

# Summative Robotic Bicycle Steer Control or Effect of Automatic Bicycle Balance Stabilization to Rider Control and Handling

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## 1 Introduction

Automatic roll stabilization of single track vehicles began in earnest after predictive motorcycle models were developed and refined throughout the 1970s. Van Zytveld [6] seems to be the first to attempt to robotically stabilize a small motorbike with a controlled inverted pendulum that mimicked rider lean, but was not successful in demonstrating what his control model predicted. It was not until the mid 1980s in which someone successfully demonstrated an automatically balanced motorcycle [4]. Ruijs and Pacejka showed that steer torque driven by roll angle feedback stabilizes the capsize mode, roll rate feedback stabilizes the weave mode of a motorcycle, and steer angular rate feedback stabilizes the wobble mode. They also show how the gains need to change with respect to vehicle speed. This is the simplest controller that can stabilize a single track vehicle above a minimum speed. Many more automatically balanced single track vehicles have been demonstrated over the last 40 years, but none of these advancements have resulted in a successful commercial product. Most of these robotic bicycles and motorcycles designers did not intend for a human rider to also control the stabilized vehicle. Although, an automatically stabilized bicycle can be controlled by a human rider if the motor controlled steer torque and the rider applied steer torque act on the steer in sum. Given that premise, I want to answer this question:

Given a bicycle that has a motor (with power, torque, speed, bandwidth limits) which can apply a torque between the rear and front frames and a set

of sensors that can give good measurements or estimates of the bicycle's steer angle, steering rate, roll angle, and rolling rate what closed loop dynamics and motion are possible and how might a rider controlling this stabilized bicycle find the behavior?

Some other notes:

- we only care about the lateral dynamics and handling qualities of
- we will only explore mathematical and computational models

## 2 Model

The linear Whipple-Carvallo bicycle model [?, 1, 3] is the simplest bicycle model that exhibits non-minimum phase behavior and self-stability. It can be described by the state space equations:

$$\dot{\bar{x}} = \mathbf{A}\bar{x} + \mathbf{B}\bar{u} \text{ where } \bar{x} = \begin{bmatrix} \phi \\ \dot{\phi} \\ \delta \\ \dot{\delta} \end{bmatrix} \text{ and } \bar{u} = \begin{bmatrix} T_\phi \\ T_\delta \end{bmatrix} \quad (1)$$

The states are the roll angle  $\phi$  and steer angle  $\delta$  along with their derivatives and the inputs are roll torque  $T_\phi$  and steer torque  $T_\delta$ . The state  $\mathbf{A}$  and input  $\mathbf{B}$  matrices are populated with expressions that are functions of 27 geometric and inertial parameters of the nonholonomic multibody system made up of four rigid bodies: two wheels, front frame, and rear frame.

I use realistic numeric values for the state and input matrices measured from a Batavus Browser Dutch-style city bicycle and a 75 kg rider whose inertia properties were estimated using the method of Yeadon. These calculations were performed with the BicycleParameters and yeadon software [2]. The following table gives the numerical values of the vehicle.

TODO: Add table of parameters.

## 3 Controllability

The Whipple-Carvallo model can be uncontrollable at specific speeds. [5] showed uncontrollable speeds for two extended Whipple-Carvallo models

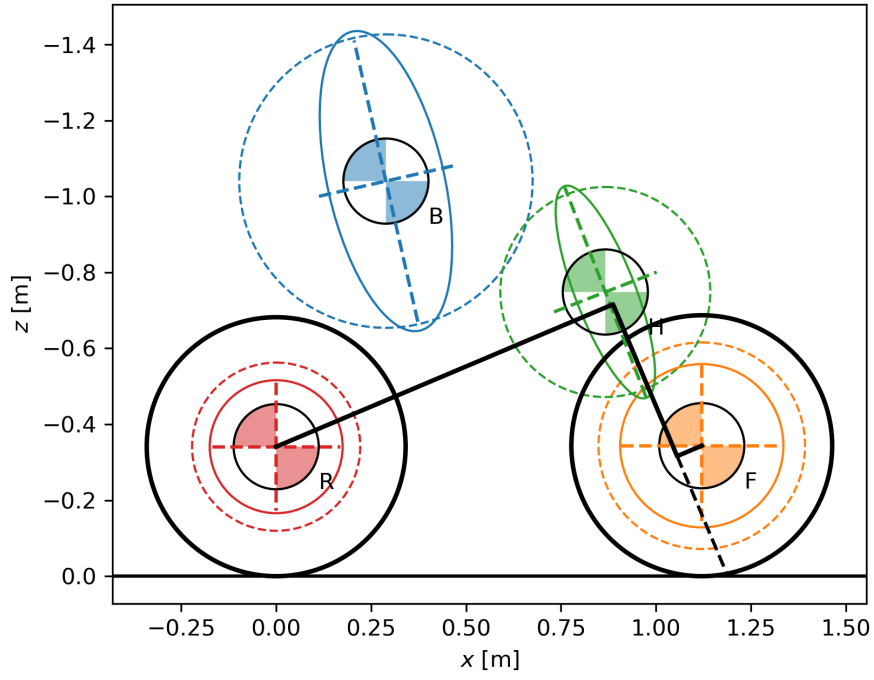


Figure 1: Geometry, mass, and inertia of the bicycle-rider system.

