Deformation and Stiffness

- Designs for high rigidity
- Design for high flexibility

Rigidity

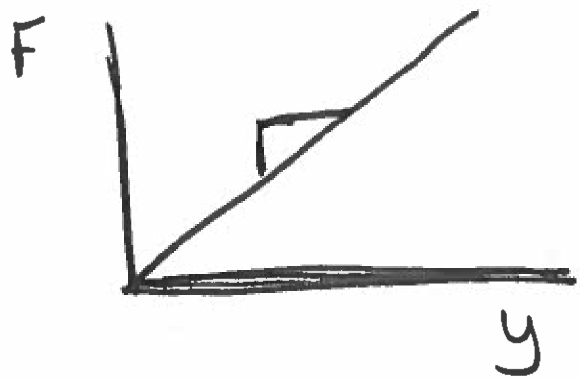
definition => $\dfrac{\text{deflexion}}{\text{load}}$ $\dfrac{x}{F}$, $\dfrac{\theta}{T}$

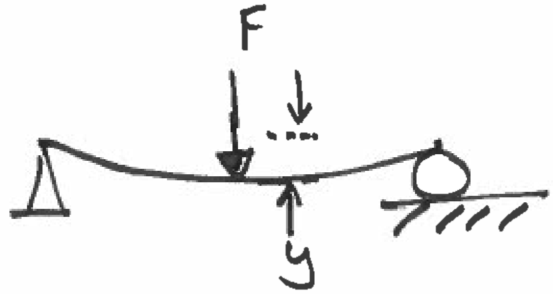
stiffness => $\dfrac{\text{load}}{\text{deflection}}$ $\dfrac{F}{x}$, $\dfrac{T}{\theta}$

Modulus of elasticity is reasonably good indicator of rigidity/stiffness

But => The actual stiffness depends on geometry!

