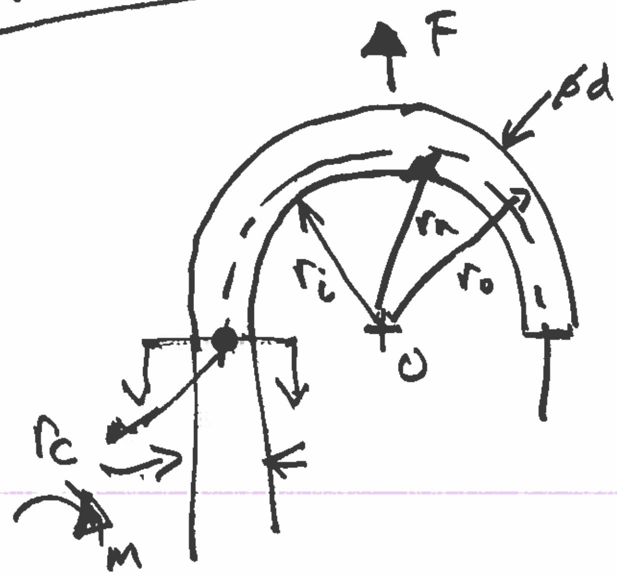
EME 150A LECTURE 11 October 19, 2015

HW 3 PROB #4

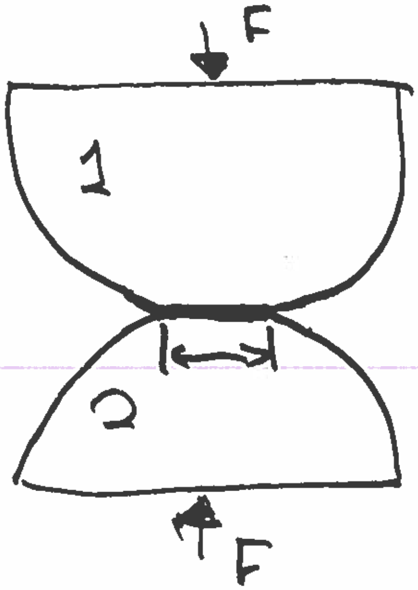


$M = F r_c$

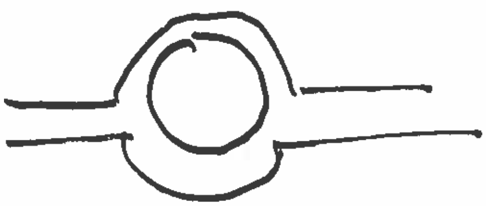
if $d << r_c$

then $r_c, r_n,$ or r_i or r_o are sufficient, ∴ Using r_o is most conservative

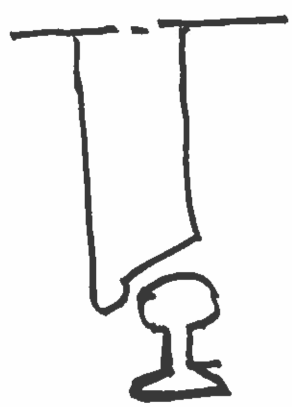
Contact Stress



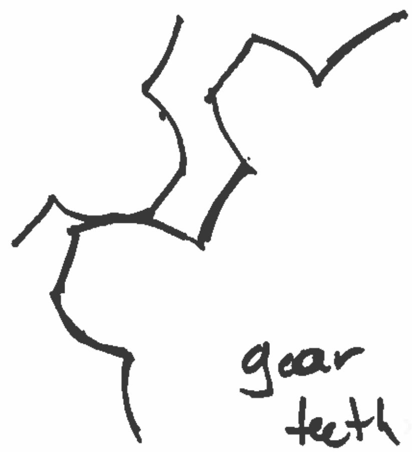
When two objects are pressed together a region of area contact develops.



