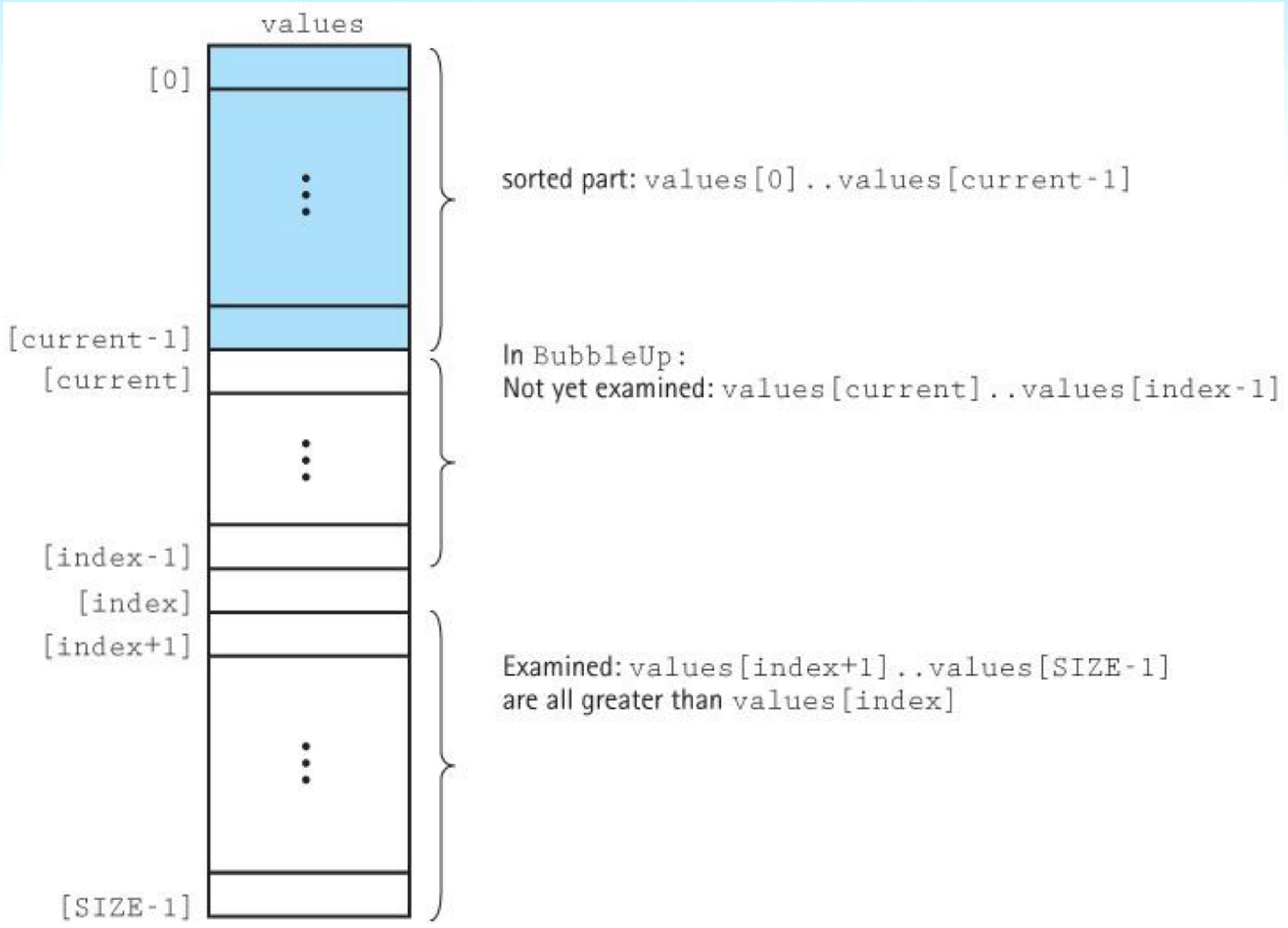
```
BubbleSort
Set current to the index of first element in the array
while more elements in unsorted part of array
    "Bubble up" the smallest element in the unsorted part,
           causing intermediate swaps as needed
  Shrink the unsorted part of the array by incrementing current

bubbleUp(startIndex, endIndex)
for index going from endIndex DOWNTO startIndex +1
    if values[index] < values[index - 1]
        Swap the value at index with the value at index - 1
```

An example is depicted on the following slide …


(a)
values
[0] 36 [1] 24 [2] 3 [3] 12 [4] 6
values
[0] 36 [1] 24 [2] 3 [3] 6 [4] 12
OK
values
[0] 36 [1] 24 [2] 3 [3] 6 [4] 12
values
[0] 36 [1] 3 [2] 24 [3] 6 [4] 12
values
[0] 3 [1] 36 [2] 24 [3] 6 [4] 12
(b)
values
[0] 3 [1] 36 [2] 24 [3] 6 [4] 12
OK
values
[0] 3 [1] 36 [2] 24 [3] 6 [4] 12
values
[0] 3 [1] 36 [2] 6 [3] 24 [4] 12
values
[0] 3 [1] 6 [2] 36 [3] 24 [4] 12
(c)
values
[0] 3 [1] 6 [2] 36 [3] 24 [4] 12
values
[0] 3 [1] 6 [2] 36 [3] 12 [4] 24
values
[0] 3 [1] 6 [2] 12 [3] 36 [4] 24
(d)
values
[0] 3 [1] 6 [2] 12 [3] 36 [4] 24
values
[0] 3 [1] 6 [2] 12 [3] 24 [4] 36

Bubble Sort Snapshot



values
[0]
[current-1]
[current]
[index-1]
[index]
[index+1]
[SIZE-1]
sorted part: values[0]..values[current-1]
In BubbleUp:
Not yet examined: values[current]..values[index-1]
Examined: values[index+1]..values[SIZE-1]
are all greater than values[index]

Bubble Sort Code

```
static void bubbleUp(int startIndex, int endIndex)
// Switches adjacent pairs that are out of order
// between values[startIndex]..values[endIndex]
// beginning at values[endIndex].
{
  for (int index = endIndex; index > startIndex; index--)
    if (values[index] < values[index – 1])
      swap(index, index – 1);
}

static void bubbleSort()
// Sorts the values array using the bubble sort algorithm.
{
  int current = 0;
  while (current < SIZE – 1)
  {
    bubbleUp(current, SIZE – 1);
    current++;
  }
}
```

Bubble Sort Analysis

- Analyzing the work required by `bubbleSort` is the same as for the straight selection sort algorithm.
- The comparisons are in `bubbleUp`, which is called $N - 1$ times.
- There are $N - 1$ comparisons the first time, $N - 2$ comparisons the second time, and so on.
- Therefore, `bubbleSort` and `selectionSort` require the same amount of work in terms of the number of comparisons.
- The Bubble Sort algorithm is $\boxed{O(N^2)}$

Insertion Sort

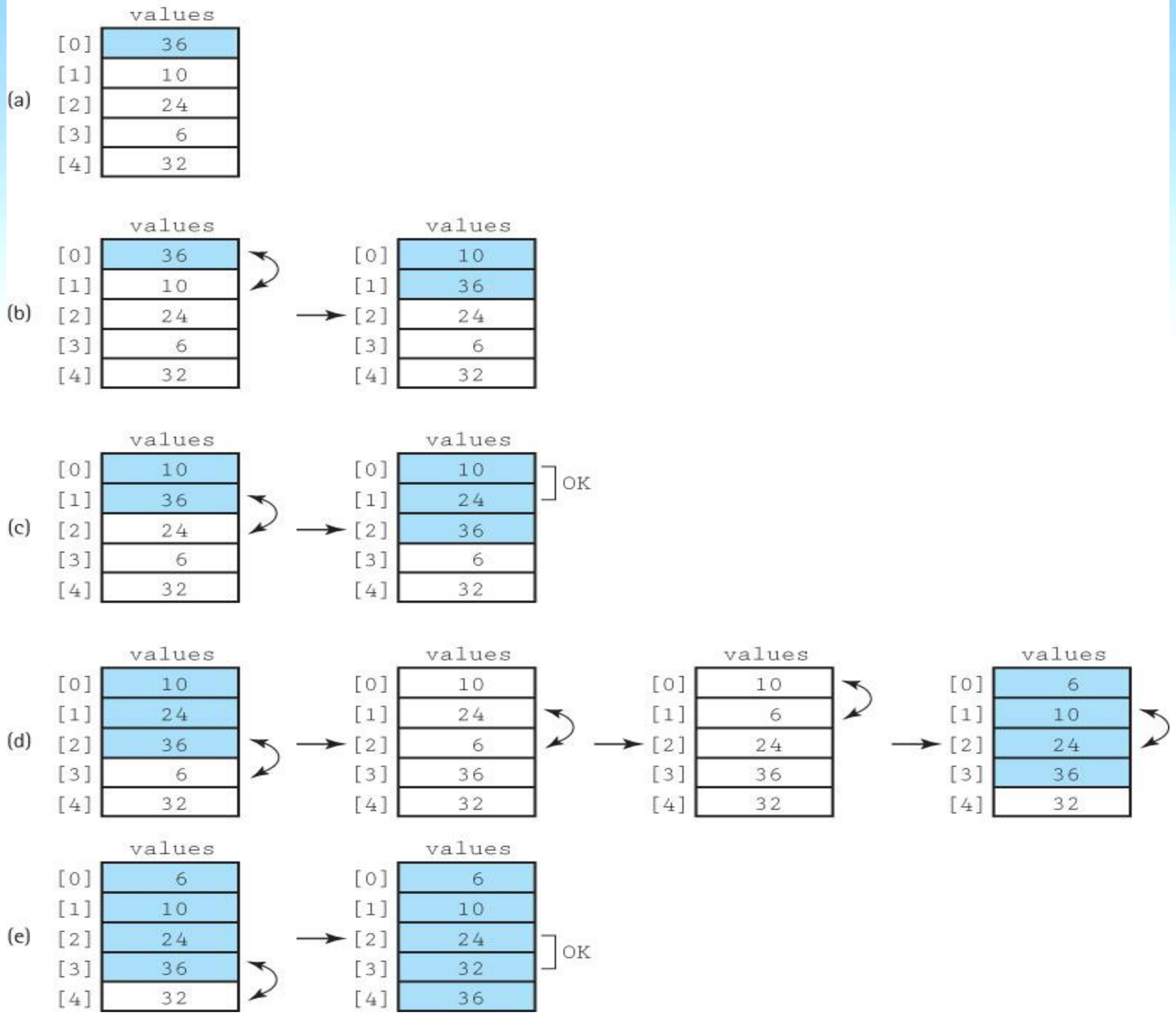
- In Section 6.4, “Sorted Array-Based List Implementation,” we described the Insertion Sort algorithm and how it could be used to maintain a list in sorted order. Here we present essentially the same algorithm.
- Each successive element in the array to be sorted is inserted into its proper place with respect to the other, already sorted elements.
- As with the previous sorts, we divide our array into a sorted part and an unsorted part.
  - Initially, the sorted portion contains only one element: the first element in the array.
  - Next we take the second element in the array and put it into its correct place in the sorted part; that is, values[0] and values[1] are in order with respect to each other.
  - Next the value in values[2] is put into its proper place, so values[0]..values[2] are in order with respect to each other.
  - This process continues until all the elements have been sorted.

