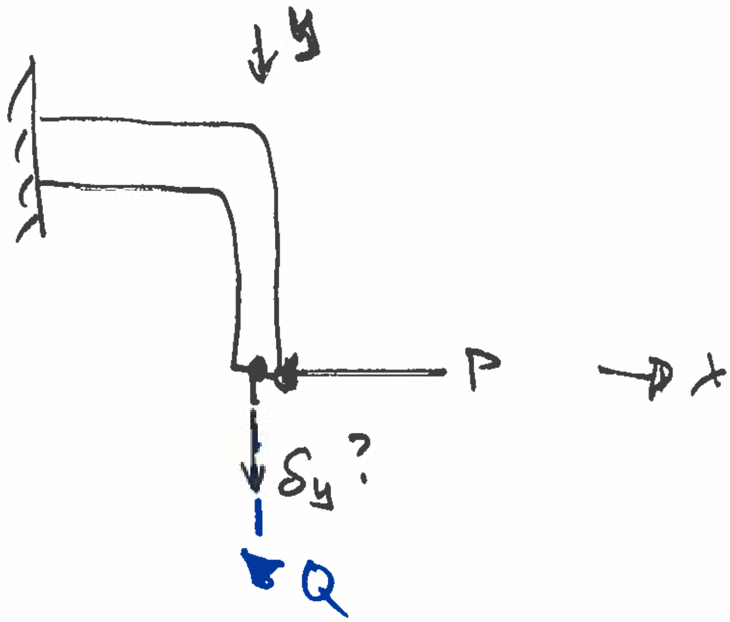
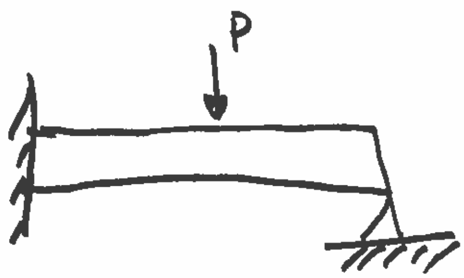
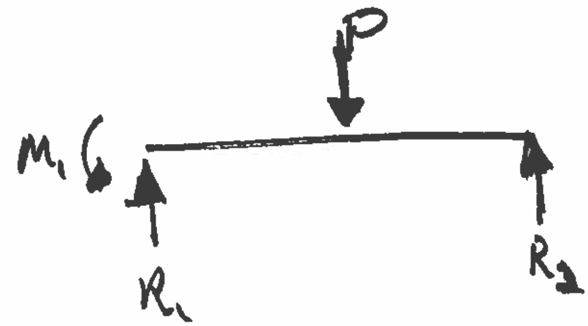


$$\delta_Q = \frac{\partial U}{\partial Q} = f(Q)\Big|_{Q=0}$$

Static Indeterminate Problems





$$\delta = \frac{PL}{AE}$$

Procedure

1. Choose one the reactions as redundant.

2. Write the static equilibrium equations for remaining reactions as functions of the applied loads and the redundant reactions.

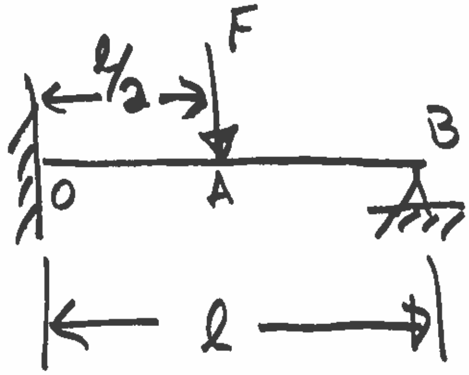
3. Apply Castigliano's Thereom to total strain energy.

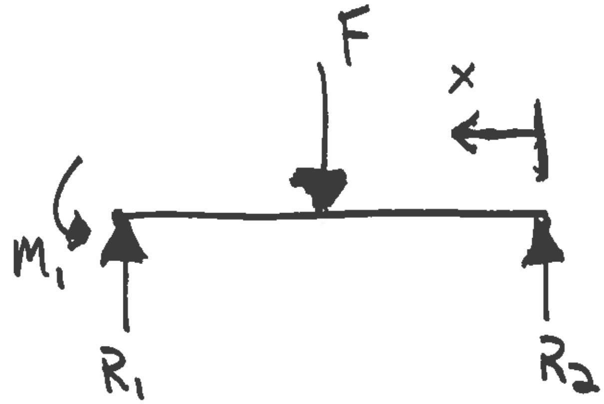
$$\frac{\partial U}{\partial R_{unk}} = 0$$