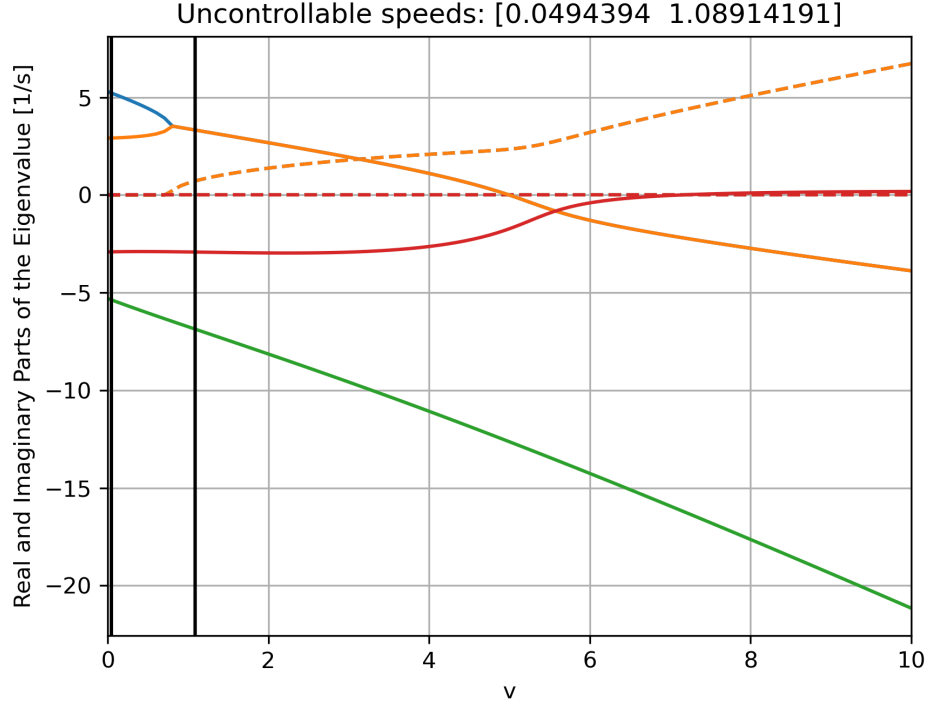


Figure 2: Real and imaginary parts of the eigenvalues as a function of speed. Vertical black lines indicate uncontrollable dynamics.

(with rider lean) and their degree of uncontrollability using modal controllability. It is also true that the non-extended Whipple-Carvallo model exhibits uncontrollable speeds when controlling with steer torque, but notably  $v = 0$  is not one of those speeds.

The controllability matrix  $\mathbf{C}$  can be formed for the steer input,  $T_\delta$ . There are speed parameter values where the linear Whipple-Carvallo bicycle model is uncontrollable. Solving the  $\det(\mathbf{C}(v)) = 0$  returns two speeds in the range  $[0, 10]$  m/s that are uncontrollable.

I have no idea why this model is uncontrollable at these specific speeds.

We can also look at the modal controllability to see the degree of controllability Fig. 3.

I don't think this model controllability calc is correct because it isn't smooth and it doesn't reveal the two uncontrollable speeds.

