

CSc 106 Fall 2012

Lab 3: Logic and Circuits, Sorting and Running times

Boolean Algebra review

- Boolean Operators:
 - AND \wedge
 - OR \vee
 - NOT \neg
 - XOR \oplus
 - NAND
 - NOR

Order of Operations

BNAO

1. Brackets

2. Not

3. And

4. Or

AND, OR, XOR, NOT

Truth Tables

AND

A	B	$A \wedge B$
0	0	0
0	1	0
1	0	0
1	1	1

OR

A	B	$A \vee B$
0	0	0
0	1	1
1	0	1
1	1	1

XOR

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

NOT

A	$\neg A$
0	1
1	0

NAND, NOR

Truth Tables

AND

A	B	$A \wedge B$
0	0	0
0	1	0
1	0	0
1	1	1

OR

A	B	$A \vee B$
0	0	0
0	1	1
1	0	1
1	1	1

NAND

A	B	$\neg(A \wedge B)$
0	0	1
0	1	1
1	0	1
1	1	0

NOR

A	B	$\neg(A \vee B)$
0	0	1
0	1	0
1	0	0
1	1	0

$$\neg(A \text{ OR } B)$$

What is the answer if: $A=1$, $B=0$?

A. 0

B. 1

Quick Quiz

$$\neg(A \text{ OR } B)$$

What is the answer if: $A=1$, $B=0$?

A. 0

B. 1

ANSWER: 0

$$(\neg A \text{ XOR } B)$$

What is the answer if: $A=1$, $B=1$?

A. 0

B. 1

Quick Quiz

$(\neg A \text{ XOR } B)$

What is the answer if: $A=1$, $B=1$?

A. 0

B. 1

ANSWER: 1


Boolean Operators and Gates

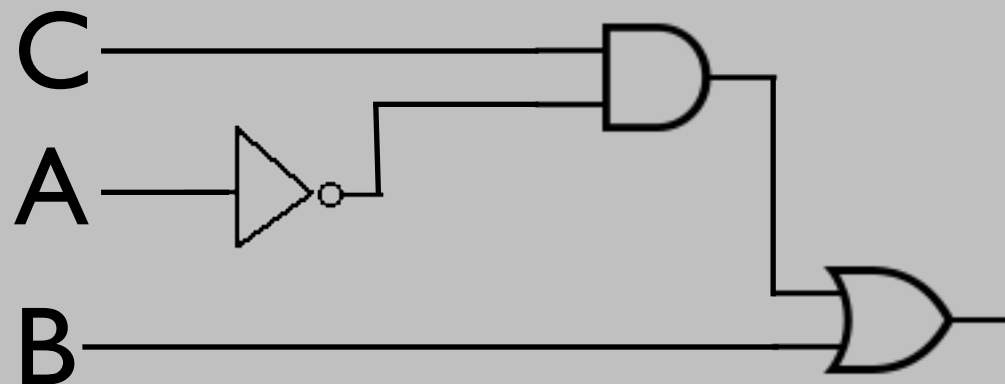
- Each Operator can be physically built as a gate to build more complex circuits
- We can represent these gates as symbols to draw circuits

