2019 Fall, Electrical and Electronic Circuits (4190.206A 002)

Homework #2

Solution

Due date: Oct. 23, 2019 3:15pm

If you hand-in after the due date, your score will be deducted by 20%.

No more submission will be accepted after Oct. 28, 2019. 3:15pm.

Name : ____________________

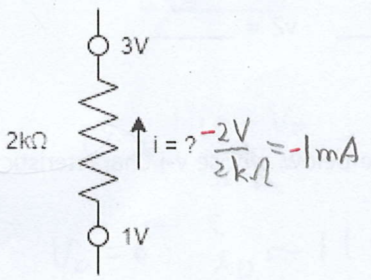
Student ID Number : ____________________

1				2	3	4	5			6		7			8				9	10			
a	b	c	d				a	b	c	a	b	a	b	c	a	b	c	d		a	b	c	d

11						12	13				Total
a	b	c	d	e	f		a	b	c	d	

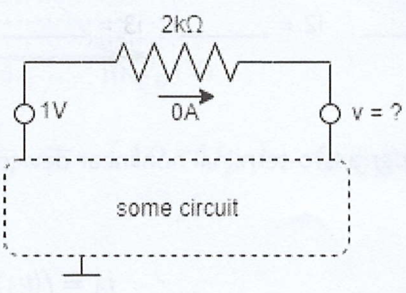
2-1. (point 9) Find out the unknown values in the following circuits, that are indicated with the question marks.

(a).(point 2)



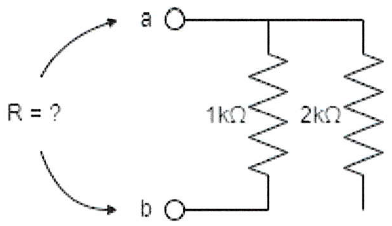
Answer : i = -1 (mA)

(b). (point 2)



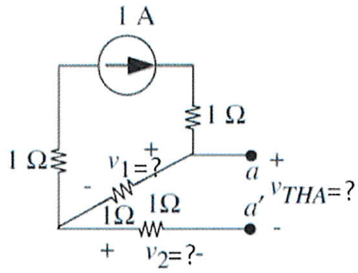
Answer : v = 1 (V)

(c). (point 2) Resistance between terminal a and b.



Answer : R = 1 (kΩ)

(d) (point 3)

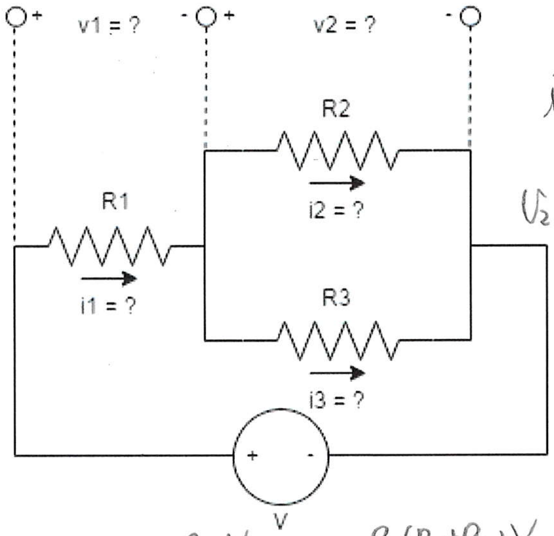


Answer : v_1 = 1 (V), v_2 = 0 (V), v_{THA} = 1 (V)

2-2. (point 10) R1, R2, R3, and V are the values of corresponding resistors and a voltage source shown in the figure below. Please find the expressions for i1, i2, i3, v1, and v2 in the figure in terms of R1, R2, R3, and V.