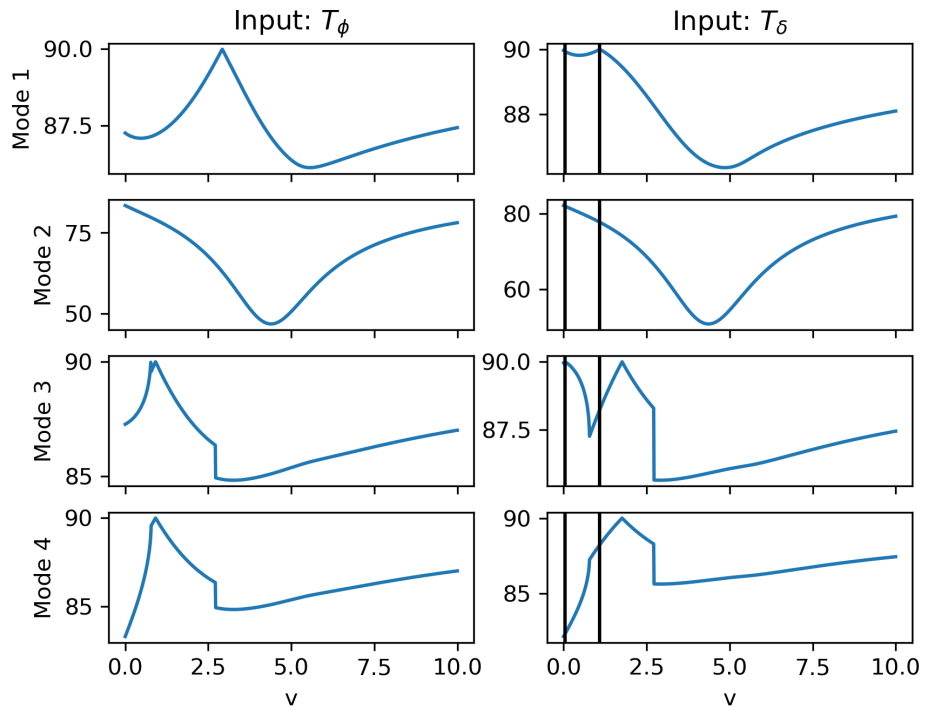


Figure 3: Testing

## 4 Stabilization with Roll and Steer Feedback

One IMU on the handlebars and one on the rear frame easily gives: roll rate and steer rate. Steer angle sensor gives steer angle. Roll angle must be estimated.

Single motor that applies a torque between the rear frame and the handlebars/fork.

Assume that the torque between the two frames is the sum of the motor and the rider:

$$T_{\delta} = T_{\delta,\text{human}} + T_{\delta,\text{motor}} \quad (2)$$

The motor is driven by a full state feedback controller:

$$T_{\delta,\text{motor}} = k_{\phi}\phi + k_{\delta}\delta + k_{\dot{\phi}}\dot{\phi} + k_{\dot{\delta}}\dot{\delta} \quad (3)$$

The motor closed loop dynamics is then:

$$\dot{\bar{x}} = (\mathbf{A} - \mathbf{BK})\bar{x} + \mathbf{B}[T_{\phi} \quad T_{\delta,\text{human}}]^T \quad (4)$$

$$(5)$$

where

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ k_{\phi} & k_{\delta} & k_{\dot{\phi}} & k_{\dot{\delta}} \end{bmatrix} \quad (6)$$

Note that the rider still applies steer torque through the original B matrix and any roll disturbances also through same B matrix. The human controlled dynamics can be manipulated by changing the motor controller gains.

## 5 Proportional Dervivative Roll Control

Proportional Derivative Roll Control

- What are the achievable closed loop dynamics?

- Vary speed and controller gain, show 3D root locus
- Pick a speed and vary the controller gain, show 2D root locus
- Difference with and without a rigid rider
- Could be PD on roll rate criteria (overshoot, decay time) and then pole placement with the two parameters for each speed to get similar behavior across speeds.
- What if we only have rate feedback (raw IMU data)? Does steer rate feedback help?

It is well known that at low speeds simple proportional positive feedback of roll rate stabilize the bicycle. This is positive roll derivative feedback.

Roll rate feedback alone is not sufficient to stabilize the bicycle at very low speeds ( $< 0.5$  m/s), even with infinite gains, and speeds where the capsize mode is unstable. But if roll angle and roll rate are available to feedback, the bicycle can also be stabilized at the higher speeds. If you gain schedule roll angle and roll rate feedback to both stabilize sufficiently while minimizing the weave frequency exponential functions provide a good model for gain scheduling with respect to speed.

The roll PD gain scheduling with respect to speed stabilizes at all speeds above about 0.75 m/s and retains similar dynamics except that the weave frequency is significantly higher.

At very low speeds large steer torques and steer angles are required to stabilize the system.

## 6 Full State Feedback

If you now assume full state feedback and we drive the state to zero as before, an LQR controller can be realized at each speed. Assumed  $\mathbf{Q}$  and  $\mathbf{R}$  to be identity for simplicity. I also scale the gains proportionally to limit the maximum torque that can be applied for some expected maximum motion. If you do that, you get some LQR gain scheduling over speed. The required gains at the uncontrollable points go to infinity, so you basically can't stabilize below the largest uncontrollable speed.