unk

Example 4-14 but also read $\underline{\text{Ex}}$ 4-15

Example 4-14





1. Choose M_1 to be redundant.

2. $R_1 = \dfrac{F}{2} + \dfrac{M_1}{\ell}$ $\qquad$ $\Sigma F = 0$

   $R_2 = \dfrac{F}{2} - \dfrac{M_1}{\ell}$ $\qquad$ $\Sigma M = 0$

3. $\Theta_0 = \dfrac{\partial U}{\partial M_1} = 0$ $\qquad$ $\boxed{U \Rightarrow \text{total strain Energy}}$

$$\begin{cases} M = \left(\dfrac{F}{2} - \dfrac{M_1}{\ell}\right)x & 0 \le x \le \dfrac{\ell}{2} \\ M = \left(\dfrac{F}{2} - \dfrac{M_1}{\ell}\right)x - F\left(x - \dfrac{\ell}{2}\right) & \dfrac{\ell}{2} \le x \le \ell \end{cases}$$

$$\Theta_i = \int \frac{1}{EI}\left(M \frac{\partial M}{\partial M_i}\right) dx$$

USE FORM WITH PARTIAL MOVED INSIDE THE INTEGRAL.

$$\frac{\partial M}{\partial M_1} = -\frac{x}{l}$$

$$\theta_0 = \frac{\partial u}{\partial M_1}$$

$$\theta_0 = \frac{1}{EI}\left[\int_0^{l/2}\left(\frac{F}{2} - \frac{M_1}{l}\right)x\left(-\frac{x}{l}\right)dx + \int_{l/2}^{l}\left[\left(\frac{F}{2} - \frac{M_1}{l}\right)x - F\left(x - \frac{l}{2}\right)\right]\left(-\frac{x}{l}\right)dx\right]$$

$$= 0$$

$$\left(\frac{F}{2} - \frac{M_1}{l}\right)\int_0^{l} x^2 dx - F\int_{l/2}^{l}\left(x - \frac{l}{2}\right)x\,dx = 0$$

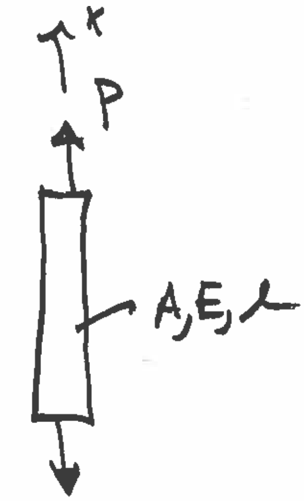
<u>Integrate</u>

$$\left(\frac{F}{2} - \frac{M_1}{l}\right)\frac{l^3}{3} - \frac{F}{3}\left[l^3 - \left(\frac{l}{2}\right)^3\right] + \frac{Fl}{4}\left[l^2 - \left(\frac{l}{2}\right)^2\right] = 0$$

$$\boxed{M_1 = \frac{3FL}{16} \Rightarrow R_1 = \frac{11F}{16} \quad R_2 = \frac{5F}{16}}$$

Compressive Loading

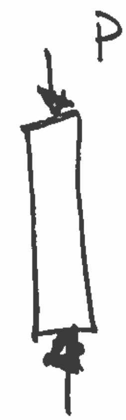
Tensile: $\underline{\sigma_x} = \frac{P}{A}$ $\delta = \frac{PL}{AE}$



σ_x can't be used to find the max stress and factor of safety wrt yield strength.

Compressive:

Above doesn't generally apply. The member will actually fail at a much lower load.



This is called buckling.

Buckling occurs suddenly.

Very dangerous!!

Short columns — fails in compressive

intermediate colums — may fail in buckling

long columns — definately fails in buckling categorized "slenderness ratio"