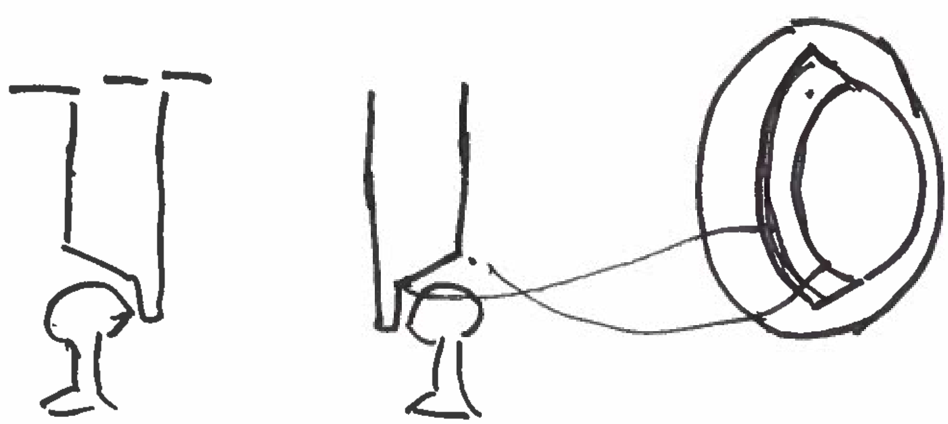
roller bearing



train wheel



gear teeth



Theory developed 1882 Hertz

Hertzian

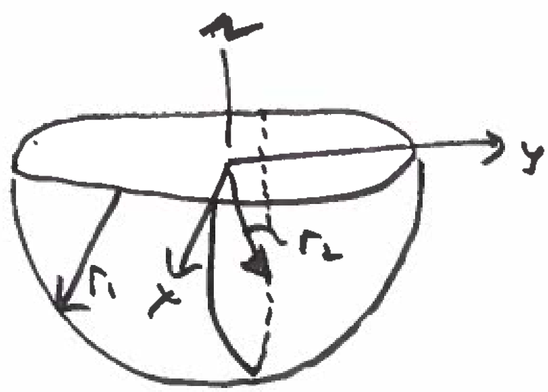
~~Hertian~~ contact stresses

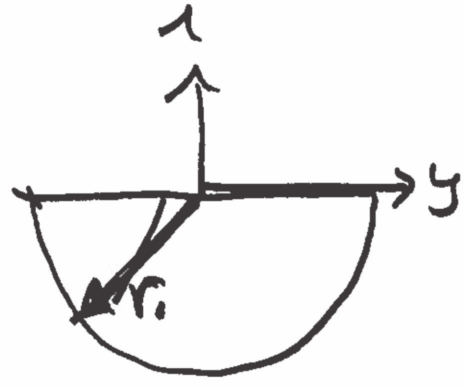
Assumptions:

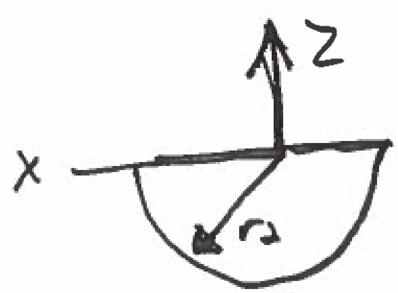
- loads are perpendicular to the surface
- no friction, smooth, continous surfaces
- small strain theory