$\frac{R_2R_3}{R_2+R_3}$

Total resistance: $R_1 + R_2 // R_3$



$i_1 = \frac{V}{R_1+R_2//R_3} = \frac{(R_2+R_3)V}{R_1R_2+R_1R_3+R_2R_3}$

$V_2 = (R_2//R_3)\, i_1 = \frac{(R_2//R_3)V}{R_1+R_2//R_3} = \frac{R_2R_3V}{R_1R_2+R_1R_3+R_2R_3}$

$i_2 = \frac{V_2}{R_2} = \frac{\frac{R_3}{R_2+R_3}V}{R_1+R_2//R_3} = \frac{R_3V}{R_1R_2+R_1R_3+R_2R_3}$

$i_3 = \frac{V_1}{R_3} = \frac{\frac{R_2}{R_2+R_3}V}{R_1+R_2//R_3} = \frac{R_2V}{R_1R_2+R_1R_3+R_2R_3}$

$V_1 = R_1 i_1 = \frac{R_1V}{R_1+R_2//R_3} = \frac{R_1(R_2+R_3)V}{R_1R_2+R_1R_3+R_2R_3}$

Answer : i1 = ______ i2 = ______ i3 = ______ v1 = ______ v2 = ______

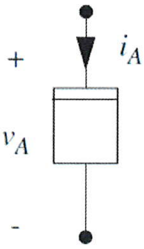
2.3. (point 6) Consider a two-terminal nonlinear device on the figure below, whose v-i characteristic is given by:

$$i_A = f(v_A)$$

Show that, if the voltage is changed by the small amount ($\Delta v_A = v_a$) around the DC operating point V_A and I_A, the incremented change in the current ($\Delta i_A = i_a$) is given by:

$$i_a = \frac{df(v_A)}{dv_A}\bigg|_{v_A=V_A} v_a.$$

(Hint: Substitute $i_A = I_A + i_a$ and $v_A = V_A + v_a$ in equation $i_A = f(v_A)$, expand using Taylor series, ignore second order and higher terms in v_a, and equate corresponding DC and small signal terms.)



$$i_A = I_A + i_a = f(V_A + v_a)$$

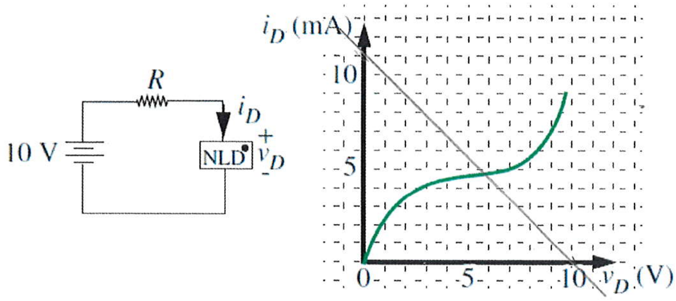
$$= f(V_A) + \frac{df(v_A)}{dv_A}\bigg|_{v_A=V_A} v_a + \frac{1}{2!}\frac{d^2 f(v_A)}{dv_A^2}\bigg|_{v_A=V_A} v_a^2 + \cdots$$

Answer :

$$I_A = f(V_A), \quad i_a \approx \frac{df(v_A)}{dv_A}\bigg|_{v_A=V_A} v_a$$

2.4. (point 10) The nonlinear device (NLD) in the circuit in the figure below has the v-i characteristics shown. Find the operating point i_D and v_D for $R = 910\ \Omega$. (Mark on the graph.)

Answer : $i_D \approx 5$ (mA), $V_D \approx 6$ (V)



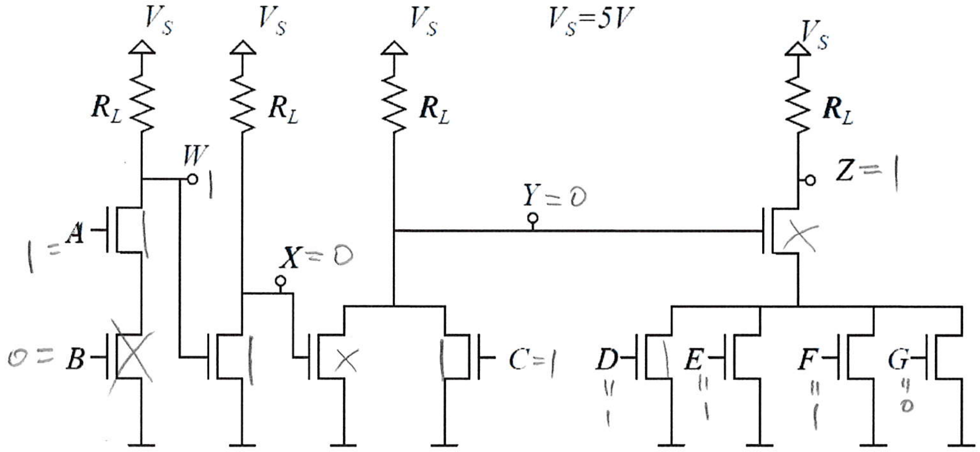
$$i_D = \frac{10 - V_D}{R} = \frac{10 - V_D}{910(\Omega)} \approx (10 - V_D)\times 0.0011\ (A) = (10 - V_A)\times 1.1\ (mA)$$