Insertion Sort Algorithm

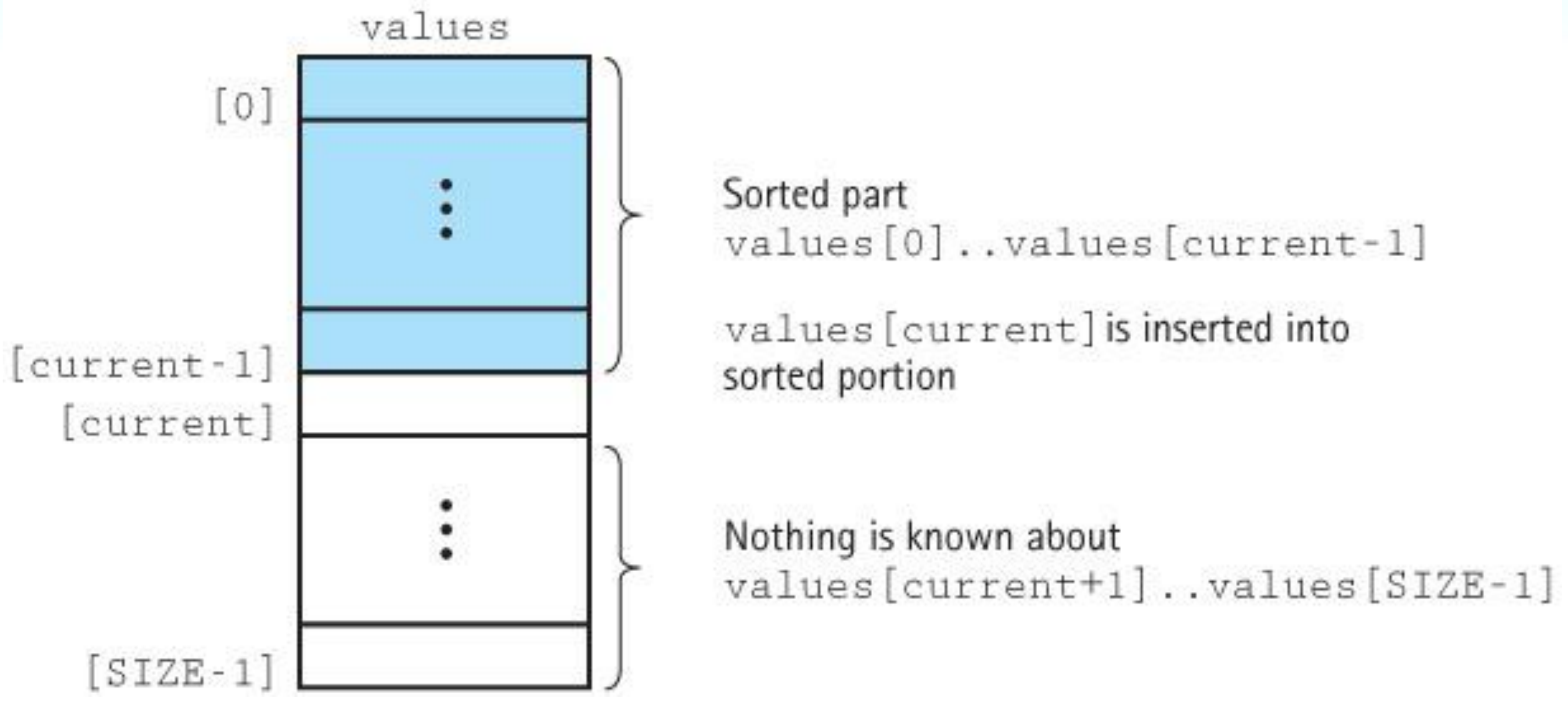
```
insertionSort
for count going from 1 through SIZE - 1
    insertElement(0, count)

InsertElement(startIndex, endIndex)
Set finished to false
Set current to endIndex
Set moreToSearch to true
while moreToSearch AND NOT finished
    if values[current] < values[current - 1]
        swap(values[current], values[current - 1])
        Decrement current
        Set moreToSearch to (current does not equal startIndex)
    else
        Set finished to true
```

An example is depicted on the following slide …


(a)
values
[0] 36
[1] 10
[2] 24
[3] 6
[4] 32
(b)
values
[0] 36 [1] 10 [2] 24 [3] 6 [4] 32
values
[0] 10 [1] 36 [2] 24 [3] 6 [4] 32
(c)
values
[0] 10 [1] 36 [2] 24 [3] 6 [4] 32
values
[0] 10 [1] 24 [2] 36 [3] 6 [4] 32
OK
(d)
values
[0] 10 [1] 24 [2] 36 [3] 6 [4] 32
values
[0] 10 [1] 24 [2] 6 [3] 36 [4] 32
values
[0] 10 [1] 6 [2] 24 [3] 36 [4] 32
values
[0] 6 [1] 10 [2] 24 [3] 36 [4] 32
(e)
values
[0] 6 [1] 10 [2] 24 [3] 36 [4] 32
values
[0] 6 [1] 10 [2] 24 [3] 32 [4] 36
OK

Insertion Sort Snapshot


values
[0]
[current-1]
[current]
[SIZE-1]
Sorted part
values[0]..values[current-1]
values[current] is inserted into sorted portion
Nothing is known about
values[current+1]..values[SIZE-1]

Insertion Sort Code

```
static void insertElement(int startIndex, int endIndex)
// Upon completion, values[0]..values[endIndex] are sorted.
{
  boolean finished = false;
  int current = endIndex;
  boolean moreToSearch = true;
  while (moreToSearch && !finished)
  {
    if (values[current] < values[current - 1])
    {
      swap(current, current - 1);
      current--;
      moreToSearch = (current != startIndex);
    }
    else
      finished = true;
  }
}

static void insertionSort()
// Sorts the values array using the insertion sort algorithm.
{
  for (int count = 1; count < SIZE; count++)
    insertElement(0, count);
}
```

Insertion Sort Analysis

- The general case for this algorithm mirrors the `selectionSort` and the `bubbleSort`, so the general case is $\boxed{O(N^2}$) .
- But `insertionSort` has a "best" case: The data are already sorted in ascending order
  - `insertElement` is called *N* times, but only one comparison is made each time and no swaps are necessary.
- The maximum number of comparisons is made only when the elements in the array are in reverse order.

11.3 O(N $\log_2$N) Sorts

- O(N^2) sorts and are very time consuming for sorting large arrays.
- Several sorting methods that work better when *N* is large are presented in this section.
- The efficiency of these algorithms is achieved at the expense of the simplicity seen in the straight selection, bubble, and insertion sorts.

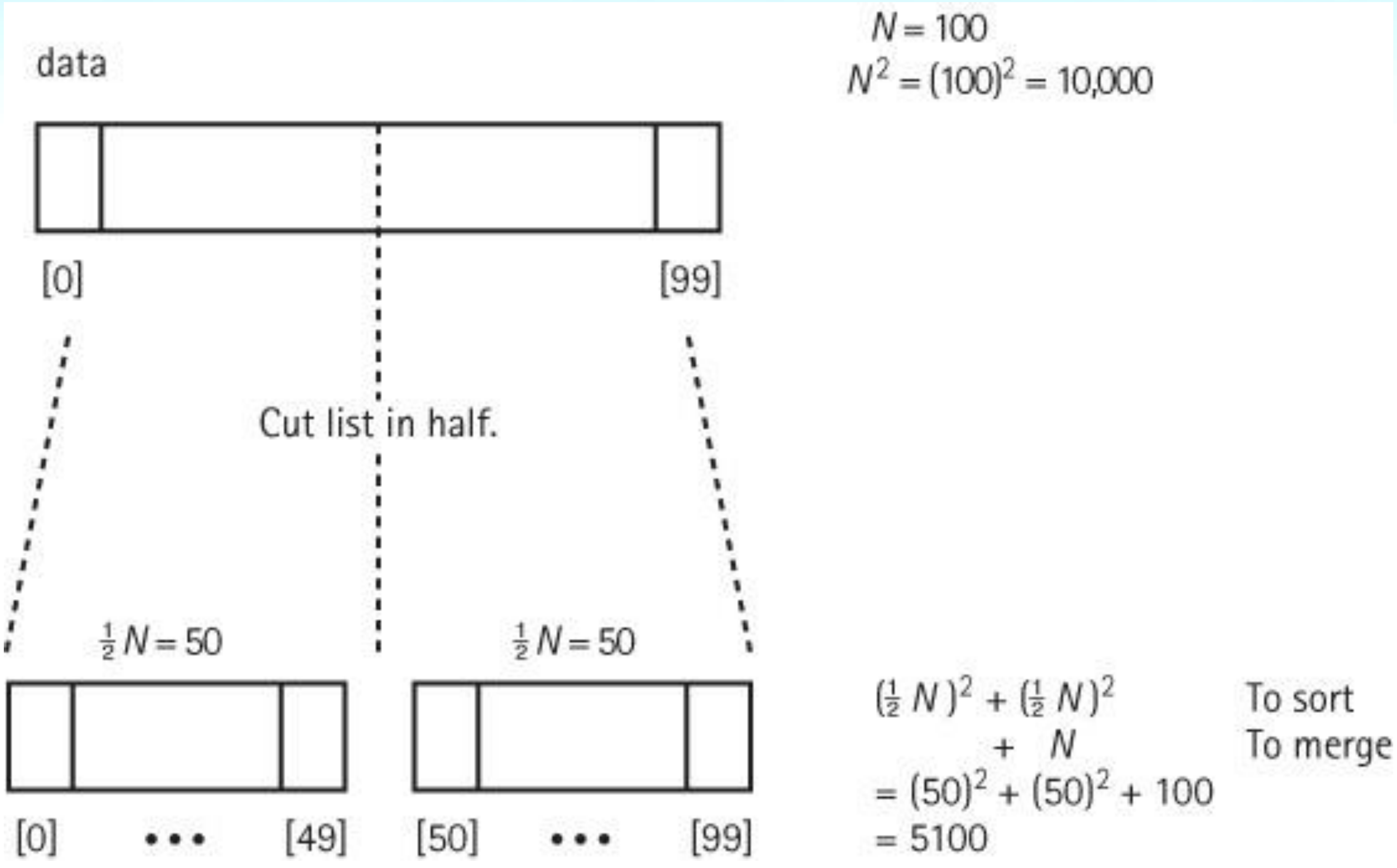
The Merge Sort

- The sorting algorithms covered in Section 10.2 are all $O(N^2)$.
- Note that N^2 is a lot larger than

$$(1/2N)^2 + (1/2N)^2 = 1/2N^2$$

- If we can cut the array into two pieces, sort each segment, and then merge the two back together, we should end up sorting the entire array with a lot less work.

Rationale for Divide and Conquer



$N = 100$

$N^2 = (100)^2 = 10{,}000$

$(\frac{1}{2}N)^2 + (\frac{1}{2}N)^2$ To sort

$+ \quad N$ To merge

$= (50)^2 + (50)^2 + 100$

$= 5100$

Merge Sort Algorithm

```
mergeSort
Cut the array in half
Sort the left half
Sort the right half
Merge the two sorted halves into one sorted array
```

Because `mergeSort` is itself a sorting algorithm, we might as well use it to sort the two halves.