General Case

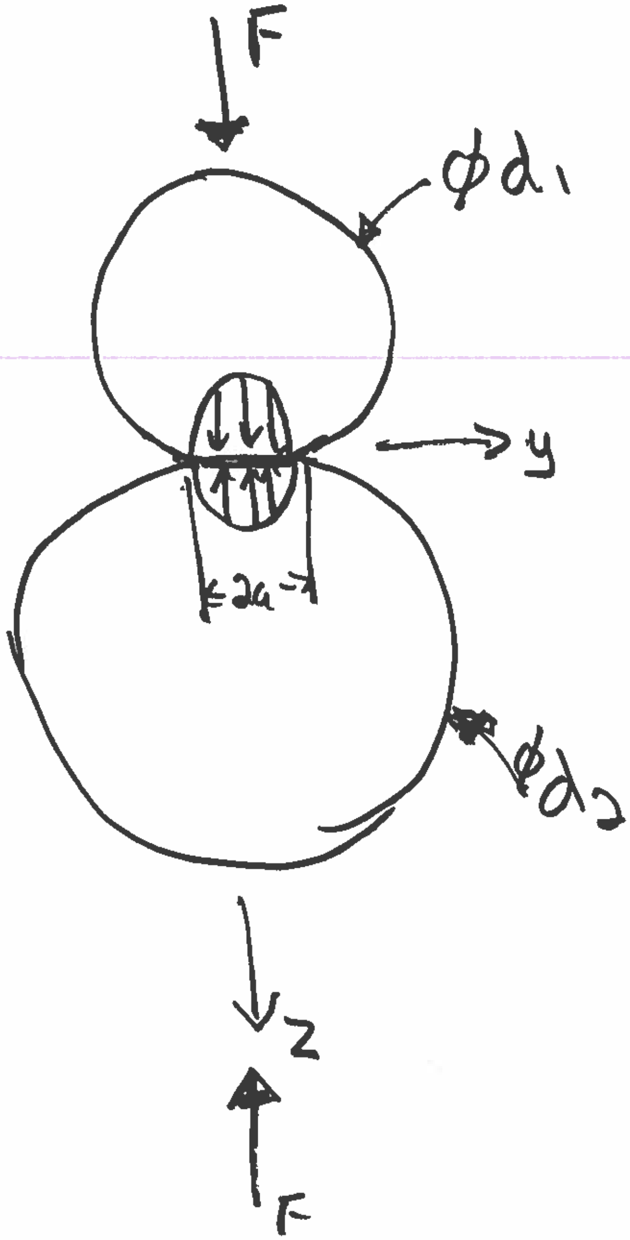
two radii of curvature



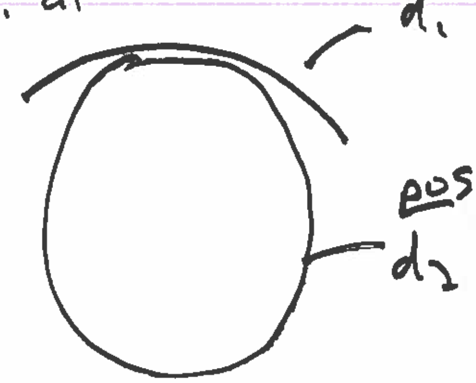




Special Case: Two Spheres



- circular area of contact
- hemispherical stress dis. at area

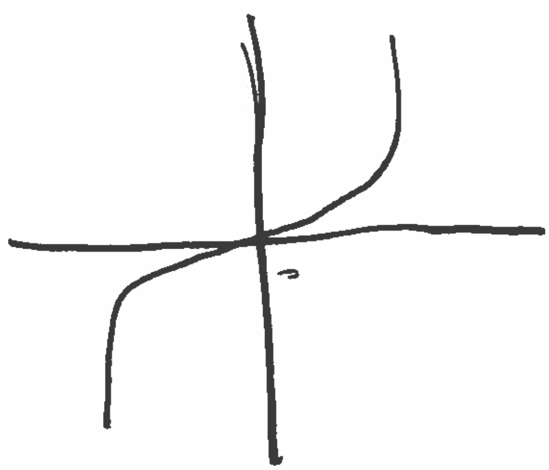


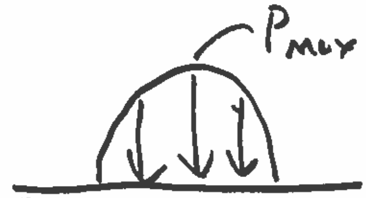
$$a = \sqrt[3]{\frac{3F}{8}\,\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}} \qquad \boxed{3\text{-}68}$$

subscripts match to potentially different diameter spheres

$d = \infty \Rightarrow$ flat surface

$d < 0 \Rightarrow$ internal spherical shape





$$P_{max} = \frac{3F}{2\pi a^2} \qquad 3\text{-}69$$

Along the z axis is max stress:

$$\sigma_1 = \sigma_2 = \sigma_x = \sigma_y =$$

$$-P_{max}\left[\left(1-\left|\frac{z}{a}\right|\tan^{-1}\frac{1}{|z/a|}\right)(1+\nu) - \frac{1}{2\left(1+\frac{z^2}{a^2}\right)}\right]$$

