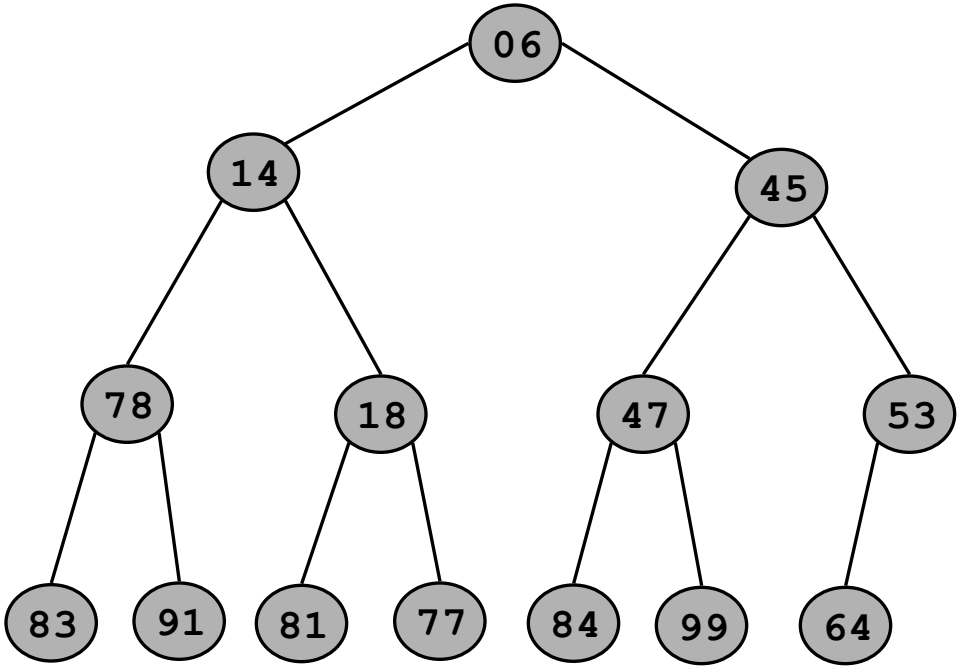
Binary Heap: Definition

Binary heap.

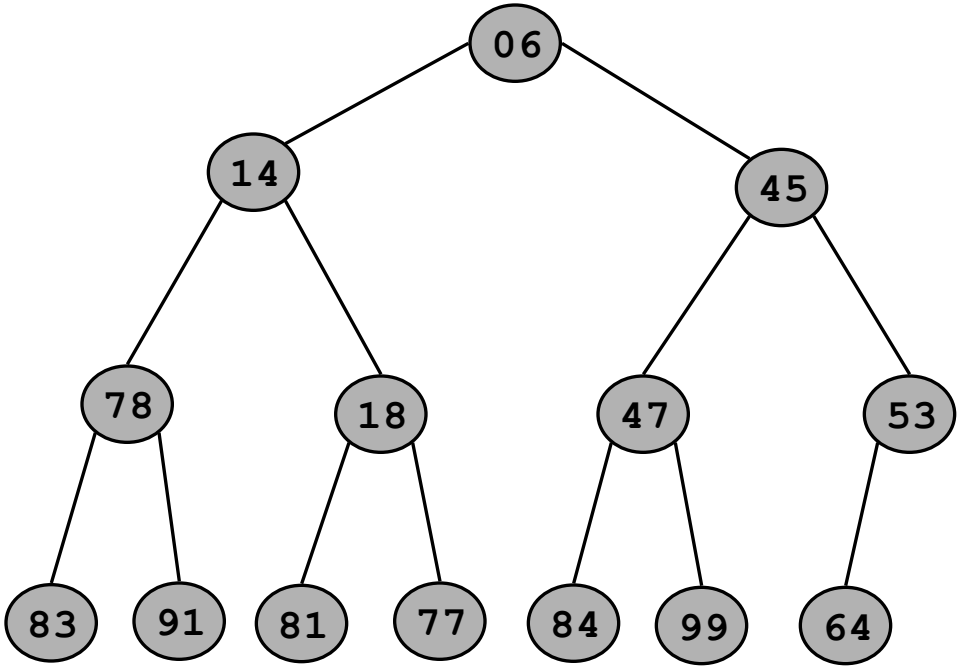
- **Almost complete binary tree.**
  - **filled on all levels, except last, where filled from left to right**
- **Min-heap ordered.**
  - **every child greater than (or equal to) parent**



Binary Heap: Properties

Properties.