We can make `mergeSort` a recursive method and let it call itself to sort each of the two subarrays:

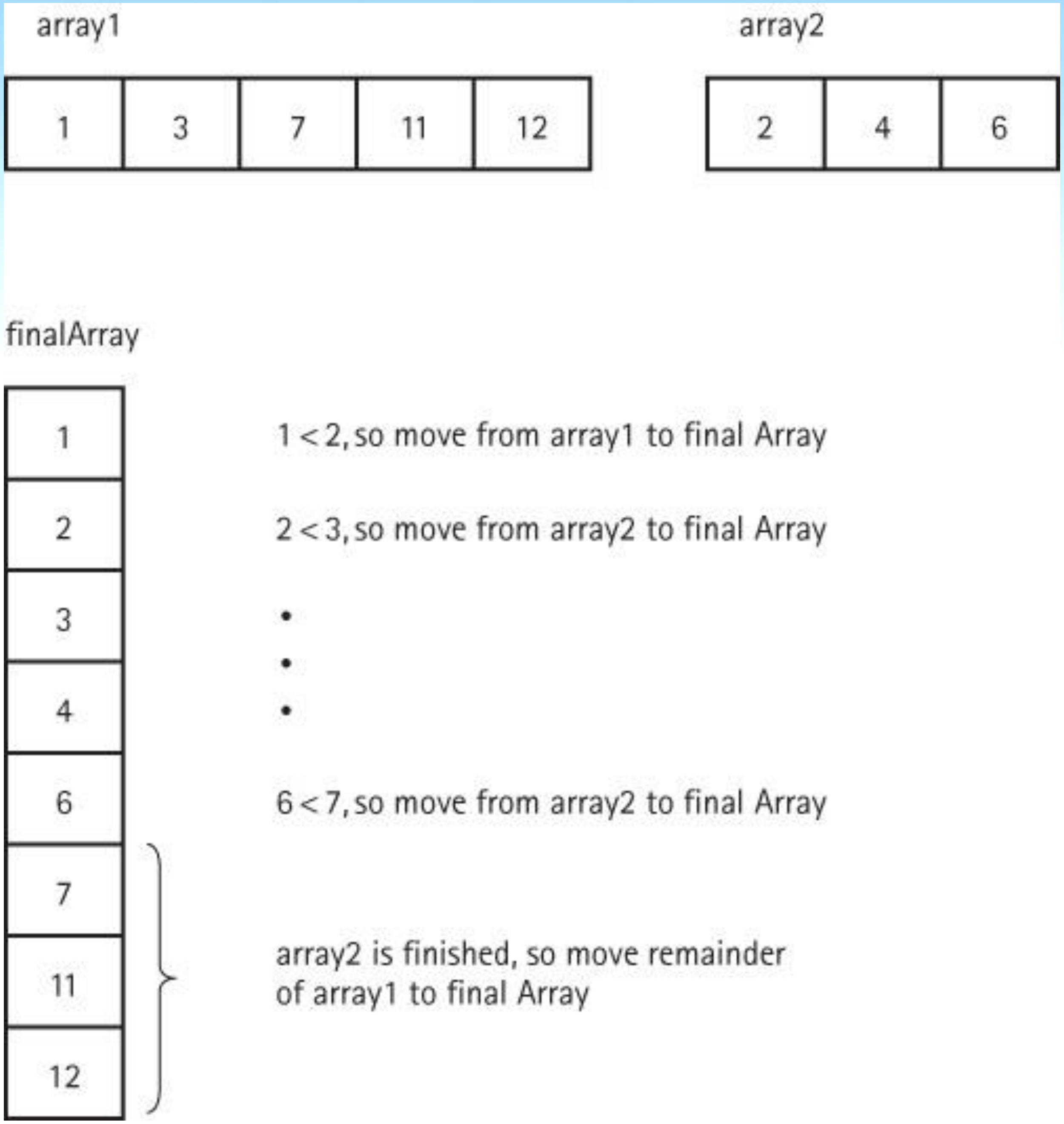
```
mergeSort—Recursive
Cut the array in half
mergeSort the left half
mergeSort the right half
Merge the two sorted halves into one sorted array
```

Merge Sort Summary

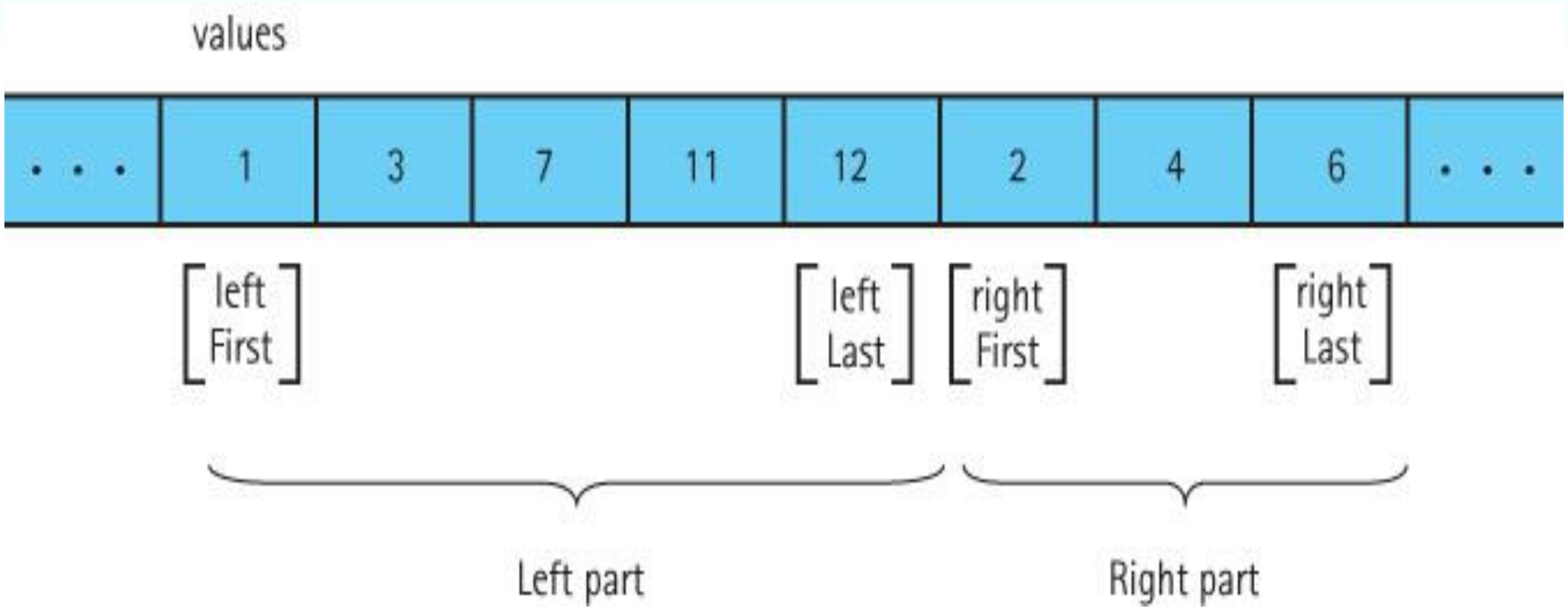
Method mergeSort(first, last)

Definition:	Sorts the array elements in ascending order.
Size:	last - first + 1
Base Case:	If size less than 2, do nothing.
General Case:	Cut the array in half.
	mergeSort the left half.
	mergeSort the right half.
	Merge the sorted halves into one sorted array.

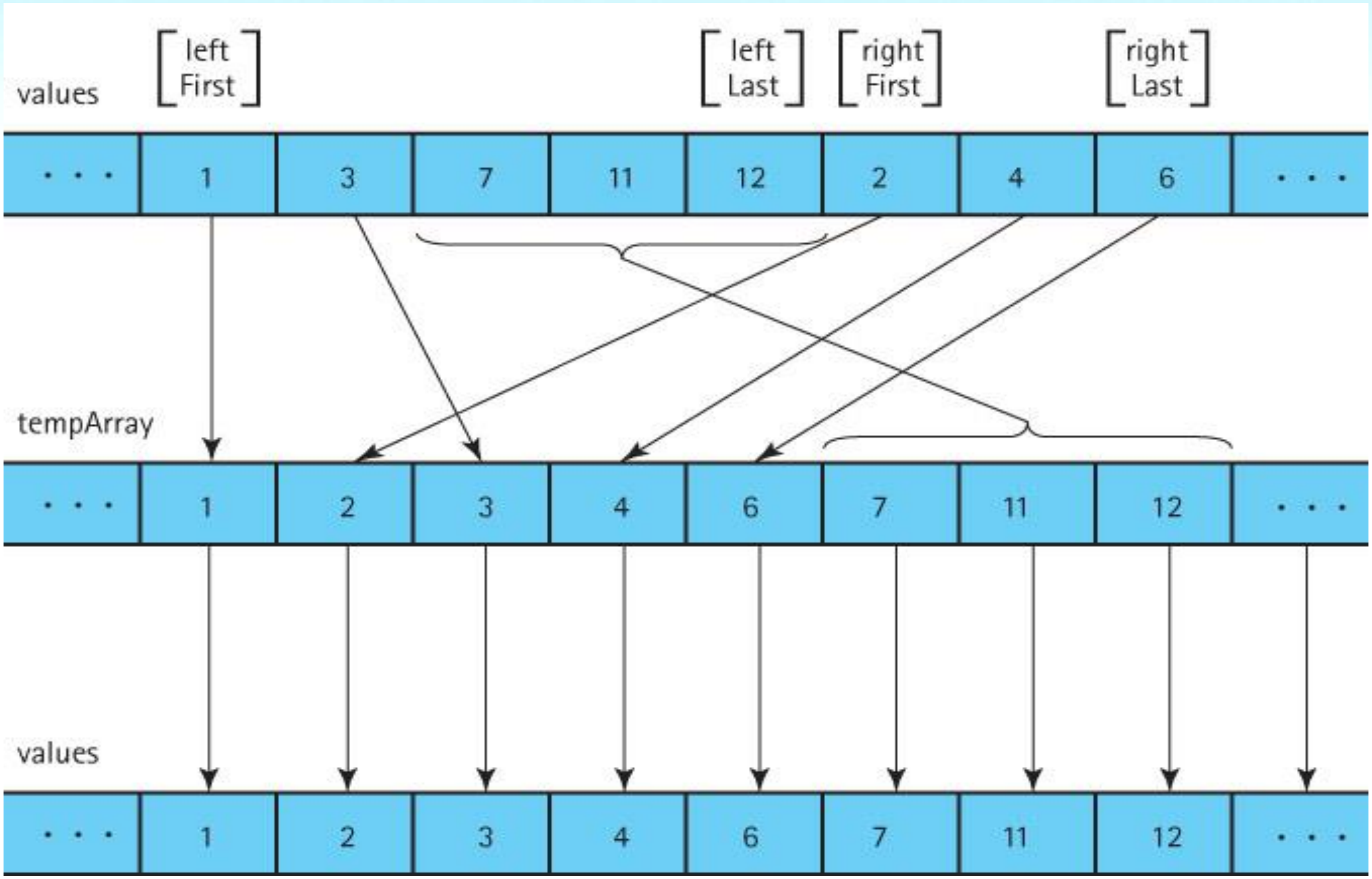
Strategy for merging two sorted arrays


array1
1
3
7
11
12
array2
2
4
6
finalArray
1
2
3
4
6
7
11
12
1 < 2, so move from array1 to final Array
2 < 3, so move from array2 to final Array
•
•
•
6 < 7, so move from array2 to final Array
array2 is finished, so move remainder of array1 to final Array

Our actual merge problem



Our solution



The `merge` algorithm

```
merge (leftFirst, leftLast, rightFirst, rightLast)

(uses a local array, tempArray)

Set index to leftFirst
while more elements in left half AND more elements in right half
    if values[leftFirst] < values[rightFirst]
        Set tempArray[index] to values[leftFirst]
        Increment leftFirst
    else
        Set tempArray[index] to values[rightFirst]
        Increment rightFirst
    Increment index
Copy any remaining elements from left half to tempArray
Copy any remaining elements from right half to tempArray
Copy the sorted elements from tempArray back into values
```

