Perspectives on bicycle and motorcycle steer torque estimation with methods to eliminate crosstalk and inertial effects

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Introduction

To control a bicycle or motorcycle, the rider's primary means of directing the vehicle is to apply forces to the handlebars which cause the front frame to rotate relative to the rear frame. The rider can, of course, also use other more subtle biomechanical motions to influence the motion of the vehicle, especially the lighter-weight vehicles, but it is known that forces applied to the handlebar give much more control authority than other means regardless of the vehicle's inertia [16, 17].

From the modeling perspective, it is generally easier to model the interface forces between the rider and the front frame as a single torque about the steer axis which acts between the front frame and the rear frame. The rider can be assumed to be part of the rear frame, regardless if the rider rigidity is an assumption or not.

Due to this modeling assumption and ease of measurement most experimental measurements of the interface forces between the rider and the front frame are obtained by measuring the torque generated in the steer axis of the front frame. These direct measurements of the rider applied steer torque are susceptible to error from two major causes: (1) inertia effects of the front frame which are located between the sensor and the rider's hands and (2) cross talk from rider applied forces other than those which generate steer torque. Accounting for the error sources are particularly important when the steer torques are of low magnitude ($<$ 20 Nm or so).

In this paper we review previous methods of steer torque measurements in bicycle and motorcycles throughout history and detail the design and implementation of a bicycle steer torque measurement system which minimizes the aforementioned cross talk errors. We then show the computations needed to correctly compensate for inertial effects of the front frame and bearing friction to obtain a more accurate estimate of the rider applied steer torque. Finally, to show the necessity of this approach, we compare the residuals between the uncompensated and compensated steer torque measurements for a large set of bicycle experiments.

History of Steer Torque Measurements

The earliest steer torque measurements were preformed by [21] in 1951. Wilson-Jones developed a set of motorcycle handlebars mounted in the rubber bushings that indicated the direction and value of torque in an analog fashion in real time. He demonstrated that you apply a negative torque with respect the steering angle to enter into a turn and measured torques in normal maneuvers in the 4 to 14 Nm range. Not long after this [13] was the first to record torque measurements on a motorcycle for post experiment analysis. Work in Japan on motorcycle dynamics grew considerably after World War II due to sanctions on aircraft research. [11] and [8] continued to improve steer torque measurements in motorcycles and studied further steady turning. [6] was the first person to measure steer torque in the United States during experiments. He attached a third handle bar above the regular handlebars with strain gages that produced voltage proportional to the applied torque around the steer axis. The rider operated the motorcycle with one hand. He measured steer torques up to 3.4 Nm for straight riding during speeds of 15 to 30 mph. This led him to conclude that most of the steer torque was due to rider remnant, as opposed to deliberate control. Not long after this, [20] developed a modular torque sensor which could be affixed to multiple motorcycles with a $\pm$ 70 Nm range and a 1% accuracy with a 10 Hz bandwidth. They were careful to reduce crosstalk from other forces applied to the handlebars. They unfortunately oversized the sensor and the signal to noise ratio was low for steady turn and straight riding maneuvers, but they measured torques of -20 to 55 Nm in lane changes. [?] also measured steer torque in high speed motorcycle lane change maneuvers and recorded torques between -20 and 20 Nm.

After years of motorcycle steer torque measurements, the first bicycle measurements were made by [5] on a downhill mountain bicycle which was fitted with a custom strain gauged handlebar which could effectively measure torque about the longitudinal axis and the vertical axis. His plot of torque measurements show maximum steer torques of 7 Nm and maximum longitudinal handlebar torques of 15 Nm which demonstrated that non-steer related forces on the handlebars can be significantly higher that those needed for steering.

Around the turn of the century, [1] designed a motorcycle steer torque transducer in which floating handlebars engage the fork through a small strain gaged cantilever beam. This design was less susceptible to crosstalk than earlier designs. They found torques to up to 20 Nm for slalom maneuvers at 40 m/s. Around the same time, [10] developed a secondary handlebar with integrated load cell to measure steer torques in an off road motorcycle.

