2019 Fall, Electrical and Electronic Circuits (4190.206A 002)

Midterm #2

Nov. 18, 2019 2:00 pm

Sol

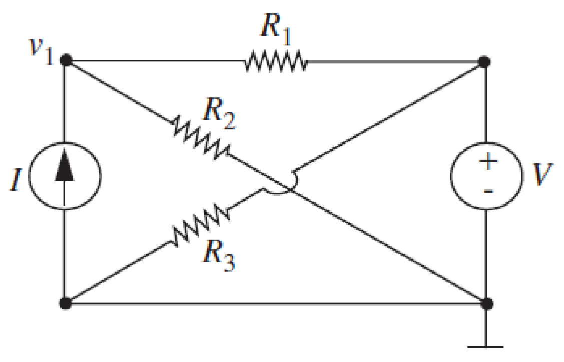
Name : ____________________

Student ID Number : ____________________

1	**2**		**3**				**4**				**5**				
	a	**b**	**a**	**b**	**c**	**d**	a	b	c	d	a	b	c	d	
6									**7**			**8**			
a	b	c	**d**	**e**	**f**	**g**	h	**i**	**a**	**b**	**c**	**a**	**b**		

Problem 1 - 10 Points

Find v_1 by superposition for the circuit below.



Problem 2 - 15 Points

There is a MOSFET amplifier circuit in Fig.(a). We use a switch-current source (SCS) MOSFET model in this problem and its characteristics are shown in Fig.(b) and Fig.(c).

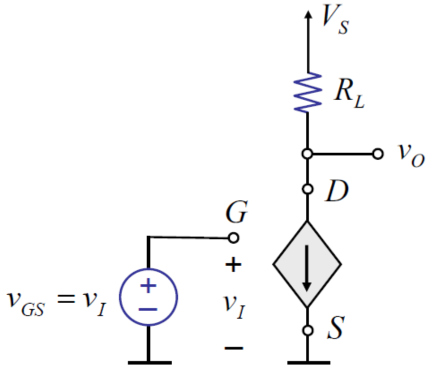


Fig.(a) MOSFET amplifier circuit.

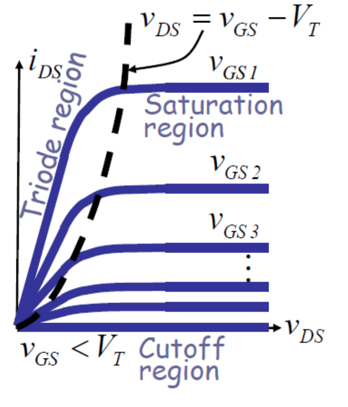
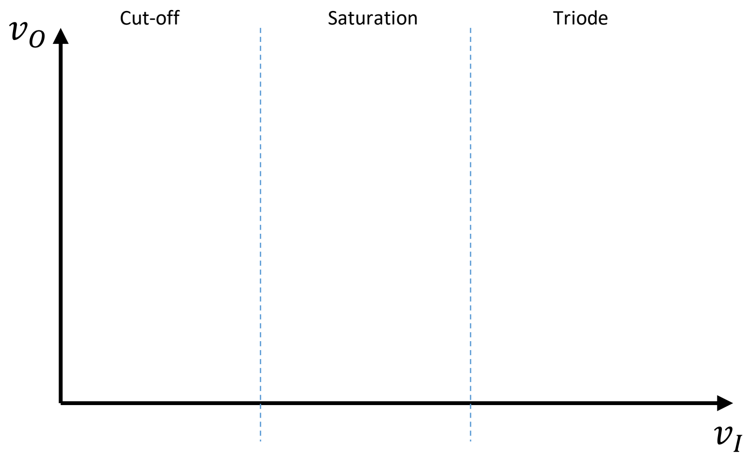


Fig.(b) Characteristics of the MOSFET showing triode, saturation, and cut-off regions.

$$
i_{DS} = \begin{cases} K\left[(v_{GS} - V_T)v_{DS} - \dfrac{v_{DS}^2}{2}\right] & \text{for } v_{GS} \geq V_T \text{ and } v_{DS} < v_{GS} - V_T \quad \text{(Triode)} \\ \dfrac{K(v_{GS} - V_T)^2}{2} & \text{for } v_{GS} \geq V_T \text{ and } v_{DS} \geq v_{GS} - V_T \quad \text{(Saturation)} \\ 0 & \text{for } v_{GS} < V_T \quad \text{(Cut-off)} \end{cases}
$$

Fig.(c) Characteristics of the MOSFET showing triode, saturation, and cut-off regions.