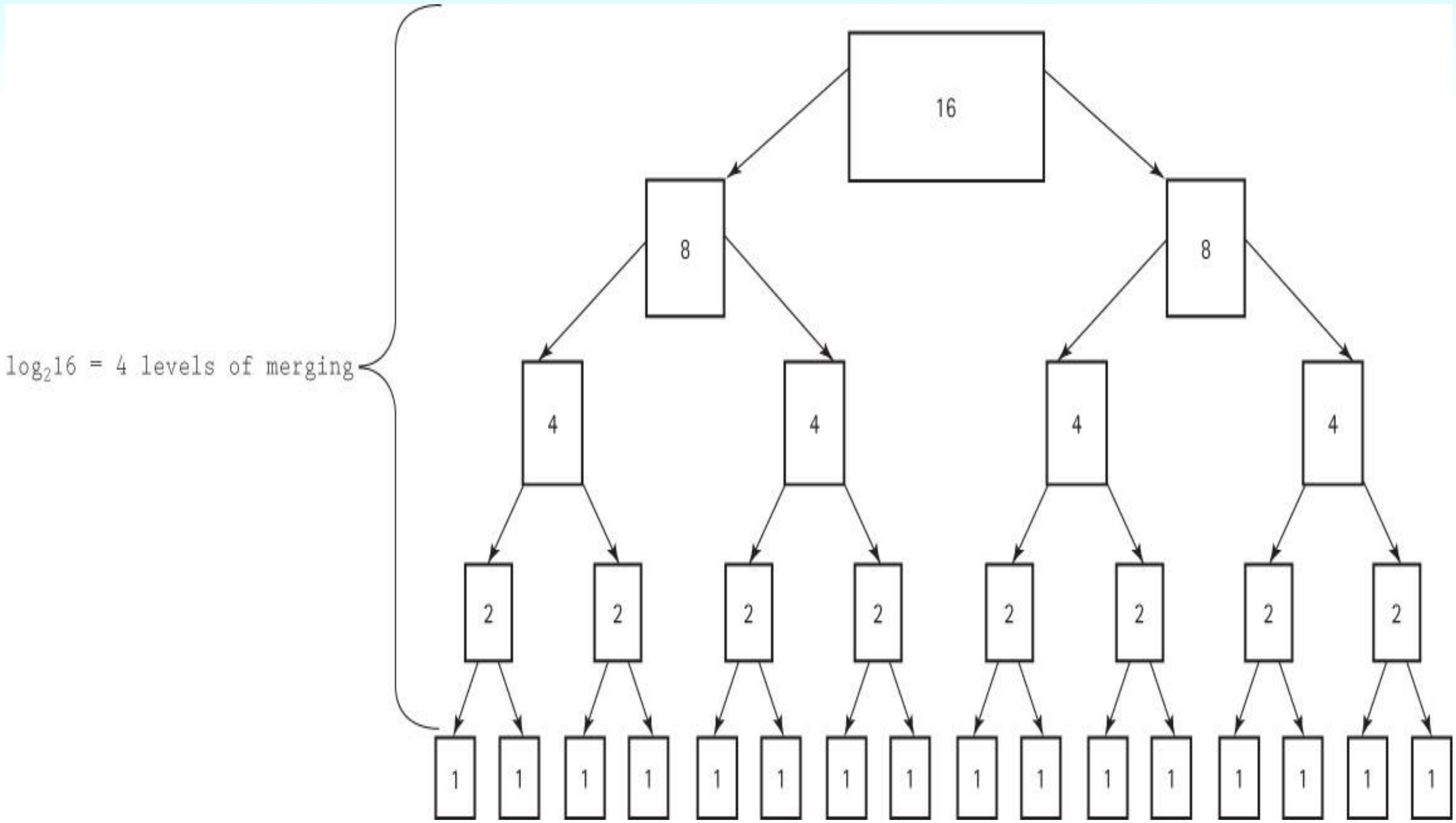
The `mergeSort` method

The code for `merge` follows the algorithm on the previous slide.
`merge` does most of the work!

Here is `mergeSort`:

```
static void mergeSort(int first, int last)
// Sorts the values array using the merge sort algorithm.
{
  if (first < last)
  {
    int middle = (first + last) / 2;
    mergeSort(first, middle);
    mergeSort(middle + 1, last);
    merge(first, middle, middle + 1, last);
  }
}
```

Analysing Merge Sort



Analyzing Merge Sort

- The total work needed to divide the array in half, over and over again until we reach subarrays of size 1, is O(N).
- It takes O(N) total steps to perform merging at each "level" of merging.
- The number of levels of merging is equal to the number of times we can split the original array in half
  - If the original array is size N, we have $\log_2 N$ levels. (This is the same as the analysis of the binary search algorithm in Section 6.6.)
- Because we have $\log_2 N$ levels, and we require O(N) steps at each level, the total cost of the merge operation is: O($N \log_2 N$).
- Because the splitting phase was only O(N), we conclude that Merge Sort algorithm is $\boxed{\text{O}(N \log_2 N)}$.

Comparing N^2 and N log_2 N

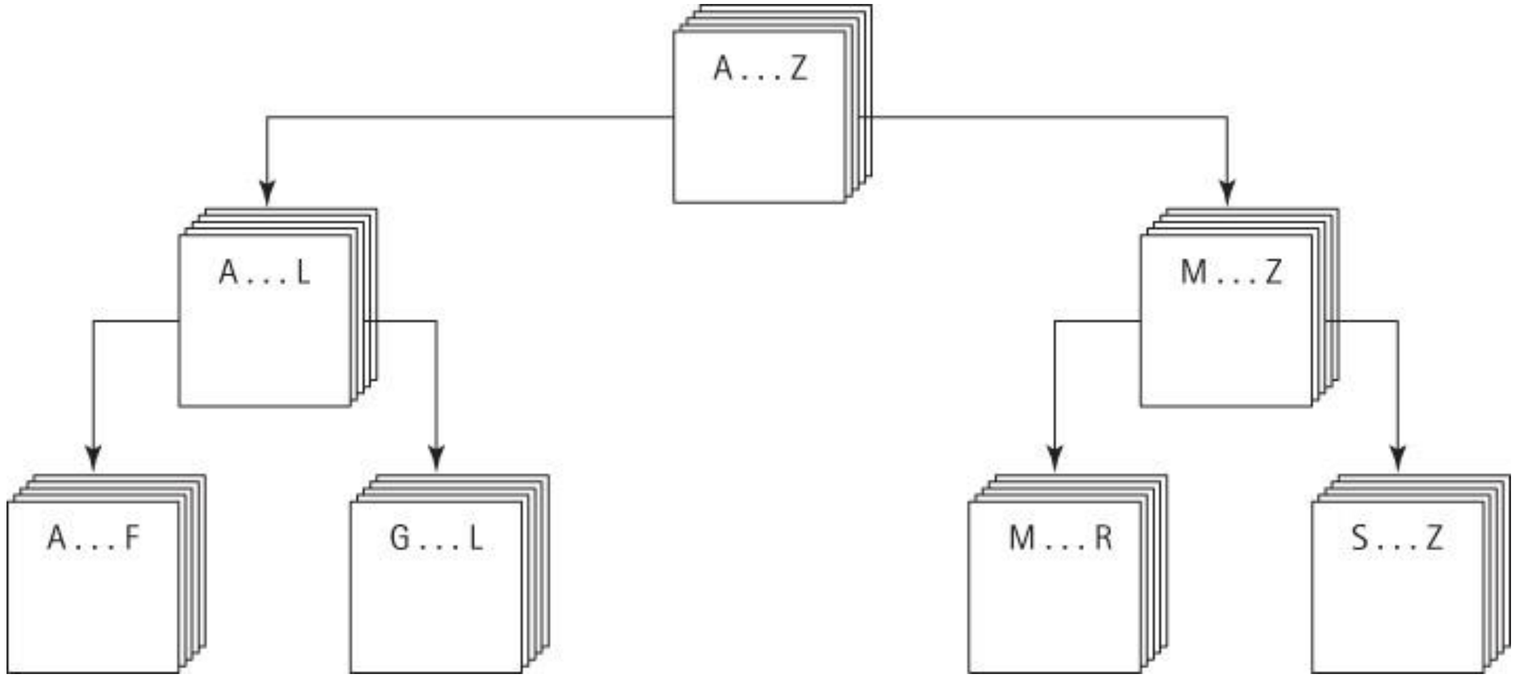
N	**$\log_2 N$**	**N^2**	**N $\log_2 N$**
32	5	1,024	160
64	6	4.096	384
128	7	16,384	896
256	8	65,536	2,048
512	9	262,144	4,608
1024	10	1,048,576	10,240
2048	11	4,194,304	22,528
4096	12	16,777,216	49,152

Drawback of Merge Sort

- A disadvantage of `mergeSort` is that it requires an auxiliary array that is as large as the original array to be sorted.
- If the array is large and space is a critical factor, this sort may not be an appropriate choice.
- Next we discuss two $O(N \log_2 N)$ sorts that move elements around in the original array and do not need an auxiliary array.

Quick Sort

- A divide-and-conquer algorithm
- Inherently recursive
- At each stage the part of the array being sorted is divided into two “piles”, with everything in the left pile less than everything in the right pile
- The same approach is used to sort each of the smaller piles (a smaller case).
- This process goes on until the small piles do not need to be further divided (the base case).



Quick Sort Summary

Method quickSort (first, last)

Definition:	Sorts the elements in sub array values[first]..values[last].
Size:	last - first + 1
Base Case:	If size less than 2, do nothing.
General Case:	Split the array according to splitting value. quickSort the elements <= splitting value. quickSort the elements > splitting value.

The Quick Sort Algorithm

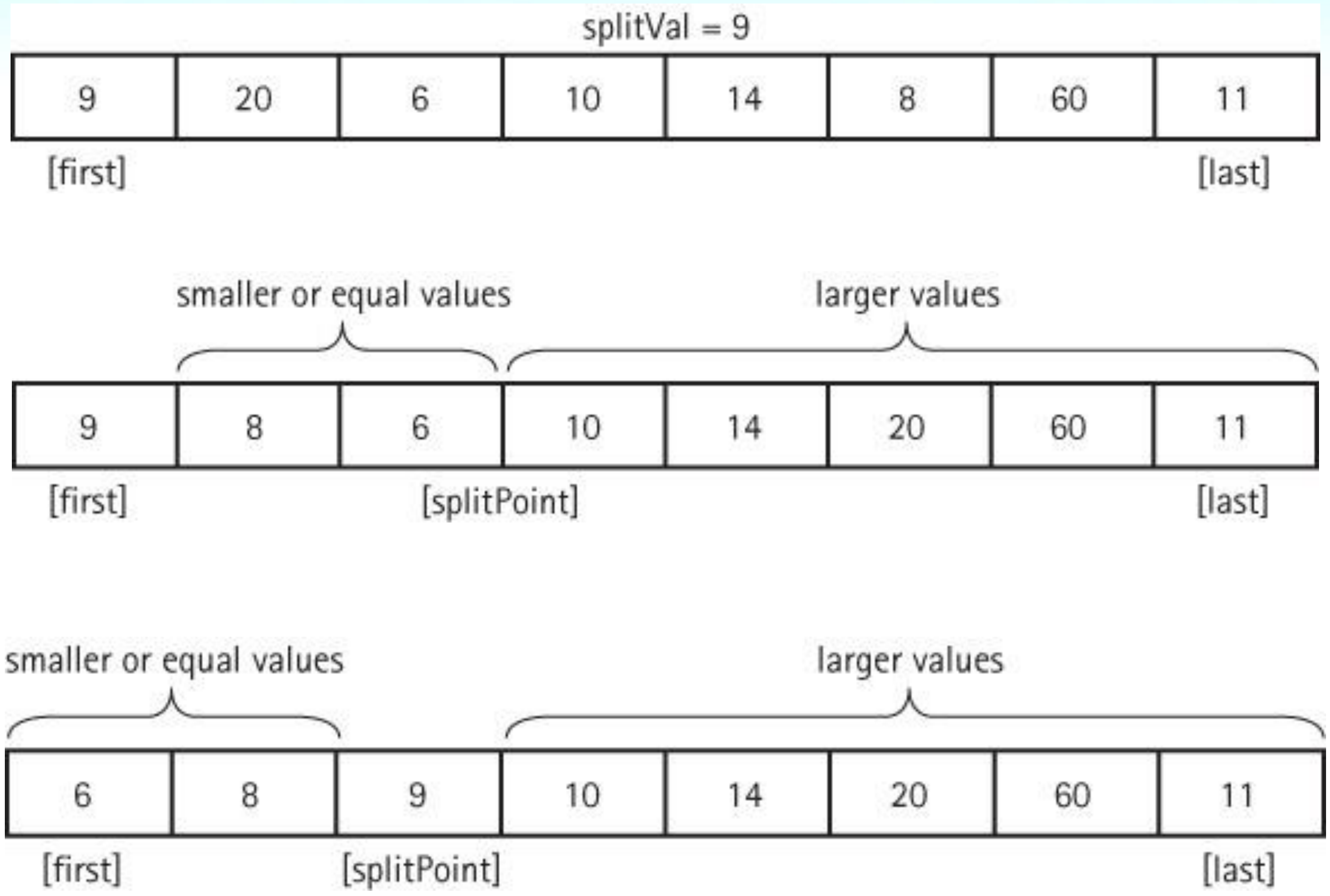
```
quickSort
if there is more than one element in values[first]..values[last]
    Select splitVal
    Split the array so that
        values[first]..values[splitPoint – 1] <= splitVal
        values[splitPoint] = splitVal
        values[splitPoint + 1]..values[last] > splitVal
    quickSort the left sub array
    quickSort the right sub array
```

The algorithm depends on the selection of a “split value”, called `splitVal`, that is used to divide the array into two sub arrays.

