

# Applied Ordinary and Partial Differential Equations

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### Tutorial Set 1

1. Determine the basic properties of the differential equations and solve the following
  - (a)  $xy' + y = x, x > 0, y(1) = 0.$
  - (b)  $y'' - y' - 6y = 4, y(0) = 2, y'(0) = 0$
  - (c)  $y' = y^2(1 + t^2), y(0) = 1.$
  - (d)  $y'' + 3ty' + y = 0$
2. Find the general solution of
  - (a)  $3\partial_x u - 2\partial_y u + u = x$
  - (b)  $y\partial_x u - x\partial_y u = 0$
3. Write the conservation law form,  $\partial_t u + \phi_x = 0$ , by finding the flux function  $\phi(u)$  of
  - (a)  $\partial_t u + c\partial_x u = 0$
  - (b)  $\partial_t u + u^2\partial_x u + \partial_{xxx} u = 0.$
4. Solve the following
  - (a)  $\partial_t u + 4\partial_x u = 0, u(x, 0) = \frac{1}{1 + x^2}.$
  - (b)  $y\partial_x u + x\partial_y u = 0, u(0, y) = e^{-y^2}$
5. Let us consider the following two equations (i)  $\partial_t u + u\partial_x u = 0, |x| < \infty, t > 0$  (ii)  $\partial_t u + u^2\partial_x u = 0, |x| < \infty, t > 0$  with initial condition  $u(x, 0) = \frac{1}{1 + x^2}.$ 
  - (a) Find the plot of characteristics.
  - (b) Analytically, determine the breaking time.
  - (c) Plot the solution  $u(x, t)$  at times before and after breaking time.
6. Let us consider the initial value problems
  - (a)  $\partial_t u + u\partial_x u = 0, |x| < \infty, t > 0$  with initial condition

$$u(x, 0) = \begin{cases} 1, & x \leq 0 \\ 2, & x > 0 \end{cases} \quad (1)$$

(b)  $\partial_t u + u \partial_x u = 0, |x| < \infty, t > 0$  with initial condition

$$u(x, 0) = \begin{cases} 2, & x \leq 0 \\ 1, & x > 0 \end{cases}$$

(c)  $\partial_t u + u^2 \partial_x u = 0, |x| < \infty, t > 0$  with initial condition

$$u(x, 0) = \begin{cases} 1, & x \leq 0 \\ 1 - x, & 0 \leq x \leq 1 \\ 0, & x \geq 1. \end{cases}$$

Answer the following

- i. Find the plot of characteristics.
- ii. Find the plot of the fan characteristics.
- iii. Write out the shock wave or rarefaction wave solution if they exist.