$$\sigma_3 = \sigma_z = -\frac{P_{max}}{1+\frac{z^2}{a^2}}$$

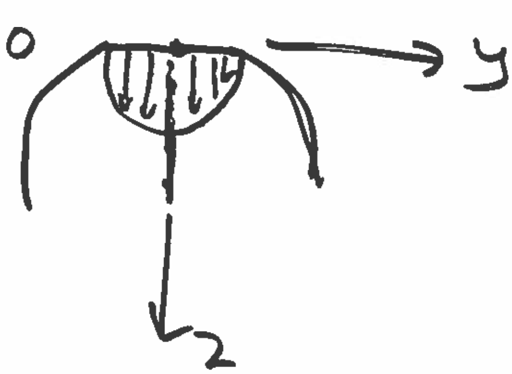


Fig 3-37 Sphere contact stress as function of depth into sphere

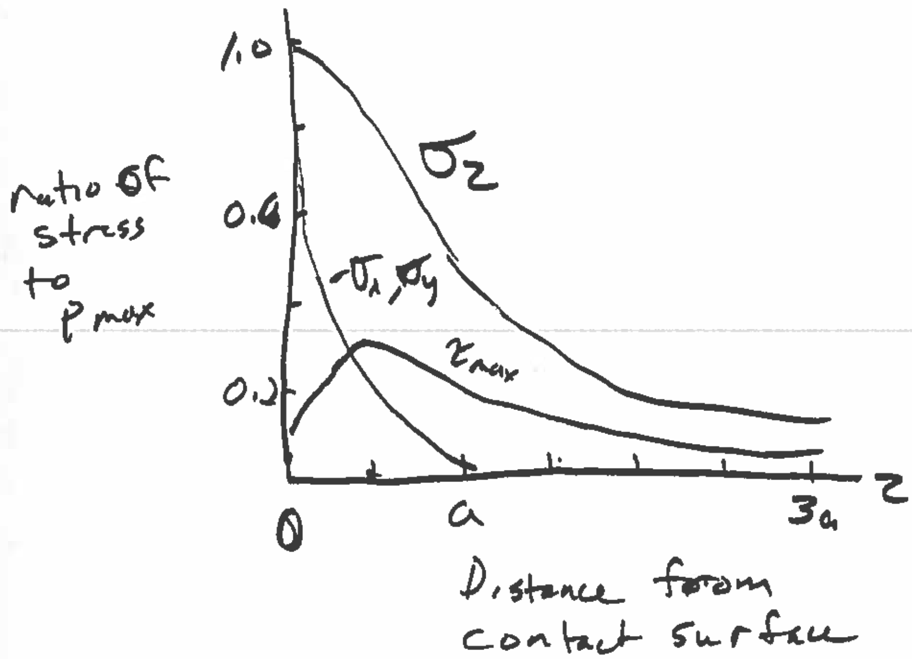
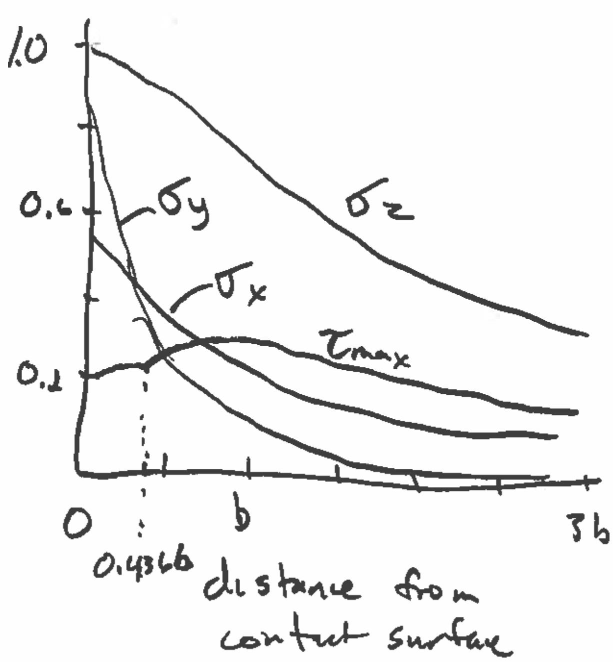


