MAE 223: L2-01

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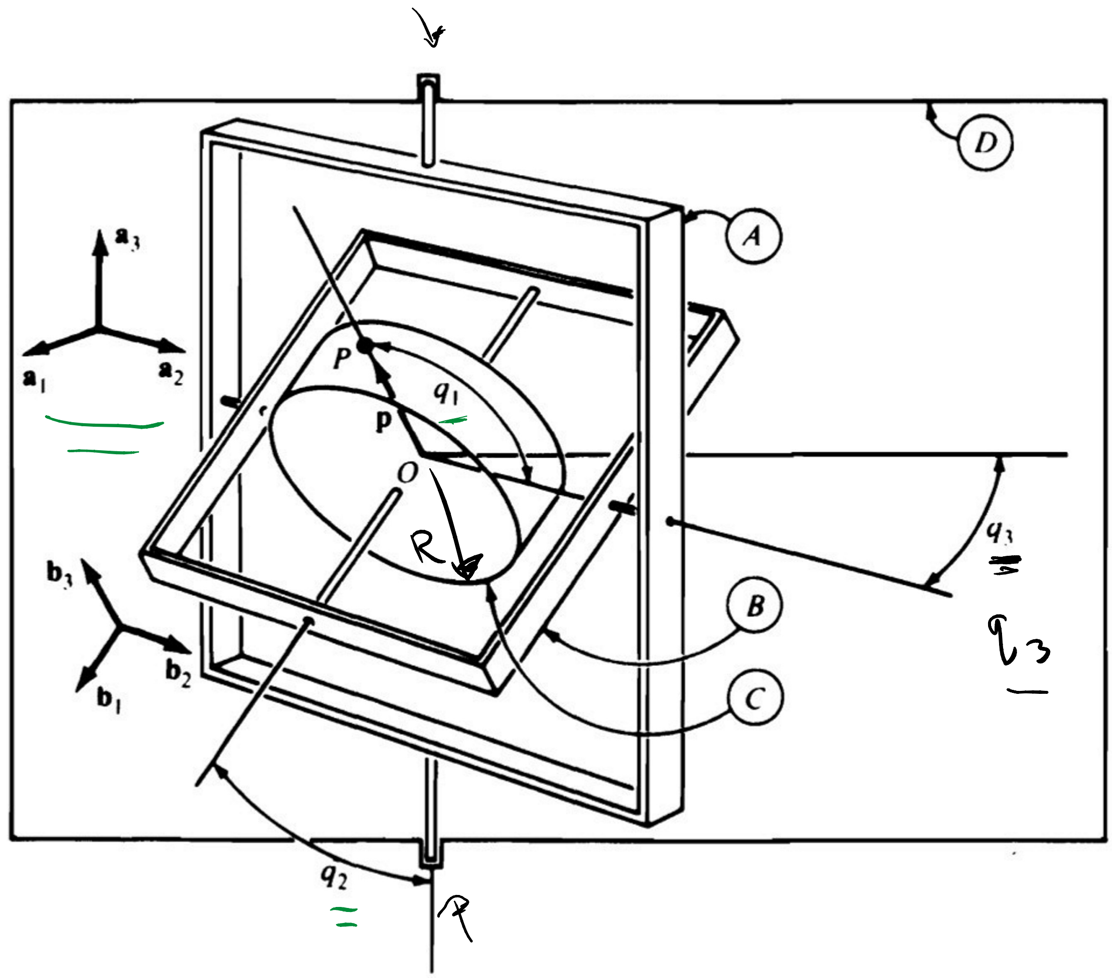

Figure 1.3.1

Same vector has different measure numbers in 2 different RF's and is a function of different scalars in different reference frames.