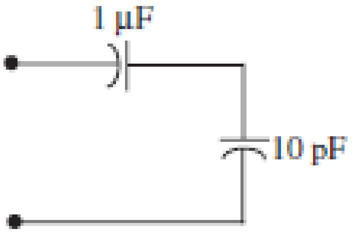
(a) Plot the relation between v_I and v_O and specify v_I and v_O value of the point between the cut-off and the saturation region, and the point between the saturation and the triode region. (10 Points)



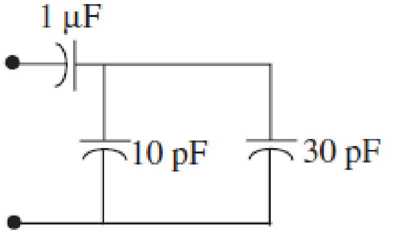
(b) When the MOSFET is being used as an amplifier in the saturation region, we analyze our circuit by considering that the input signal v_I is a sum of a DC bias V_I and a small signal v_i, i.e. $v_I = V_I + v_i$. Assuming that the small signal v_i is always negative, if we want to get the maximum gain on the amplifier circuit in Fig.(a), what is the optimal value for the DC bias V_I ? (5 Points)

Problem 3 - 20 Points

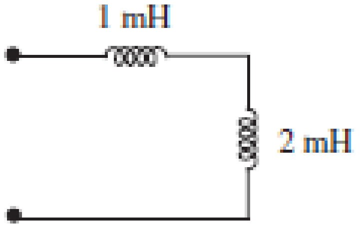
(a) Calculate the equivalent capacitance. (5 Points)



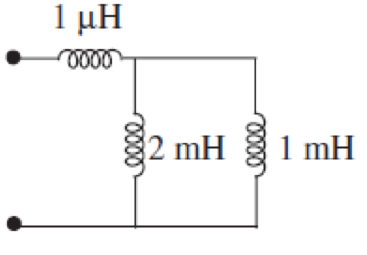
(b) Calculate the equivalent capacitance. (5 Points)



(c) Calculate the equivalent inductance. (5 Points)

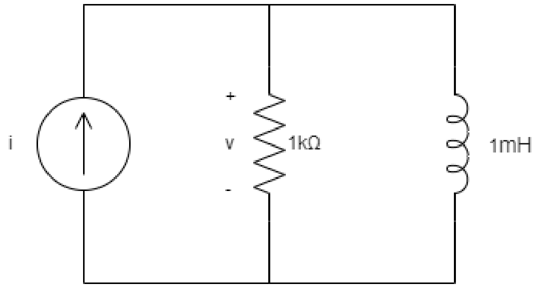


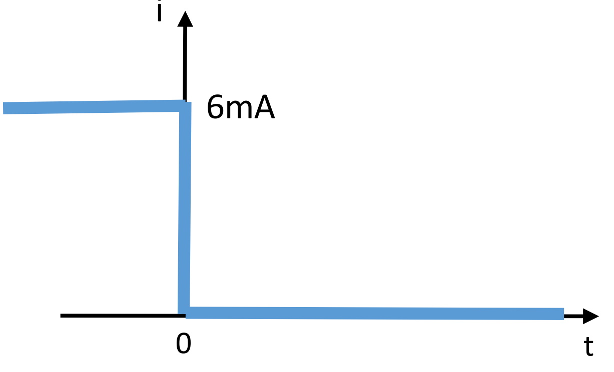
(d) Calculate the equivalent inductance. (5 Points)