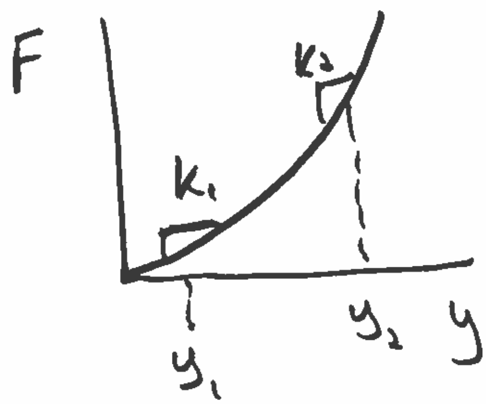
Linear Springs





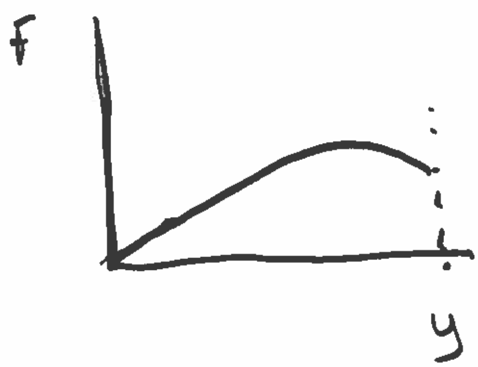
$$\frac{F}{y} = K$$

Nonlinear Springs

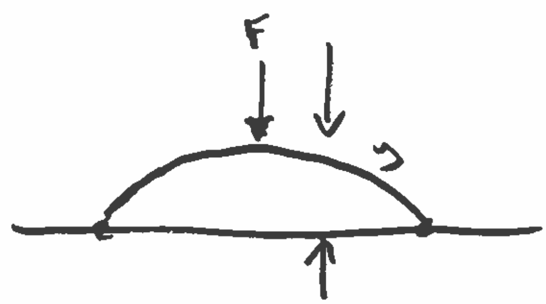


Stiffening spring

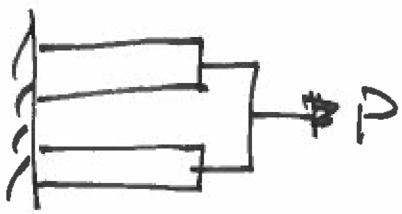
$K_2 > K_1$



Softening spring

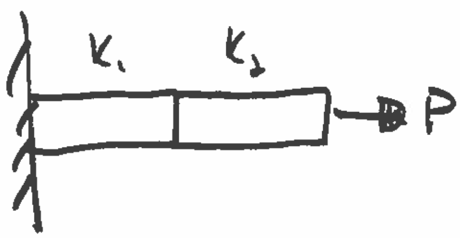


Parrallel



$$k = \sum_{i=1}^{N} k_i$$

Series



$$k = \frac{1}{\sum_{i=1}^{N} \left(\frac{1}{k_i}\right)}$$

Axial

$$\delta = \frac{F\ell}{AE} \qquad k = \frac{AE}{\ell}$$

AE: Axial rigidity

Torsional

$$\Theta = \frac{T\ell}{GJ} \Rightarrow k = \frac{GJ}{\ell}$$

GJ: Torsional rigidity

Bending (angular stiffness)

$$\theta = \frac{ML}{EI} \qquad k = \frac{EI}{L}$$

$EI \Rightarrow$ flexural rigidity

Bending (linear deflections)

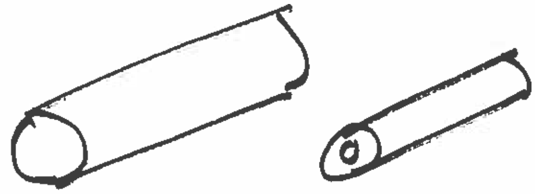
$$\delta = \frac{ML^2}{2EI} \qquad k = \frac{2EI}{L^2}$$

Cantilever load on end

$$\delta = \frac{PL^3}{3EI}$$

$$k = \frac{3EI}{L^3}$$

Relative Stiffness



① 1" OD round steel bar, 10" long tensile

$$K = \frac{AE}{l} \Rightarrow K = 23.6 \times 10^5 \frac{lb}{in}$$

② bending

$$K = \frac{3EI}{l^3} \Rightarrow K = 4.4 \times 10^3 \frac{lb}{in}$$

③ bending as cantilever w/ shear

$$K = \frac{9AG}{10\,l} \Rightarrow K = 81.3 \times 10^5 \frac{lb}{in}$$

⑤ torsion

$$K = \frac{JG}{l} \Rightarrow K = 1.13 \times 10^5 \frac{lb\,in}{rad}$$

⑥ chang to a tube w/ circular cross section

$$K = \frac{JG}{l} \Rightarrow K = \cancel{55}\; 55.3 \times 10^5 \frac{lb\,in}{rad}$$

Mid term

- Closed book
- I provide paper, you bring pencil (maybe calculator)
- Non-programmable calculator is allowed
- You'll be expected to remember equations
- 100 minutes
- True/False, Multiple choice, short answer
  - longer more like homework