- **Min element is in root.**
- **Heap with N elements has height = $\lfloor \log_2 N \rfloor$.**

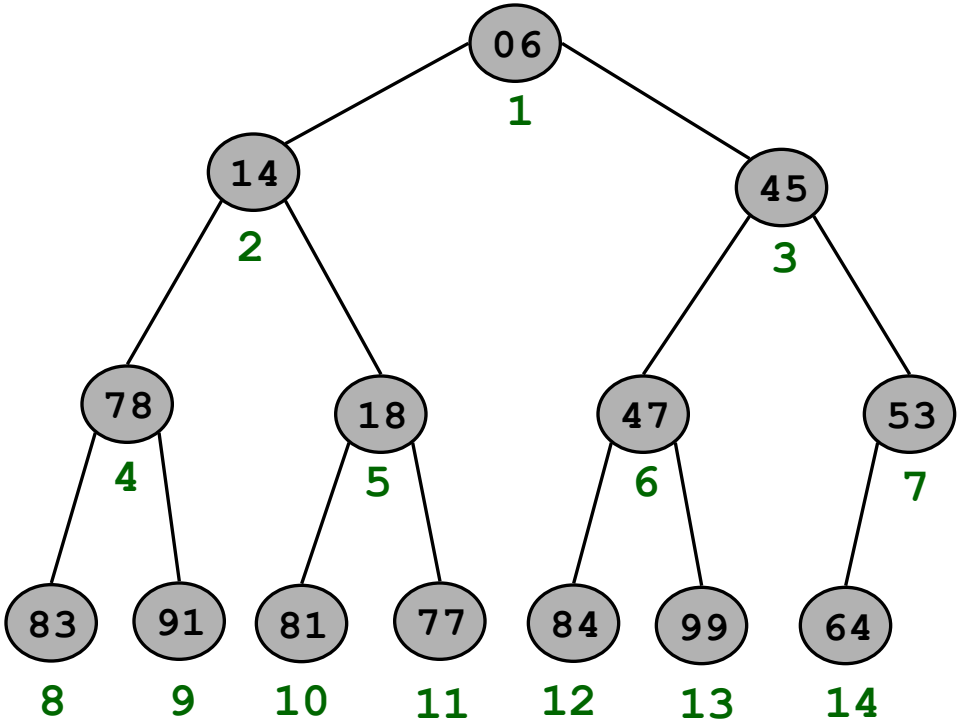


N = 14
Height = 4

Binary Heaps: Array Implementation

Implementing binary heaps.

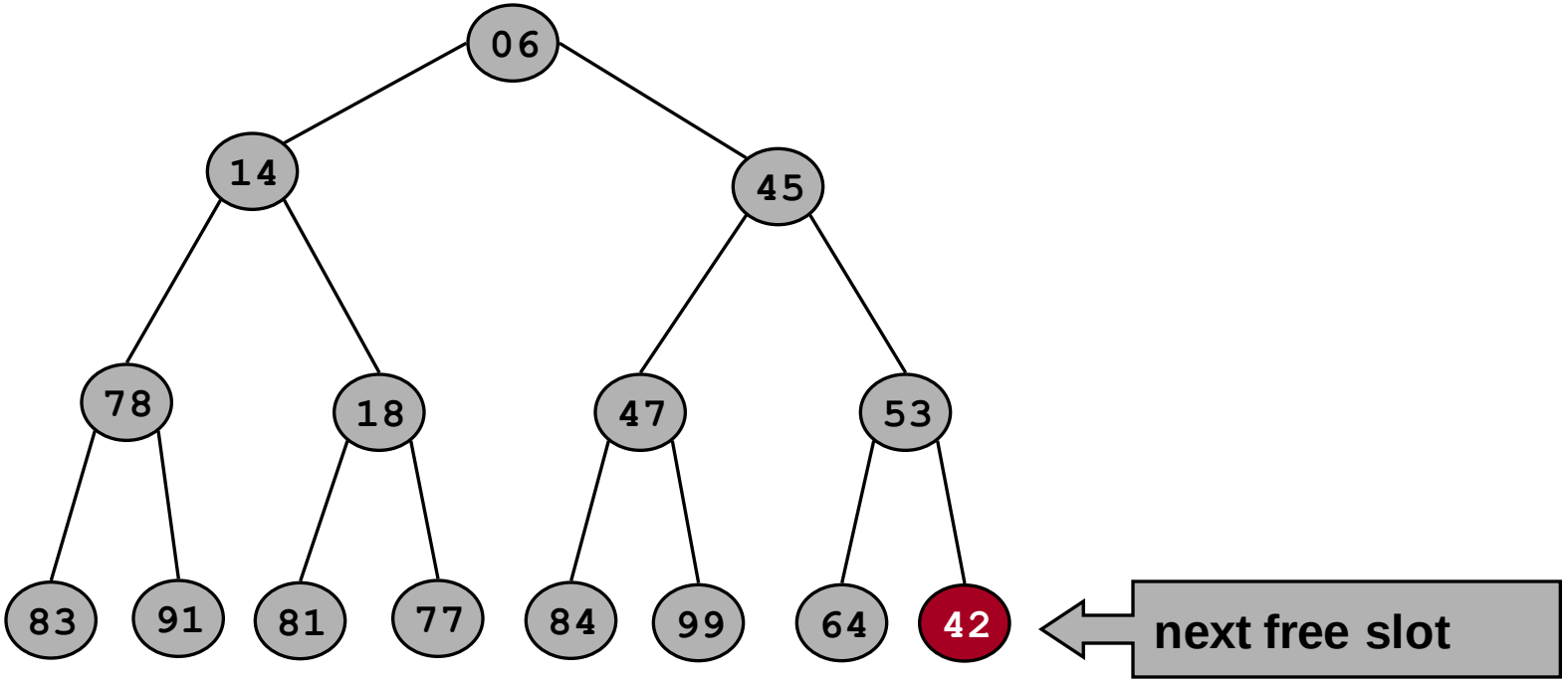
- **Use an array: no need for explicit parent or child pointers.**
  - `Parent(i) = ⌊i/2⌋`
  - `Left(i) = 2i`
  - `Right(i) = 2i + 1`



