# Methods for elimination of crosstalk and inertial effects in bicycle and motorcycle steer torque estimation

Jason K. Moore and Mont Hubbard

September 30, 2013

## Introduction

To control a bicycle or motorcycle, the rider's primary means of directing the vehicle is to apply forces to the handlebars which cause the front frame to rotate relative to the rear frame. The rider can, of course, also use other more subtle biomechanical motions to influence motion of vehicles, especially lighter-weight ones, but it is known that forces applied to the handlebar give much more control authority than other means regardless of the vehicle's inertia [16, 17].

From a modeling perspective, it is generally easier to model the interface forces as a single torque about the steer axis which acts between the front and rear frames. The rider can be assumed to be part of the rear frame, regardless of whether rider is assumed to be rigid or not.

Due to this modeling assumption and ease of measurement, most experimental measurements of the interface forces between the rider and the front frame are obtained by measuring the torque generated in the steer axis of the front frame. These direct measurements of rider applied steer torque are susceptible to error from two major causes: (1) inertia effects of the front frame which are located between the sensor and the rider's hands and (2) cross talk from rider applied forces other than those which generate steer torque. Accounting for the error sources are particularly important when the steer torques are small ($< 20$ Nm or so).

In this paper we review previous methods of steer torque measurement in bicycles and motorcycles and detail the design and implementation of a bicycle steer torque measurement system which minimizes the aforementioned cross talk errors. We then show the computations needed to correctly compensate for inertial effects of the front frame and bearing friction to obtain a more accurate estimate of the rider applied steer torque. Finally, to show the need for this approach, we compare the differences between uncompensated and compensated steer torque measurements for a large set of bicycle experiments.

## History of Steer Torque Measurements

The earliest steer torque measurements were performed in 1951. Wilson-Jones [22] developed a set of motorcycle handlebars mounted in rubber bushings that indicated the direction and value of torque in an analog fashion in real time. He demonstrated that a negative torque is applied with respect the steering angle to enter into a turn and measured torques in normal maneuvers in the 4 to 14 Nm range. Not long after this Kondo [13] was the first to record torque measurements on a motorcycle for post-experiment analysis. Work in Japan on motorcycle dynamics grew considerably after World War II due to sanctions on aircraft research. Kageyama [11] and Fu [8] continued to improve steer torque measurements in motorcycles and further studied steady turning. Eaton [6] was the first to measure and record motorcycle steer torque in the United States. He attached a third handle bar above the regular handlebars with strain gages that produced voltage proportional to the applied torque around the steer axis while the rider operated the motorcycle with one hand. He measured steer torques up to 3.4 Nm for straight riding for speeds of 15 to 30 mph. This led him to conclude that most of the steer torque was due to rider remnant, as opposed to deliberate control. Not long after this, Weir [21] developed a modular torque sensor which could be affixed to multiple motorcycles with a $\pm$ 70 Nm range and a 1% accuracy with a 10 Hz bandwidth and was careful to reduce crosstalk from other forces applied to the handlebars. They unfortunately oversized the sensor and the signal to noise ratio was low for steady turn and straight riding maneuvers, but they measured torques of -20 to 55 Nm in lane changes. Sugizaki [18] also measured steer torque in high speed motorcycle lane change maneuvers and recorded torques between -20 and 20 Nm.

After years of motorcycle steer torque measurements, the first bicycle measurements were made by de Lorenzo [5] on a downhill mountain bicycle which was fitted with a custom strain gauged handlebar that could effectively measure torque about the longitudinal axis and the vertical axis. His plot of torque measurements show maximum steer torques of 7 Nm and maximum longitudinal handlebar torques of 15 Nm which demonstrated that non-steer related forces on the handlebars can be significantly higher that those needed for steering.

Around the turn of the 21st century, Bortoluzzi [1] designed a successful motorcycle steer torque transducer in which floating handlebars engage the fork through a small strain-gaged cantilever beam. This design was less susceptible to crosstalk than earlier designs. They found torques up to 20 Nm for slalom maneuvers at 40 m/s. Around the same time, James [10] developed a secondary handlebar with integrated load cell to measure steer torques in an off-road motorcycle.