Using the scheduling in Figure 8 give the dynamics in Figure 9. The LQR solution manages to stabilize the system without drastically changing

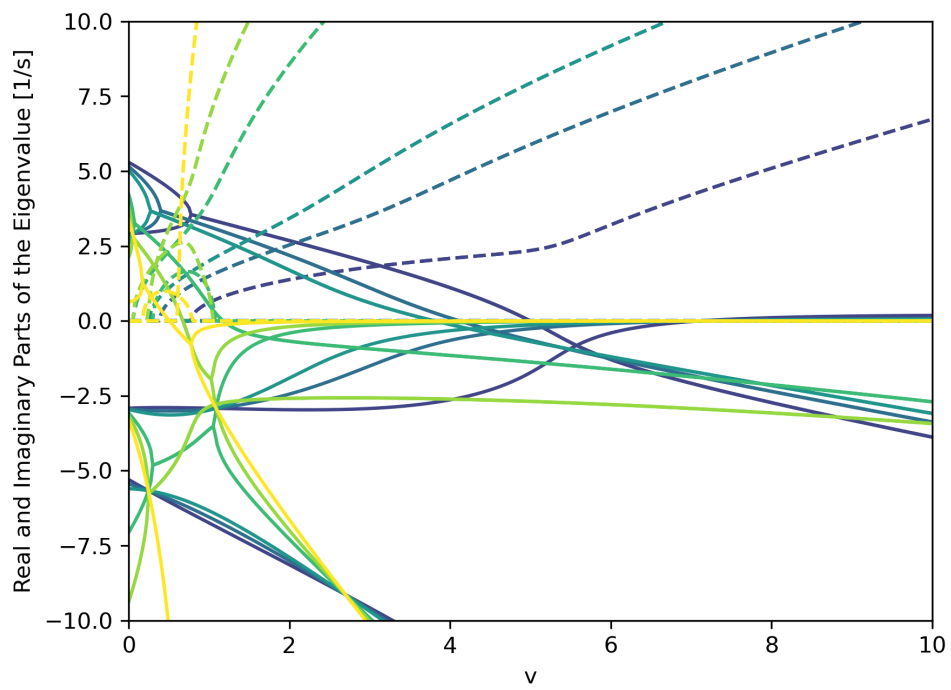


Figure 4: Effect on the motor controlled closed loop dynamics with changing  $k_{\dot{\phi}}$ . Blue to yellow varies the gain from zero to a large value.



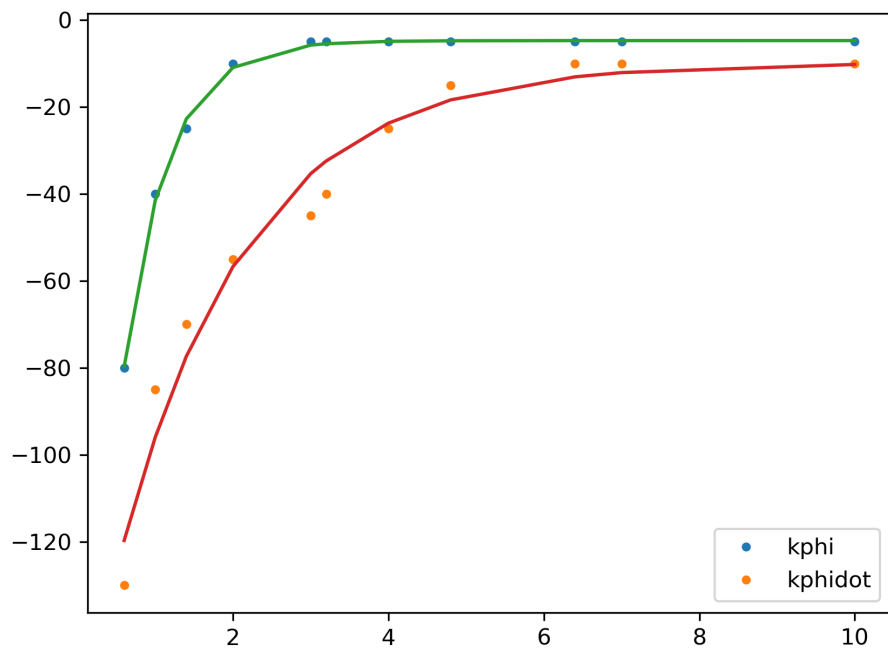


Figure 5: Exponentially scheduled roll angle and roll rate gains with respect to speed.