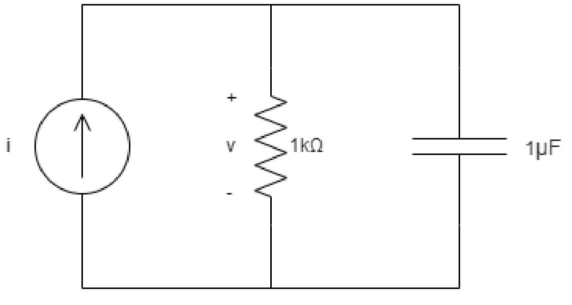
Problem 4 - 40 Points

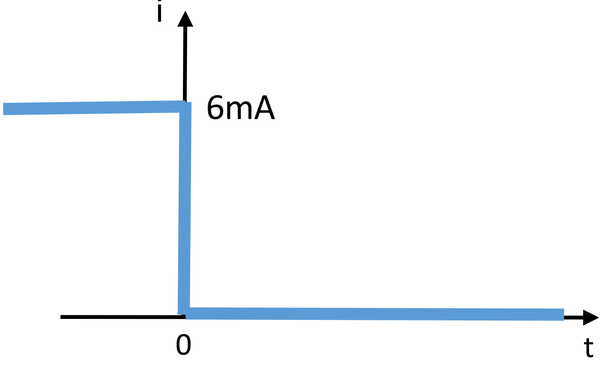
(a) Calculate v(t) in the circuit with the given condition for t > 0. (10 Points)



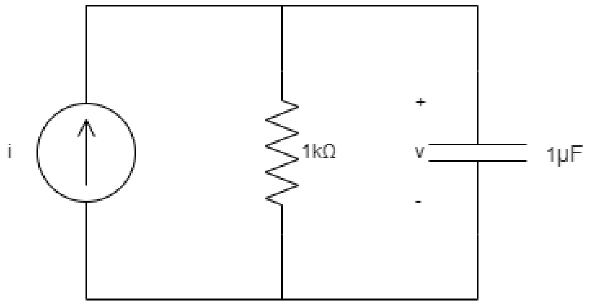


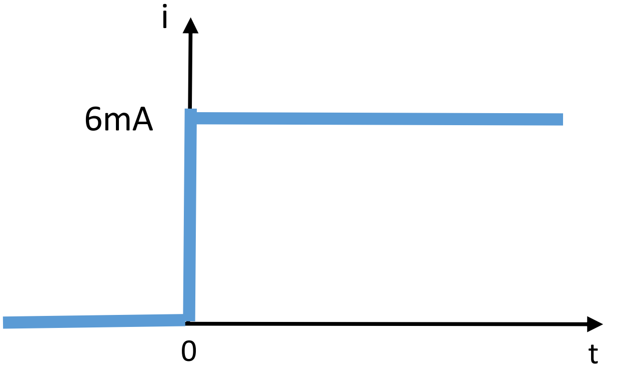
(b) Calculate v(t) in the circuit with the given condition for t > 0. (10 Points)



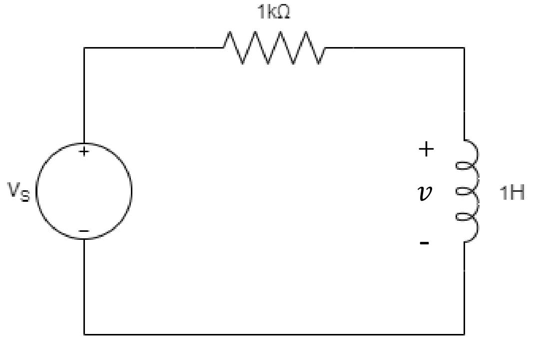


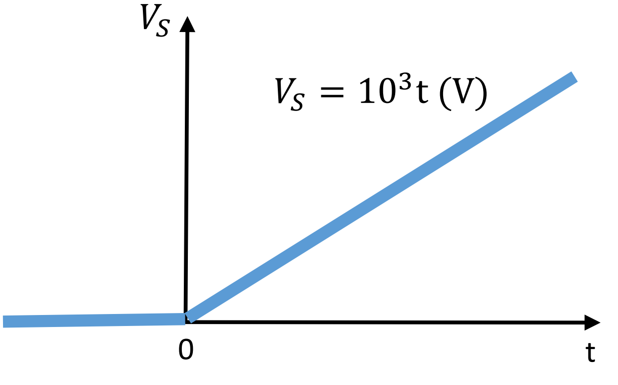
(c) Calculate v(t) in the circuit with the given condition for t > 0. (10 Points)





(d) Calculate v(t) in the circuit with the given condition for t > 0. (10 Points)





Problem 5 - 20 Points

Find the homogeneous solution.

