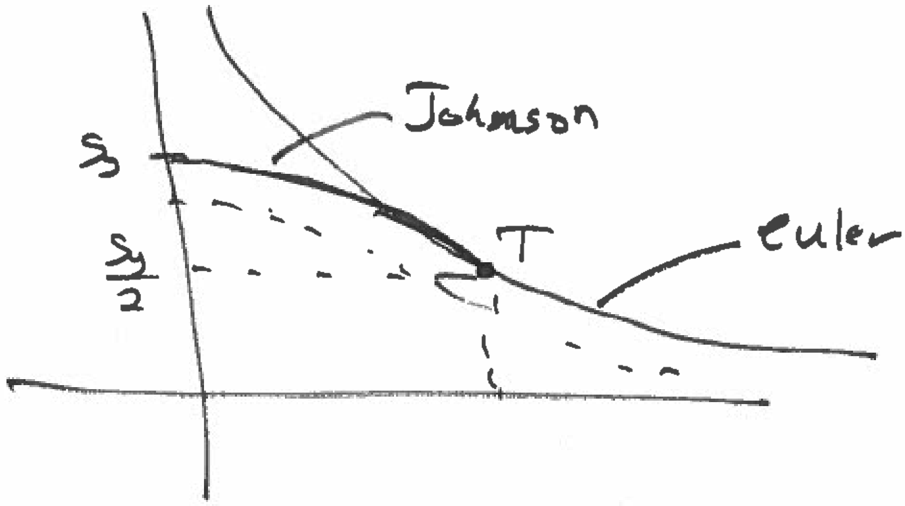

Johnson
S_y
$\frac{S_y}{2}$
T
euler

Static Failure Theories 5-1 → 5-7

Fail:

- distortion
- cracks
- rupture
- etc

Best way to know when an element will fail:

test in exact conditions
operating conditions

Expensive!!

→ human safety
→ high volume manufacturing

bear the expense

Given tensile, compressive, and/or shear strength for a given material how do we choose what to check given the state of stress in the element: $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{zx}, \tau_{yz}$

or $\sigma_1, \sigma_2, \sigma_3$ or $\tau_{1/2}, \tau_{2/3}, \tau_{1/3}$

Theories are broken into two categories:

Ductile

$\varepsilon_f \geq 0.05$

$S_{yt} = S_{yc} = S_y$

failure: S_y

ε_f : strain at ultimate failure

Brittle

$\varepsilon_f < 0.05$

S_{ut}, S_{uc}

fail: ultimate strength

- Maximum Shear Stress (MSS)
- Distortion Energy Theory (DE)
- Ductile Coulomb-Mohr Theory (DCM)

Maximum Shear Stress Theory

Yielding begins when τ_{max} at any element equals or exceeds the τ_{max} in a tensile test specimen of the same material, surface finish, ambient temperature, ...) when it begins to yield.

If $(\tau_{max})_{general} \geq (\tau_{max})_{\text{tensile specimen}} \Rightarrow$ failure due to yielding

For a tensile specimen $\Rightarrow \tau_{max} = \dfrac{S_y}{2}$

For a 3D state of stress:

$$\tau_{max} = \tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\therefore \quad \tau_{1/3} \geq \frac{S_y}{2} \Rightarrow \boxed{\sigma_1 - \sigma_3 \geq S_y}$$

↳ fails

$$\sigma_1 - \sigma_3 = \frac{S_y}{n_{mss}} \Rightarrow \boxed{n_{mss} = \frac{S_y}{\sigma_1 - \sigma_3}}$$