Logic Gates

AND 

NOT 

OR 

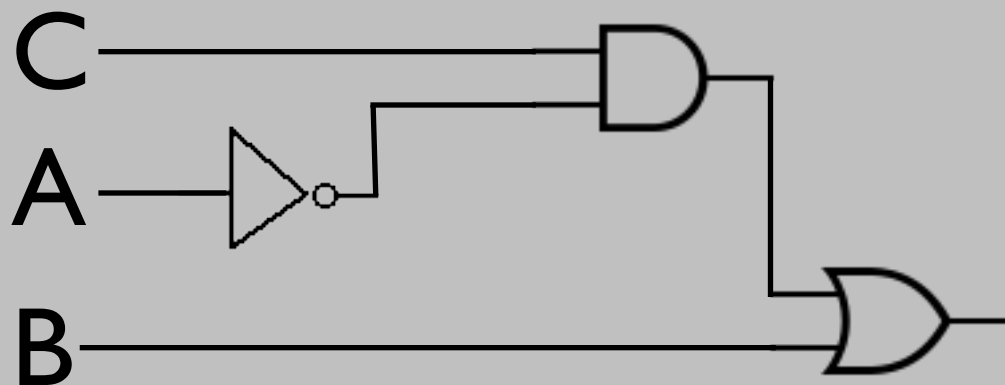


Boolean Formulas and Circuits

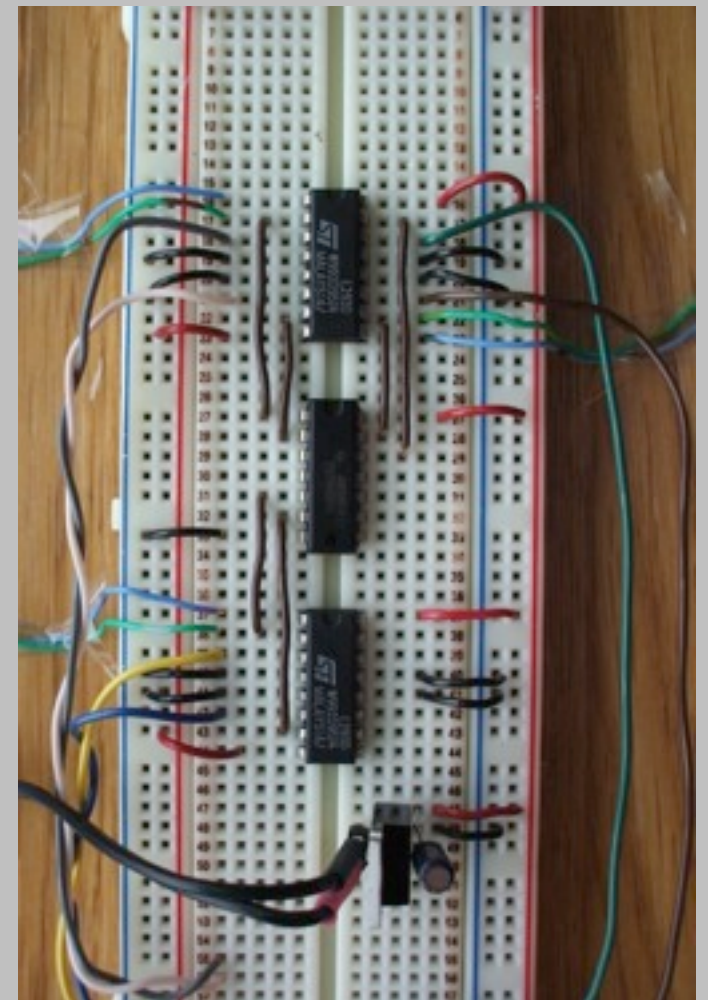
Boolean Formula

$$B \vee (C \wedge \neg A)$$

Circuit: Representation



Circuit:
Breadboard*





* <http://webhome.csc.uvic.ca/~mcheng/samples/hoole/wiring.html>

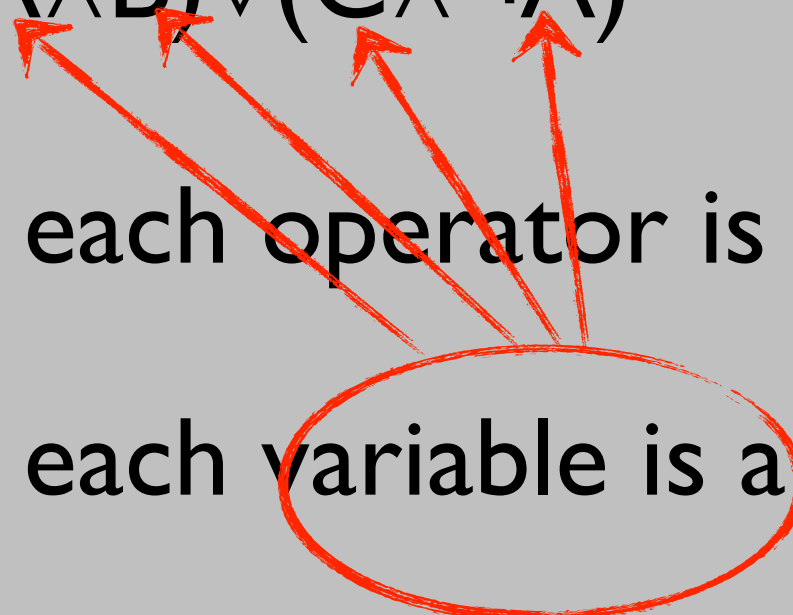
Translating from Formula to Circuit

- $(A \wedge B) \vee (C \wedge \neg A)$
- each operator is a gate

Translating from Formula to Circuit

- $(A \wedge B) \vee (C \wedge \neg A)$ 
- each operator is a gate
- each variable is an input