How do we select `splitVal`?

One simple solution is to use the value in `values[first]` as the splitting value.

Quick Sort Steps



The `quickSort` method

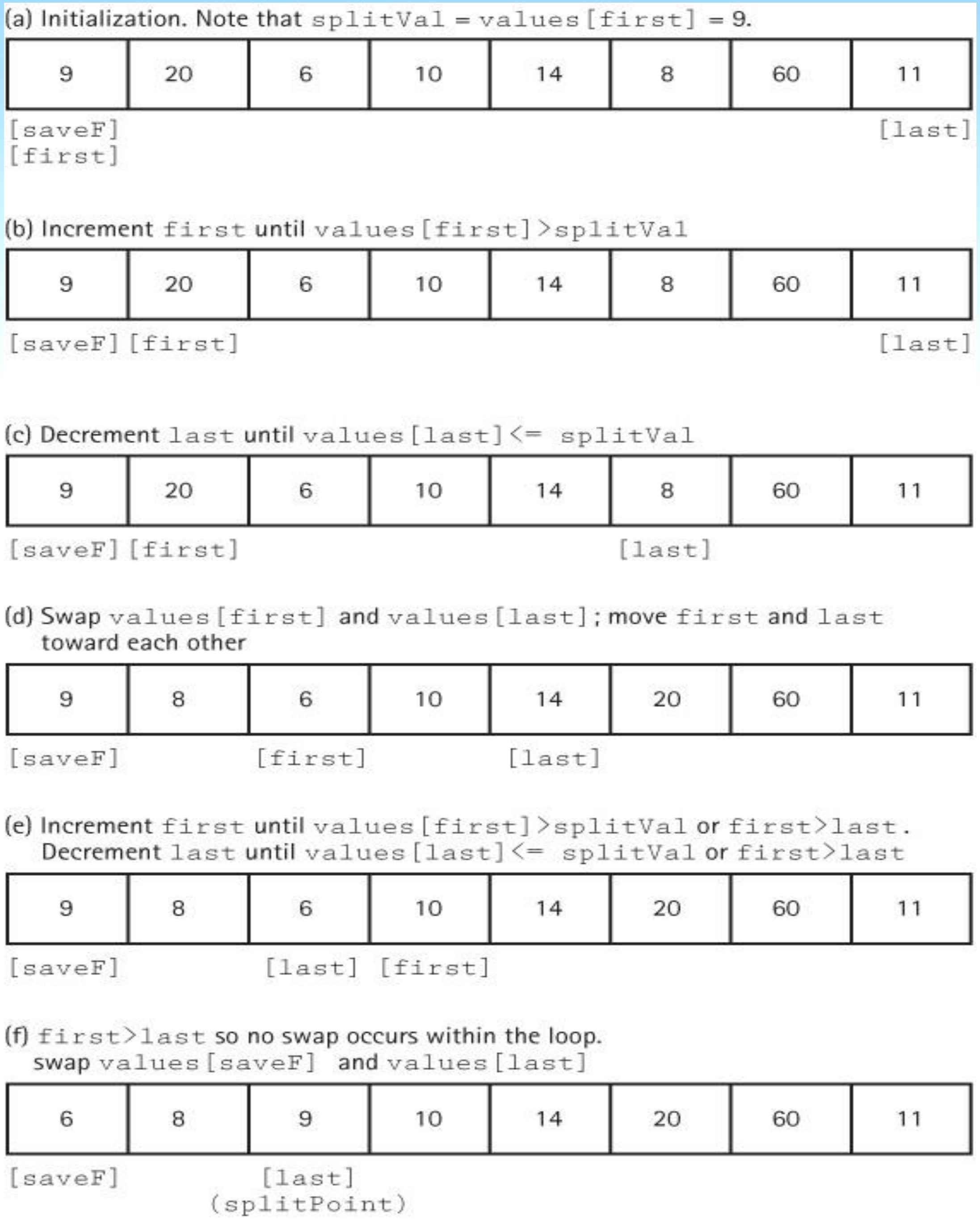
```
static void quickSort(int first, int last)
{
  if (first < last)
  {
    int splitPoint;

    splitPoint = split(first, last);
    // values[first]..values[splitPoint - 1] <= splitVal
    // values[splitPoint] = splitVal
    // values[splitPoint+1]..values[last] > splitVal

    quickSort(first, splitPoint - 1);
    quickSort(splitPoint + 1, last);
  }
}
```

The split operation

The code for split is on page 650


(a) Initialization. Note that splitVal = values[first] = 9.

(b) Increment first until values[first]>splitVal

(c) Decrement last until values[last]<= splitVal

(d) Swap values[first] and values[last]; move first and last toward each other

(e) Increment first until values[first]>splitVal or first>last. Decrement last until values[last]<= splitVal or first>last

(f) first>last so no swap occurs within the loop. swap values[saveF] and values[last]

Analyzing Quick Sort

- On the first call, every element in the array is compared to the dividing value (the “split value”), so the work done is O(*N*).
- The array is divided into two sub arrays (not necessarily halves)
- Each of these pieces is then divided in two, and so on.
- If each piece is split approximately in half, there are $O(\log_2 N)$ levels of splits. At each level, we make O(*N*) comparisons.
- So Quick Sort is an $\boxed{O(N \log_2 N)}$ algorithm.

Drawbacks of Quick Sort

- Quick Sort isn't always quicker.
  - There are $\log_2 N$ levels of splits if each split divides the segment of the array approximately in half. As we've seen, the array division of Quick Sort is sensitive to the order of the data, that is, to the choice of the splitting value.
  - If the splits are very lopsided, and the subsequent recursive calls to quickSort also result in lopsided splits, we can end up with a sort that is $O(N^2)$.
- What about space requirements?
  - There can be many levels of recursion "saved" on the system stack at any time.
  - On average, the algorithm requires $O(\log_2 N)$ extra space to hold this information and in the worst case requires $O(N)$ extra space, the same as Merge Sort.

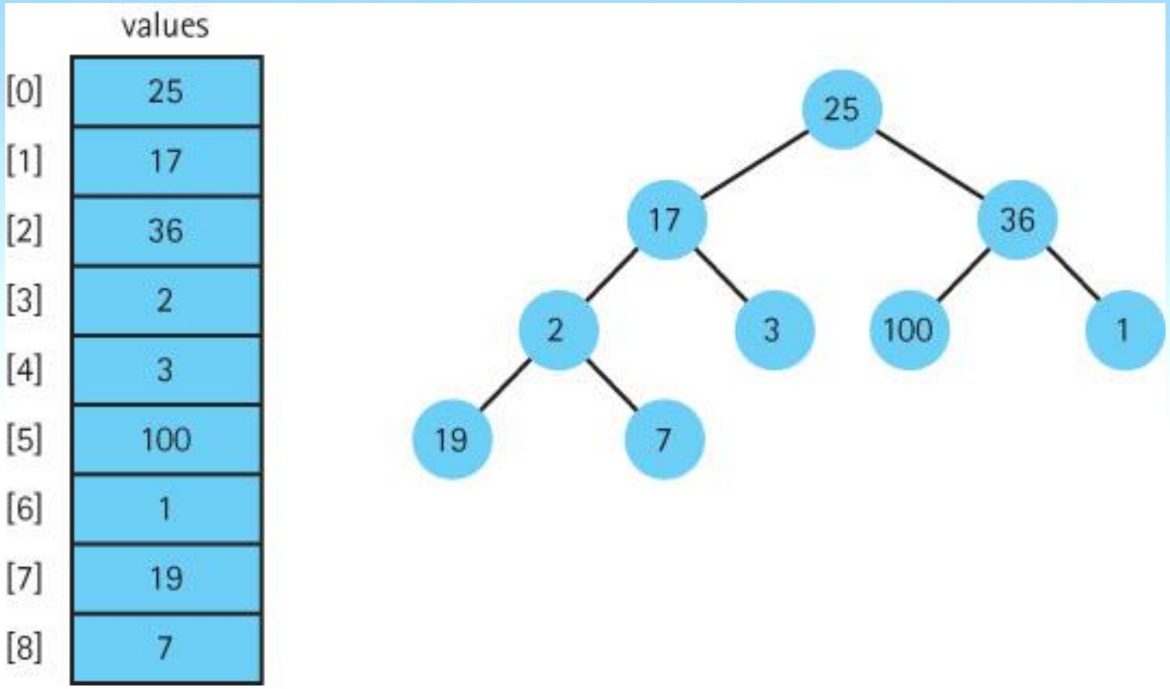
Quick Sort

- Despite the drawbacks remember that Quick Sort is VERY quick for large collections of random data

Heap Sort

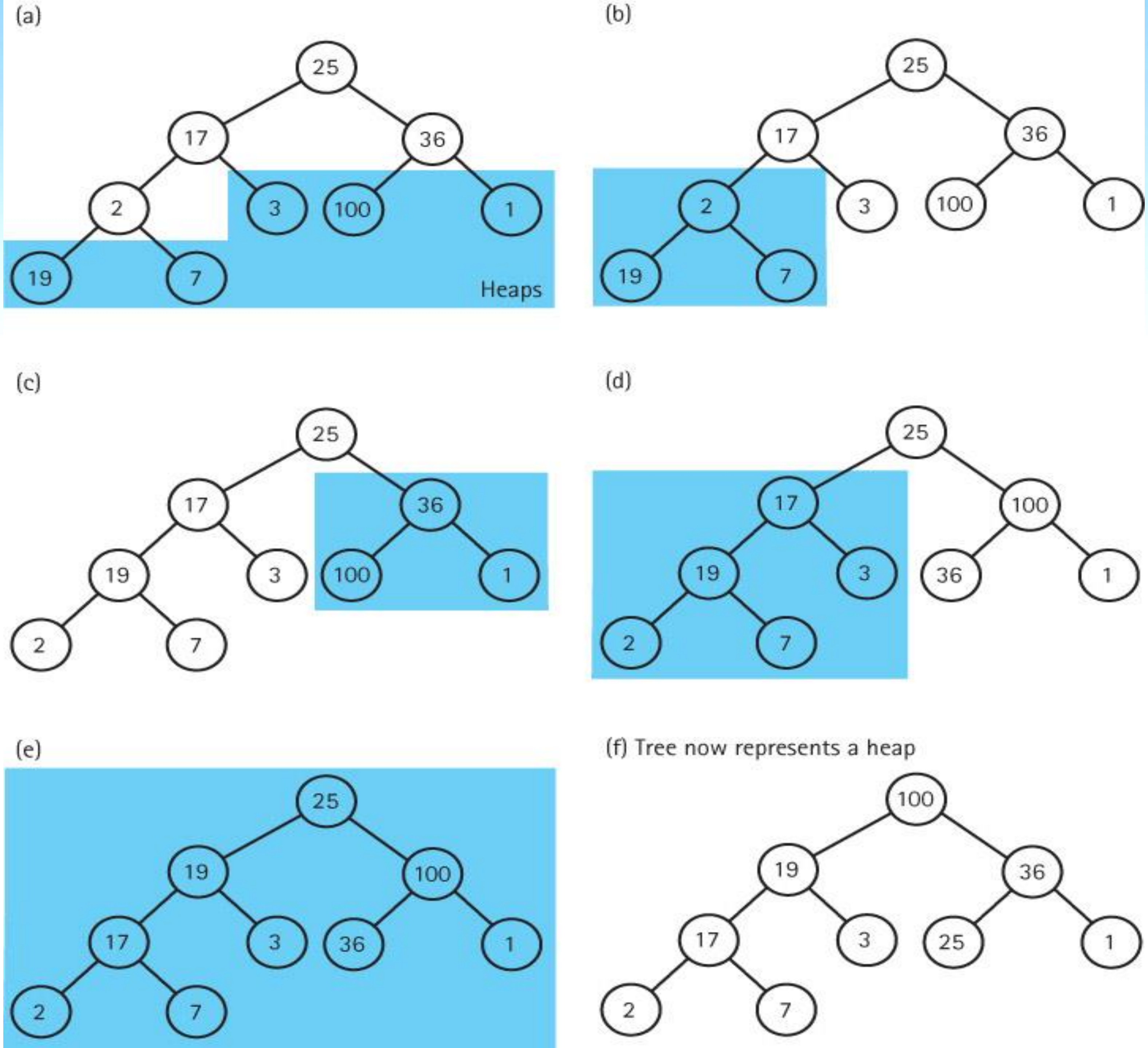
- In Chapter 9, we discussed the *heap - b*ecause of its order property, the maximum value of a heap is in the root node.
- The general approach of the Heap Sort is as follows:
  - take the root (maximum) element off the heap, and put it into its place.
  - reheap the remaining elements. (This puts the next-largest element into the root position.)
  - repeat until there are no more elements.
- For this to work we must first arrange the original array into a heap