In 2003, Cheng [4] completed a comprehensive study on bicycle steer torque for an undergraduate project. Cheng started by simply attaching a torque wrench to a bicycle and made left turns at speeds from 0 to 13 m/s and found that most steering torques were under 5 Nm. He then designed a floating handlebar which engaged the steer tube via a linear load cell, configured to measure torques of 0 to 84 Nm. Cheng found torques up to 1 Nm for steady turning at 4.5 m/s and up to 10 Nm for sharp turns, confirming that bicycles require much lower torques for maneuvering.

Iuchi [9] constructed a bicycle with a steer motor that "senses" the rider's input for use in additive control. The rider applied steer torque was estimated from the motor torque and the handlebar and motor moments of inertia. Capitani [3] found measured steer torques from an instrumented scooter between -15 and 40 Nm. Evertse [7] was perhaps the only person to estimate steer torque from sensors in the handle grips of a motorcycle that give force measurements directly at the human-vehicle interface. During the test maneuvers, a maximum of 40 Nm was observed. In 2010, Teerhuis [19] shows measured torques just under 20 Nm for a motorcycle in slalom maneuvers.

Recently, Cain [2] developed an in-the-steer-tube torque sensor for a bicycle. The measured steer torques in steady turns never exceed 2.4 Nm but he admits that his sensor was 90% oversized. And most recently, van den Ouden [20] developed a steer torque sensor sensor that was susceptible to cross talk from other handlebar loads but had an appropriate measurement range of $\pm$7.5 Nm.

Steering torque has been measured in relatively few instances of bicycle experiments and not many more for motorcycles. Of these, very few of the designs may actually measure the true rider applied steer torque. This is more consequential for bicycles than motorcycles because the small torques used in typical bicycle control are of the order of 5-10 Nm. van den Ouden [20], in particular, showed how sensitive the torque measurements are to other handlebar loads. Also, most of these designs measure the torque somewhere between the rider hands and the ground contact point. This is a physically ideal way to measure the steer torque, but apparently no one has accounted for the dynamic inertial effects of the front frame above or below the sensor, except Iuchi [9]. Evertse [7] may have the only design which mitigates this inertial compensation issue completely.

With this information in hand we designed a steer torque measurement system for a bicycle that accounts for the deficiencies in previous designs.

# Isolated Steer Torque Measurement Design

Our design is based on a Futek 150 in-lb ($\pm$17 Nm) TFF350 torque sensor to ensure high accuracy for the low torques used in normal bicycle maneuvering. To guarantee that torques are measured only about the steer axis we isolated the steer torque sensor from any of the non-axial torques and all forces transmitted through the handlebar or ground contact with a zero backlash telescoping double universal joint, Figure 1.

# Steer Dynamics

The final design measured the torque in the steer tube along the steer axis, but this measured torque $T_M$ is not the same as the effective input torque applied by the rider. The rider applied steer torque $T_\delta$ can be shown to be a function of the kinematics of the front and rear frame and the friction torques generated by the bearings.

A free body diagram can be drawn of the portion of the front frame assembly above the torque sensor, Figure 2. The torques acting on the handlebar about the steer axis are the measured torque $T_M$ the rider applied steer torque $T_\delta$ and the friction from the upper bearing set $T_U$ which we describe by the sum of Coulomb $T_{U_F}$ and viscous friction $T_{U_V}$.

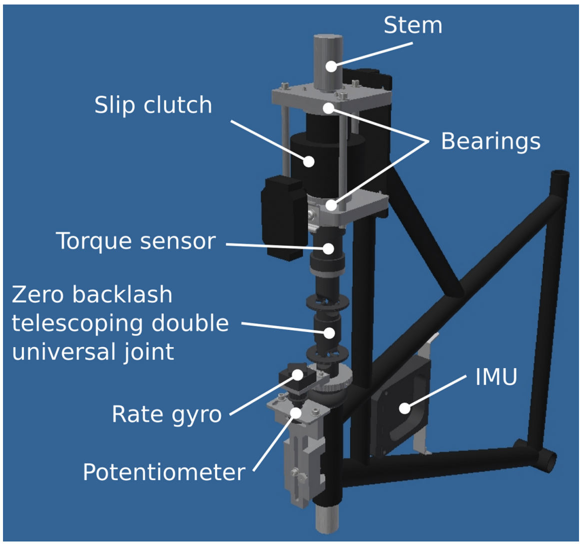


Figure 1: The steer torque sensor isolation design. The handlebars attach to the stem which is mounted in the upper bearings. The fork and steer tube are mounted in the normal headset of the bicycle. Between these two sets of bearings the stem is connected to the steer tube via the torque sensor and a zero backlash telescoping double universal joint. The steer angle and rate are measured at the steer tube and rate and acceleration of the rear frame are collected with the IMU.

