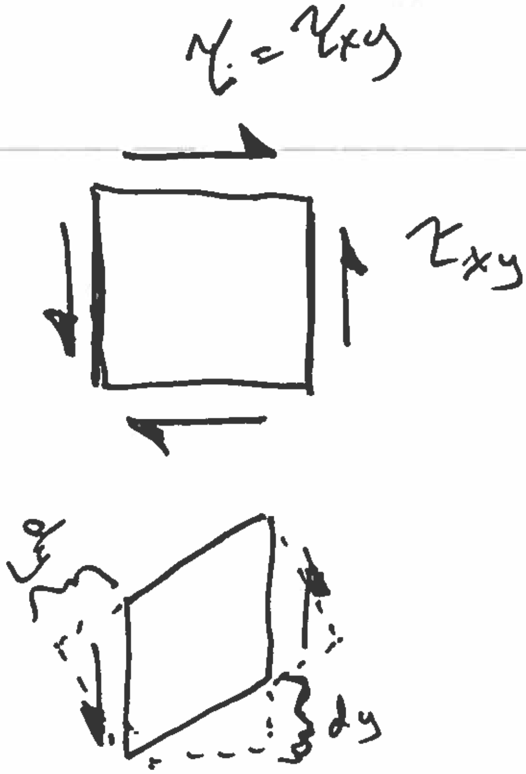
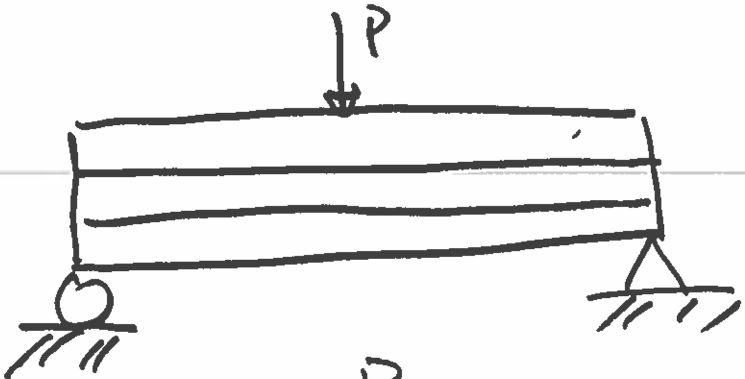
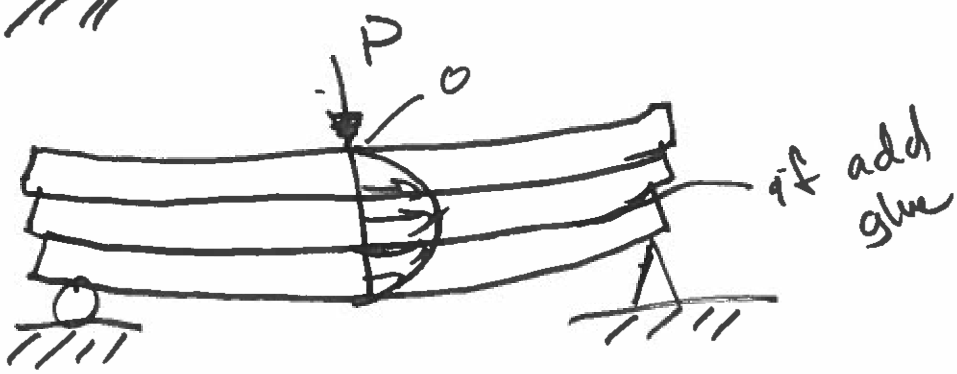
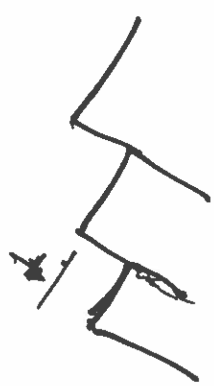
Complementary Shear



