MAE 223: L1-01

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Course objs: Increase ability to analyze (write and solve EoM) for complex multibody systems.

1. Direct dynamics $\bar{F} \Rightarrow m\bar{a}$ (known)
2. Inverse dynamics $m\bar{a} \Rightarrow \bar{F}$

3 facets to solution:

1. generation of a conceptual model
2. use of principles of mechanics to generate differential equations of motion
3. extraction of desired info (how model evolves/changes in time)

Systems are made up of <u>particles</u> and coupled <u>rigid bodies</u>.

Particle: object with mass but no dimensions or extent

Rigid Body: has mass but also extent (distributed mass in space)

<u>Newton's 2nd Law</u>: EoM

$$\bar{F} = m\bar{a} = \frac{d}{dt}\bar{P} \qquad \bar{M}_P = \frac{d}{dt}\bar{H}_P$$

<u>Newton's</u> <u>Euler</u>

Kane's method to derive the EoM.

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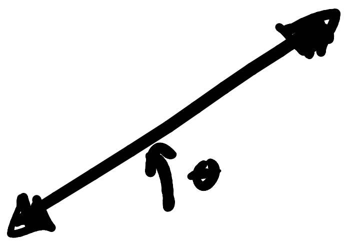
Vectors

3 characteristics:

1 magnitude
2 orientation
3 sense

$\left.\begin{matrix} \text{2 orientation} \\ \text{3 sense} \end{matrix}\right\}$ direction



Notation: scalars a, b, c
vector $\bar{a}, \bar{b}, \bar{c}$

Vector is equal if all 3 characteristics are the same.

Product of scalar & vector

$\bar{b} = k\bar{a}$ $\Rightarrow$ orientiation of $\bar{a}$ and $\bar{b}$ are the same

Unit vector: mag $= 1$

$|\bar{n}| = 1$

write $\bar{n}$ as $\hat{n}$ — hat

$$\frac{\bar{a}}{|\bar{a}|} = \hat{a}$$

commutative $\bar{a} + \bar{b} = \bar{b} + \bar{a}$
distributive $\lambda(\bar{a} + \bar{b}) = \lambda\bar{a} + \lambda\bar{b}$
associative $(\bar{a} + \bar{b}) + \bar{c} = \bar{a} + (\bar{b} + \bar{c})$

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Reference Frames

1. references frames and rigid bodies are interchangeable
2. Every RB can serve as a RF and every RF can be a rigid body
3. RF $\neq$ coordinate system (there many CS in a RF)

→ How is a vector a "function" of a scalar variable?

$\bar{\omega}$ is a vector function of scalar variable, q, in a reference frame A if, when q changes, $\bar{\omega}$ changes when viewed in A.

if I change q does $\bar{\omega}$ change?

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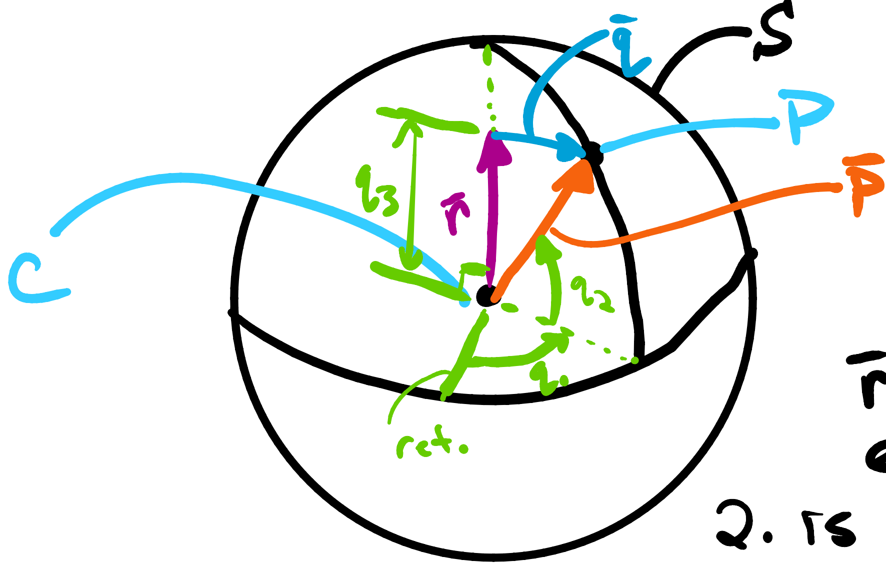
Example sphere, contain point P



1. is $\bar{p}$ a vector function of q_1, q_2 in S?

   Yes

   $\bar{r}$ is normal to equatorial plane

2. Is $\bar{p}$ a function of q_3 in S?

3. $\bar{q}$ a vector function of q_1, q_2, q_3 in S?

$\bar{r}$ a vector function of q_3 in S but independent of q_1, q_2 in S

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Scalar Functions

Given a RF A and vector $\overline{V}$ which is a function of n scalars $q_1, q_2, \ldots, q_n$ in A.

Let $\hat{a}_1, \hat{a}_2, \hat{a}_3$ be a set of non-parallel non-coplanar unit vectors fixed in RF A. $\hat{a}_1, \hat{a}_2, \hat{a}_3$ are constant wrt A

Then there are three unique scalar functions V_1, V_2, V_3 of $q_1, \ldots, q_n$ such that $\overline{V} = \underbrace{V_1}_{\text{component of } \overline{V}} \hat{a}_1 + V_2 \hat{a}_2 + \underbrace{V_3}_{\text{measure number of } \hat{a}_3 \text{ in vector } \overline{V}} \hat{a}_3$

Says how $\overline{V}$ can be a function of the scalars q_i in A only through measure numbers.

$$V_i = f(q_1, \ldots, q_n)$$

$$\boxed{\text{If any measure number } V_i \text{ is a function of } q_j \text{ then } \overline{V} \text{ is a function of } q_j \text{ in A}}$$