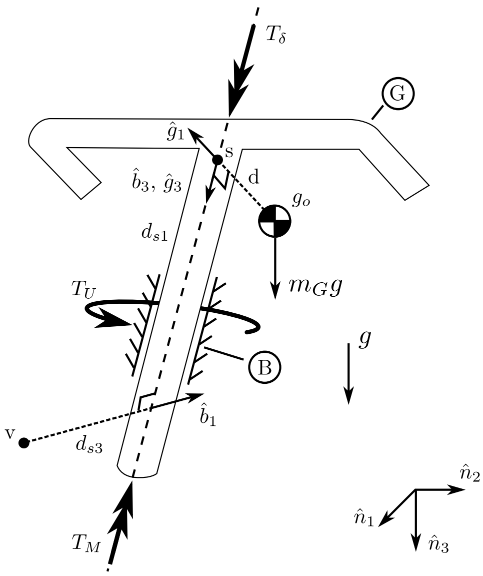


Figure 2: A free body diagram of the handlebar, $G$, with all axial torques shown. The rear frame $B$ has arbitrary orientation with respect to the Newtonian reference frame $N$. The handlebar rotates about $\hat{b}_3$ with respect to the rear frame. Gravity $g$ is in the $\hat{n}_3$ direction. The rider applies forces to the handlebars, resulting in a component of torque $T_\delta$ about the steer axis. This torque is resisted by the upper bearing friction $T_U$, the measured torque at the sensor $T_M$, the inertia of the handlebar, and the gravitational force acting on the handlebar center of mass $g_o$. The three distances, $d$, $d_s1$, and $d_s3$ locate the handlebar center of mass with respect to the center of the IMU.

We measure three components of body fixed angular rate of the rear frame $B$ in the Newtonian reference frame $N$ with three rate gyros. This is written as

$$^N\bar{\omega}^B = w_{b1}\hat{b}_1 + w_{b2}\hat{b}_2 + w_{b3}\hat{b}_3 \tag{1}$$

The handlebar $G$ is connected to the bicycle frame $B$ by a revolute joint that rotates through the steering angle $\delta$ and we measure a component of the body fixed inertial angular rate of the handlebar $w_{h3}$ about the steer axis with a rate gyro. The angular velocity of the handlebar can be written as

$$^N\bar{\omega}^G = (w_{b1}c_\delta + w_{b2}s_\delta)\hat{g}_1 + (-w_{b1}s_\delta + w_{b2}c_\delta)\hat{g}_2 + w_{h3}\hat{g}_3 \tag{2}$$

where $c_\delta$ and $s_\delta$ are shorthand for $\cos(\delta)$ and $\sin(\delta)$ respectively.

The steer rate, $\dot{\delta}$, can be computed by subtracting the angular rate of the bicycle frame about the steer axis from the angular rate of the handlebar about the steer axis.

$$\dot{\delta} = w_{h3} - w_{b3} \tag{3}$$

Now we define a point $s$ on the steer axis a minimum distance $d$ from the center of mass of the handlebar $g_o$.

$$\bar{r}^{g_o/s} = d\hat{g}_1 \tag{4}$$

We also measure the body fixed acceleration of a point $v$ on the bicycle frame which includes the acceleration due to gravity.