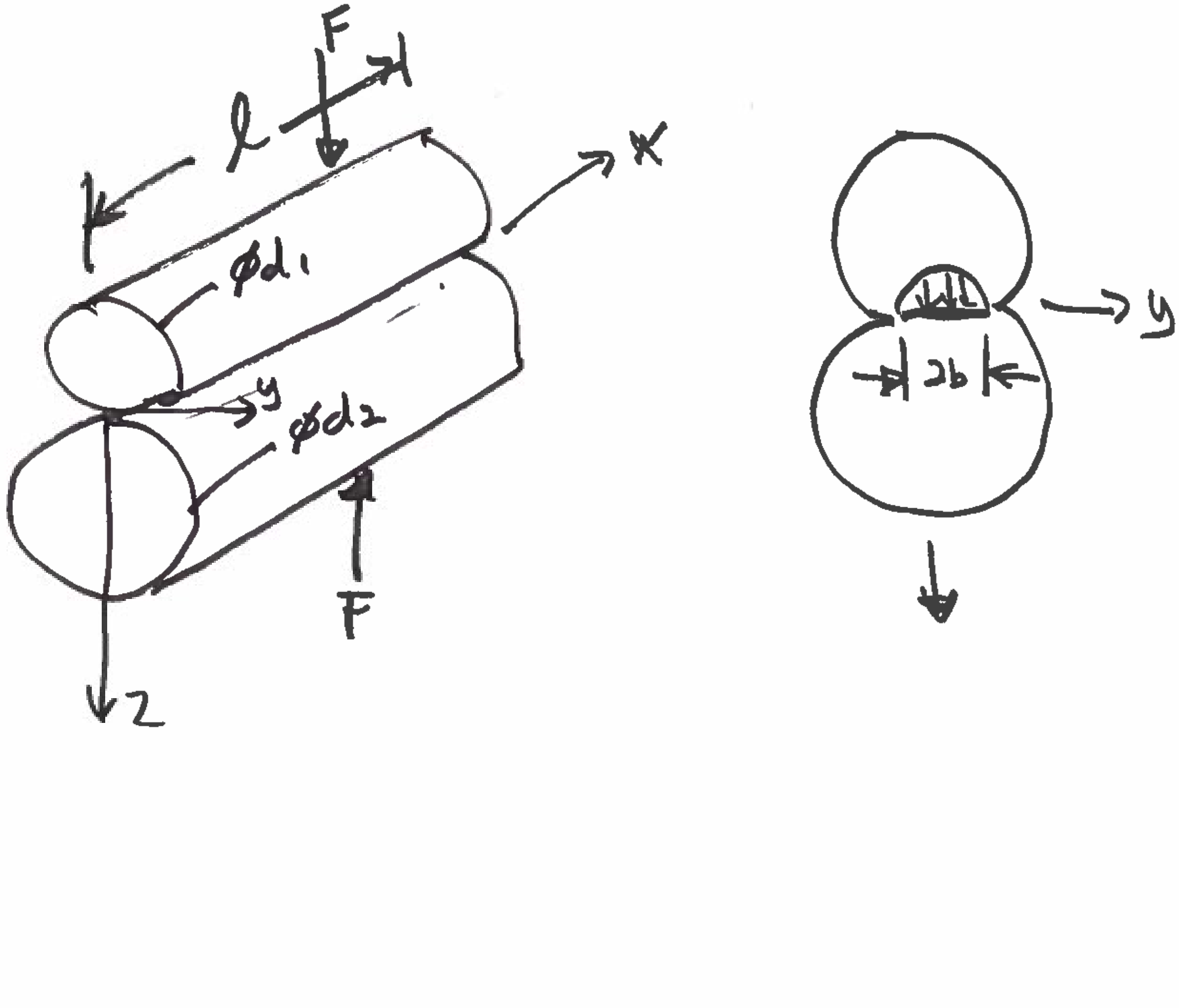
Fig 3-39 Cylindrical



$$\sigma_1 = \sigma_2 \Rightarrow \tau_{1/2} = 0$$

$$\tau_{max} = \tau_{1/3} = \tau_{2/3} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_2 - \sigma_3}{2}$$

Special Case Cylindrical Contact



$$b = \sqrt{\frac{2F}{\pi l} \frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}}$$ 3-73

$$P_{max} = \frac{2F}{\pi b l}$$

$$\sigma_x = -2\nu P_{max}\left(\sqrt{1+\frac{z^2}{b^2}} - \left|\frac{z}{b}\right|\right)$$ 3-75

$$\sigma_y = -P_{max}\left(\frac{1+2\frac{z^2}{b^2}}{\sqrt{1+\frac{z^2}{b^2}}} - 2\left|\frac{z}{b}\right|\right)$$ 3-76

$$\sigma_3 = \sigma_z = \frac{-P_{max}}{\sqrt{1+\frac{z^2}{b^2}}}$$

$0 \le z \le 0.436b$ $\quad \sigma_1 = \sigma_x$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

$z \geq 0.436b$ $\quad \sigma_1 = \sigma_y$

$$\tau_{max} = \frac{(\sigma_y - \sigma_z)}{2}$$

pitting: spalling due to fatigue in roller bearings

brinelling: permanent indentation of hard surface

spalling: flaking of material from surface

See course website for links to photos and more info