$V_D = 0$: $i_D \approx 11$ (mA)

$V_D = 10$: $i_D = 0$ (mA)

2.5. (point 12)

(a) (point 4) Assuming that the circuit shown below satisfies the static discipline, determine the logical values of W, X, Y and Z given that the inputs are set as follows: A=1, B=0, C=1, D=1, E=1, F=1, G=0.

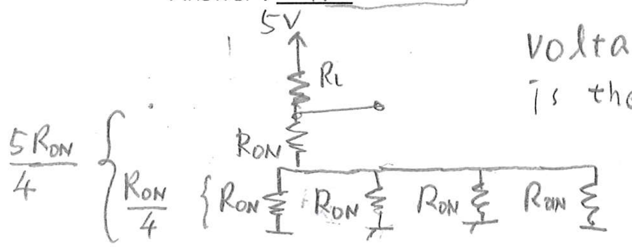


Answer : W=1, X=0, Y=0, Z=1

(b) (point 4) Using the SR model for the MOSFETs with on Resistance R_ON , determine the lowest voltage possible at Z.

Answer : $\boxed{\frac{5R_{ON}\cdot V_s}{4R_L+5R_{ON}}}$

Lowest voltage is possible when the voltage drop caused by R_ON resistance is the minimum.

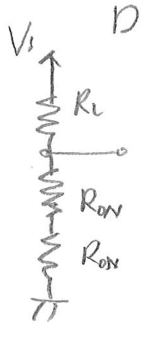


$$\frac{\frac{5R_{ON}}{4}\cdot V_s}{R_L+\frac{5}{4}R_{ON}} = \frac{5R_{ON}\,V_s}{4R_L+5R_{ON}}$$

(c) (point 4) If R_L=10Ω and R_{ON}=1Ω, does the output at Z satisfy the static discipline if V_{OL}=1V? Please explain your answer.

Answer : Yes.

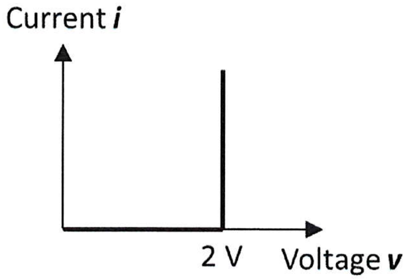
The largest low output voltage occurs when only one of D, E, F, G is on.

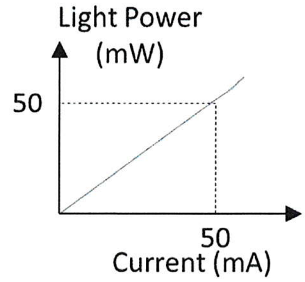


$$Z=\frac{2R_{ON}\cdot V_s}{R_L+2R_{ON}} = \frac{2\cdot1\cdot5}{10+2} = \frac{10}{12} = \frac{5}{6} < 1V = V_{OL}$$

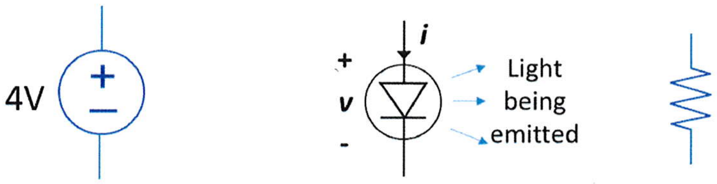
∴ The largest low output voltage is less than V_{OL} and satisfy the static discipline.

2.6. (point 12) We want to control the light power emitted by the following LED (light-emitting diode). Assume that the given diode (LED) has an ideal i-v relation shown in the left plot. Right plot shows the light power as a function of the current flowing through LED.

