ME EN 541 – HW 3

Problem	Points	Topic
1	15	Richardson extrapolation
2	20	Models with source terms
3	20	Models with source terms

1. Explore Richardson extrapolation on HW 2 Problem 5 as follows.
   a. Calculate the base heat transfer simulated using 5, 10, and 20 control volumes. Estimate the order of the simulation using these values.
   b. Predict the grid-independent base heat transfer value using the estimated simulation order from part (a) and the base heat transfer values from the 10- and 20-control volume cases.
   c. Plot the base heat transfer values for 5, 10, 20, 40, …, 5120, 10240 control volumes. Show the exact solution on this plot. Also generate a table that lists, for these control volumes, the number of control volumes, the simulated base heat transfer, and the percent difference between the simulated solutions and the grid-independent solution (the latter from part b).

Problem Setup for Problems 2 and 3

Consider cooling of a circular fin by means of convective and radiative heat transfer along its length. For constant diameter, the steady state conservation equation is

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)-\frac{4}{D}\left[h\left(T-T_{\infty}\right)+\varepsilon\sigma\left(T^{4}-T_{surr}^{4}\right)\right]=0$$

where k is the thermal conductivity, D the diameter, T_{∞} the ambient temperature, T_{surr} the temperature of the surroundings, ε the fin surface emissivity, and σ the Stefan-Boltzmann constant (5.67×10^{-8} W/m^{2}K^4).

Formulate this problem for solution via the control-volume method, and apply the two cases specified below. For both cases, the temperature at the base is fixed, the tip of the fin can be considered to be adiabatic, and the following parameters apply:

L = 2 cm, D = 3 mm
k = 401 W/mk (copper), h = 10 W/m^{2}K
T_B = 400 K, T_{∞} = 273 K, T_{surr} = 273 K

Note the following in developing your model:

- An iterative method is required since the source term is nonlinear. Use the TDMA method to solve the linear system at each iteration.
- Iterate until the maximum temperature change at any node between iterations is < 0.0001.
- Appropriately linearize the source term.

2. Validate your model by predicting the temperature distribution $T(x)$ and the fin heat transfer q_f for the convection-only case ($\varepsilon = 0$). For this case there is an analytical solution:

$$\frac{T - T_\infty}{T_B - T_\infty} = \frac{\cosh(n(L - x))}{\cosh(nL)}$$

where $n^2 = 4h/Dk$. Plot the maximum error vs. number of control volumes.

3. Predict and plot the steady-state temperature distribution for the combined convection and radiation case with $h = 10$ W/m^{2}K and $\varepsilon = 1$. Use an appropriate number of CVs as determined from the previous problem. Comparing this problem with the previous problem, comment on the relative importance of convective vs. radiative heat transfer on the tip temperature.