Translating from Formula to Circuit

- $(A \wedge B) \vee (C \wedge \neg A)$ 
- each operator is a gate
- each variable is an input
- lines represent wires that connect them

$$(A \wedge B) \vee (C \wedge \neg A)$$

Let's draw the circuit representation
of this boolean formula

$$(A \wedge B) \vee (C \wedge \neg A)$$

First: inputs

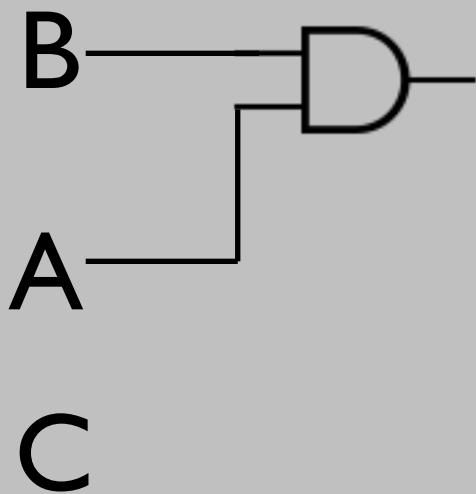
B

A

C

$$(A \wedge B) \vee (C \wedge \neg A)$$

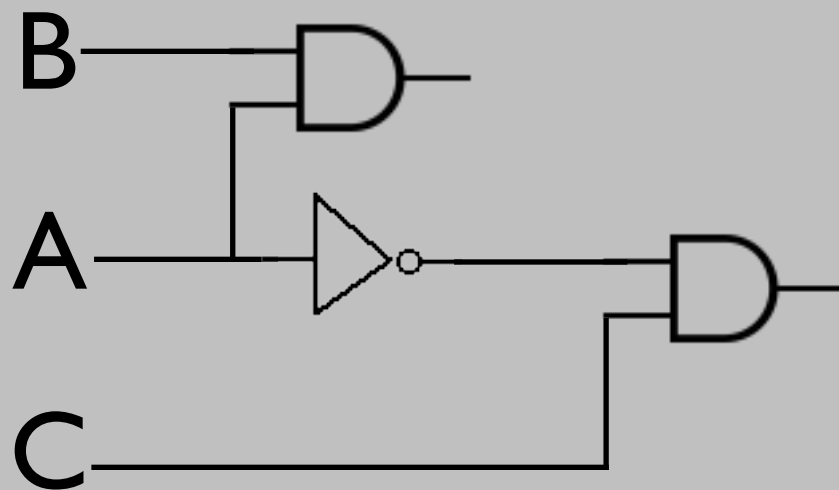
$$(A \wedge B)$$



$$(A \wedge B) \vee (C \wedge \neg A)$$

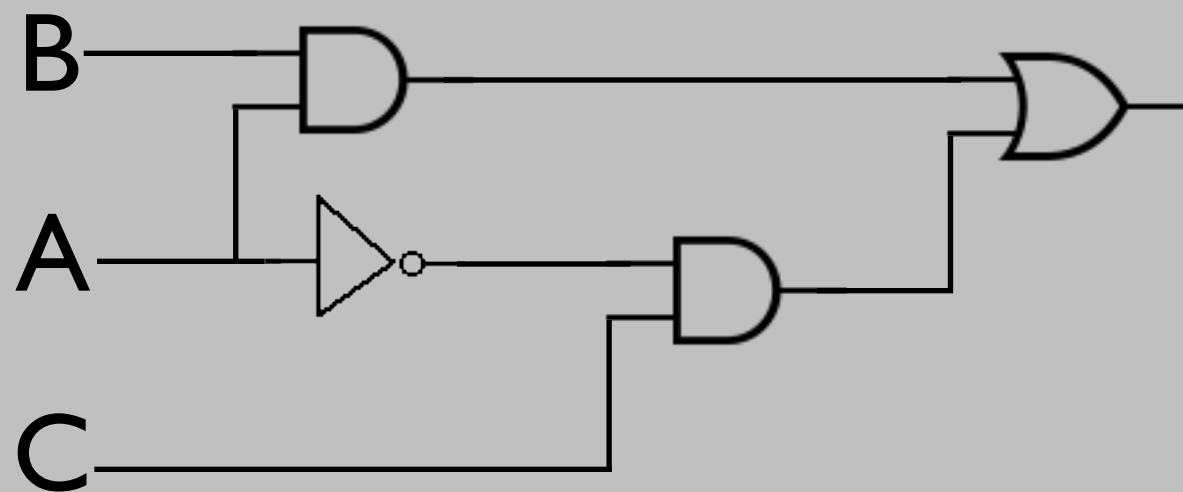
$$(A \wedge B)$$

$$(C \wedge \neg A)$$

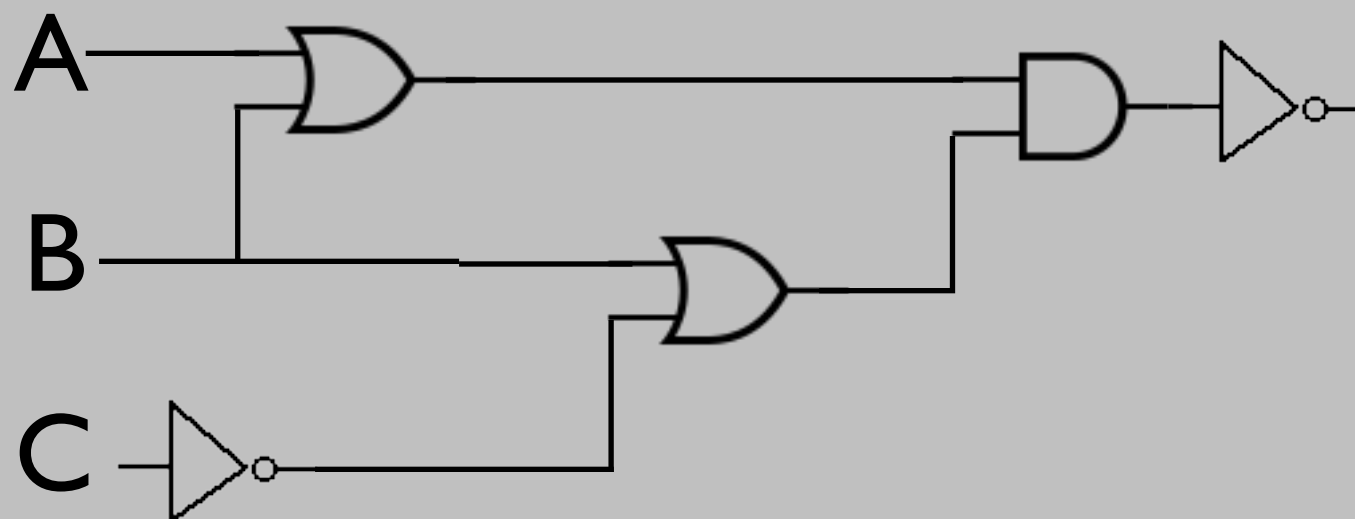


$$(A \wedge B) \vee (C \wedge \neg A)$$

$$(A \wedge B) \vee (C \wedge \neg A)$$

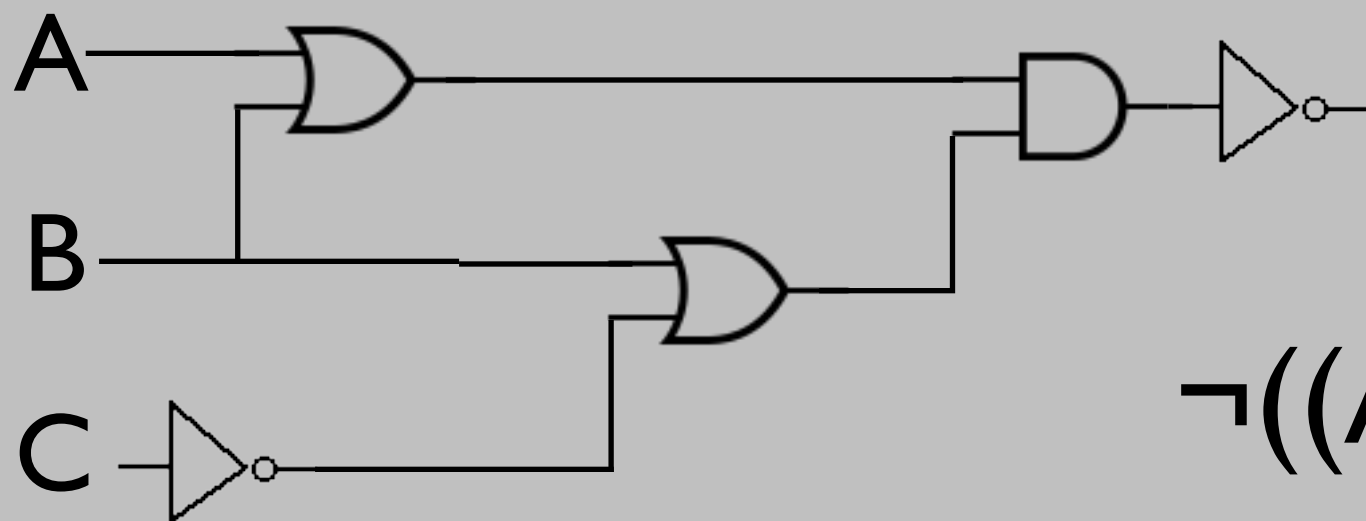


Translating from Circuit to Formula



Work it out...