Building a heap

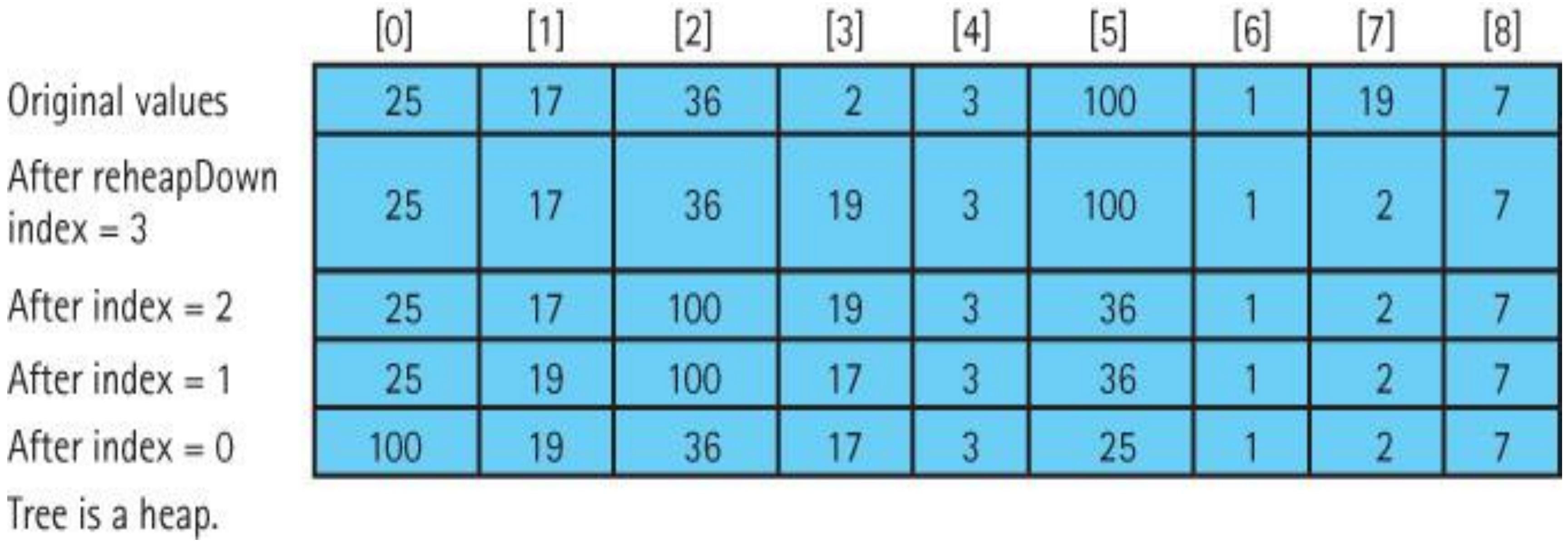


```
buildHeap
for index going from first nonleaf node up to the root node
    reheapDown(values[index], index)
```

See next slide …


(a)
Heaps
(b)
(c)
(d)
(e)
(f) Tree now represents a heap

The changing contents of the array



	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
Original values	25	17	36	2	3	100	1	19	7
After reheapDown index = 3	25	17	36	19	3	100	1	2	7
After index = 2	25	17	100	19	3	36	1	2	7
After index = 1	25	19	100	17	3	36	1	2	7
After index = 0	100	19	36	17	3	25	1	2	7

Tree is a heap.

The Sort Nodes algorithm

```
Sort Nodes
for index going from last node up to next-to-root node
    Swap data in root node with values[index]
    reheapDown(values[0], 0, index 2 1)
```

The `heapSort` method

```
static void heapSort()
// Post: The elements in the array values are sorted by key
{
  int index;
  // Convert the array of values into a heap
  for (index = SIZE/2 – 1; index >= 0; index--)
    reheapDown(values[index], index, SIZE – 1);

  // Sort the array
  for (index = SIZE – 1; index >=1; index--)
  {
    swap(0, index);
    reheapDown(values[0], 0, index – 1);
  }
}
```

Analysis of Heap Sort

- Consider the sorting loop
  - it loops through $N - 1$ times, swapping elements and reheaping
  - the comparisons occur in `reheapDown` (actually in its helper method `newHole`)
  - a complete binary tree with N nodes has $O(\log_2(N + 1))$ levels
  - in the worst cases, then, if the root element had to be bumped down to a leaf position, the `reheapDown` method would make $O(\log_2 N)$ comparisons.
  - so method `reheapDown` is $O(\log_2 N)$
  - multiplying this activity by the $N - 1$ iterations shows that the sorting loop is $O(N \log_2 N)$.
- Combining the original heap build, which is $O(N)$, and the sorting loop, we can see that Heap Sort requires $O(N \log_2 N)$ comparisons.

The Heap Sort

- For small arrays, `heapSort` is not very efficient because of all the “overhead.”
- For large arrays, however, `heapSort` is very efficient.
- Unlike Quick Sort, Heap Sort’s efficiency is not affected by the initial order of the elements.
- Heap Sort is also efficient in terms of space – it only requires constant extra space.
- Heap Sort is an elegant, fast, robust, space efficient algorithm!

Comparison of Sorting Algorithms

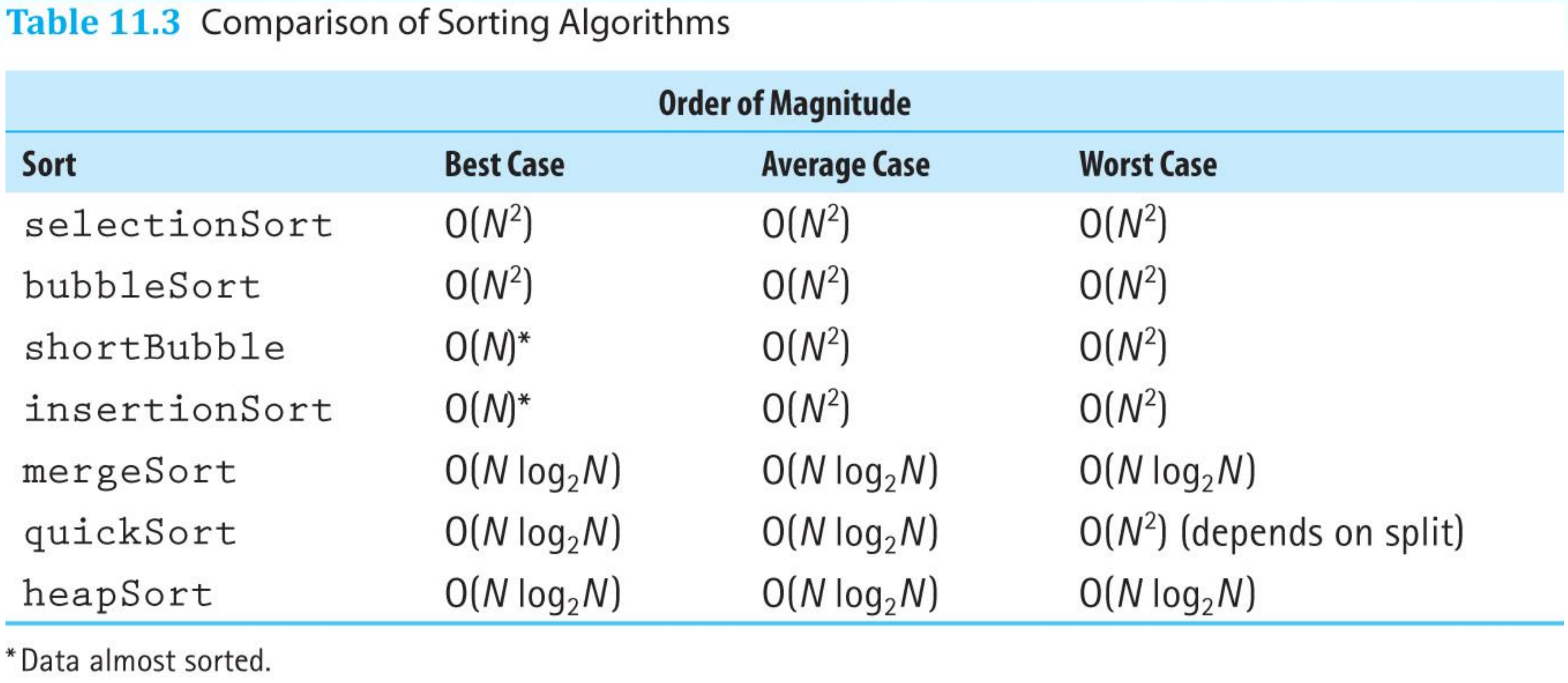


Table 11.3 Comparison of Sorting Algorithms

	Order of Magnitude		
Sort	**Best Case**	**Average Case**	**Worst Case**
`selectionSort`	$O(N^2)$	$O(N^2)$	$O(N^2)$
`bubbleSort`	$O(N^2)$	$O(N^2)$	$O(N^2)$
`shortBubble`	$O(N)$*	$O(N^2)$	$O(N^2)$
`insertionSort`	$O(N)$*	$O(N^2)$	$O(N^2)$
`mergeSort`	$O(N \log_2 N)$	$O(N \log_2 N)$	$O(N \log_2 N)$
`quickSort`	$O(N \log_2 N)$	$O(N \log_2 N)$	$O(N^2)$ (depends on split)
`heapSort`	$O(N \log_2 N)$	$O(N \log_2 N)$	$O(N \log_2 N)$

*Data almost sorted.