In 2003, [4] completed a comprehensive study on bicycle steer torque for an undergraduate project. Cheng started by simply attaching a torque wrench to a bicycle and made left turns at speeds from 0 to 13 m/s and found that most steering torques were under 5 Nm. He then designed a floating handlebar which connected engaged the steer tube via a linear load cell. They configured the load cell to measure 0 to 84 Nm. Cheng found torques up to 1 Nm for steady turning at 4.5 m/s and up to 10 Nm for sharp turns. Confirming that bicycles require much lower torques for maneuvering.

[9] constructed a bicycle with a steer motor that "senses" the rider's input and for additive control of steer torque. The rider applied steer torque was estimated from the motor torque and the handlebar and motor moments of inertia. [3] shows measured steer torques from an instrumented scooter between -15 and 40 Nm. [7] may be the only person to estimate steer torque from sensors in the handle grips of a motorcycle that give force measurements directly at the human-vehicle interface. During the test maneuvers, a maximum of 40 Nm was observed. [18] shows measured torques just under 20 Nm for a motorcycle in slalom maneuvers.

Recently, [2] developed an in-the-steer tube torque sensor for a bicycle. The measured steer torques in steady turns never exceed 2.4 Nm but he admits that his sensor was 90% oversized. And most recently, [19] developed a steer torque sensor sensor that was susceptible to cross talk from other handlebar loads but had an appropriate measurement range of $\pm$7.5 Nm.

Steering torque has been measured in relatively few instances of bicycle experiments and not many more for motorcycles. Of these, very few of the designs may actually measure the true rider applied steer torque. This is more consequential for bicycles than motorcycles because the small torques used in typical bicycle control are of the order of 5 Nm. [19], in particular, showed how sensitive the torque measurements are to other handlebar loads. Also, most of these designs measure the torque somewhere between the rider hands and the ground contact point. This is a physically ideal way to measure the steer torque, but no one has accounted for the dynamic inertial effects of the front frame above or below the sensor. [7] may be the only design which mitigates this inertial compensation issue completely.

With this information in hand we designed a steer torque measurement system for a bicycle that accounts for the deficients in previous designs mentioned above.

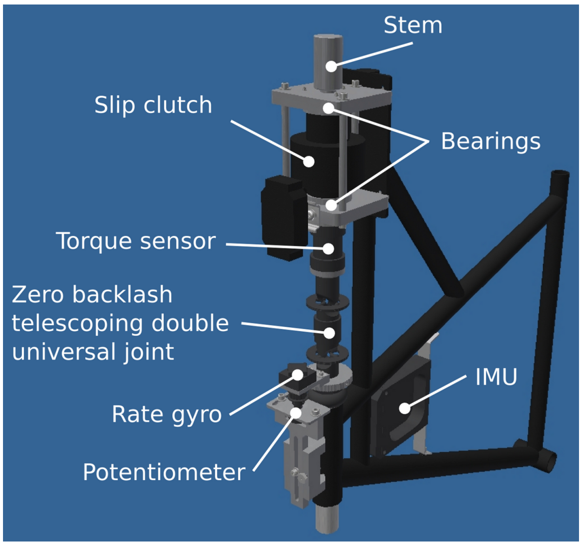

Figure 1: The steer torque sensor isolation design. The handlebars attach to the stem which is mounted in the upper bearings. The fork and steer tube are mounted in the normal headset of the bicycle. Between these two sets of bearing the stem is connected to the steer tube via the torque sensor and a zero backlash telescoping double universal joint. The steer angle and rate are measured at the steer tube and rate and acceleration of the rear frame are collected with the IMU.