(a) $\left(\frac{d}{dt} - 2\right)\left(\frac{d}{dt} + 3\right) y(t) = 0$ (4 Points)

(b) $\left(\frac{d}{dt} - 1\right)^2 y(t) = 0$ (4 Points)

Find the particular solution.

(c) $\boldsymbol{\frac{d}{dt} y + 2y = 2t^2}$ (6 Points)

(d) $\frac{d}{dt} y + 2y = e^{-2t}$ (6 Points)

Problem 6 - 40 Points

All the analog amplifier circuit we learned in the class is designed to amplify the small signal change around some DC voltage called as bias. However, the real signal like a sound is changing around 0 V as shown in the following Fig. (a), and therefore we need to shift the center of input voltage level from 0 to non-zero DC voltage. Let's see whether the circuit shown in the following Fig. (b) can shift the center of the input voltage level.

For simplicity of our analysis, assume that our input voltage (v_I) is 0 V when t < 0, and it starts to change in the form of a sine wave between +1 V and -1V as shown in Fig. (c). Fig. (d) shows the expected waveform of the signal $v_g(t)$ at the gate pin. The following sub questions will walk you through the calculation of $v_g(t)$ and therefore find out the DC Bias and the amplitude A. Also we assume the MOSFET is ideal, so there is no input capacitance between the gate and the source.

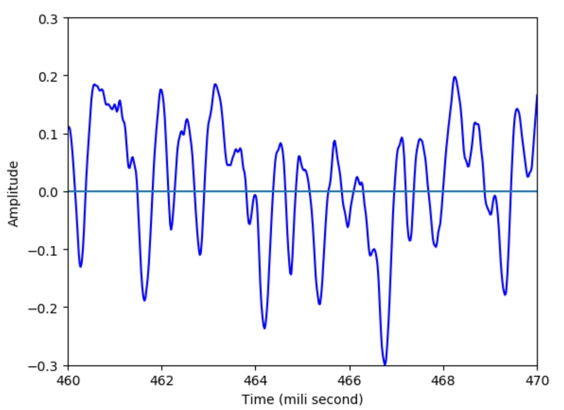


Fig. (a) Example signal of sound

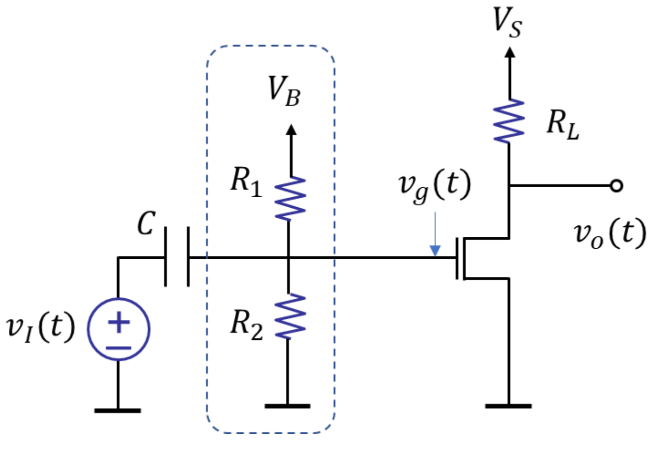


Fig. (b) C and voltage divider is added to shift the DC input bias.