Binary Heap: Insertion

Insert element x into heap.

- **Insert into next available slot.**
- **Bubble up until it's heap ordered.**
  - **Peter principle: nodes rise to level of incompetence**



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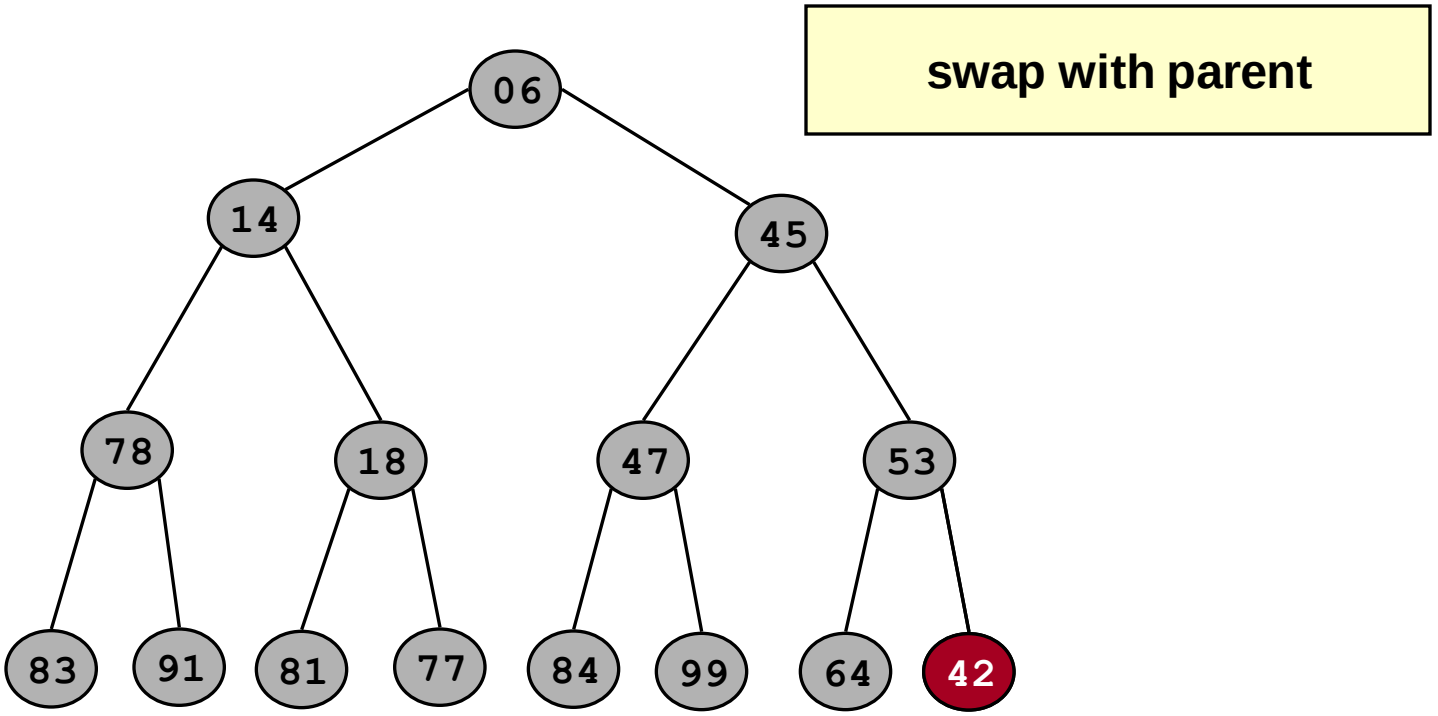