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$$\bar{P} = \alpha_1 \hat{a}_1 + \alpha_2 \hat{a}_2 + \alpha_3 \hat{a}_3$$

$$= \beta_1 \hat{b}_1 + \beta_2 \hat{b}_2 + \beta_3 \hat{b}_3$$

$$\bar{P} = R \cos q_1 \hat{b}_2 + R \sin q_1 \hat{b}_3$$

$\sin(q_1)$

$\beta_1 = 0 \qquad \beta_2 = R\, c_1 \qquad \beta_3 = R\, s_1$

$c_1 \to \cos q_1$

$\beta_1 = \bar{P} \cdot \hat{b}_1 \qquad \beta_2 = \bar{P} \cdot \hat{b}_2 \qquad \beta_3 = \bar{P} \cdot \hat{b}_3$

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to calculate α_i

$$\alpha_1 = \bar{p} \cdot \hat{a}_1$$

$$= R c_1 \underbrace{\cancel{\hat{b}_2 \cdot \hat{a}_1}}_{0} + R s_1 \underbrace{\hat{b}_3 \cdot \hat{a}_3}$$

$$= R s_1 c_2$$

$$\bar{p} = \alpha_1 \hat{a}_1 + \alpha_2 \hat{a}_2 + \alpha_3 \hat{a}_3$$

$$\alpha_2 = R C_1$$

$$\alpha_3 = R s_1 s_2$$

$$\bar{p} = R s_1 c_2 \hat{a}_1 + R C_1 \hat{a}_2 + R s_1 s_2 \hat{a}_3$$

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Derivatives

Let $\bar{V}$ be vector function of n scalar variables $q_1, q_2, \ldots, q_n$ in the reference frame A.

The n vectors called <u>first partial derivatives of $\bar{V}$ in A</u> and denoted by $\frac{{}^A\partial \bar{V}}{\partial q_r}$, $\frac{{}^A\partial}{\partial q_r}(\bar{V})$ or ${}^A\partial\bar{V}/\partial q_r$.

Partial derivative is defined as follows:

$$\frac{{}^A\partial \bar{V}}{\partial q_r} \triangleq \sum_{i=1}^{3} \frac{\partial v_i}{\partial q_r}\hat{a}_i \quad \text{for } r=1,\ldots,n$$

$\hat{a}_i$ are unit vectors fixed in A

v_i are measure numbers

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if $\bar{V}$ is function only of one variable (e.g. time)

Then

$$\frac{{}^A\partial \bar{V}}{\partial t} = \frac{{}^A d\bar{V}}{dt} = \frac{dv_1}{dt}\hat{a}_1 + \dot{v}_2\hat{a}_2 + \dot{v}_3\hat{a}_3$$

If $\bar{V}$ is not a function of q in A the $\frac{{}^A\partial \bar{V}}{\partial q} = 0$.

xample:

$${}^E\bar{P} = R s_1 c_2 \hat{a}_1 + R c_1 \hat{a}_2 + R s_1 s_2 \hat{a}_3$$

$$\frac{{}^A\partial \bar{p}}{\partial q_1} = R c_1 c_2 \hat{a}_1 - R s_1 \hat{a}_2 + R c_1 s_2 \hat{a}_3$$

$$\frac{{}^C\partial \bar{p}}{\partial q_i} \qquad \bar{P} = \underbrace{R}_{\text{no } q\text{'s !!}}\hat{c}_2$$