Isolated Steer Torque Measurement Design

Our design is based around a Futek 150 in-lb (± 17 Nm) TFF350 torque sensor to ensure high accuracy for the low torques used in normal bicycle maneuvering. To guarantee that we only measured torque about the steer axis we isolated the steer torque sensor from any of the non-axial torques and all forces transmitted through the handlebar or ground contact with a zero backlash telescoping double universal joint, Figure 1. This design ensured that the torque about the steer axis was the only load the sensor detected.

Steer Dynamics

The final design measured the torque in the steer tube along the steer axis, but this measured torque, T_M, is not the same as the effective input torque applied by the rider. The rider applied steer torque, T_δ, can be shown to be a function of the kinematics of the front and rear frame and the friction torques generated by the bearings.

A free body diagram can be drawn of the portion of the front frame assembly above the torque sensor, Figure 2. The torques acting on the handlebar about the steer axis are the measured torque, T_M, the rider applied steer torque, T_δ, and the friction from the upper bearing set, T_U, which we describe by the sum of Coulomb, T_{U_F}, and viscous friction, T_{U_V}.

We measure three components of body fixed angular rate of the rear frame, B, in the Newtonian reference frame N with three rate gyros. This is described by

$$^N\bar{\omega}^B = w_{b1}\hat{b}_1 + w_{b2}\hat{b}_2 + w_{b3}\hat{b}_3 \tag{1}$$

The handlebar, G, is connected to the bicycle frame, B, by a revolute joint that rotates through the steering angle, δ, and we measure a component of the body fixed angular rate of the handlebar, w_{h3} about the steer axis with a rate gyro. The angular velocity of the handlebar can be written as follows

$$^N\bar{\omega}^G = (w_{b1}c_\delta + w_{b2}s_\delta)\hat{g}_1 + (-w_{b1}s_\delta + w_{b2}c_\delta)\hat{g}_2 + w_{h3}\hat{g}_3 \tag{2}$$

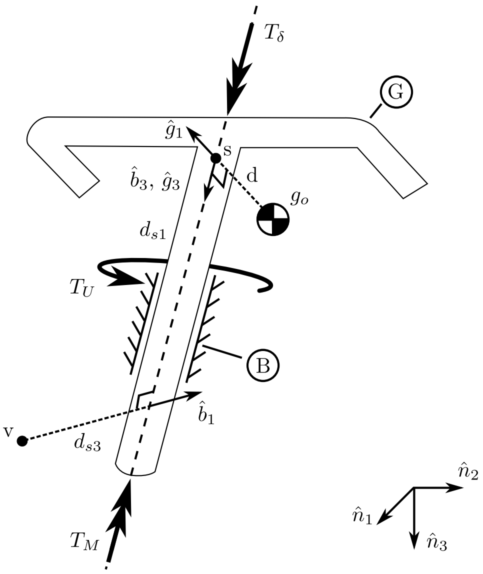

Figure 2: A free body diagram of the handlebar with all of the axial torques shown. The rear frame, B, is at an arbitrary orientation with respect to the Newtonian reference frame.

where c_δ and s_δ are shorthand for $\cos(\delta)$ and $\sin(\delta)$ respectively.

The steer rate, $\dot{\delta}$, can be computed by subtracting the angular rate of the bicycle frame about the steer axis from the angular rate of the handlebar about the steer axis.

$$\dot{\delta} = w_{h3} - w_{b3} \tag{3}$$

Now we define a point, s, on the steer axis a minimum distance d from the center of mass of the handlebar, g_o.

$$\bar{r}^{g_o/s} = d\hat{g}_1 \tag{4}$$

We also measure the body fixed acceleration of a point, v, on the bicycle frame.