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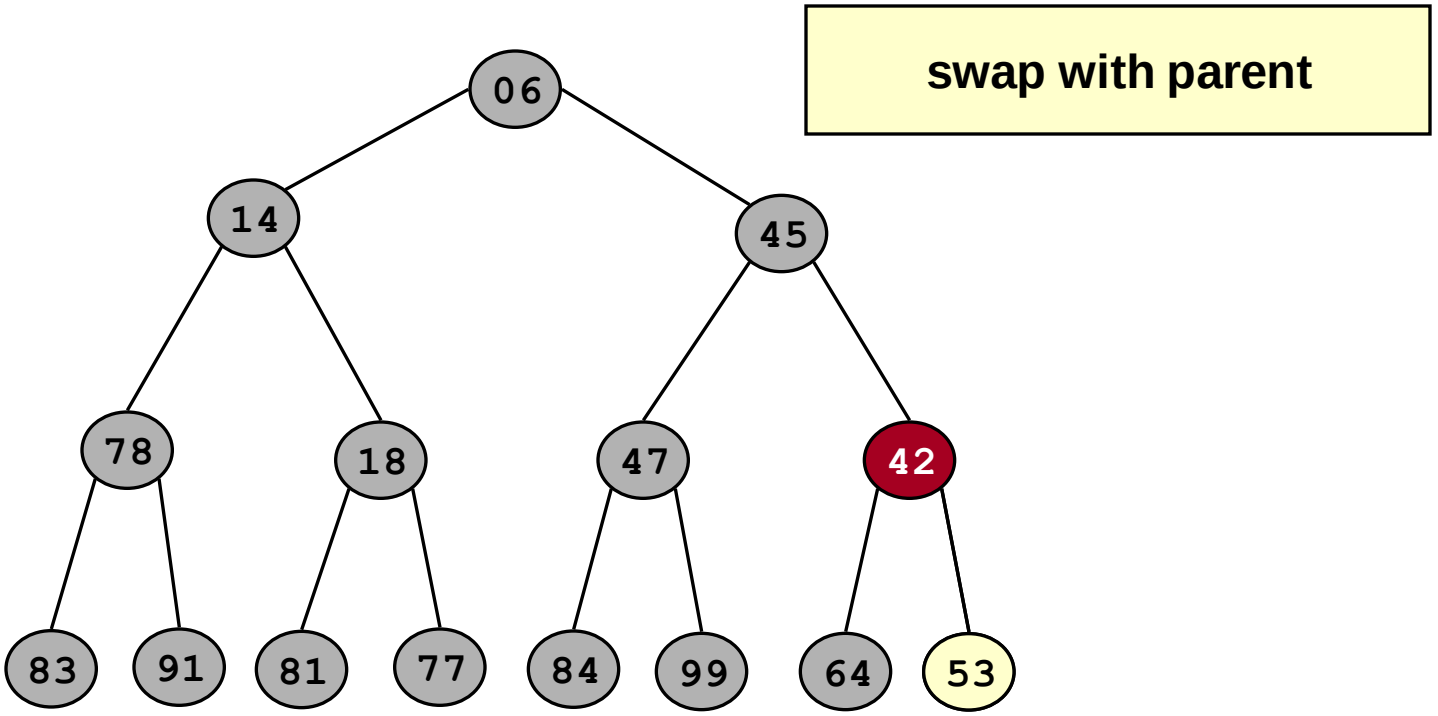