Chapters

1-1 ~~[illegible]~~ 1-17

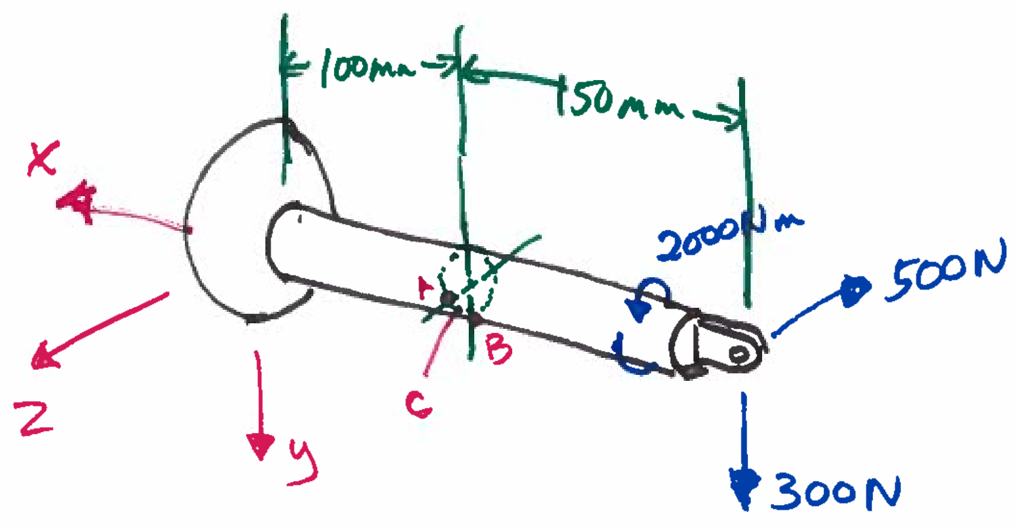
3-1 to 3-11

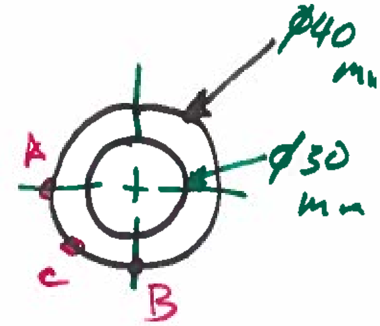
3-12 but exclude thin walled tubes

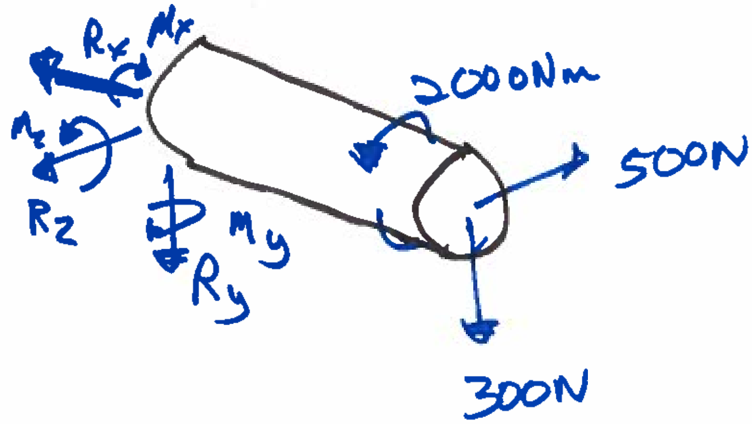
3-13 to 3-14

3-18 to 3-19

4-3 to 4-6







$R_x = 0$ axial

$R_y = -300N$ shear

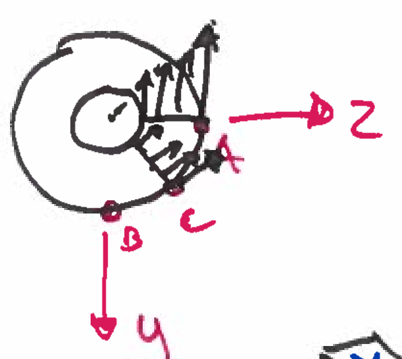
$R_z = 500N$ shear

$M_x = 2000Nm$ torsion

$M_y = 75Nm$ bending

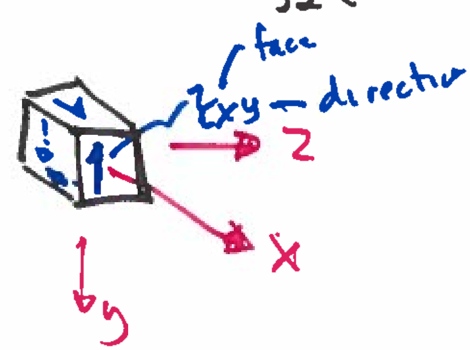
$M_z = 45m$ bending

Torsion

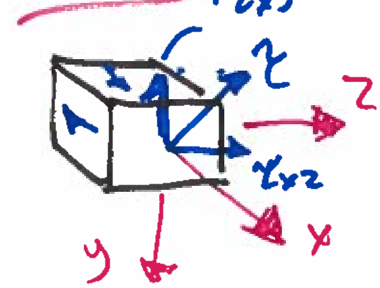


$$\tau = \frac{Tr}{J}$$