$$^N\bar{a}^v = a_{v1}\hat{b}_1 + a_{v2}\hat{b}_2 + a_{v3}\hat{b}_3 \tag{5}$$

The location of point v is known with respect to s

$$\bar{r}^{s/v} = d_{s1}\hat{b}_1 + d_{s3}\hat{b}_3 \tag{6}$$

$^N\bar{a}^{g_o}$ can now be calculated using the two point theorem for acceleration [12] twice starting at the point v

$$^N\bar{a}^s = {}^N\bar{a}^v + {}^N\dot{\bar{\omega}}^B \times \bar{r}^{s/v} + {}^N\bar{\omega}^B \times ({}^N\bar{\omega}^B \times \bar{r}^{s/v}) \tag{7}$$

$$^N\bar{a}^{g_o} = {}^N\bar{a}^s + {}^N\dot{\bar{\omega}}^G \times \bar{r}^{g_o/s} + {}^N\bar{\omega}^G \times ({}^N\bar{\omega}^G \times \bar{r}^{g_o/s}) \tag{8}$$

The angular momentum of the handlebar about its center of mass is

$$^N\bar{H}^{G/g_o} = I^{G/g_o} \cdot {}^N\bar{\omega}^G \tag{9}$$

where I^{G/g_o} is the inertia dyadic with reference to the center of mass which exhibits symmetry about the 1-3 plane.

Now, the dynamic equations of motion of the handlebar can be written: the sum of the torques on the handlebar about point s is equal to the derivative of the angular momentum of G in N

about g_o plus the cross product of the vector from s to g_o with the mass times the acceleration of g_o in N [14]

$$\sum \bar{T}^{G/s} = {}^{N}\dot{\bar{H}}^{G/g_o} + \bar{r}^{g_o/s} \times m_G \, {}^{N}\bar{a}^{g_o} \tag{10}$$

We are only interested in the components of the previous equation in which the steer torque appears, so only the torques about the steer axis are examined.

$$\sum T_3^{G/s} = T_\delta - T_U - T_M = \left({}^{N}\dot{\bar{H}}^{G/g_o} + \bar{r}^{g_o/s} \times m_G \, {}^{N}\bar{a}^{g_o} \right) \cdot \hat{g}_3 \tag{11}$$

Finally, T_δ can be written as

$$\begin{aligned}
T_\delta = & I_{G_{22}} \left[(-w_{b1} s_\delta + w_{b2} c_\delta) \, c_\delta + w_{b2} s_\delta \right] + I_{G_{33}} \dot{w}_{g3} + \\
& I_{G_{31}} \left[(-w_{g3} + w_{b3}) w_{b1} s_\delta + (-w_{b3} + w_{g3}) w_{b2} c_\delta + s_\delta \dot{w}_{b2} + c_\delta \dot{w}_{b1} \right] + \\
& \left[I_{G_{11}} (w_{b1} c_\delta + w_{b2} s_\delta) + I_{G_{31}} w_{g3} \right] \left[-w_{b1} s_\delta + w_{b2} c_\delta \right] + \\
& d m_G \left[d(-w_{b1} s_\delta + w_{b2} c_\delta)(w_{b1} c_\delta + w_{b2} s_\delta) + d \dot{w}_{g3} \right] - \\
& d m_G \left[-d_{s1} w_{b2}^2 + d_{s3} \dot{w}_{b2} - (d_{s1} w_{b3} - d_{s3} w_{b1}) w_{b3} + a_{v1} \right] s_\delta + \\
& d m_G \left[d_{s1} w_{b1} w_{b2} + d_{s1} \dot{w}_{b3} + d_{s3} w_{b2} w_{b3} - d_{s3} \dot{w}_{b1} + a_{v2} \right] c_\delta + \\
& T_U + T_M
\end{aligned} \tag{12}$$

All of the time varying terms in T_δ are measured by the on-board sensors or can be calculated with numerical differentiation except for the upper bearing frictional torque, T_U. We estimate this torque contribution through experiments described in the following section. The distance, mass, and inertial values can be measured as described in [15].