Binary Heap: Insertion

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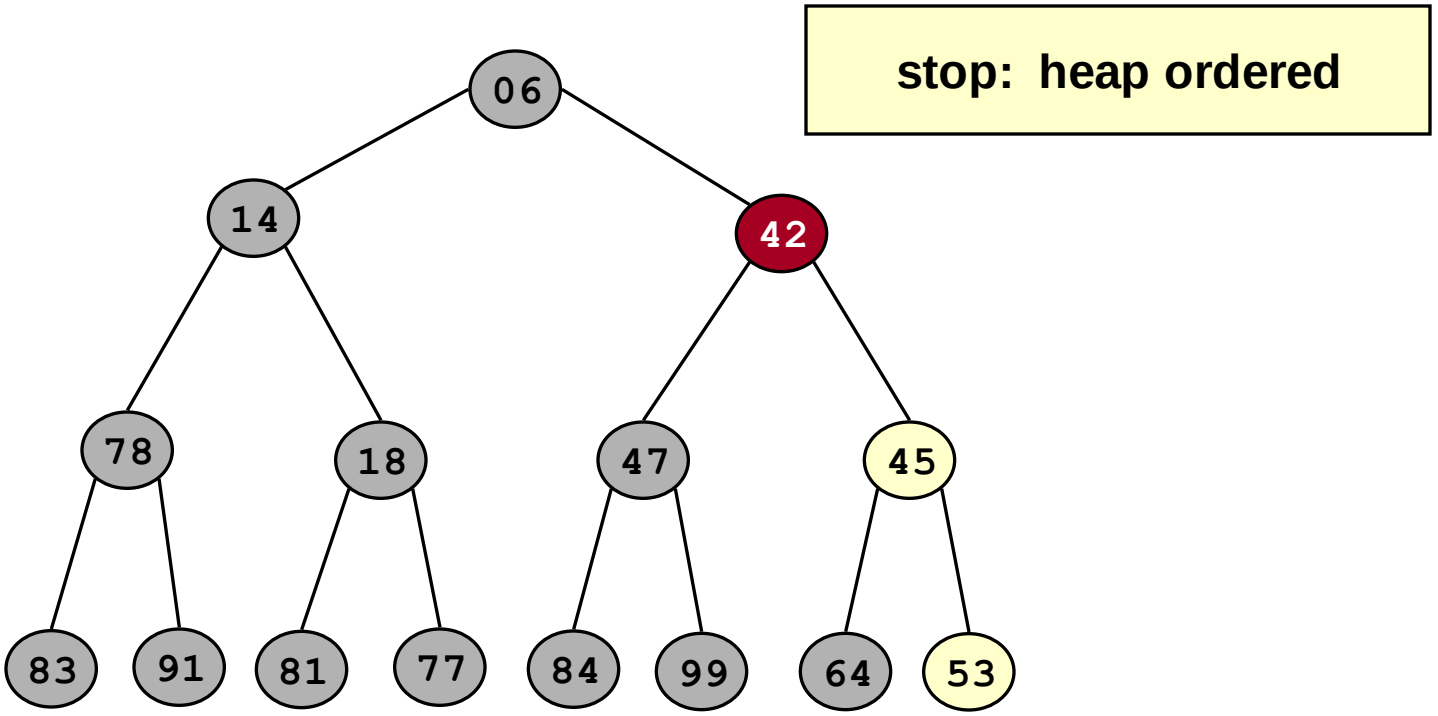
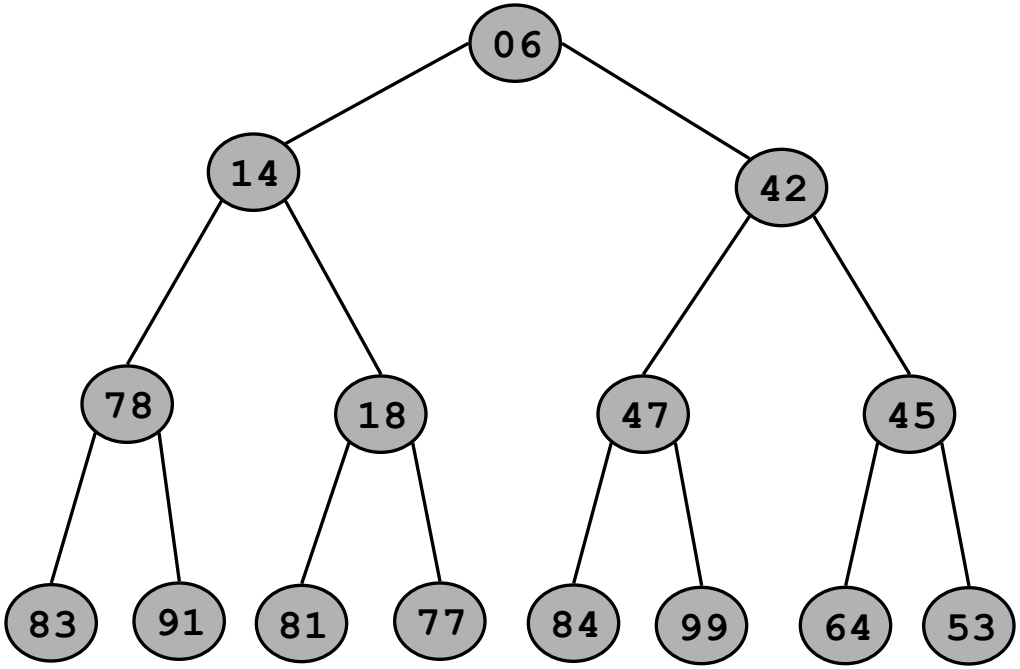
- **O(log N) operations.**



Binary Heap: Decrease Key

Decrease key of element x to k.