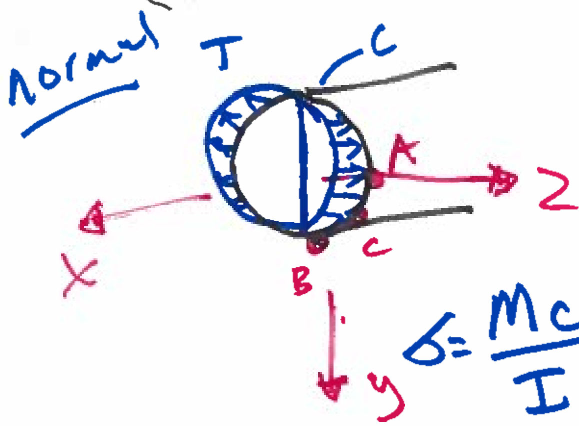
$$J = \frac{\pi}{32}(D^4 - d^4)$$



Point C



Bending y



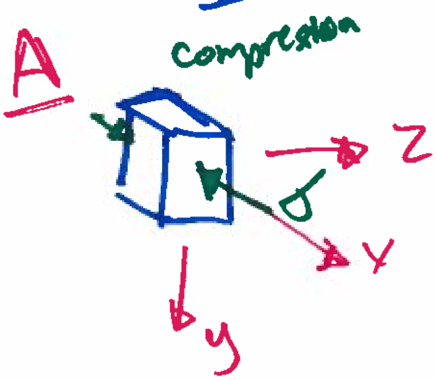
$$\sigma = \frac{Mc}{I}$$

A: max compression $\frac{Mr}{I}$

B: ~~zero~~ zero stress

C: $\frac{Mc_c}{I}$

<u>A</u>

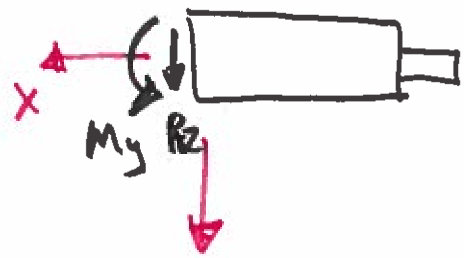


~~Bending Z~~

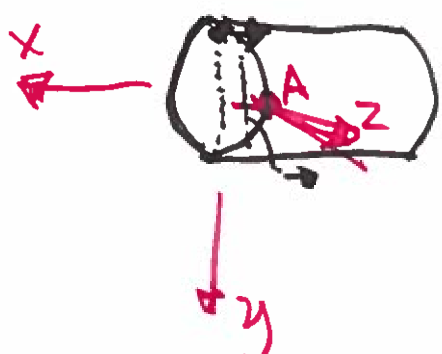
Shear Z

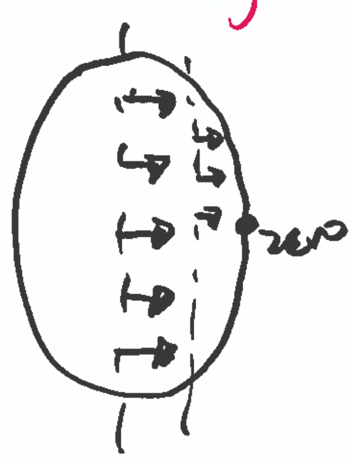
~~R_z = 500N~~ $R_z = 500N$

~~M_y~~ $M_y = 75Nm$



~~z~~





<u>Point A</u>

$$\tau_{xz}\big|_A = 0$$