Exercises: Neuron Models

It is recommended to work through Part 1 of *Introduction to PyNEST* before starting.

1) PyNEST working styles

a) Working interactively within ipython

Start `ipython`, import the `nest` module and start the `helpdesk`. Look at the documentation for the neuron models `iaf_psc_exp`, `iaf_psc_alpha`, `iaf_cond_exp` and `iaf_cond_alpha`. How would you explain the difference between them? Create a neuron of one of these models and look at its status dictionary. Set one of its parameters differently from its initial setting and look at its status dictionary again. Use the in-console help system to find out about the functions `ResetKernel()` and `ResetNetwork()`.

b) Working with scripts

Copy one of the examples from the handout into a text file. Run it directly from the command line using `python`.

c) A hybrid working style

Run the script from the last question from within `ipython` using the command `run`. Now examine the status dictionary of the neuron and simulate for another second.

2) Postsynaptic Potentials

a) Amplitude and rise time

Set up a simulation with a `spike_generator` connected to an `iaf_psc_exp` neuron and an `iaf_psc_alpha` neuron - use the default synaptic strengths and delays for each connection. The `spike_generator` should be set to generate one spike at $200\mathrm{ms}$. Add a `multimeter` to record the membrane potentials of the two neurons.

Plot the postsynaptic potentials (PSPs) of the two neurons in the same figure. Calculate the maximum amplitude and the rise time of the PSP for each neuron and print them out. Definition of terms: *maximum amplitude* means the height of the PSP *relative* to the membrane potential before the PSP started, not the *absolute* highest value recorded for the membrane potential. Likewise, *rise time* means the time the PSP takes to reach maximum from the start of the PSP (i.e. where it moves away from the baseline membrane potential), not the start of the simulation. To complete this task, you will have to numerically analyse the membrane potentials recorded by the `multimeter`.

Manually adjust the strength of the synapse to the `iaf_psc_exp` neuron so that both PSPs have the same amplitude and adjust the synaptic time constant of the `iaf_psc_exp` neuron so that both PSPs have the same rise time. Now forget your answer and start again, this time first adjusting the time constant and then the amplitude. Is it easier to get the times right first and then adjust the amplitude, or the other way around? Why do you think this is?

b) Dependence on the membrane potential

Repeat the previous simulation, this time using an `iaf_psc_exp` neuron and an `iaf_cond_exp` neuron. By analysing the recorded membrane potentials, calculate the maximum amplitude of the PSP for each neuron and print them out.

Now set positive background currents for the neurons such that they saturate at around the same potential approximately halfway between the resting potential and the threshold before the spike from the `spike_generator` is sent at $200\mathrm{ms}$.

How does the amplitude of the PSP of the `iaf_psc_exp` neuron compare between the simulation with and without positive background current? How about for the `iaf_cond_exp` neuron?

Now repeat the simulation for a negative background current. How do the maximum amplitudes compare now for the two neurons? Can you explain this?

3) Firing rates

Let us consider a linear neuron model defined by the following dynamics:

$$C\frac{dU}{dt} = -\frac{U}{R} + I$$

where C is the membrane capacity and R the membrane resistance.

a) Rheobase current

The rheobase current is defined as the current that would bring the neuron to fire at $t = \infty$. Assuming an initial condition $U(0) = 0$, integrating the model dynamics for a step current:

$$I(t) = \begin{cases} 0 & \text{for } t < 0 \\ I & \text{for } t \geq 0 \end{cases}$$

allows us to determine the membrane potential of the neuron for $t > 0$:

$$U(t) = RI\left(1 - \mathrm{e}^{-t/\tau_\mathrm{m}}\right)$$

with membrane time constant $\tau_\mathrm{m} = RC$. Setting $U(\infty) = \Theta$ for spiking threshold Θ gives us the rheobase current $I_\mathrm{rh} = \Theta C/\tau_\mathrm{m}$.

What is the problem with determining this quantity empirically using a simulation? Write a simulation using the `iaf_psc_exp` neuron model that shows a reasonable approximation, using $C = 250$ pF, $\tau_\mathrm{m} = 10$ ms and $\Theta = 15$ mV (relative to the resting membrane potential). A reasonable approximation would be that you can show that a current of x pA does not cause the neuron to fire within, say, 2 s, but a current of $x + 0.1$ pA does cause the neuron to fire.

b) F-I curve

By setting $U(t) = \Theta$ in the membrane potential equation given in the previous question and solving for t, calculate the time a neuron takes to fire from the initial condition $U(0) = 0$ for $I > I_\mathrm{rh}$. Assuming that after firing, a neuron is clamped at $U = 0$ for a refractory period $t_\mathrm{ref} = 2$ ms, calculate the firing rate of the neuron as a function of the input current. Plot the theoretical value and an empirically determined value for a range of input currents.

Hint: Make a loop over a range of input currents and set parameters and simulate in each iteration. Use `ResetNetwork()` to reset the membrane potential of the neuron and empty the `spike_detector`.

c) Variance of the synaptic current

The mean μ_I and the variance σ_I^2 of the synaptic current received by a neuron are given by:

$$\begin{aligned} \mu_I &= \left(\lambda_\mathrm{e} J_\mathrm{e} + \lambda_\mathrm{i} J_\mathrm{i}\right) F^1 \\ \sigma_I^2 &= \left(\lambda_\mathrm{e} J_\mathrm{e}^2 + \lambda_\mathrm{i} J_\mathrm{i}^2\right) F^2 \end{aligned}$$

where λ_e and λ_i are the rates of the excitatory and inhibitory processes, respectively, and J_e and J_i the strengths of their synapses. F^k are form factors defined by:

$$F^k = \int_{-\infty}^{\infty} I_s^k\left(t\right) dt$$

The `iaf_psc_exp` neuron model has a postsynaptic current that is exponential in shape (hence the name), i.e.

$$I_s\left(t\right) = e^{-t/\tau_s}$$

for $t > 0$, where τ_s is the time constant of the post-synaptic current. This results in $F^1 = \tau_\mathrm{s}$ and $F^2 = \tau_\mathrm{s}/2$.

Create an `iaf_psc_exp` neuron, parameterised as above, and provide it with a direct current a little less than I_rh. Connect your neuron to two `poisson_generator` devices, one to provide an excitatory spike train and the other to provide an inhibitory spike train. Choose the rates and strengths of the poisson processes such that the mean synaptic current is zero. A good choice for this is $\lambda_\mathrm{i} = \lambda_\mathrm{e}$ and $J_\mathrm{i} = -J_\mathrm{e}$. Now set up a loop, similarly to the previous exercise, in which you systematically vary the strengths of the connections from the `poisson_generator`s (such that the *mean synaptic current remains zero*). For each synaptic strength, calculate the corresponding variance of the synaptic current and record the firing rate of the neuron. Finally, create a plot which shows the dependence of the neuron's firing rate on the variance of the synaptic current.