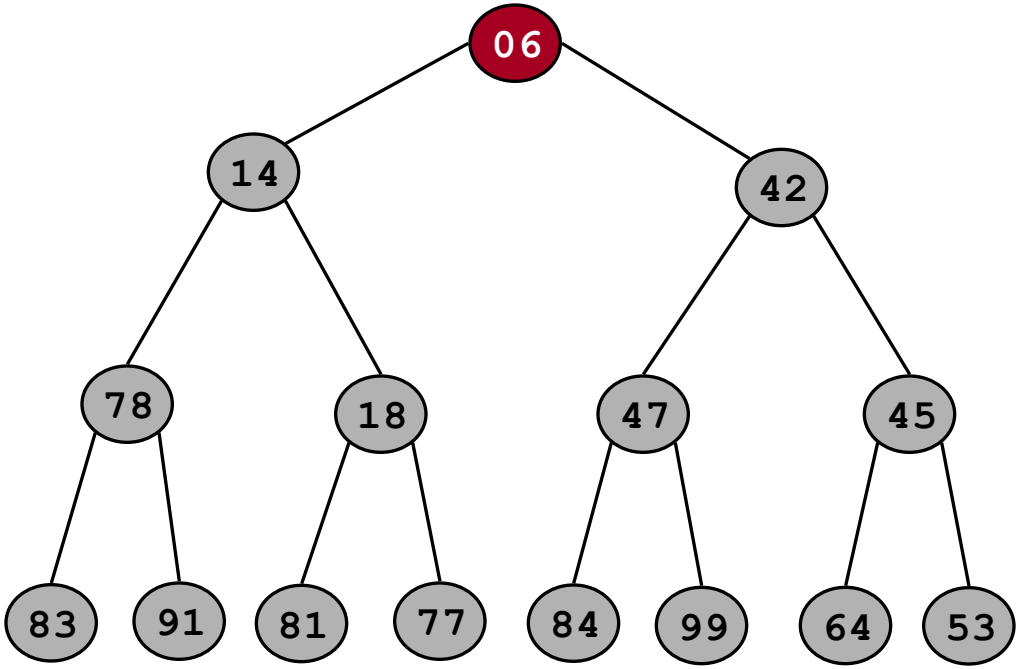
- **Bubble up until it's heap ordered.**
- **O(log N) operations.**



Binary Heap: Delete Min

Delete minimum element from heap.

- **Exchange root with rightmost leaf.**
- **Bubble root down until it's heap ordered.**
  - **power struggle principle: better subordinate is promoted**



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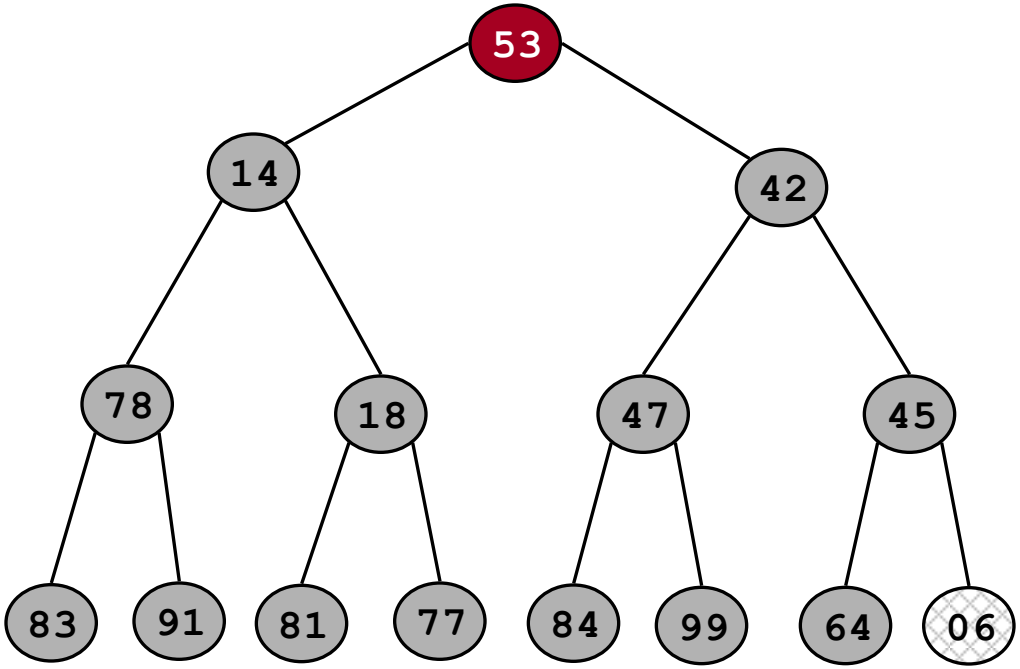