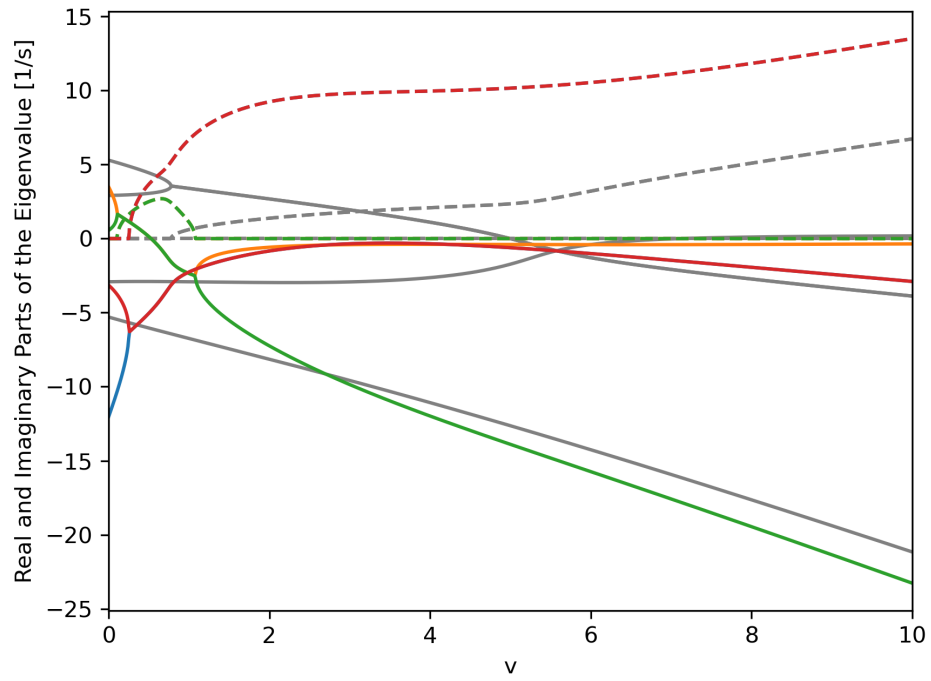


Figure 6: Root locus of the eigenvalues components with respect to speed when the gain scheduling in Figure 5 are applied. Grey lines are the uncontrolled bicycle.

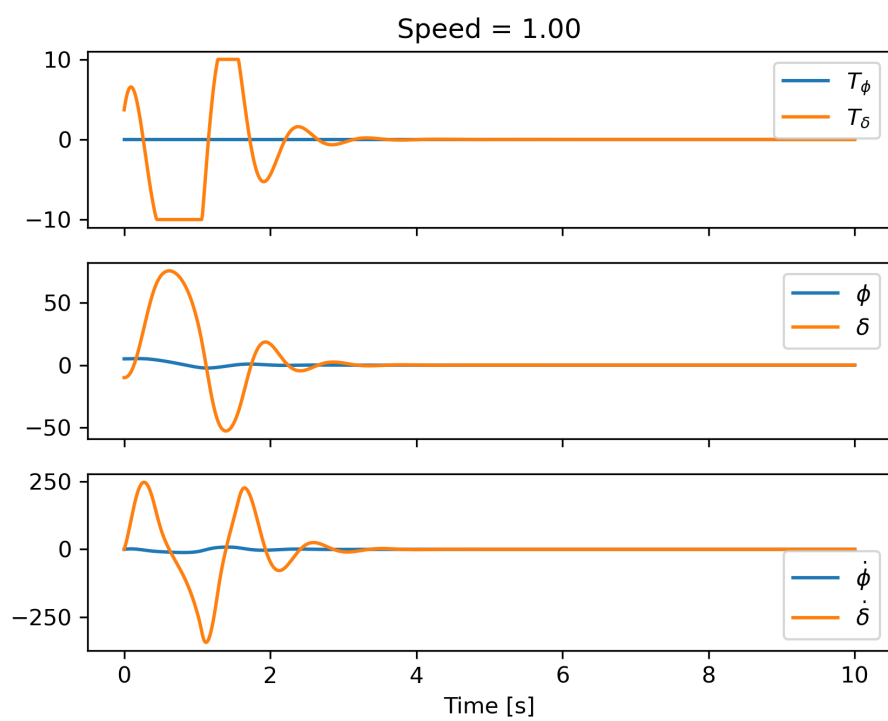


Figure 7: Simulation with saturated steer torque.