$$^N\bar{a}^v = a_{v1}\hat{b}_1 + a_{v2}\hat{b}_2 + a_{v3}\hat{b}_3 \tag{5}$$

The location of point $v$ is known with respect to $s$

$$\bar{r}^{s/v} = d_{s1}\hat{b}_1 + d_{s3}\hat{b}_3 \tag{6}$$

The acceleration $^N\bar{a}^{g_o}$ can now be calculated using the two point theorem for acceleration [12] twice starting at the point $v$

$$^N\bar{a}^s = {}^N\bar{a}^v + {}^N\dot{\bar{\omega}}^B \times \bar{r}^{s/v} + {}^N\bar{\omega}^B \times ({}^N\bar{\omega}^B \times \bar{r}^{s/v}) \tag{7}$$

$$^N\bar{a}^{g_o} = {}^N\bar{a}^s + {}^N\dot{\bar{\omega}}^G \times \bar{r}^{g_o/s} + {}^N\bar{\omega}^G \times ({}^N\bar{\omega}^G \times \bar{r}^{g_o/s}) \tag{8}$$

The angular momentum of the handlebar about its center of mass is

$$^N\bar{H}^{G/g_o} = I^{G/g_o} \cdot {}^N\bar{\omega}^G \tag{9}$$

where $I^{G/g_o}$ is the inertia dyadic with reference to the center of mass which exhibits symmetry about the 1-3 plane.

Now the dynamic equations of motion of the handlebar can be written: the sum of the torques on the handlebar about point $s$ equals the derivative of the angular momentum of $G$ in $N$ about $g_o$ plus the cross product of the vector from $s$ to $g_o$ with the mass times the acceleration of $g_o$ in $N$ [14]. We neglect the gravitational torque because it is already accounted for in the measured linear acceleration, Equation 5.

$$\sum \bar{T}^{G/s} = {}^N\dot{\bar{H}}^{G/g_o} + \bar{r}^{g_o/s} \times m_G \, {}^N\bar{a}^{g_o} \tag{10}$$

We are only interested in the components of the previous equation in which the steer torque appears, so only the torques about the steer axis are examined.

$$\sum T_3^{G/s} = T_\delta - T_U - T_M = \left({}^N\dot{\bar{H}}^{G/g_o} + \bar{r}^{g_o/s} \times m_G \, {}^N\bar{a}^{g_o}\right) \cdot \hat{g}_3 \tag{11}$$

Finally, $T_\delta$ can be written as

$$
\begin{aligned}
T_\delta = & I_{G_{22}}\left[\left(-w_{b1}s_\delta + w_{b2}c_\delta\right)c_\delta + w_{b2}s_\delta\right] + I_{G_{33}}\dot{w}_{g3} + \\
& I_{G_{31}}\left[(-w_{g3} + w_{b3})w_{b1}s_\delta + (-w_{b3} + w_{g3})w_{b2}c_\delta + s_\delta\dot{w}_{b2} + c_\delta\dot{w}_{b1}\right] + \\
& \left[I_{G_{11}}(w_{b1}c_\delta + w_{b2}s_\delta) + I_{G_{31}}w_{g3}\right]\left[-w_{b1}s_\delta + w_{b2}c_\delta\right] + \\
& dm_G\left[d(-w_{b1}s_\delta + w_{b2}c_\delta)(w_{b1}c_\delta + w_{b2}s_\delta) + d\dot{w}_{g3}\right] - \\
& dm_G\left[-d_{s1}w_{b2}^2 + d_{s3}\dot{w}_{b2} - (d_{s1}w_{b3} - d_{s3}w_{b1})w_{b3} + a_{v1}\right]s_\delta + \\
& dm_G\left[d_{s1}w_{b1}w_{b2} + d_{s1}\dot{w}_{b3} + d_{s3}w_{b2}w_{b3} - d_{s3}\dot{w}_{b1} + a_{v2}\right]c_\delta + \\
& T_U + T_M
\end{aligned} \tag{12}
$$

All time varying terms in $T_\delta$ are measured by on-board sensors or can be calculated with numerical differentiation except for the upper bearing frictional torque, $T_U$. We estimate this torque contribution through experiments described in the following section. The distance, mass, and inertia values are measured as described in [15].