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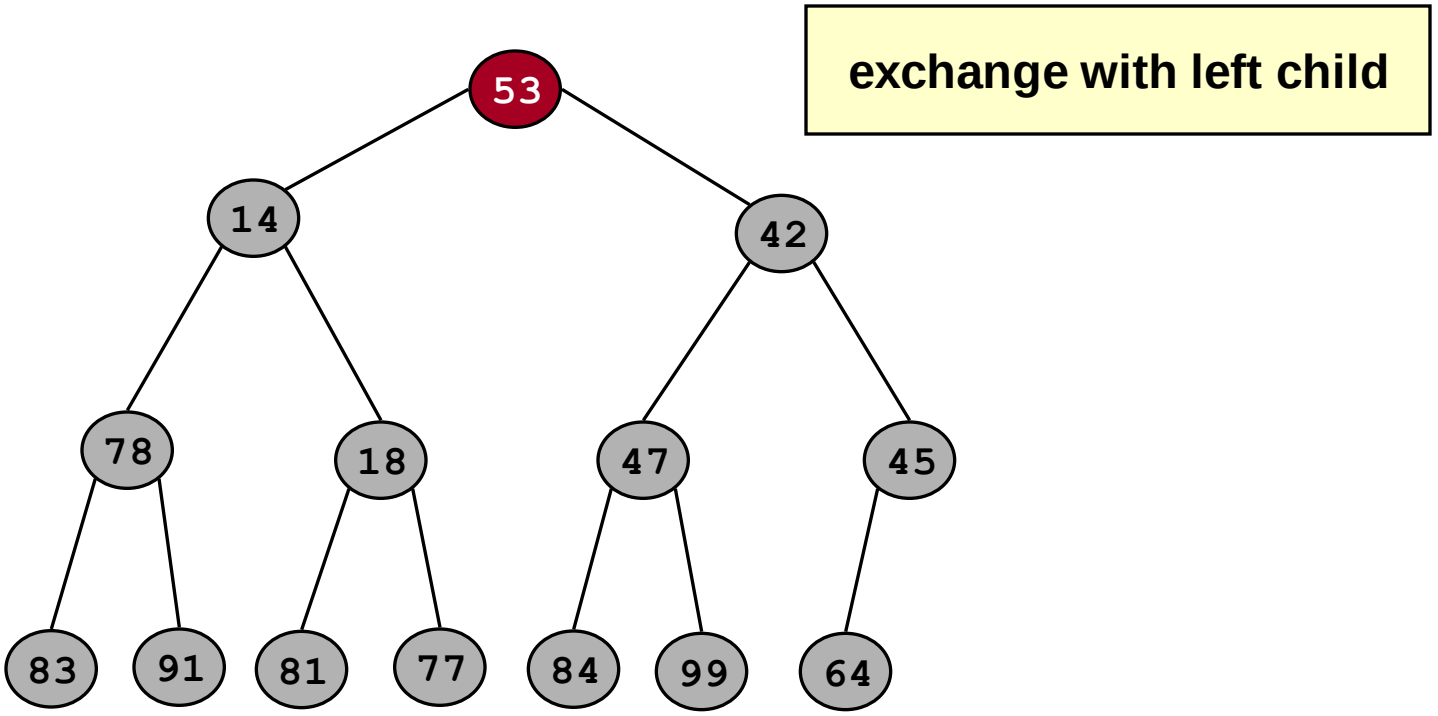

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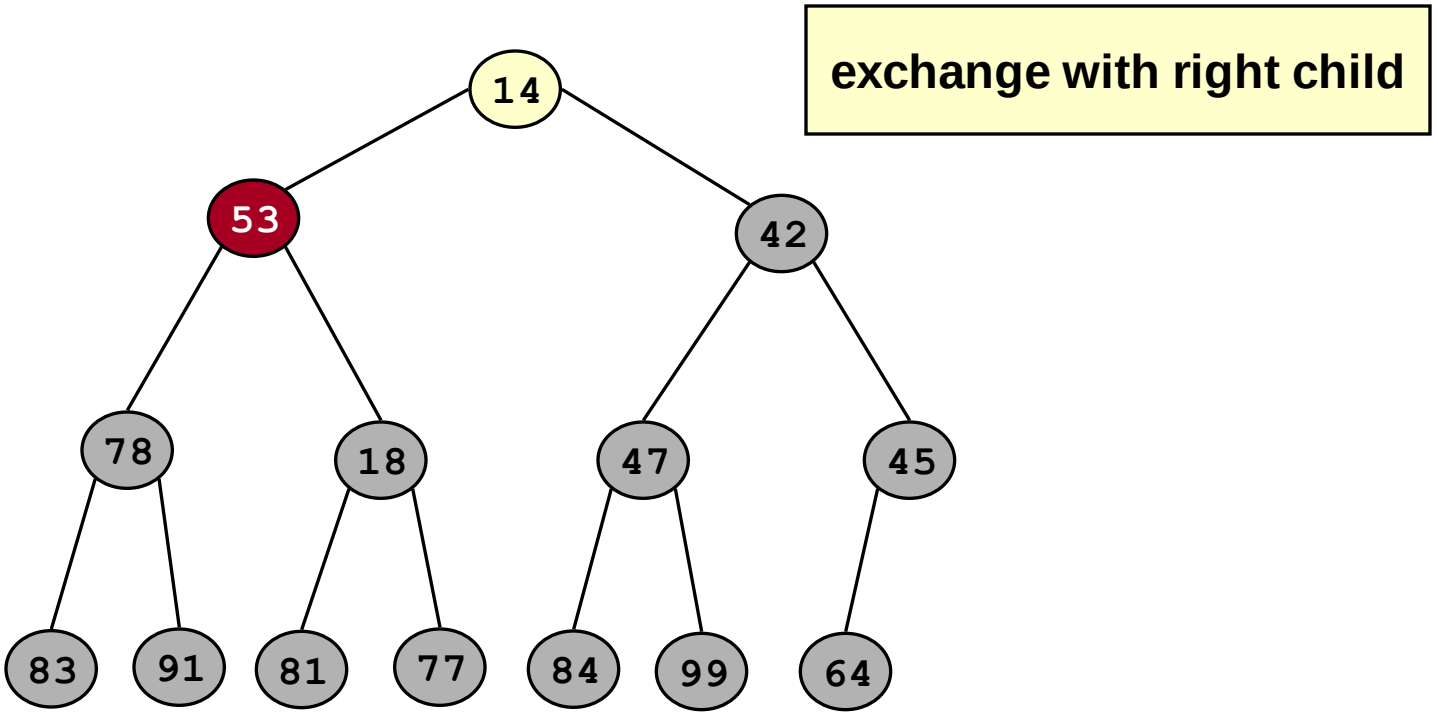