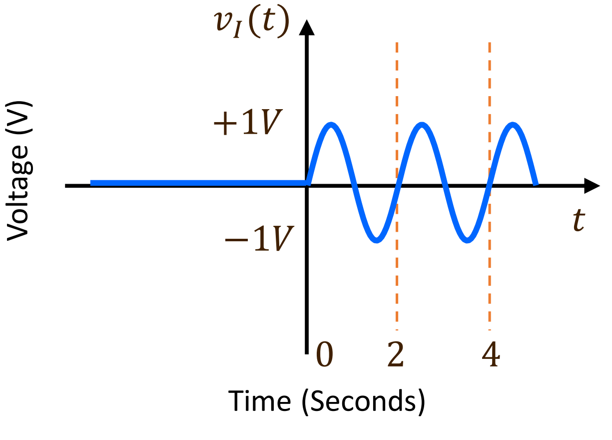


Fig. (c) $V_I(t)$ signal changing around 0V

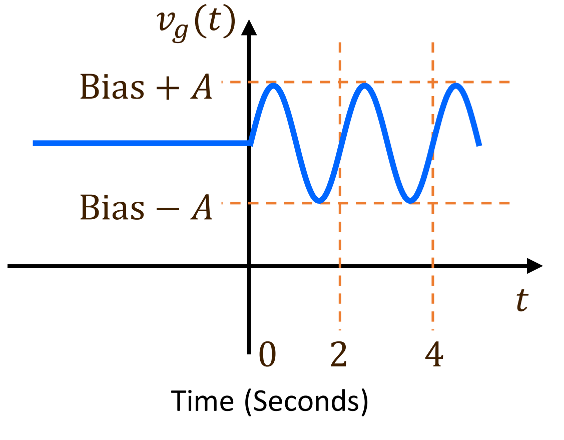
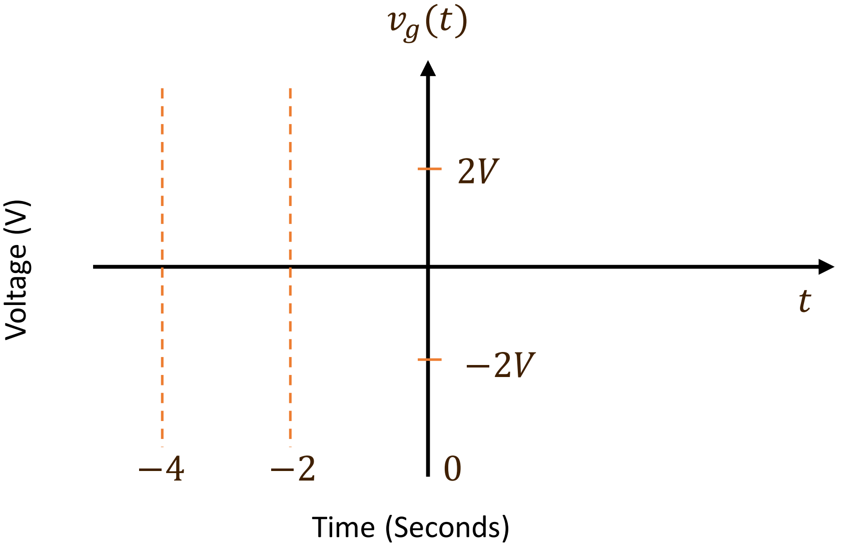


Fig. (d) Expected result at $v_g(t)$

(a) Find out Thevenin equivalent circuit of the circuit in the box drawn with dashed line. (4 Points)

(b) Redraw the circuit on the left-hand side of $v_g(t)$ as a simple RC circuit composed of $v_I(t)$, C, R_{Th}, V_{Th}, and label the position of $v_g(t)$ in this circuit. For the rest of the problem, assume that $V_B = 6\ V$, $R_1 = 6M\Omega$, $R_2 = 3\ \mathrm{M\Omega}$, $C = 200\mu F$. (4 Points)

(c) Find out the circuit before $t = 0$. As shown in Fig. (b), $v_I(t) = 0$ for $t \leq 0$, which means the input voltage didn't change for a very long time.
Plot the voltage of $v_g(t)$ for $-4\, sec < t < 0\, sec$. (4 Points)