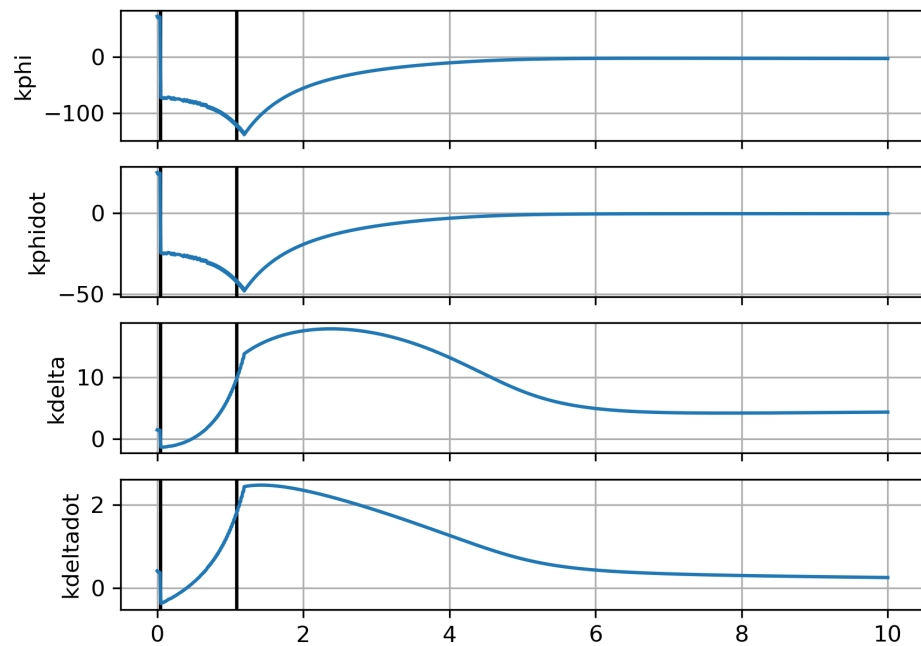


Figure 8: Gain scheduled LQR gains.

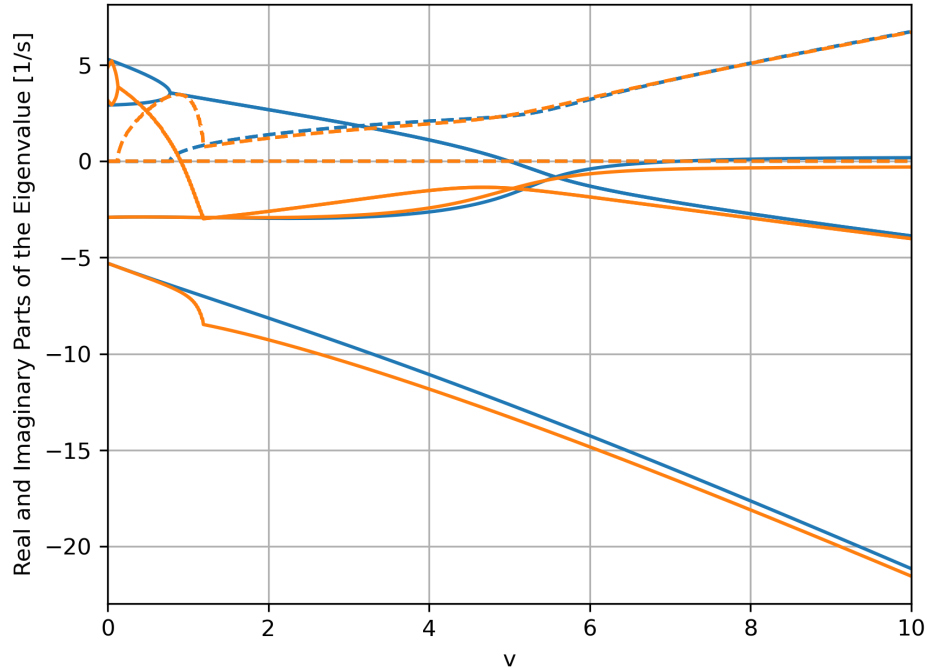


Figure 9: Root locus of eigenvalue components with the LQR gain scheduling.

the dynamics, in particular the weave frequency is unchanged. If large gains are permitted, the LQR controller theoretically can stabilize the bicycle at  $v=0$  speed.

Here is a low speed simulation with LQR solution.

What do the transfer functions from human steer to roll look like when they are controlling the closed loop motor controlled bike? For a very low speed, there is a damped resonant peak around 3 rad/s which changes the way the vehicle feels in the human control bandwidth.

Would it be better/possible to ask for the gains that make the Bode plot of controlled bike match the uncontrolled bike at high speeds? This would give matching feeling to the rider.