$$S_r = \frac{l}{k}$$

l — length of column

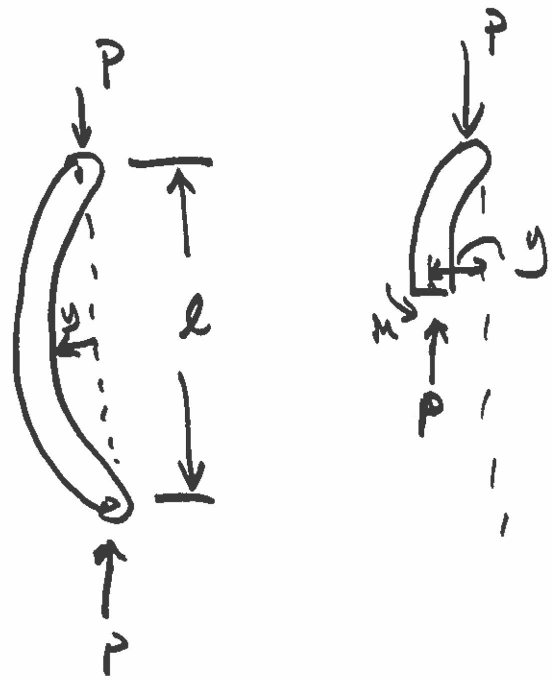
k — radius of gyration

$$k = \sqrt{\frac{I_{min}}{A}}$$

Second moment of area

for $S_r \lesssim 10 \Rightarrow$ short

Long Columns



$M = Py$

small deflections

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\frac{d^2y}{dx^2} + \left(\frac{P}{EI}\right)y = 0 \Rightarrow$$ second order differential equation

$$y = A\sin\left[\sqrt{\frac{P}{EI}}\,x\right] + B\cos\left[\sqrt{\frac{P}{EI}}\,x\right]$$

Boundary Conditions

$y=0$ @ $x=0$

$y=0$ @ $x=l$

nontrivial sol:

$$\sin\sqrt{\frac{P}{EI}}\,l = 0$$

$$\sqrt{\frac{P}{EI}}\cdot l = n\pi \qquad n = 1, 2, 3, \ldots$$

$$\boxed{P_{cr} = \frac{\pi^2 EI}{l^2}}$$

<u>Euler Column Formula</u>

Normalize wrt to A

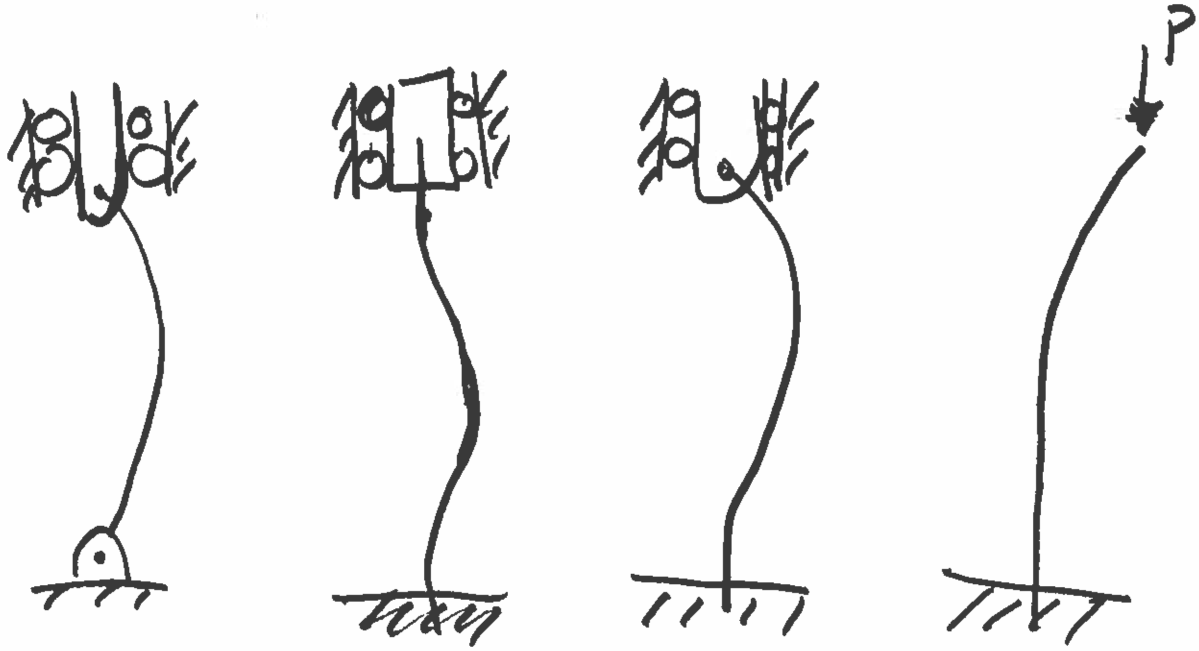
$$\boxed{\frac{P_{cr}}{A} = \frac{C\pi^2 E}{S_r^2}}$$