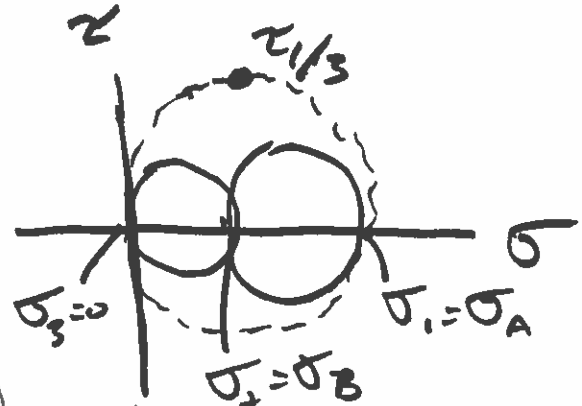
Plane stress

- one principal stress is equal to zero

Given two prin. stresses: σ_A, σ_B

where $\sigma_A > \sigma_B$

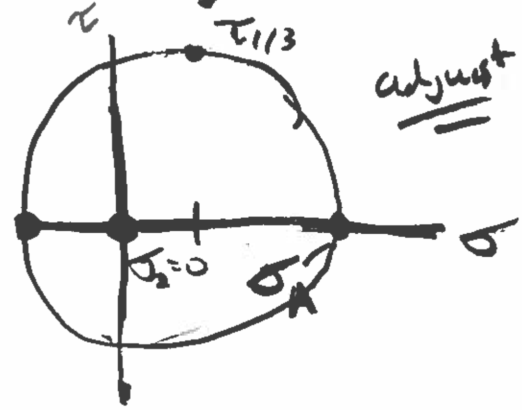
1) $\sigma_A \geq \sigma_B \geq 0$



$\boxed{\sigma_A \geq S_y \Rightarrow \text{failure}}$

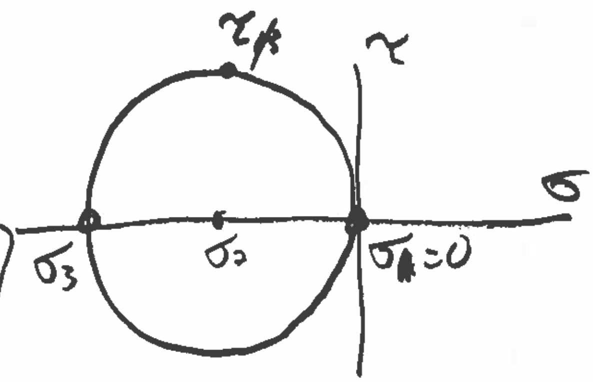
2) $\sigma_A \geq 0 \geq \sigma_B$

$\boxed{\sigma_A - \sigma_B \geq S_y}$ $\Rightarrow$ failure $\bar{\sigma}_B$

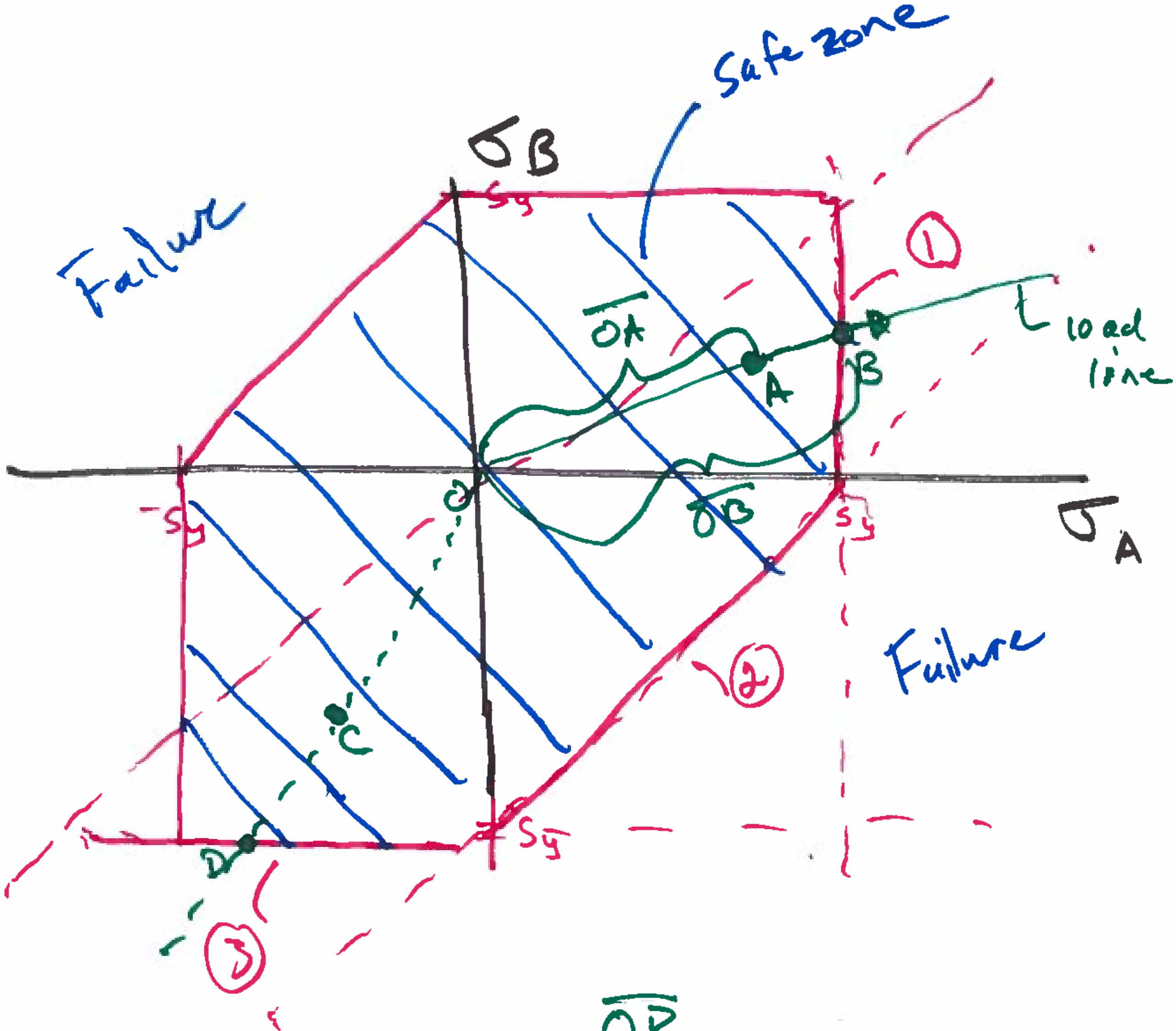


adjust

