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CS325 – HW Group 11
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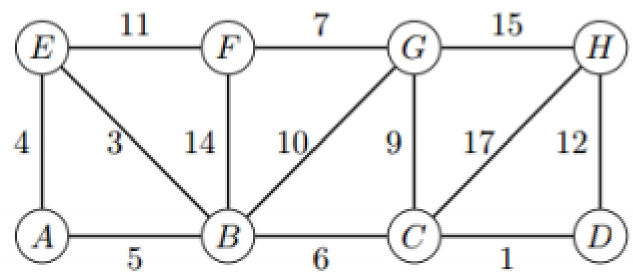
Homework Assignment 3

1. **Consider the weighted graph below:**



a. **Demonstrate Prim's algorithm starting from vertex A. Write the edges in the order they were added to the minimum spanning tree**

Here is the order they are added to the MST: (AE, BE, BC, CD, CG, FG, DH)

b. **Demonstrate Dijkstra's algorithm on the graph, using vertex A as the source. Write the vertices in the order which they are marked and compute all distances at each step.**

Order they are being evaluated:

Step	Vertex
1	A
2	E
3	B
4	C
5	D
6	F
7	G
8	H

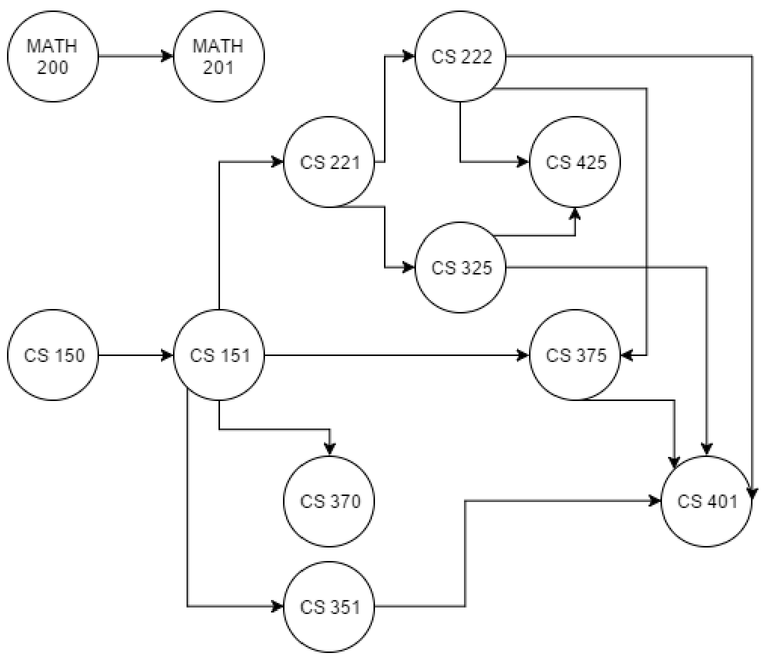
Distance at each step:

Step	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
1	0	5	∞	∞	4	∞	∞	∞
2	0	5	∞	∞	4	15	∞	∞
3	0	5	11	∞	4	15	15	∞
4	0	5	11	12	4	15	15	28
5	0	5	11	12	4	15	15	24
6	0	5	11	12	4	15	15	24
7	0	5	11	12	4	15	15	24
8	0	5	11	12	4	15	15	24

2. **Below is a list of courses and prerequisites for a factious CS degree.**

Course	Prerequisite
CS 150	None
CS 151	CS 150
CS 221	CS 151
CS 222	CS 221
CS 325	CS 221
CS 351	CS 151
CS 370	CS 151
CS 375	CS 151, CS 222
CS 401	CS 375, CS 351, CS325, CS 222
CS 425	CS 325, CS 222
MATH 200	None
MATH 201	MATH 200

a. Draw a directed acyclic graph (DAG) that represents the precedence among the courses.



b. Give a topological sort of the graph.

MATH 200, CS 150, CS 151, MATH 201, CS 221, CS 370, CS351, CS 222, CS 325, CS 425, CS 375, CS 401

c. **Find an order in which all the classes can be taken. (Assuming you take 2 classes a quarter)**

Quarter	Classes
1	MATH 200, CS 150
2	MATH 201, CS 151
3	CS 221, CS 351
4	CS 222, CS 325
5	CS 375, CS 425
6	CS 370, CS 401

d. **Determine the length of the longest path in the DAG. How did you find it? What does this represent?**

The longest path in the DAG is 4. This is found by following the path from CS 150 → CS 151 → CS 221 → CS 325 → CS 401. This represents the least amount of quarters require to obtain a degree, simply due to the amount of prerequisites for the classes. Even if you did 3 classes a quarter, you would need to take at least 4 quarters to complete them all due to the levels of prerequisites for all of the classes leading to CS 401.

3. **Suppose you have an undirected graph $G = (V, E)$ and you want to determine if you can assign two colors (blue and red) to the vertices such that adjacent vertices are different colors. This is the graph Two-Color problem. If the assignment of two colors is possible, then a 2-coloring is a function C: $V -> \{blue, red\}$ such that $C(u) \neq C(v)$ for every edge $(u, v) \in E$. Note: a graph can have more than one 2-coloring.**

   Give an O(V + E) algorithm to determine the 2-coloring of a graph if one exists or terminate with the message that the graph is not Two-Colorable. Assume that the input graph G=(V,E) is represented using adjacency lists.

   a. **Give a verbal description of the algorithm and provide detailed pseudocode.**

```
TwoColor(InputVertices V)
    For all vertices in V
            color[V] = unknown
    For all vertices in V
            If color[V] = unknown
                    Color_vertex(V, blue, red)
Color_vertex(Vertex V, color1, color2)
    color[V] = color1
    For all edge (V, V2)
```

If color[V2] == color1

Possible = return “Not Colorable”

If color[V2] == UNKNOWN

Color_vertex(V2, color2, color1)

b. Analyze the running time.

Given a proper adjacency list, this algorithm should run in O(V + E). This is because for every vertex it will give the appropriate color, and if it is not possible, it will return false since you have a non-2-colorable graph.

4. A region contains a number of towns connected by roads. Each road is labeled by the average number of minutes required for a fire engine to travel to it. Each intersection is labeled with a circle. Suppose that you work for a city that has decided to place a fire station at location G. (While this problem is small, you want to devise a method to solve much larger problems).

a. What algorithm would you recommend be used to find the fastest route from the fire station to each of the intersections? Demonstrate how it would work on the example above if the fire station is placed at G. Show the resulting routes.

b. Suppose one “optimal” location (maybe instead of G) must be selected for the fire station such that it minimizes the distance to the farthest intersection. Devise an algorithm to solve this problem given an arbitrary road map. Analyze the time complexity of your algorithm when there are f possible locations for the fire station (which must be at one of the intersections) and r possible roads.

c. In the above graph what is the “optimal” location to place the fire station? Why?