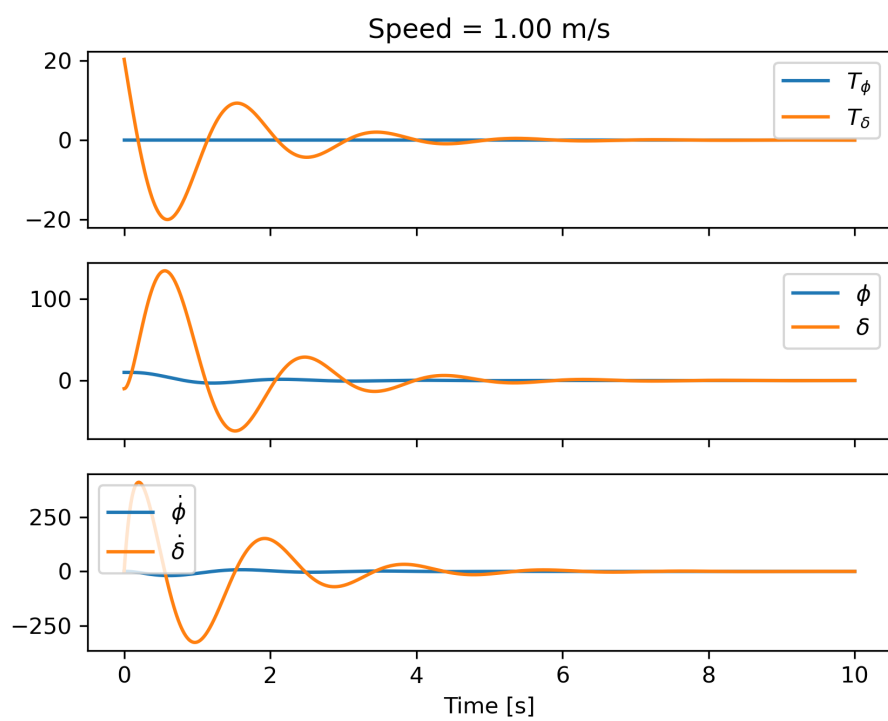


Figure 10: Testing

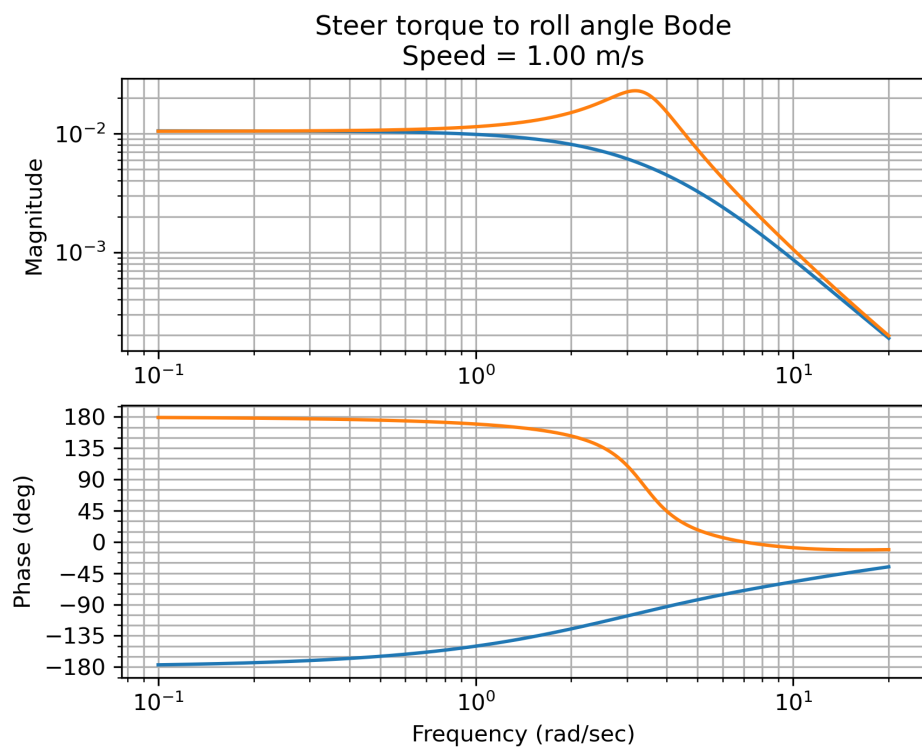


Figure 11: Orange is with the motor control on, blue is without.