(d) Please find out the differential equation for $v_g(t)$ when $t > 0$. As shown in Fig. (b), $v_I(t) = \sin(\pi t)$ for $t > 0$. Therefore if the differential equation includes $\frac{d}{dt}v_I(t)$ term, then you need to calculate $\frac{d}{dt}v_I(t)$ term explicitly using $v_I(t) = \sin(\pi t)$ for $t > 0$. (6 Points)

(e) Please find out the particular solution for $t > 0$. The particular solution will be composed of multiple components, and simplify the solution by using the following approximation, if applicable. (6 Points)

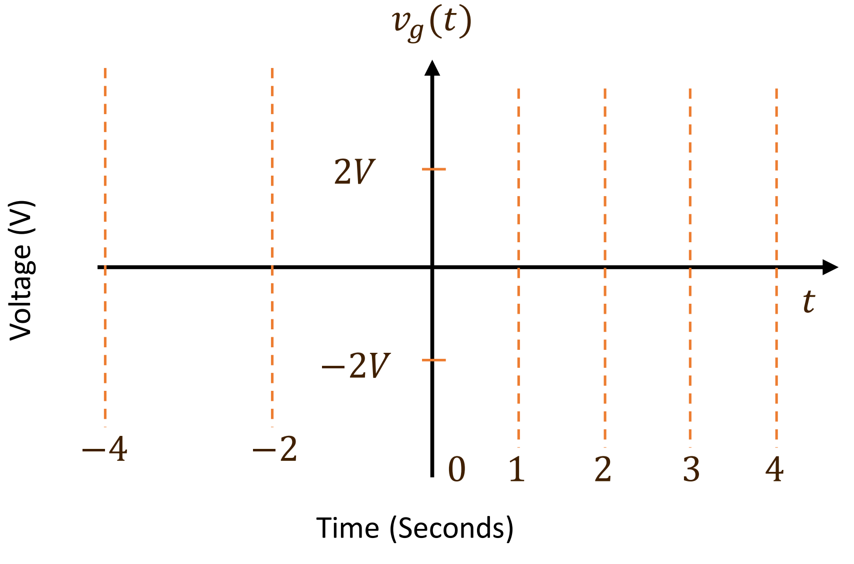
- If $\frac{A}{B} > 100, A + B \approx A$.
- If $\frac{A}{B} > 100, A \sin x + B \cos x \approx A \sin x$.

(f) Please find out the homogeneous solution for $t > 0$. (4 Points)

(g) Please find out the initial value for $v_g(t)$ at $t = 0$. (4 Points)

(h) By combining the answers from (E), (F), (G), Please determine the unknown coefficient of homogeneous solution, and write down the total solution for $t > 0$. (4 Points)

(i) Combine the result from (C) and (G) and plot the voltage of $v_g(t)$ between -4 seconds and 4 seconds. If you didn't make any mistake, $v_g(t)$ should look very similar to Fig. (d). Please specify the value of DC Bias and the amplitude A. (4 Points)



Problem 7 - 10 Points

From the previous problem 6, assume that you obtained $v_g(t)$ shown in Fig. (a). (Note that this signal might be different from the answer of the previous problem.) We want to find out the expected waveform of $v_o(t)$ at the output of the MOSFET amp.

The following is the information about the MOSFET.

$$i_{DS} = \begin{cases} K\left[(v_{GS} - V_T)v_{DS} - \frac{v_{DS}^2}{2}\right] & \text{for } v_{GS} \geq V_T \text{ and } v_{DS} < v_{GS} - V_T \quad \text{(Triode)} \\ \frac{K}{2}(v_{GS} - V_T)^2 & \text{for } v_{GS} \geq V_T \text{ and } v_{DS} \geq v_{GS} - V_T \quad \text{(Saturation)} \\ 0 & \text{for } v_{GS} < V_T \quad \text{(Cutoff)} \end{cases}$$

Circuit parameters are $V_S = 8\text{V}$, $V_T = 1\text{V}$, $K = 1\text{mA/V}^2$, $R_L = 2\text{k}\Omega$.