11.4 More Sorting Considerations

- In this section we wrap up our coverage of sorting by
  - revisiting testing
  - revisiting efficiency
  - discussing special concerns involved with sorting objects rather than primitive types
  - considering the “stability” of sorting algorithms

Testing

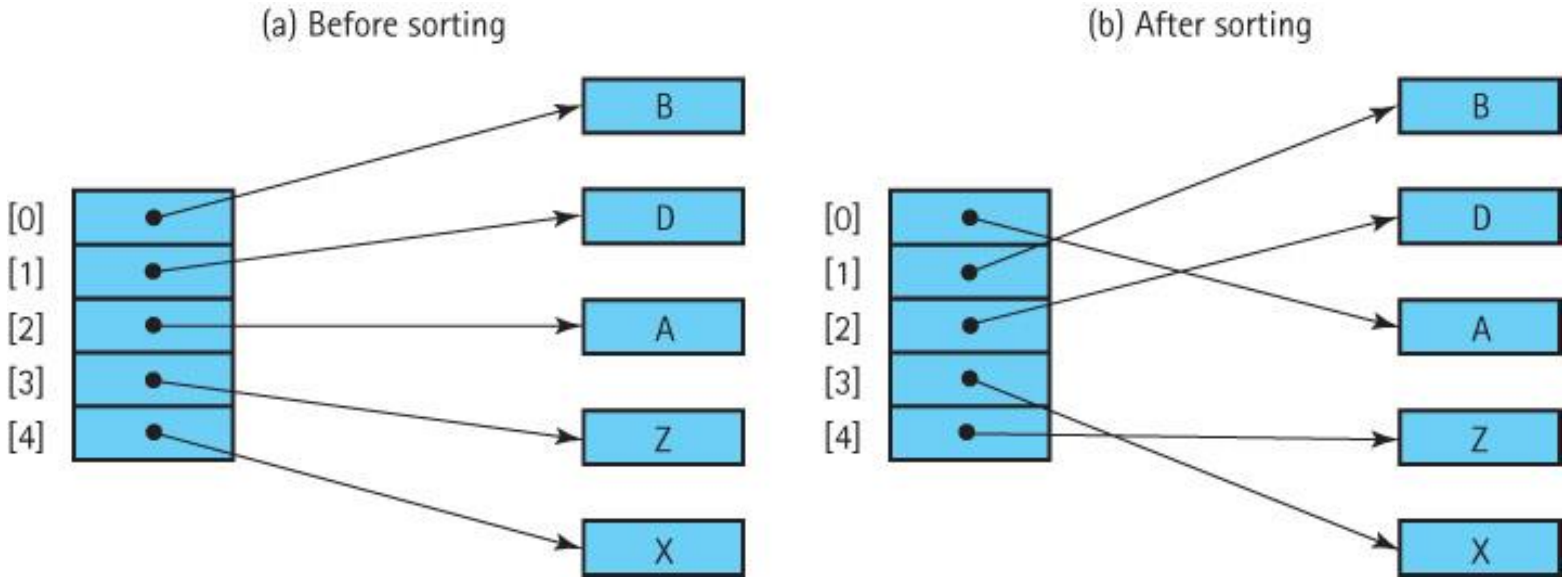
- To thoroughly test our sorting methods we should
  - vary the size of the array
  - vary the original order of the array
    - Random order
    - Reverse order
    - Almost sorted
    - All identical elements

Efficiency

- When N is small the simple sorts may be more efficient than the “fast” sorts because they require less overhead.
- Sometimes it may be desirable, for efficiency considerations, to streamline the code as much as possible, even at the expense of readability. For instance, instead of using a `swap` method directly code the swap operation within the sorting method.

Special Concerns when Sorting Objects

- When sorting an array of objects we are manipulating references to the object, and not the objects themselves



Stability of a Sorting Algorithm

- Stable Sort: A sorting algorithm that preserves the order of duplicates
- Of the sorts that we have discussed in this book, only `heapSort` and `quickSort` are inherently unstable

11.5 Searching

- This section reviews material scattered throughout the text related to searching.
- Here we bring these topics together to be considered in relationship to each other to gain an overall perspective.
- Searching is a crucially important information processing activity. Options are closely related to the way data is structured and organized.

Sequential Searching

- If we want to add elements as quickly as possible to a collection, and we are not as concerned about how long it takes to find them we would put the element
  - into the last slot in an array-based collection
  - into the first slot in a linked collection
- To search this collection for the element with a given key, we must use a simple *linear* (or *sequential*) *search*
  - Beginning with the first element in the collection, we search for the desired element by examining each subsequent element’s key until either the search is successful or the collection is exhausted.
  - Based on the number of comparisons this search is O(N)
  - In the worst case we have to make N key comparisons.
  - On the average, assuming that there is an equal probability of searching for any element in the collection, we make $N/2$ comparisons for a successful search

High-Probability Ordering

- Sometimes certain list elements are in much greater demand than others. We can then improve the search:
  - Put the most-often-desired elements at the beginning of the list
  - Using this scheme, we are more likely to make a hit in the first few tries, and rarely do we have to search the whole list.
- If the elements in the list are not static or if we cannot predict their relative demand, we can
  - move each element accessed to the front of the list
  - as an element is found, it is swapped with the element that precedes it
- Lists in which the relative positions of the elements are changed in an attempt to improve search efficiency are called *self-organizing* or *self-adjusting* lists.

Sorted collections

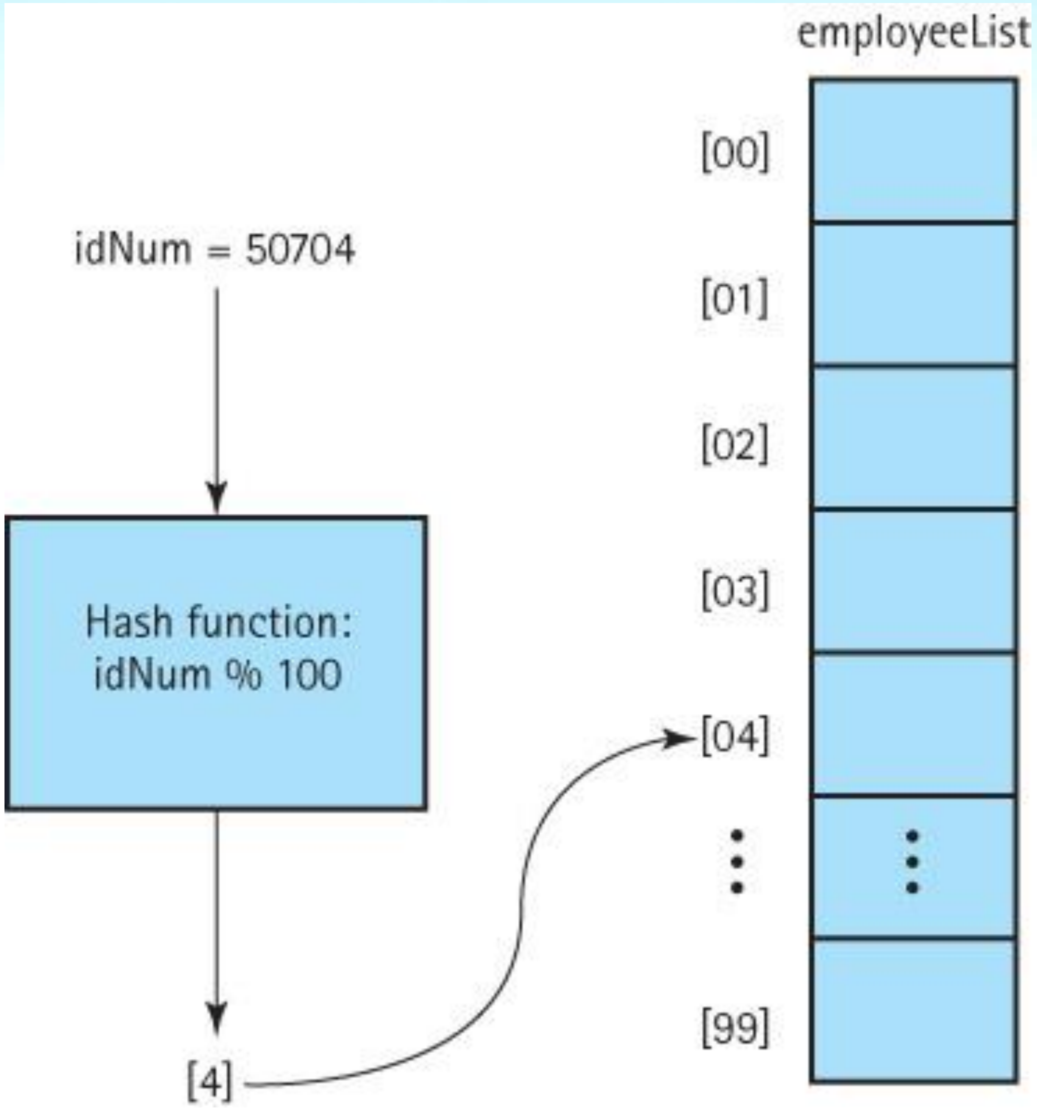
- If the collection is sorted, a sequential search no longer needs to search the whole collection to discover that an element does *not* exist. It only needs to search until it has passed the element’s logical place in the collection—that is, until an element with a larger key value is encountered.
- Another advantage of linear searching is its simplicity.
- The binary search is usually faster, however, it is not guaranteed to be faster for searching very small collections.
- As the number of elements increases, however, the disparity between the linear search and the binary search grows very quickly.
- The binary search is appropriate only for collection elements stored in a sequential array-based representation.
- However, the binary search tree allows us to perform a binary search on a linked data representation

Fast Searching

- The best searching scheme is BST so far
- Can we have faster searching than O($\log_2 N$)?
- YES… think how.

•Think about a scheme with O(1)

10.6 Hashing


employeeList
idNum = 50704
Hash function:
idNum % 100
[4]
[00]
[01]
[02]
[03]
[04]
[99]

Definitions

- **Hash Functions:** A function used to manipulate the key of an element in a list to identify its location in the list
- **Hashing:** The technique for ordering and accessing elements in a list in a relatively constant amount of time by manipulating the key to identify its location in the list
- **Hash table:** Term used to describe the data structure used to store and retrieve elements using hashing

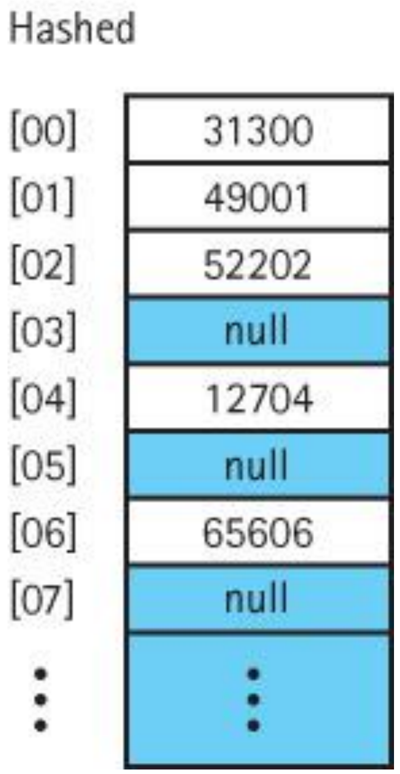
Our `Hashable` interface

```
public interface Hashable
// Objects of classes that implement this interface can be used
// with lists based on hashing.
{
  // A mathematical function used to manipulate the key of an element
  // in a list to identify its location in the list.
  int hash();
}
```

Using the `hash` method

```
public void add (Hashable element)
// Adds element to this list at position element.hash().
{
  int location;
  location = element.hash();
  list[location] = element;
  numElements++;
}


public Hashable get(Hashable element)
// Returns an element e from this list such
// that e.equals(element).
{
  int location;
  location = element.hash();
  return (Hashable)list[location];
}
```



Collisions

- **Collision:** The condition resulting when two or more keys produce the same hash location
- **Linear probing:** Resolving a hash collision by sequentially searching a hash table beginning at the location returned by the hash function