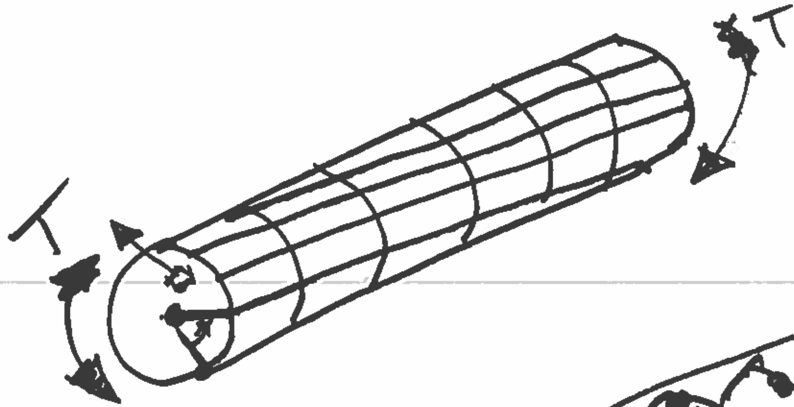
Transverse Shear in Bending

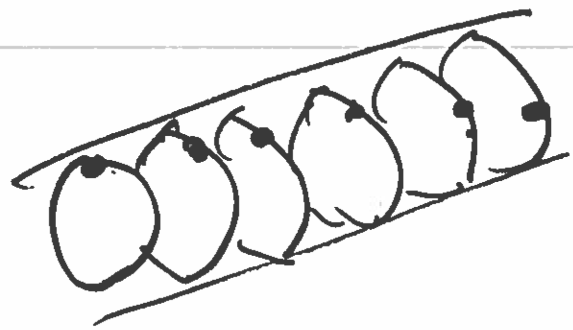


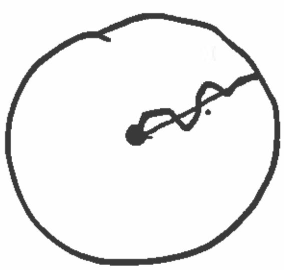


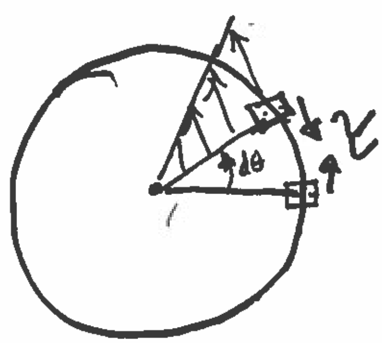


Torsion and Shear



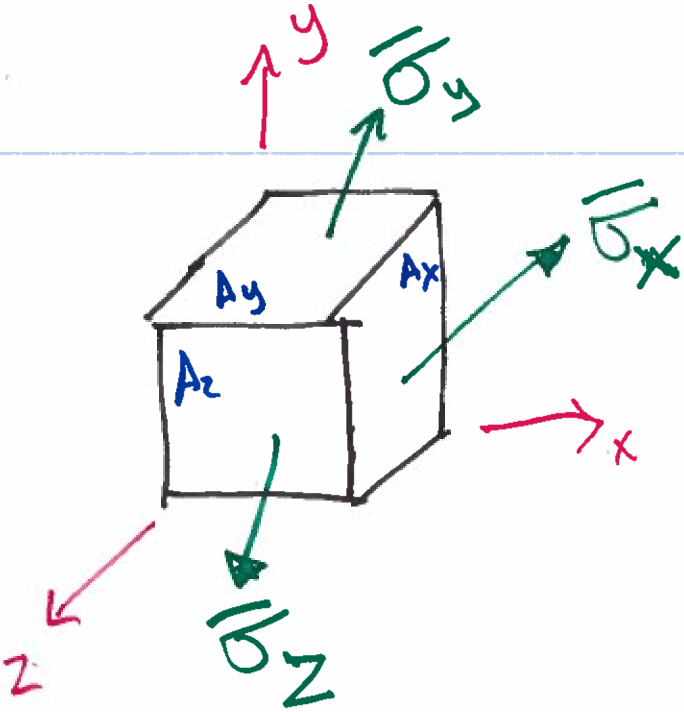




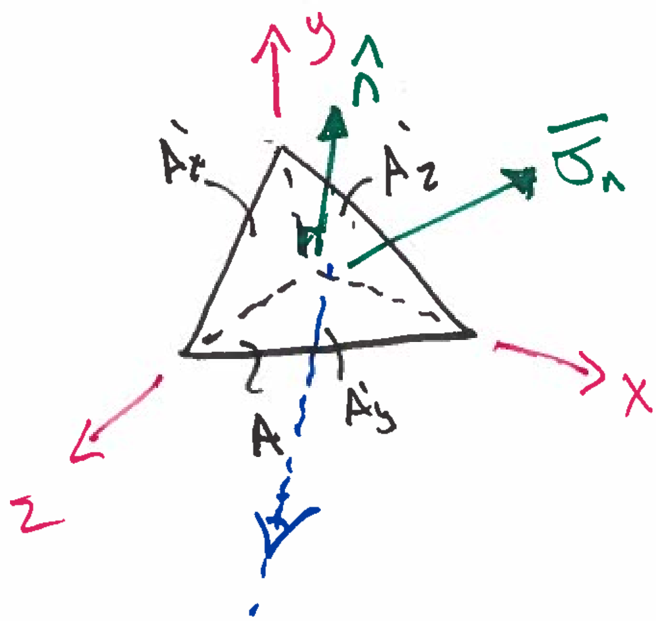


Computing stress on an arbitrary plane

$$\bar{\sigma}_x = \underbrace{\sigma_x \hat{i}}_{\text{normal}} + \underbrace{\bar{\tau}}_{\text{shear}}$$