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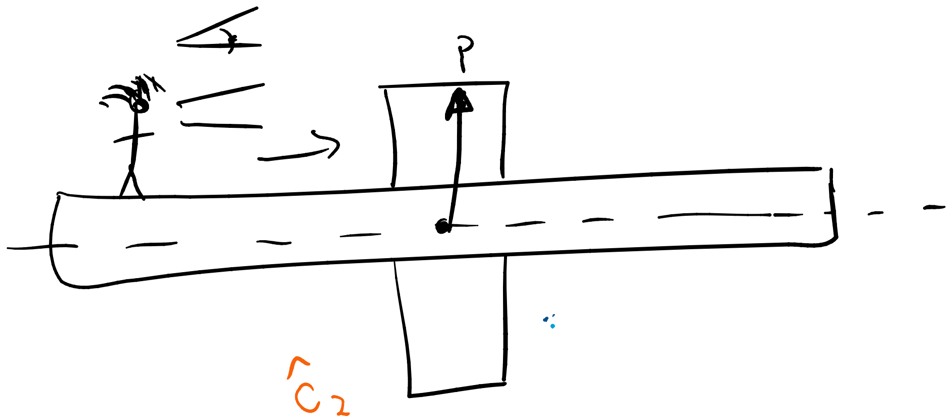
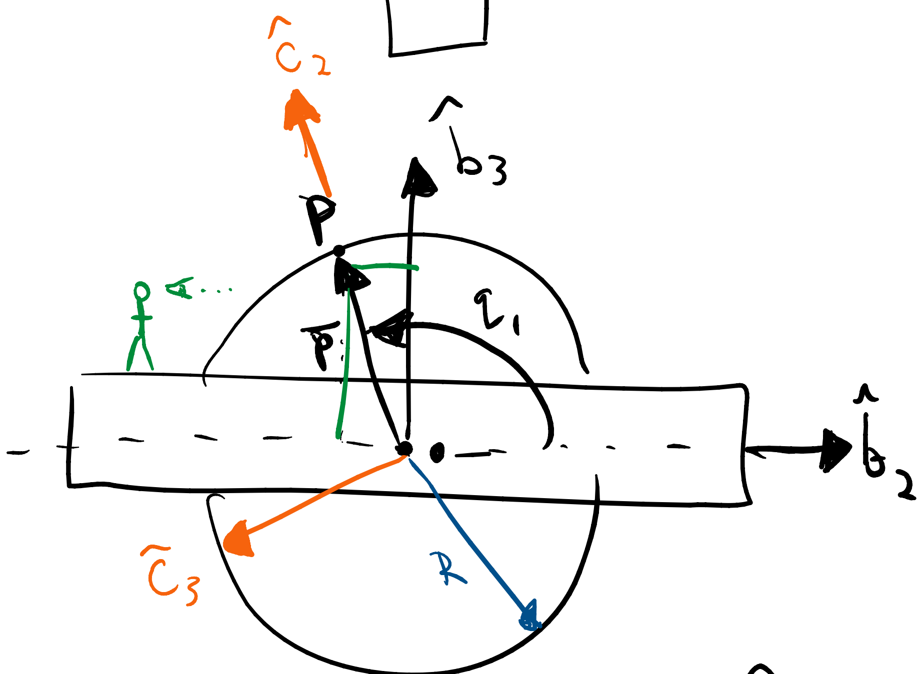
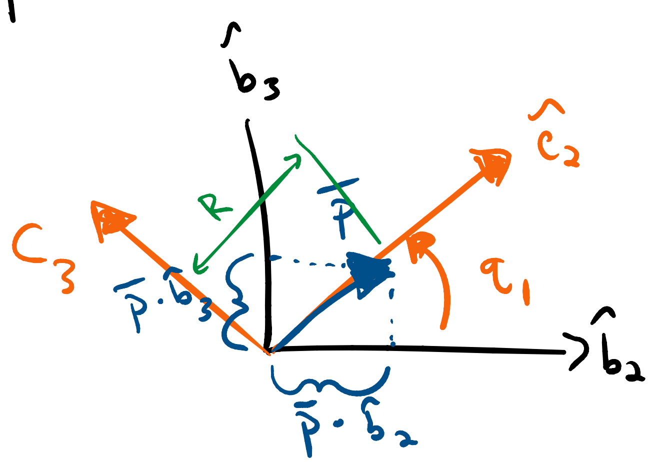




$$\bar{p} = R c_1 \hat{b}_2 + R s_1 \hat{b}_3$$



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$$\frac{{}^A\partial \bar{p}}{\partial q_1}$$ can be expressed in different RFs.

$$\frac{{}^A\partial \bar{p}}{\partial q_1} = \ldots \hat{b}_1 + \hat{b}_2 + \hat{b}_3$$

$\hat{a}_1 \quad \hat{a}_2 \quad \hat{a}_3$

$d_1 \quad d_2 \quad d_3$

$$\frac{\partial \bar{V}}{\partial q_3} \Rightarrow$$ meaningless!! no RF specified!!