## Estimation of Bearing Friction

In our design, the torque sensor is mounted between two sets of bearings. The upper set for the handlebars are tapered roller bearings and the lower are typical bicycle headset bearings. Each are preloaded a nominal amount during installation. We assume that the rotary friction due to each bearing set can be described as the sum of viscous $T_{Bv}$ and Coulomb friction $T_{Bc}$. The Coulomb friction can be described as a piecewise-constant function of the steering rate, Equation 13, and viscous friction as linear in the steer rate, Equation 14.

$$
T_{Bc} = t_B \operatorname{sgn}(\dot{\delta}) = \begin{cases} t_B & \text{if } \dot{\delta} > 0 \\ 0 & \text{if } \dot{\delta} = 0 \\ -t_B & \text{if } \dot{\delta} < 0 \end{cases} \tag{13}
$$

$$
T_{Bv} = c_B\dot{\delta} \tag{14}
$$

The total friction due to all of the bearings is

$$
T_B = T_{Bc} + T_{Bv} \tag{15}
$$

To estimate the coefficients $t_B$ and $c_B$, we mounted the bicycle with the steer axis vertical, the front wheel off the ground, and the rear frame rigidly fixed in inertial space. We then attached two parallel springs of stiffness $k$ to the left handlebar so that the force from the springs acted through lever arm $l$ relative to the steer axis.

This configuration allowed application of small perturbations to the handlebars and subsequent measurement of the damped vibrations in the steer angle, steer rate, and steer tube torque. The equations of motion governing the system then become

$$
I_{HF}\ddot{\delta} + c_B\dot{\delta} + t_B \operatorname{sgn}(\dot{\delta}) + 2kl^2\delta = 0 \tag{16}
$$

We measured the lever arm and spring stiffness as 0.213 meters and $904.7 \pm 0.6$ N/m respectively. The inertia of the handlebar, fork, and front wheel about the steer axis, $I_{HF}$, was estimated based on the measurements described in [15] and found to be $0.1297 + / - 0.0005\ kg \cdot m^2$

We estimated the friction coefficients with a non-linear grey box identification based on the measured steer angle over 15 trials in which the steering assembly was perturbed from equilibrium. The identified viscous coefficient is $c_B = 0.34 \pm 0.04\ N \cdot m \cdot s^2$ and the Coulomb coefficient is $t_B = 0.15 \pm 0.05\ N \cdot m$.

To calculate the applied steer torque $T_\delta$ we need an estimate of the upper bearing friction $T_U$. We made the simple assumption that the friction in the upper and lower bearings are equal, $T_U = T_B/2$, due to indeterminacy of the upper and lower bearing friction individually; see [15] for details.