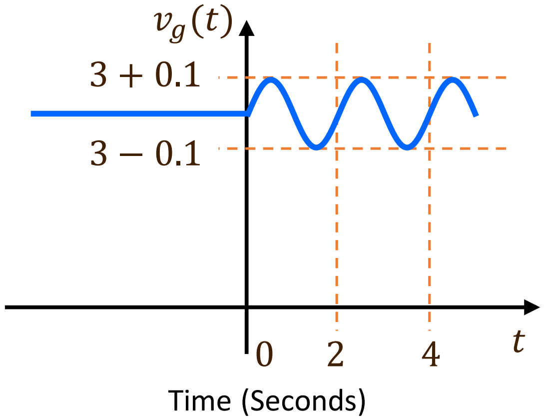


Fig. (a) Input signal at $v_g(t)$. Not drawn to scale.

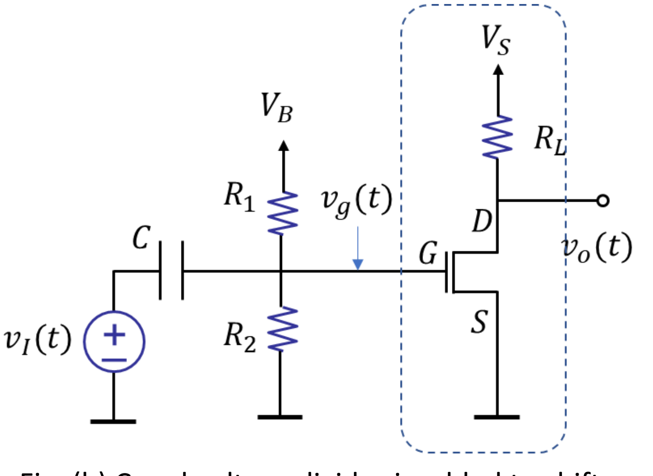
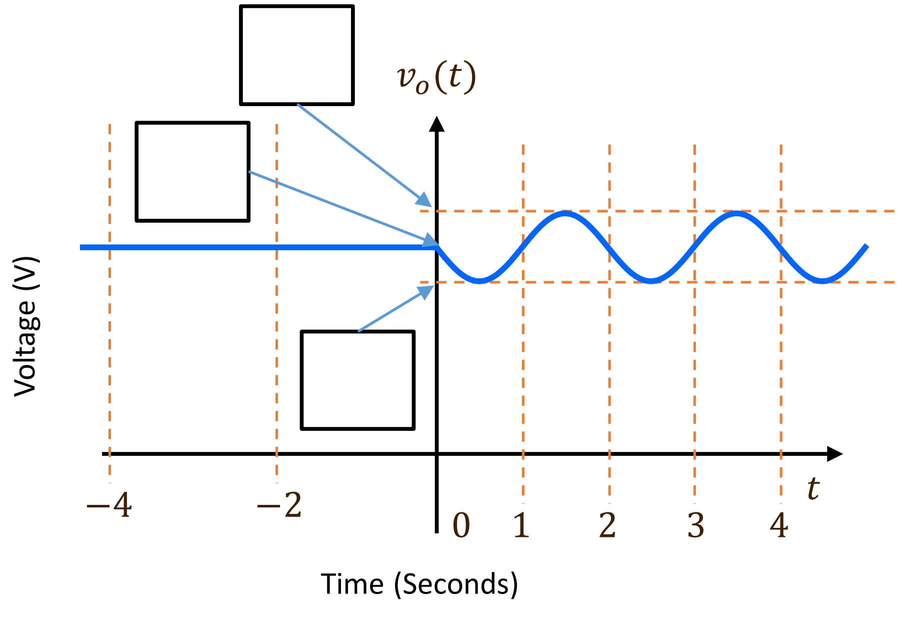


Fig. (b) C and voltage divider is added to shift the DC input bias.