normal & shear stress on the face



$$\hat{n} = l\hat{i} + m\hat{j} + n\hat{k}$$

$$|\hat{n}| = 1$$

$$\left.\begin{matrix} l \\ m \\ n \end{matrix}\right\} \text{direction cosines}$$

$$\sigma_n = \hat{n} \cdot \bar{\sigma}_n$$

$$\bar{\sigma}_x = \sigma_x \hat{i} + \tau_{xy} \hat{j} + \tau_{xz} \hat{k}$$

$$\vdots$$

$$\bar{\sigma}_z$$

$$\boxed{\begin{aligned} \sigma_n = {} & l^2 \sigma_x + m^2 \sigma_y + n^2 \sigma_z \\ & + 2lm\tau_{xy} + 2ln\tau_{xz} \\ & + 2mn\tau_{yz} \end{aligned}}$$

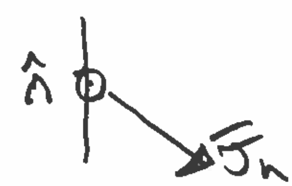
$$\Sigma F = 0 = \underbrace{A\bar{\sigma}_n - A'_x\bar{\sigma}_x - A'_y\bar{\sigma}_y - A'_z\bar{\sigma}_z}$$

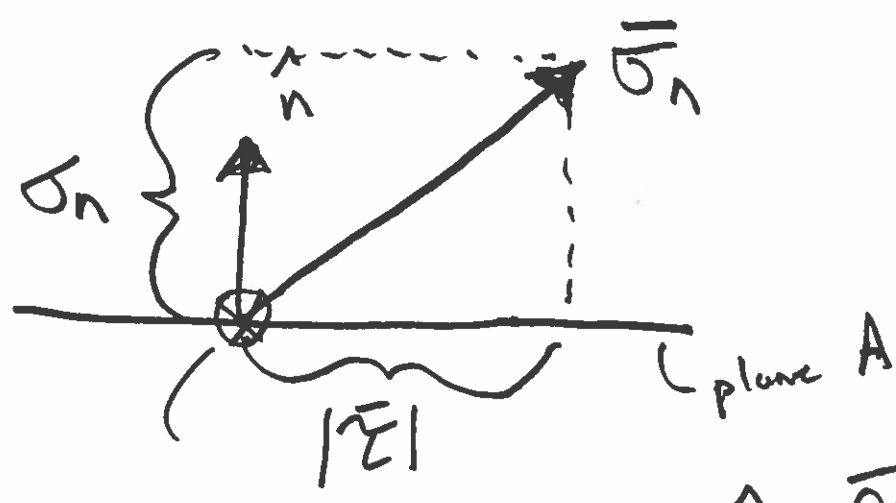
Project A onto xy, xz, yz planes

$$A'_x = lA \quad A'_y = mA \quad A'_z = nA$$

$$\boxed{\therefore \bar{\sigma}_n = l\bar{\sigma}_x + m\bar{\sigma}_y + n\bar{\sigma}_z}$$

$\bar{\sigma}_n$: stress associated with the face A (defined by $\hat{n}$), contains ~~the normal and shear stress~~

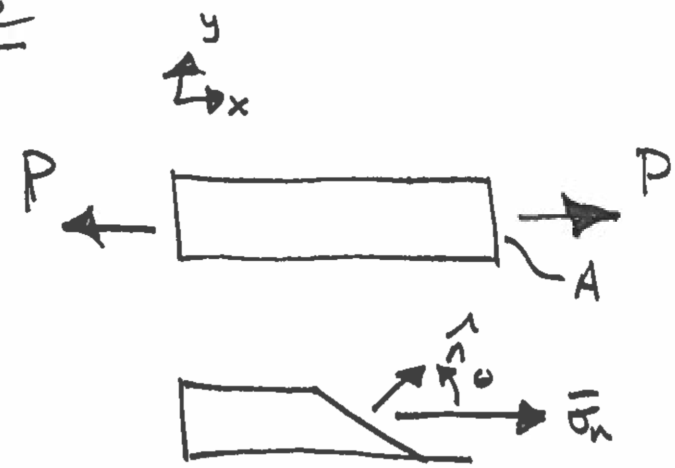




$$(\hat{n} \times \bar{\sigma}_n) \times \hat{n} = \bar{\tau}$$

$$|\bar{\sigma}_n|^2 = \sigma_n^2 + |\bar{\tau}|^2$$

Example



Find a) $\bar\sigma_n$ as function of θ

b) σ_n and $|\bar\tau|$

$$\hat n = \underbrace{\cos\theta}_{l}\,\hat i + \underbrace{\sin\theta}_{m}\,\hat j \qquad n=0$$