factor that accounts for end condition

Slenderness ratio

"buckling strength" for a specific column

Table 4-2 Fig 4-18



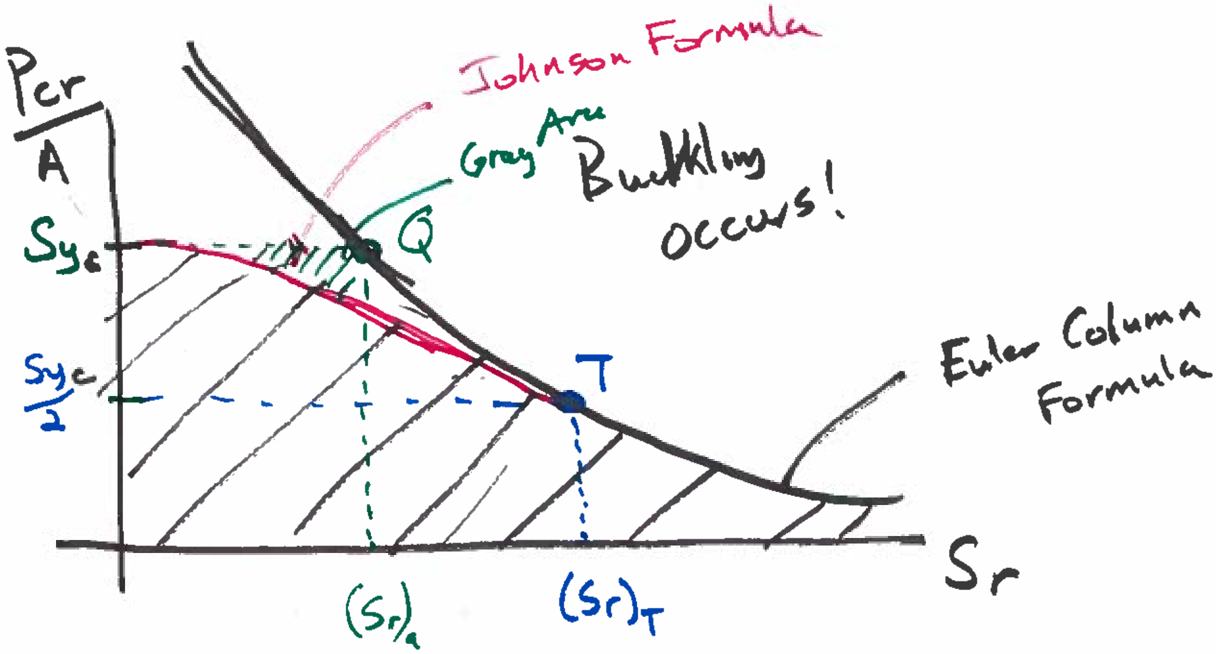
C=1 C=4 C=2 $C=\frac{1}{4}$

conservative

C= 1.2 ⇐ can use

Centrally loaded!

What about intermediate?



$S_r \geq (S_r)_T$: Euler Column formula applies

Johnson Formula

Buckling will generally not occur below the complete composite curve

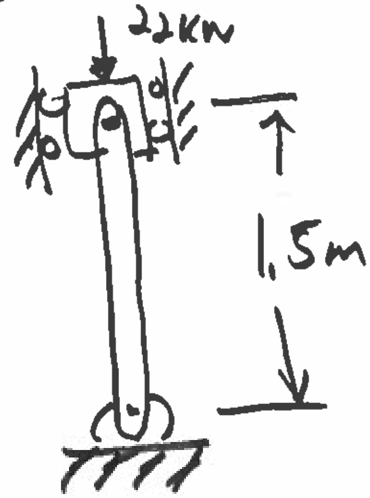
$$\frac{P_{cr}}{A} = S_{y_c} - \frac{1}{CE}\left(\frac{S_{y_c} S_r}{2\pi}\right)$$

Example 4-17

Find diameter of round column 1.5m long that is carrying a compressive load of 22KN.

~~To ensure factor of safety of~~

To ensure the design factor is 4.



$c = 1$

$S_y = 500\ MPa$

$E = 207\ GPa$

$$P_{cr} = n_d P = 4(22) = 88\ kN$$

Euler Formula

$$d = \left(\frac{64 P_{cr} l^2}{\pi^2 C E} \right) = 37.48\ mm$$

preffered size table => 40 mm

$$S_r = \frac{l}{k} = \frac{l}{d/4} = 150$$

$k = \sqrt{\frac{I}{A}}$

Is Euler Formula valid?

$$\left(\frac{l}{k}\right)_T = \left(\frac{2\pi^2 C E}{S_y}\right)^{1/2} = 90.4$$

$$\frac{l}{k} > \left(\frac{l}{k}\right)_T \checkmark$$ Euler is ok

$$\boxed{d = 40 \text{ mm}}$$