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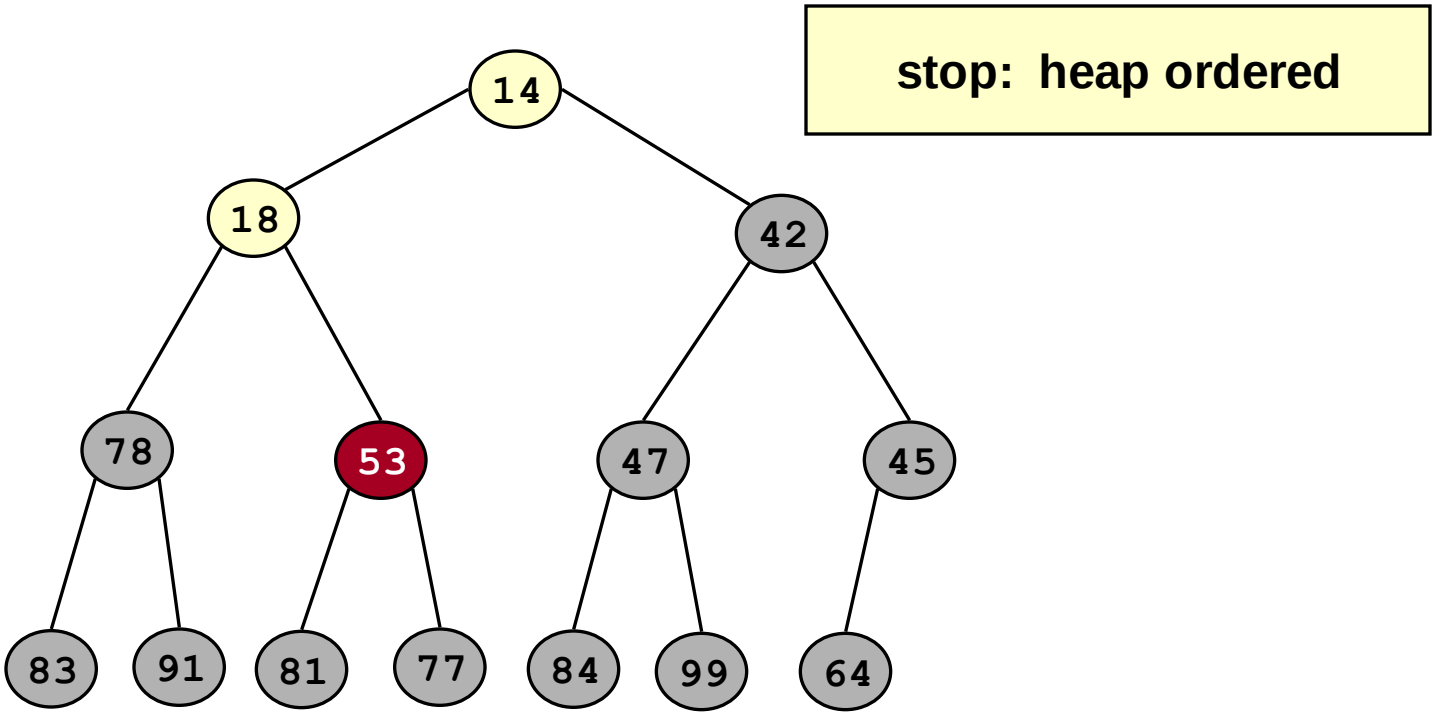
- O(log N) operations.



Binary Heap: Heapsort

Heapsort.

- Insert N items into binary heap.
- Perform N delete-min operations.
- O(N log N) sort.
- No extra storage.

Binary Heap: Union

Union.

- Combine two binary heaps H_1 and H_2 into a single heap.
- No easy solution.
  - $\Omega(N)$ operations apparently required
- Can support fast union with fancier heaps.