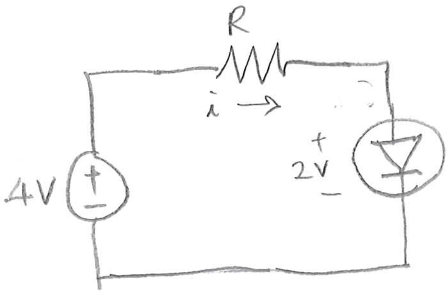




(a) (point 6) Assume that we only have a 4V voltage source, an LED and resistors with many different values as shown below, and we want 10 mW of light power emitted from LED. Please make a circuit composed of 4V voltage source, a resistor, and LED. Explicitly specify the value of the resistor.



Answer : 200 (Ω)

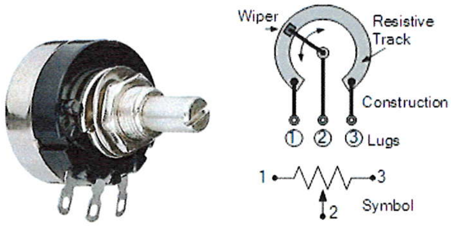


When any current is flowing through LED, the voltage drop across LED is constant 2 (V). Therefore the voltage drop across the resistor is 2(V).
To obtain 10 mW of light power, we need 10 mA of current through LED.

$\therefore R = \frac{V}{I} = \frac{2\,(V)}{10\,(mA)} = 0.2\,(k\Omega)$

(b) (point 6) Assume that we only have a 4V voltage source, an LED, many 50 Ω resistors, and many variable resistors, as shown below. The value of the variable resistors can be change from 0 ~ 300 Ω. Design a circuit which can generate any light power between 5mW and 20mW.

(The pictures below are for the explanation about the variable resistor. As shown in the second figure, we can rotate the wiper which is adjacent to the resistive materials. How much we rotate determines the resistance between the terminal 1 and 2. In this question, we can imagine using the terminal 1 and 2 as a resistor, which can be changed from 0Ω to 300Ω.)

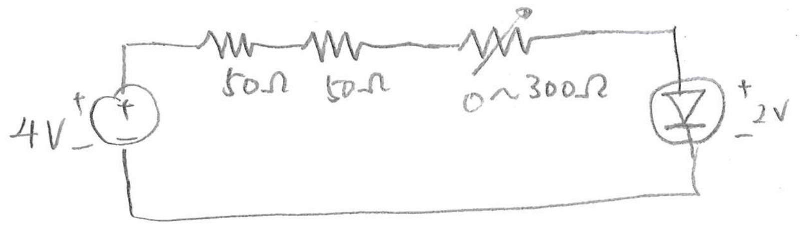


Answer :

To allow 5 mA through LED, $R = \frac{2(V)}{5(mA)} = 0.4\ (k\Omega) = 400\ (\Omega)$

To allow 20 mA " " , $R = \frac{2(V)}{20(mA)} = 0.1\ (k\Omega) = 100\ (\Omega)$

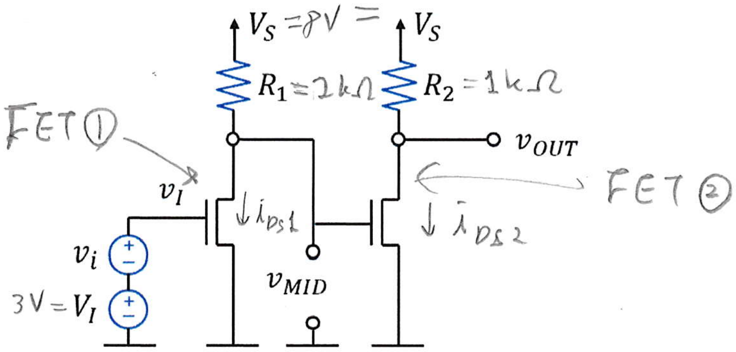
We can create variable resistor by combining two 50 Ω and a single variable resistor changing from 0 to 300Ω.