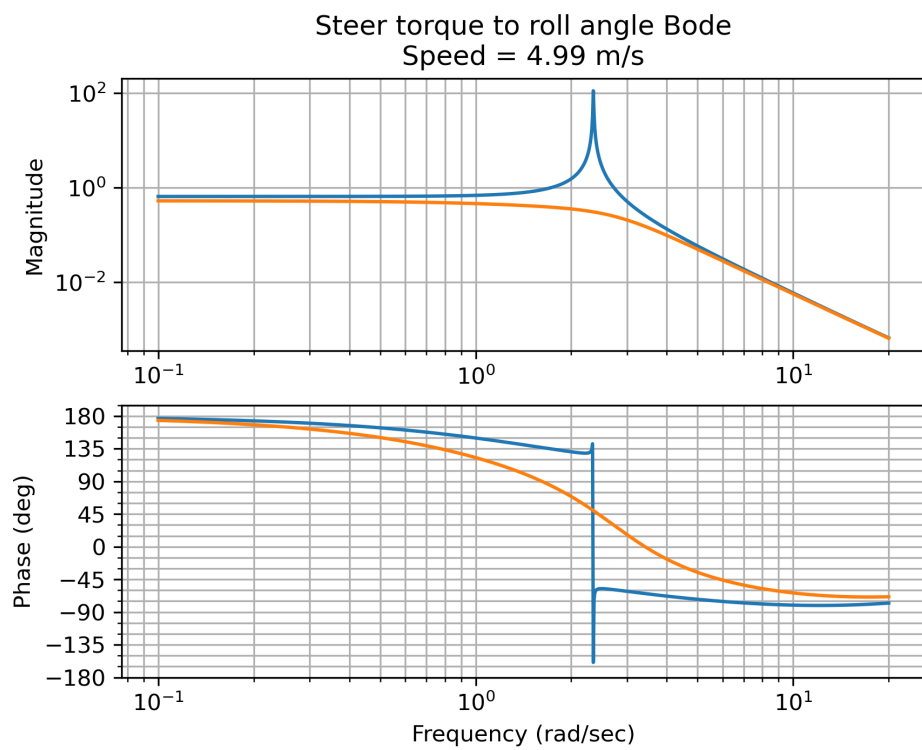


Figure 12: Orange is with the motor control on, blue is without.



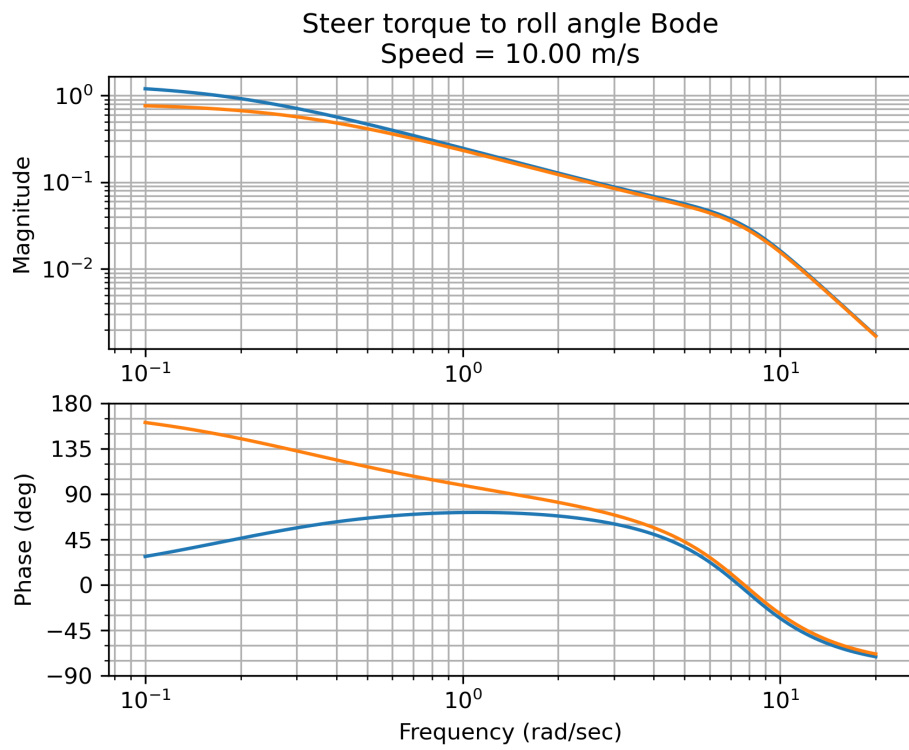


Figure 13: Orange is with the motor control on, blue is without.

## 7 Motor limitations

Set bounds on steer torque, power, and even bandwidth then solve for the open loop steer torque needed to stabilize at various speeds.

Set bounds on steer torque, power, and even bandwidth then solve for the open loop steer torque and solve for the four gains that stabilize the system with under maximal performance bounds.

Is it of value to add a motor dynamics equation into the mix? A full state feedback would then require measuring motor current.

## 8 Can we mimic other real bicycles?

Optimization that finds controller parameters for matching dynamics?

## 9 Comparison of the gyrobike and the steer motor

Show how the same dynamics can be produced

Energy costs comparison

## 10 Add human control

What human steer torques are required for normal maneuvers? Is it higher for the stabilized bikes?

Is the stability detrimental to maneuverability?

Handling quality metric for the gain scheduled controllers.

## References

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