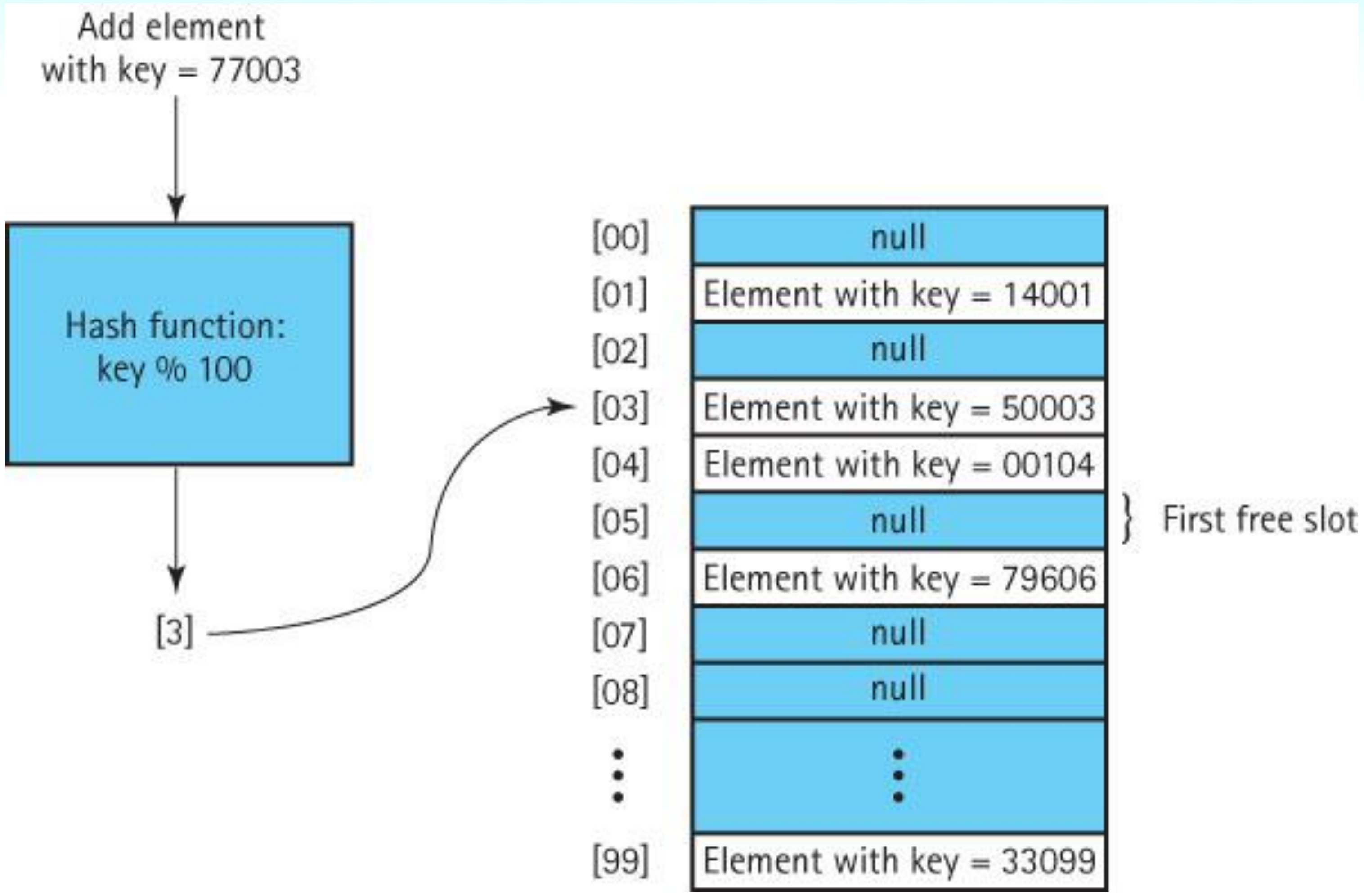
Revised methods

```
public static void add (Hashable element)
// Adds element to this list at position element.hash(),
// or the next free array slot.
{
  int location;
  location = element.hash();
  while (list[location] != null)
    location = (location + 1) % list.length;
  list[location] = element;
  numElements++;
}

public static Hashable get(Hashable element)
// Returns an element e from this list such that e.equals(element).
{
  int location;
  location = element.hash();
  while (!list[location].equals(element))
    location = (location + 1) % list.length;

  return (Hashable)list[location];
}
```

Handling collisions with linear probing


Add element
with key = 77003
Hash function:
key % 100
[3]
[00] null
[01] Element with key = 14001
[02] null
[03] Element with key = 50003
[04] Element with key = 00104
[05] null
First free slot
[06] Element with key = 79606
[07] null
[08] null
[99] Element with key = 33099

Removing an element

- An approach could be

```
***remove (element)***
Set location to element.hash( )
Set list[location] to null
```

- Collisions, however, complicate the matter. We cannot be sure that our element is in location element.hash().
- We must examine every array element, starting with location element.hash(), until we find the matching element.
- We cannot stop looking when we reach an empty location, because that location may represent an element that was previously removed.
- This problem illustrates that hash tables, in the forms that we have studied thus far, are not the most effective data structure for implementing lists whose elements may be deleted.

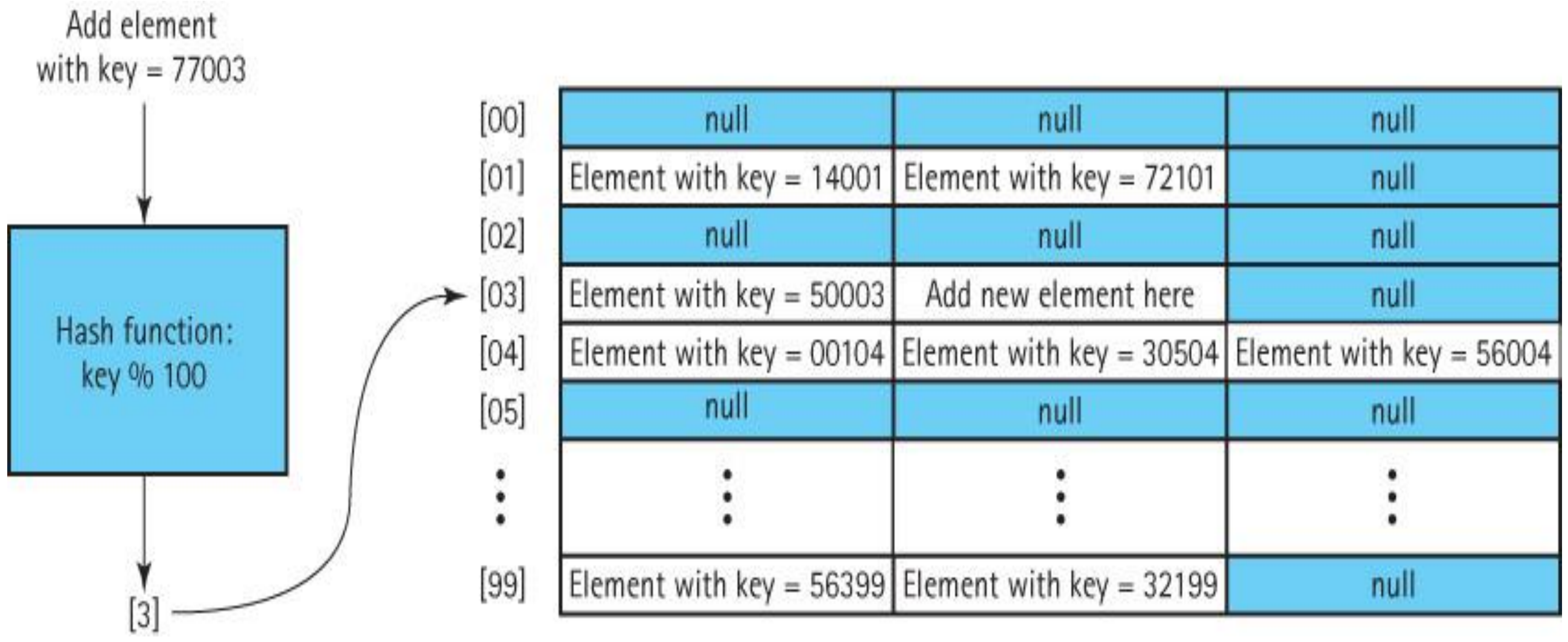
More Considerations

- **Clustering:** The tendency of elements to become unevenly distributed in the hash table, with many elements clustering around a single hash location
- **Rehashing:** Resolving a collision by computing a new hash location from a hash function that manipulates the original location rather than the element’s key
- **Quadratic probing** Resolving a hash collision by using the rehashing formula (HashValue +/- I^2) % array-size
- **Random probing** Resolving a hash collision by generating pseudo-random hash values in successive applications of the rehash function

Buckets and Chaining

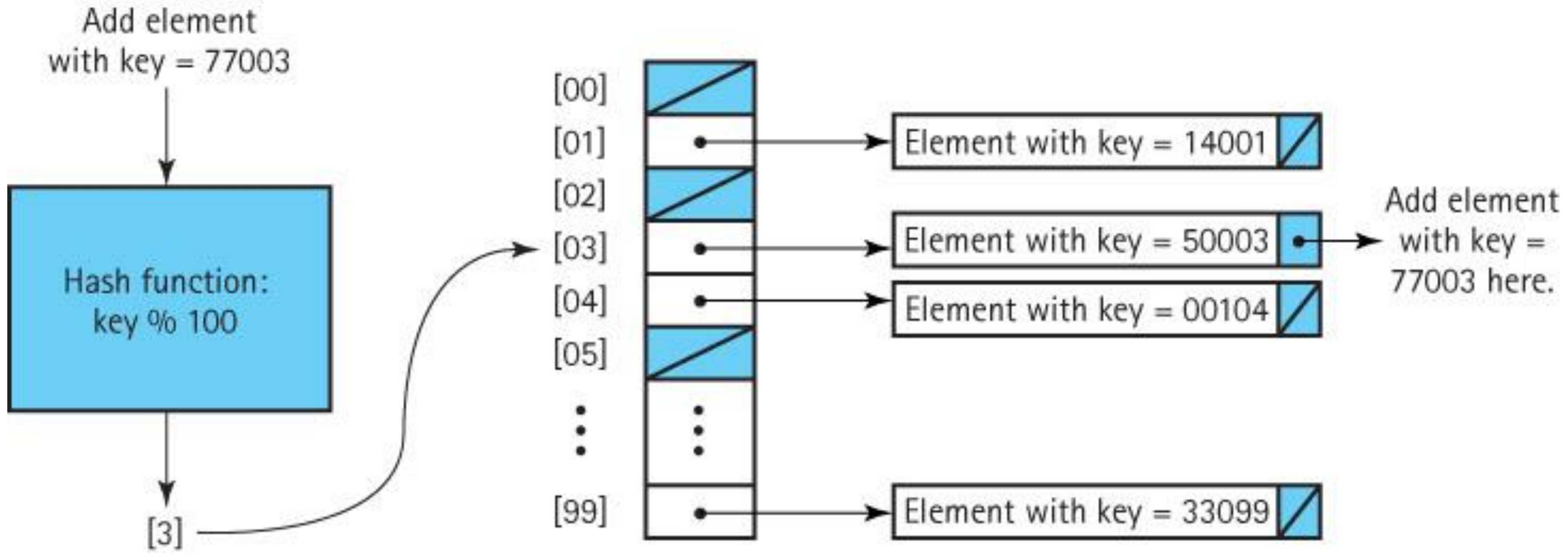
- **Bucket** A collection of elements associated with a particular hash location
- **Chain** A linked list of elements that share the same hash location

Handling collisions by hashing with buckets



[00]	null	null	null
[01]	Element with key = 14001	Element with key = 72101	null
[02]	null	null	null
[03]	Element with key = 50003	Add new element here	null
[04]	Element with key = 00104	Element with key = 30504	Element with key = 56004
[05]	null	null	null
⋮	⋮	⋮	⋮
[99]	Element with key = 56399	Element with key = 32199	null

Handling collisions by hashing with chaining



Java’s Support for Hashing

- The Java Library includes a `HashTable` class that uses hash techniques to support storing objects in a table.
- The library includes several other collection classes, such as `HashSet`, that provide an ADT whose underlying implementation uses the approaches described in this section.
- The Java Object class exports a `hashCode` method that returns an `int` hash code. Therefore all Java objects have an associated hash code.
  - The standard Java hash code for an object is a function of the object’s memory location.
- For most applications, hash codes based on memory locations are not usable. Therefore, many of the Java classes that define commonly used objects (such as `String` and `Integer`), override the Object class’s `hashCode` method with one that *is* based on the contents of the object.
- If you plan to use hash tables in your programs, you should do likewise.