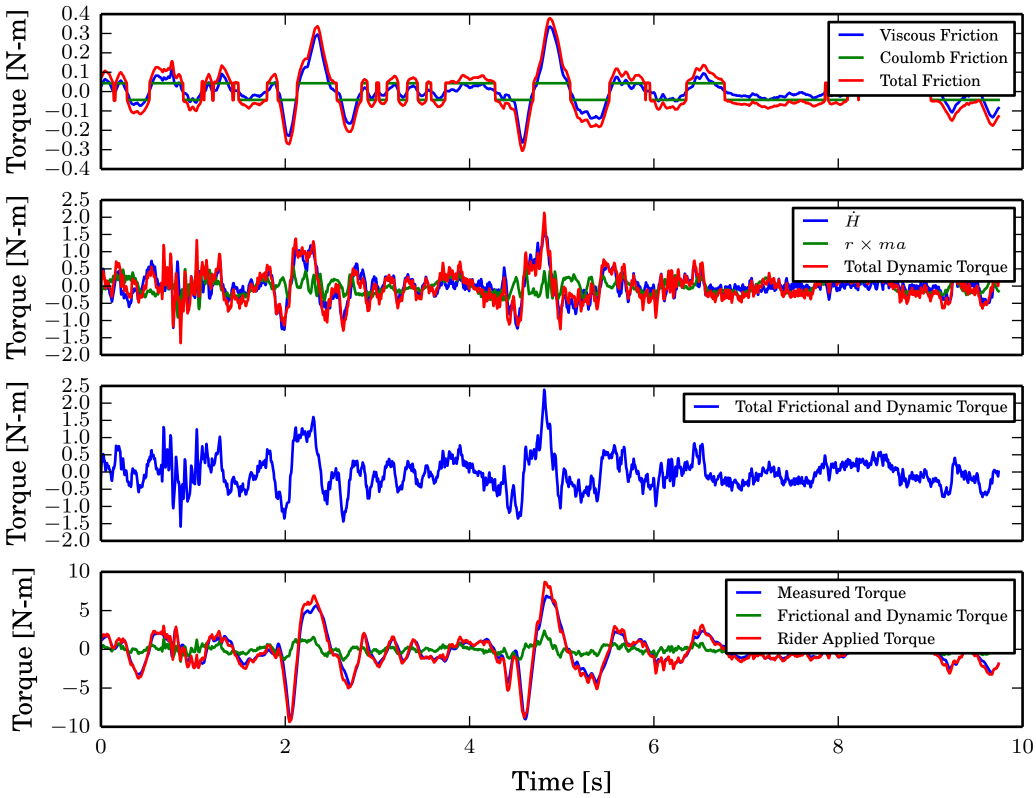


Figure 3: Steer torque measurements and the computed compensation for Trial # 700.

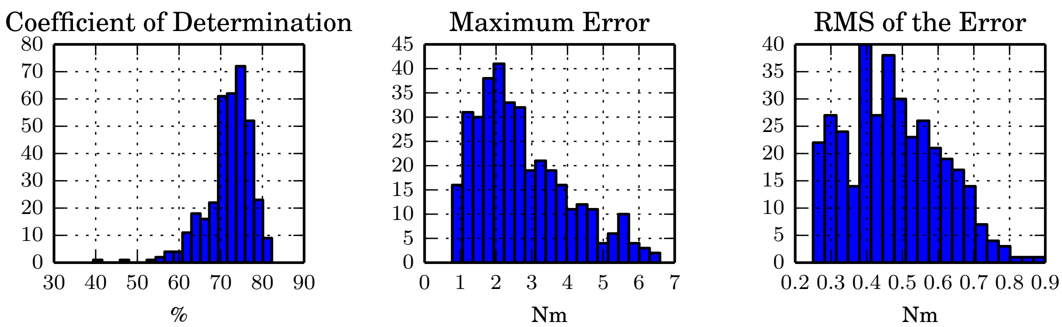


Figure 4: Histograms of the three statistics for all 359 trials.

Table 1: The median and maximum value of the error statistics.

| Statistic | Median | Maximum |
|---|---|---|
| Coefficient of Determination | 0.728814 | 0.822647 |
| Maximum Error | 2.446387 | 6.588228 |
| RMS of the Errors | 0.466733 | 0.899118 |

# Steer Torque Predictions

Using the equations described in section  and the estimates for the upper bearing friction in  we compute the compensated steer torque for 359 trials from the data collected from the instrumented bicycle set presented in [15]. Figure 3 gives results from an example trial. We then compute the root mean square of the error between the torque from the sensor and the compensated torque for each trial. We also compute the maximum of the absolute value of the error for each trial and the coefficient of determination (i.e. $R^2$) between the compensated and uncompensated torques. Outliers outside of $\pm 2\sigma$ were excluded from the results. Figure 4 shows the distribution of these statistics. The median values of the three statistics are given in Table 1.

# Discussion

For the bicycle and maneuvers performed in the experiments herein we have shown that neglecting to compensate for inertial effects can have a large influence on the accuracy of the results. In particular, on median 28% of the actual torque applied by the rider in our experiments would be neglected. This may be less important for motorcycles because the nominal steer torques are usually much larger, but this error will always be significant for measurements of low torque ($<$ 20 Nm or so) in any vehicle. Steer torque sensor designs should account for the inertial effects of the handlebars and eliminate crosstalk for high accuracy. Ideally, one would measure the forces at the hand/handlebar interface with very accurate six component load cells and inertial compensation would not be necessary. The closer the sensor is to the rider's hands the less important inertial compensation becomes. But if more traditional direct steer torque measurements are used, both inertial compensation and cross talk mitigation (mechanically or computationally) will be needed. We have found only a couple of designs that mitigate the inertial issue before us, namely [7] and [9], and many previous design were aware of crosstalk, but no design shows complete elimination as we have. Maneuvers with high steer accelerations and high handlebar axial moments of inertia are especially susceptible, due to the dominance of the effects of angular acceleration on torque. This is clearly shown in Figure 3. Our design gave very accurate torque measurements, but there is still much room for improvement, especially in terms of complexity and cost. Creating a simple, inexpensive, and accurate steer torque measurement system could play an important role in assistive control system design in production vehicles.