$$\bar\sigma_n = l\bar\sigma_x + m\cancel{\bar\sigma_y}^{\,0} + n\cancel{\bar\sigma_z}^{\,0} \qquad \bar\sigma_x = \frac{P}{A}\hat i$$

$$\bar\sigma_n = l\bar\sigma_x = \boxed{\cos\theta\,\frac{P}{A}\,\hat i}$$

$$\bar\sigma_n \cdot \hat n = \boxed{\sigma_n = \frac{P}{A}\cos^2\theta}$$

$$|\bar\tau|^2 + \sigma_n^2 = |\bar\sigma_n|^2$$

$$|\bar\tau| = \sqrt{|\bar\sigma_n|^2 - \sigma_n^2}$$

$$= \sqrt{\cos^2\theta\,\frac{P^2}{A^2} - \frac{P^2}{A^2}\cos^4\theta}$$

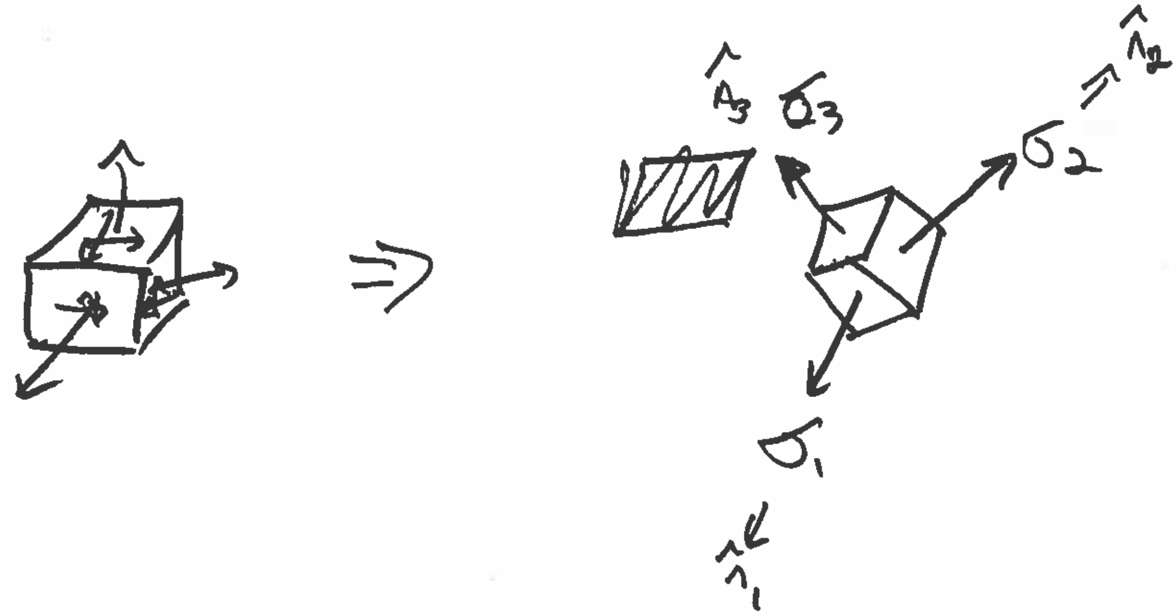
$$= \frac{P}{A}\cos\theta\sqrt{\underbrace{1-\cos^2\theta}_{\sin^2\theta}}$$

$$\boxed{= \frac{P}{A}\cos\theta\sin\theta}$$

Principal Stresses

From the theory of Elasticity

At any point with a general state of stress, an element can be oriented such that the shear components vanish.



principal stresses: $\sigma_1 \geq \sigma_2 \geq \sigma_3$

principal directions: $\hat{n}_1, \hat{n}_2, \hat{n}_3$

Eigenvalue and eignenvector problem

A

$$\begin{bmatrix} & & \\ & 3 \times 3 & \\ & & \end{bmatrix} \rightarrow \begin{bmatrix} \sigma_1 & & 0 \\ 0 & \sigma_2 & \\ & & \sigma_3 \end{bmatrix}$$

symmetric

$$A\,x = 0$$

$\uparrow$

$\hat{n}_1$

$\hat{n}_2$

$\hat{n}_3$