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An Accurate Method of Measuring and Comparing a Bicycle's Physical Parameters

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Abstract

Accurate measurements of a bicycle's physical parameters are required for realistic dynamic simulations. For the most basic models the geometry, mass, mass center location and mass distributions must be measured. More complex models require measurements of tire characteristics, stiffness, damping, etc. This paper concerns the measurement of bicycle parameters required for simulations of the benchmark bicycle presented in [4]. This model is composed of four rigid bodies, has ideal rolling and frictionless joints, and is assumed to be laterally symmetric. A set of 25 physical parameters is used to describe the geometry, mass, mass location and mass distribution of each of the rigid bodies. These parameters are presented relative to a reference frame when the bicycle is in the nominal upright configuration. The methods described herein are based primarily on the work done in [2] but have been refined for improved accuracy and methodology. Previously, [5] measured an experimental bicycle frame in a similar fashion and both [1] and [6] have used similar techniques with scooters. We measured the characteristics of six different bicycles, two of which were set up in two different configurations. This is a total of eight different parameter sets that can be used with, but not limited to, the benchmark bicycle model. The parameters set is also complete for non-linear benchmark bicycle model.

The six bicycles, chosen for both variety and convenience, are as follows:

Batavus Browser The Dutch style city bicycle was measured in two configurations. One with the instrumentation described in [3] and one as received from the manufacturer.

Batavus Stratos Deluxe A Dutch style sporty city bicycle.

Batavus Crescendo Deluxe A Dutch style city bicycle with a suspension fork.

Gary Fisher Mountain Bike A hardtail mountain bicycle.

Bianchi Pista A modern steel frame track racing bicycle.

Yellow Bicycle A stripped down aluminum frame road bicycle. This bike was measured in two configurations, the second with the fork rotated in the headtube 180 degrees for larger trail.

Geometry

The benchmark bicycle requires five basic geometric measurements: wheelbase w, steer axis tilt λ, trail c, and wheel radii r_{F} and r_{R}. The wheelbase and steer axis tilt were measured directly with a meter scale and a digital level, respectively. The trail and wheel radii were measured indirectly using fork offset and loaded wheel circumference.

Table 1. PARAMETER VALUES FOR THE BATAVUS BROWSER.

Parameter	Symbol	Value
wheel base	w	1.121 m
trail	c	0.069 m
steer axis tilt	λ	0.400 rad
gravity	g	9.81 N kg^{-1}
forward speed	v	various m s^{-1}
R*ear wheel R*		
radius	r_{R}	0.341 m
mass	m_{R}	3.11 kg
mass moments of inertia	$(I_{\mathrm{R}xx},\ I_{\mathrm{R}yy})$	(0.088, 0.152) kg m^2
rear **B***ody and frame B*		
position center of mass	$(x_{\mathrm{B}},\ z_{\mathrm{B}})$	(0.276, −0.538) m
mass	m_{B}	9.86 kg
mass moments of inertia	$\begin{bmatrix} I_{\mathrm{B}xx} & 0 & I_{\mathrm{B}xz} \\ 0 & I_{\mathrm{B}yy} & 0 \\ I_{\mathrm{B}xz} & 0 & I_{\mathrm{B}zz} \end{bmatrix}$	$\begin{bmatrix} 0.527 & 0 & -0.114 \\ 0 & 1.317 & 0 \\ -0.114 & 0 & 0.759 \end{bmatrix}$ kg m^2
front **H***andlebar and fork assembly H*		
position center of mass	$(x_{\mathrm{H}},\ z_{\mathrm{H}})$	(0.867, −0.748) m
mass	m_{H}	3.22 kg
mass moments of inertia	$\begin{bmatrix} I_{\mathrm{H}xx} & 0 & I_{\mathrm{H}xz} \\ 0 & I_{\mathrm{H}yy} & 0 \\ I_{\mathrm{H}xz} & 0 & I_{\mathrm{H}zz} \end{bmatrix}$	$\begin{bmatrix} 0.253 & 0 & -0.072 \\ 0 & 0.246 & 0 \\ -0.072 & 0 & 0.096 \end{bmatrix}$ kg m^2
F*ront wheel F*		
radius	r_{F}	0.344 m
mass	m_{F}	2.02 kg
mass moments of inertia	$(I_{\mathrm{F}xx},\ I_{\mathrm{F}yy})$	(0.090, 0.149) kg m^2

Mass, Mass Center Location and Mass Distribution

The mass of each of the rigid bodies were measured with a precision scale. The centers of mass of each of the rigid bodies were measured by hanging each rigid body at the mass center and assuming lateral symmetry. The mass centers were calculated from the orientation angle of the headtube and the location of the point of suspension relative to the wheel center. The wheel centers of mass were assumed to be in their ideal geometric center.

The moments of inertia of the rigid bodies were measured in both torsional pendulum and compound pendulum configurations. Again the assumed symmetry of the bicycle was utilized to reduce the number of measurements needed. The in-symmetric-plane moments of inertia were calculated from measurements of the rigid bodies' periods of oscillation when hung as a torsional pendulum at different orienatation angles. The out-of-plane moments of inertia were measured by swinging the rigid bodies as a compound pendulum.

Results

Parameter sets for all six bicycles and all eight configurations, example in Table 1, will be presented, together with basic comparisons of the open loop dynamics of the different types of bicycles. The physical parameters, the mass, damping and stiffness matrices of the system, the eigenvalues and open loop transfer functions at different speeds will be compared with typical anedoctal descriptions of handling.

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