(a) Please find out the output bias voltage V_O. (4 Points)

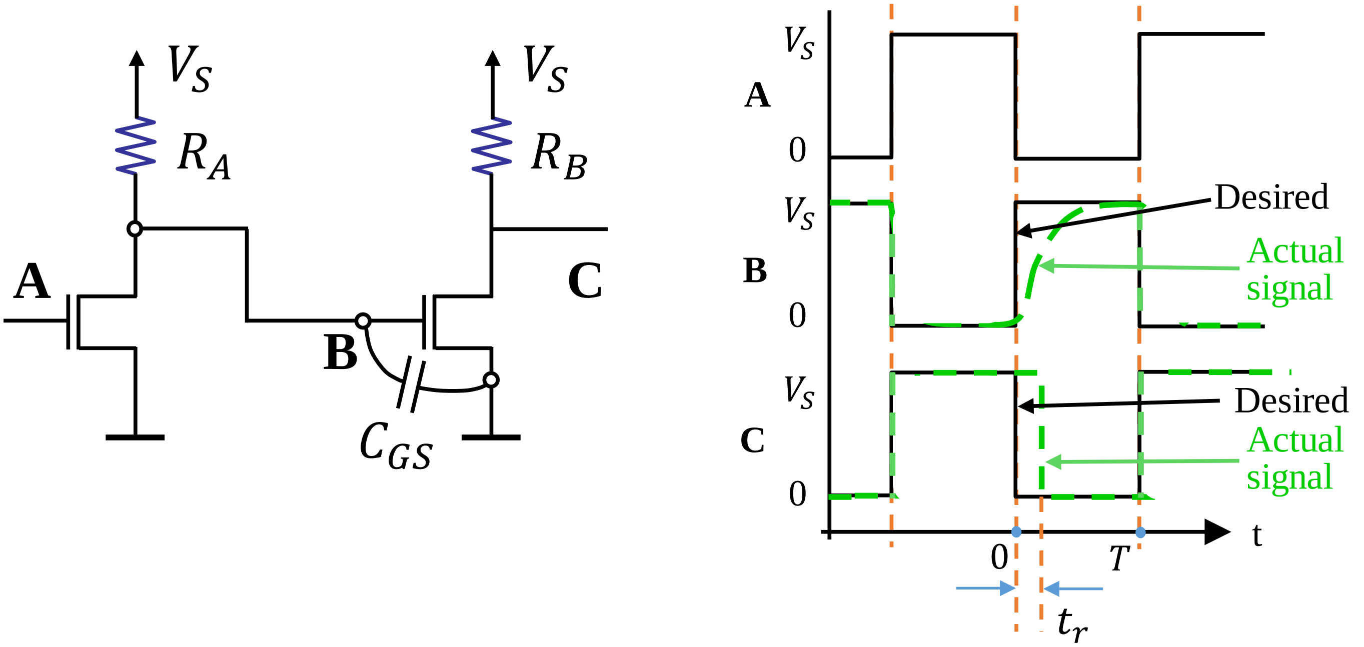
(b) Please find out the small signal gain of the amplifier. (4 Points)

(c) The following figure shows the waveform of the output voltage $v_o(t)$. Please fill out the boxes with the correct values (Note that it is not drawn to the scale). (2 Points)



Problem 8 - 10 Points

In the lecture, we learned that the output of the double inverter will be delayed by the parasitic capacitance in the 2nd inverter. We want to find out the amount of the rising-time delay t_r in terms of V_T of the 2nd MOSFET. As shown in the circuit diagram, we assume that the 1st MOSFET does not have any parasitic capacitance, so the 1st MOSFET will respond to the input voltage at $\mathbf{A}$ immediately. Also for simplicity of the problem, we assume S-model for both MOSFETs, meaning $R_{ON} = 0$.



(a) Find out the expression for the voltage at $\mathbf{B}$ when $0 < t < T$. You can either solve the full differential equation or simply obtain the solution using intuitive method. Note that R_A and R_B are different values, so please make sure to indicate the subscript of the resistance value. (5 Points)

(b) By using the fact that the 2nd MOSFET will change the output voltage at $\mathbf{C}$ only when the voltage at $\mathbf{B}$ becomes larger than the threshold voltage V_T, please find out the expression for the rising-time delay t_r. (5 Points)