Estimation of Bearing Friction

In our design, the torque sensor is mounted between two sets of bearings. The upper set for the handlebars are tapered roller bearings and the lower are typical bicycle headset bearings. Each are preloaded a nominal amount during installation. We assume that the rotary friction due to each bearing set can be described as the sum of viscous T_{Bv} and Coulomb friction T_{Bc}. The Coulomb friction can be described as a piecewise function of the steering rate, Equation 13, and viscous friction as a function linear in the steer rate, Equation 14.

$$T_{Bc} = t_B \operatorname{sgn}(\dot{\delta}) = \begin{cases} t_B & \text{if } \dot{\delta} > 0 \\ 0 & \text{if } \dot{\delta} = 0 \\ -t_B & \text{if } \dot{\delta} < 0 \end{cases} \tag{13}$$

$$T_{Bv} = c_B \dot{\delta} \tag{14}$$

The total friction due to all of the bearings is

$$T_B = T_{Bc} + T_{Bv} \tag{15}$$

To estimate the coefficients t_B and c_B, we mounted the bicycle such that the steer axis was vertical, the front wheel was off the ground, and the rear frame was rigidly fixed in inertial space. We then attached two springs of in parallel stiffness k to the left handlebar such that the force from the springs acted on a lever arm, l, relative to the steer axis.

This configuration allowed us to apply small perturbations to the handlebars and measure the dampened vibrations in the steer angle, steer rate, and steer tube torque. The equations of motion governing the system then become

$$I_{HF} \ddot{\delta} + c_B \dot{\delta} + t_B \operatorname{sgn}(\dot{\delta}) + 2kl^2 \delta = 0 \tag{16}$$

Table 1: The mean and maximum value of the residual statistics.

Statistic	Median	Maximum
Coefficient of Determination	0.728814	0.822647
Maximum Residual	2.446387	6.588228
RMS of the Residuals	0.466733	0.899118

We measured the lever arm and spring stiffness as 0.213 meters and 904.7±0.6 N/m respectively. The inertia of the handlebar, fork, and front wheel about the steer axis, I_{HF}, was estimated based on the measurements described in [15] and found to be $0.1297 + / - 0.0005$ $kg \cdot m^2$

We estimated the friction coefficients with a non-linear grey box identification based on the measured steer angle over 15 trials where the steering assembly was perturbed from equilibrium. The identified viscous coefficient is $c_B = 0.34 \pm 0.04$ $N \cdot m \cdot s^2$ and the Coulomb coefficient is $t_B = 0.15 \pm 0.05$ $N \cdot m$.

To calculate the applied steer torque, T_δ, we need an estimate of the upper bearing friction, T_U. We made the simple assumption is that the friction in the upper bearings equals the friction in the lower bearings, $T_U = T_B/2$ due to an inconclusive results from independently identifying the upper and lower bearing friction, see [15] for details.

Steer Torque Predictions

Using the equations described in section and the estimates for the upper bearing friction in we compute the compensated steer torque for 359 trials from the data collected from the instrumented bicycle set presented in [**?**]. Figure 3 gives an example trial. We then compute the root mean square of the residuals between the torque from the sensor and the compensated torque for each trial. We also compute the maximum of the absolute value of the residuals for each trial and the coefficient of determination (i.e. R^2/VAF) between the compensated and uncompensated torques. Outliers outside of $\pm 2\sigma$ were excluded from the results. Figure shows the distribution of these statistics.

The median values of the three statistics are given in 1.

Discussion

For the bicycle and maneuvers preformed in the experiments herein we've shown that neglecting to compensate for inertial effects can a large influence on the accuracy of the measurements. In particular, on median 28% of the actual torque applied by the rider is neglected. This may be less of a consequence for motorcycles because the nominal steer torques are on mean much larger, but this error will always be significant for measurements of low torque ¡ 20 Nm or so in any vehicle. Steer torque sensor designs should account for the inertial effects of the handlebars and eliminate crosstalk to keep the accuracy high. From the review we have only found a couple designs that mitigate this issue before us. Maneuvers with high steer accelerations and high handlebar axial moments of inertia are especially susceptible.