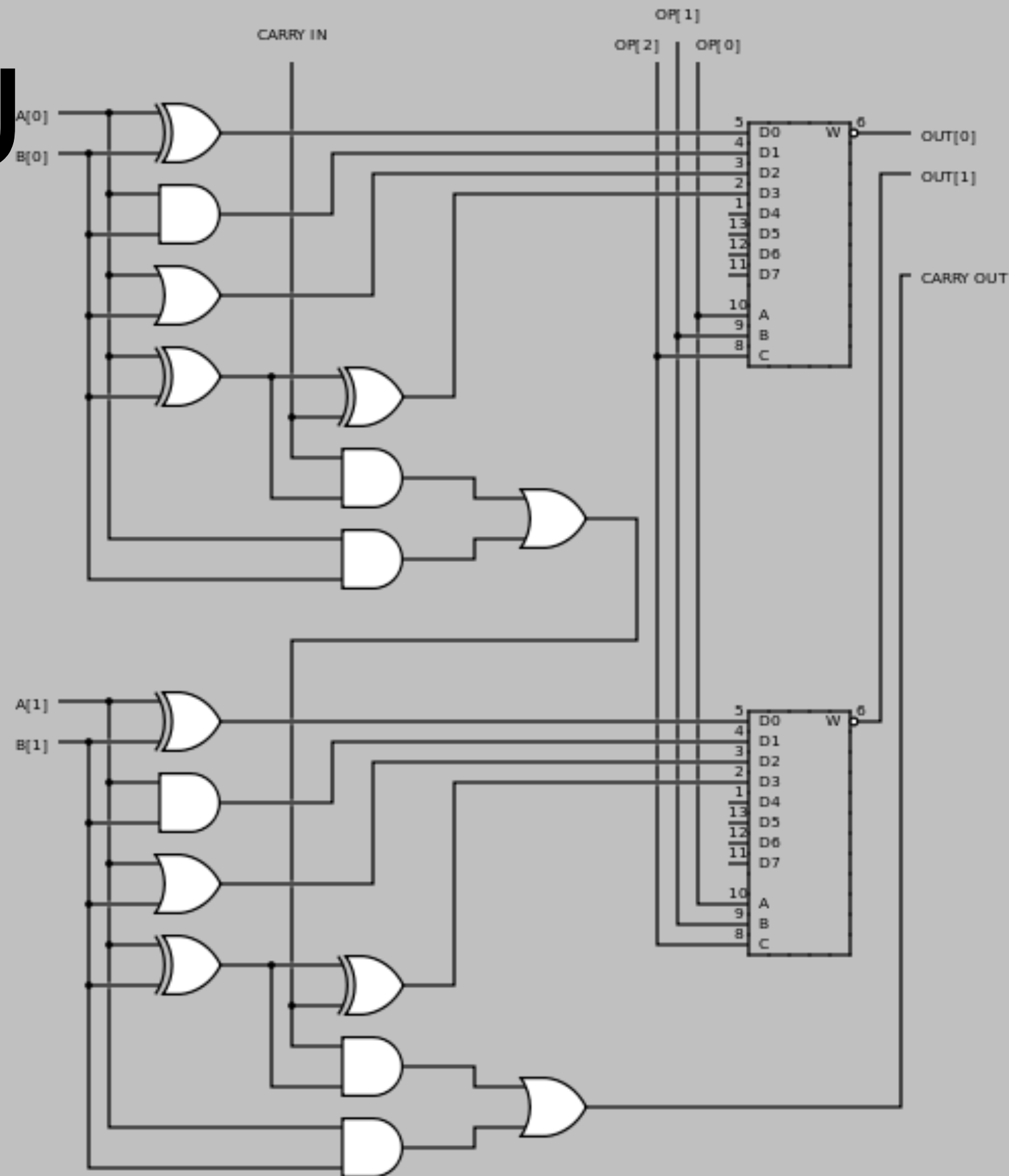
Translating from Circuit to Formula



$$\neg((A \vee B) \wedge (\neg C \vee B))$$

Simple ALU

- 2 2-bit inputs + carry
- control input
- 2-bit output + carry
- can perform AND, OR, XOR and addition



Multiplexors

Simple ALU

- 2 2-bit inputs + carry
- control input
- 2-bit output + carry
- can perform AND, OR, XOR and addition