# Acknowledgements

This paper is based on work supported by the National Science Foundation under Grant No. 0928339. Thanks to Peter de Lange for developing the bearing friction experimental protocol and analysis and to Ton van den Bogert for comments and review.

# References

[1] D. Bortoluzzi, A. Doria, and R. Lot. Experimental investigation and simulation of motorcycle turning performance. In *3rd International Motorcycle Conference*, 2000.

[2] Stephen M. Cain and Noel C. Perkins. Comparison of experimental data to a model for bicycle steady-state turning. *Vehicle System Dynamics*, 50(8):1341–1364, 2012.

[3] R. Capitani, G. Masi, A. Meneghin, and D. Rosti. Handling analysis of a two-wheeled vehicle using MSC.ADAMS/motorcycle. *Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility*, 44:698–707, 2006.

[4] Kok Y. Cheng, David Bothman, and Karl J. Åström. Bicycle torque sensor experiment. Technical report, University of California, Santa Barbara, 2003.

[5] David S. de Lorenzo. Quantification of structural loading during off-road cycling. Master's thesis, Univeristy of California, Davis, 1997.

[6] David J. Eaton. *Man-Machine Dynamics in the Stabilization of Single-Track Vehicles*. PhD thesis, University of Michigan, 1973.

[7] M. V. C. Evertse. Rider analysis using a fully instrumented motorcycle. Master's thesis, Delft University of Technology, 2010.

[8] Hiroyasu Fu. Fundamental characteristics of single-track vehicles in steady turning. *JSME Bulletin*, 9(34):284–293, 1965.

[9] K. Iuchi and T. Murakami. An approach to fusion control of stabilization control and human input in electric bicycle. In *32nd Annual Conference on IEEE Industrial Electronics*, pages 3211–3216, Paris, France, 2006.

[10] Stephen R. James. Lateral dynamics of an offroad motorcycle by system identification. *Vehicle System Dynamics*, 38(1):1–22, July 2002.

[11] Katumi Kageyama and Hiroyasu Fu. Experiments on control characteristics of a motor-cycle in steady turning, especially on the effects of lean in and lean out. *Jour. SAE Japan*, 13(10):41–45, 1959. 596009.

[12] Thomas R. Kane and David A. Levinson. *Dynamics: Theory and Applications*. McGraw Hill, New York, NY, 1985.

[13] M. Kondo. Experimental study on the stability and control of single-track vehicles. *JSME*, 58(442):827–833, 1955.

[14] J.L. Meriam. *Dynamics*. Wiley, 1975.

[15] Jason K. Moore. *Human Control of a Bicycle*. PhD thesis, University of California, Davis, Davis, CA, August 2012.

[16] Robin S. Sharp. Motorcycle steering control by road preview. *Journal of Dynamic Systems, Measurement, and Control*, 129(4):373–382, 2007.

[17] Robin S. Sharp. On the stability and control of the bicycle. *Applied Mechanics Reviews*, 61(6):24, November 2008.

[18] Masamori Sugizaki and Akira Hasegawa. Experimental analysis of transient response in motorcycle-rider systems. *Society of Automotive Engineers*, November 1988. SAE Paper 881783.

[19] A. P. Teerhuis and S. T. H. Jansen. Motorcycle state estimation for lateral dynamics. In *Bicycle and Motorcycle Dynamics 2010, Symposium on the Dynamics and Control of Single Track Vehicles*, 2010.

[20] J. H. van den Ouden. Inventory of bicycle motion for the design of a bicycle simulator. Master's thesis, Delft University of Technology, 2011.

[21] David H. Weir, John W. Zellner, and Gar Teper. Motorcycle handling. Technical Report Volume II, U.S. Department of Transportation National Highway Traffic Safety Administration and Systems Technology, Inc., Washington, D.C., May 1979.

[22] R. A. Wilson-Jones. Steering and stability of single-track vehicles. In *Proceedings of the Institute of Mechanical Engineers (Auto Div)*, pages 191–199, 1951. Part 4.