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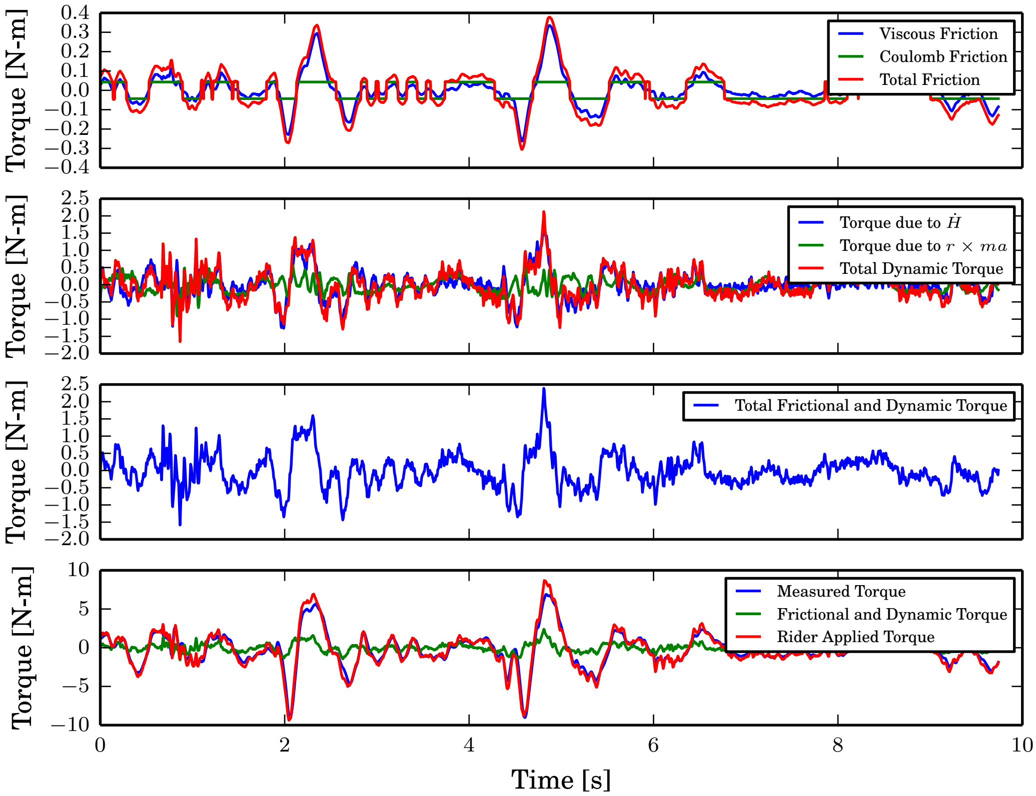
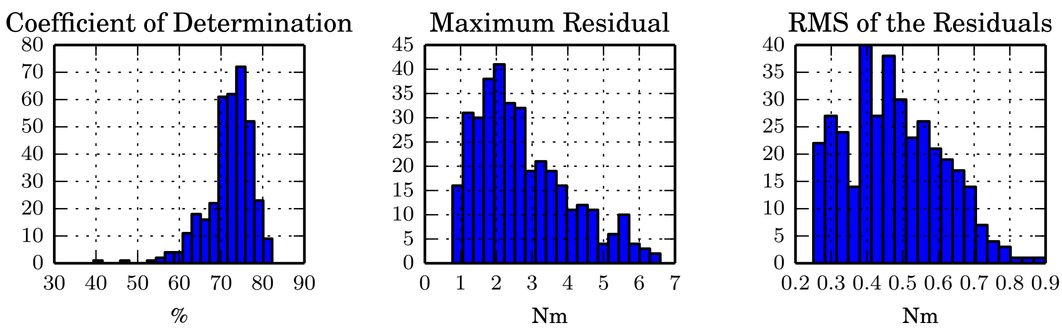


Figure 3: Steer torque measurements and the computed compensation for Trail # 700.

Figure 4: Histograms of the three statistics with respect to each of the 359 trials.



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