ex: $$\bar{p} \cdot \frac{\partial \bar{p}}{\partial q_r} = 0$$

$$\bar{p} = R s_1 c_2 \hat{a}_1 + R c_1 \hat{a}_2 + R s_1 s_2 \hat{a}_3$$

$$\frac{{}^A\partial \bar{p}}{\partial q_1} = R c_1 c_2 \hat{a}_1 - R s_1 \hat{a}_2 + R c_1 s_2 \hat{a}_3$$

$$\bar{p} \cdot \frac{{}^A\partial \bar{p}}{q_1} = R^2 s_1 c_1 c_2^2 - R^2 s_1 c_1 + R^2 s_1 c_1 s_2^2$$

$$R^2 s_1 c_1 [1] - R^2 s_1 c_1 = 0$$

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Differentiation of Sums & Products

$$\frac{\partial}{\partial q_r} \sum_{i=1}^{N} \bar{V}_i = \sum_{i=1}^{N} \frac{\partial \bar{V}_i}{\partial q_r}$$

If S is a scalar function of $q_1, \ldots, q_n$ then

$$\frac{\partial}{\partial q_r}(S\bar{V}) = \frac{\partial S}{\partial q_r}\bar{V} + S\frac{\partial \bar{V}}{\partial q_r}$$

Product Rule

$$\frac{\partial}{\partial q_r}(\bar{V}\cdot\bar{W}) = \frac{\partial \bar{V}}{\partial q_r}\cdot \bar{W} + \bar{V}\cdot\frac{\partial}{\partial q_r}\bar{W}$$

$$\frac{\partial}{\partial q_r}(\bar{V}\times\bar{W}) = \frac{\partial \bar{V}}{\partial q_r}\times\bar{W} + \bar{V}\times\frac{\partial \bar{W}}{\partial q_r}$$

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More generally:

$$P = F_1, \ldots, F_N$$

$$\frac{\partial P}{\partial q_r} = \frac{\partial F_1}{\partial q_r} F_2 \cdots F_N + F_1 \frac{\partial F_2}{\partial q_r} F_3 \cdots F_N + \ldots + F_1 F_2 \cdots \frac{\partial F_N}{\partial q_r}$$

__Second order and higher derivatives__

since $\frac{\partial \bar{V}}{\partial q_r}$ is a vector function, both in A and any other frame, it can change and be differentiated wrt to any variable, q_i in any frame (A or another). Result is __second partial derivative__.

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Notes: order is important!

$$\frac{{}^{B}\partial}{\partial q_s}\left(\frac{{}^{A}\partial \bar{v}}{\partial q_r}\right) \neq \frac{{}^{A}\partial}{\partial q_r}\left(\frac{{}^{B}\partial \bar{v}}{\partial q_s}\right)$$

even if $q_r = q_s$

$$\frac{{}^{B}\partial}{\partial t}\left(\frac{{}^{A}\partial \bar{v}}{\partial t}\right) \neq \frac{{}^{A}\partial}{\partial t}\left(\frac{{}^{B}\partial \bar{v}}{\partial t}\right)$$

If RF A = RF B

the order doesn't matter.

i.e. mixed partials commute.

Total and partial derivatives

$\frac{{}^{A}d\bar{v}}{dt}$ If $\bar{v}$ is a vector function of n scalars: $q_1, \dots, q_n$ and of time t. in RF A.

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$$\frac{{}^A d\overline{V}}{dt} = \underbrace{\sum_{r=1}^{n} \frac{{}^A \partial \overline{V}}{\partial q_r} \underbrace{\frac{d q_r}{dt}}_{\dot{q}_r}}_{\text{chain rule to deal with } q_r \text{ being implicitly a func of time}} + \underbrace{\frac{{}^A \partial \overline{V}}{\partial t}}_{\text{if time is explicitly in } \overline{V}}$$

$$\overline{V} = \underbrace{2t}_{\text{time is explicit}} \hat{a}_1 + \cos(q_1(t))\, \hat{a}_2$$

q_1

q_1 is implicitly func of time

$$\frac{{}^A d\overline{V}}{dt} = -\sin(q_1)\, \dot{q}_1 \hat{a}_2 + 2\hat{a}_1$$