3) $0 \geq \sigma_A \geq \sigma_B$



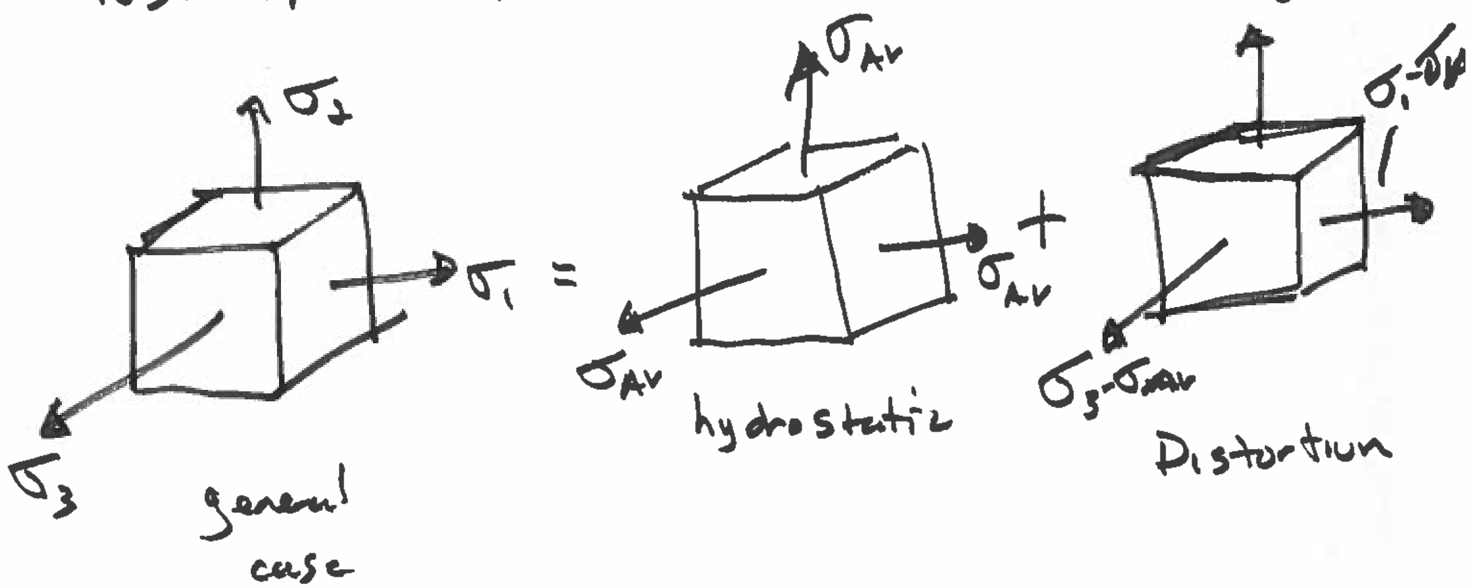
$\boxed{\sigma_B \leq -S_y \Rightarrow \text{failure}}$



$$n_{MSS} = \frac{\overline{OB}}{\overline{OA}}$$

Distortion Energy Theory

Yielding begins when the distortion strain energy per unit volume exceeds the distortion strain energy per unit volume for yield in a simple tensile test specimen.



$$\sigma_{AV} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

The distortion portion fails long before the hydrostatic portion.

Strain Energy

$$U = \frac{1}{2}\epsilon\sigma$$

$$U = \frac{1}{2}\left[\epsilon_1\sigma_1 + \epsilon_2\sigma_2 + \epsilon_3\sigma_3\right]$$

$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

Volumetric strain energy

$$U_v = \frac{3}{2E}\,\sigma_{Av}^2(1-2\nu)$$

$$U_d = U - U_v =$$

$$\frac{1+\nu}{3E}\left[\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}\right]$$

Distortion Strain for tensile test, $\sigma_1 = S_y$, $\sigma_2 = \sigma_3 = 0$

$$U_d \geq \frac{1+\nu}{3E}\,S_y^2$$

$$\underbrace{\left[\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}\right]^{1/2}}_{\sigma'} \geq S_y$$

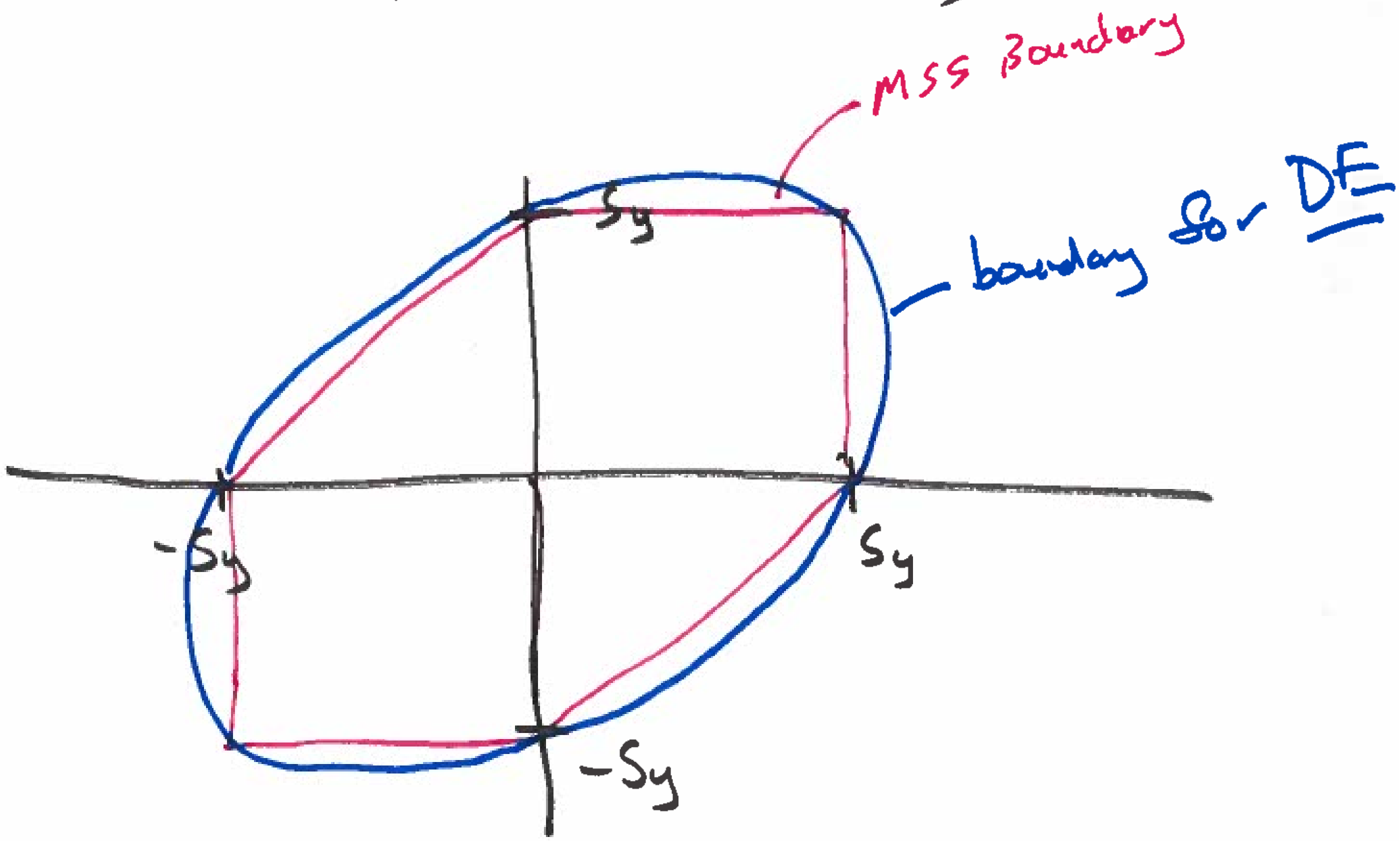
σ': Von Mises effective stress

$$n_{de} = \frac{S_y}{\sigma'}$$

Plane stress

$\sigma_1 = \sigma_A$, $\sigma_2 = \sigma_B$

$$\sigma' = (\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2)^{1/2}$$



MSS is more conservative than DE.

Assume pure shear :

$$\text{MSS} \Rightarrow \quad \underbrace{S_{sy}}_{\text{shear yield strength}} = 0.5\, S_y$$

$$\text{DE} \Rightarrow \quad S_{sy} = 0.577\, S_y$$