2.7 (point 12) The following figure shows two-stage amplifiers. Both MOSFET devices have the same typical properties as we learned in the class and it is summarized in the following:

$$i_{DS} = \begin{cases} K\left[(v_{GS} - V_T)v_{DS} - \frac{v_{DS}^2}{2}\right] & \text{for } v_{GS} \geq V_T \text{ and } v_{DS} < v_{GS} - V_T \quad \text{(Triode)} \\ \frac{K}{2}(v_{GS} - V_T)^2 & \text{for } v_{GS} \geq V_T \text{ and } v_{DS} \geq v_{GS} - V_T \quad \text{(Saturation)} \\ 0 & \text{for } v_{GS} < V_T \quad \text{(Cutoff)} \end{cases}$$

Circuit parameters are $V_S = 8\text{V}$, $V_T = 1\text{V}$, $K = 1\text{mA/V}^2$, $R_1 = 2\text{k}\Omega$, $R_2 = 1\text{k}\Omega$. Let's analyze when input DC bias is $V_I = 3\text{V}$. We assume that both MOSFETs have no capacitance between gate terminal and source terminal.



(a) (point 4) Find out the bias voltage at v_{MID} and v_{OUT}. Hint: first assume saturation region. Once the bias is obtained, check whether the result satisfies the requirement for saturation region.

With $V_I = 3V$, $I_{DS1} = \frac{1(mA/V^2)}{2}(3(V)-1(V))^2 = 2(mA)$, $V_{MID} = 8-2\cdot 2 = 4(V)$

With $V_{MID} = 4V$, $I_{DS2} = \frac{1(mA/V^2)}{2}(4-1)^2 = 4.5(mA)$, $V_{OUT} = 8-4.5 = 3.5(V)$

For FET ①, $V_{DS} = 4V$, $V_{GS} - V_T = 2(V)$ ⇒ saturation region.

For FET ②, $V_{DS} = 3.5V$, $V_{GS} - V_T = 3(V)$ ⇒ saturation

(b) (point 4) Find out the small signal gain (amplification ratio) of the 1st stage amplifier and the 2nd stage amplifier.

$$A_1 = \left.\frac{dV_{MID}}{dV_I}\right|_{V_I=3V} = -R_1\frac{di_{DS1}}{dV_I} = -2(k\Omega)\cdot K\cdot\left.(V_{GS}-V_T)\right|_{V_I=3V}$$

$$= -2(k\Omega)\cdot 1(mA/V^2)\cdot(3(V)-1(V)) = -4$$

$$A_2 = \left.\frac{dV_{OUT}}{dV_{MID}}\right|_{V_{MID}=4V} = -R_2\cdot\frac{di_{DS2}}{dV_{MID}} = -1(k\Omega)\cdot K\cdot\left.(V_{GS}-V_T)\right|_{V_{MID}=4V}$$

$$= -1(k\Omega)\cdot 1(mA/V^2)\cdot(4(V)-1(V)) = -3$$

(c) (point 4) Please write down expression for total output voltage v_{OUT} as a sum of DC bias voltage and amplified output of small signal input v_i.

Output bias voltage: 3.5 (V)

Amplification of small signal: $V_{out} = (-3)\cdot(-4)\,V_i = 12V_i$

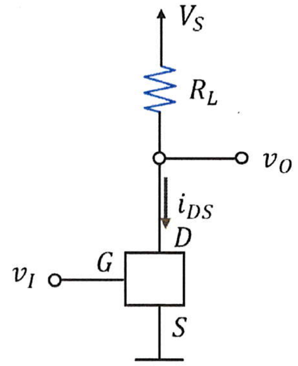
Total output voltage: $V_{OUT} \approx 3.5 + 12V_i$

2.8. (point 15) Assume that someone recently developed a new transistor which has the following properties.

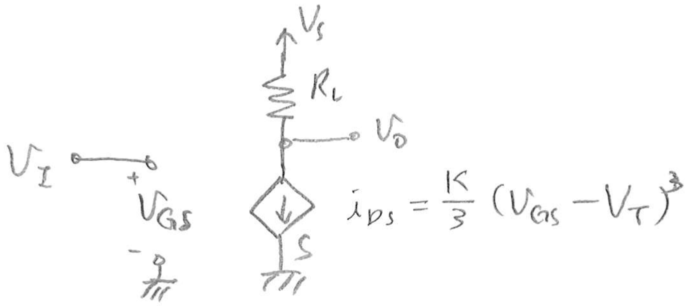
$$i_{DS} = \begin{cases} 0 & \text{when } v_{GS} < V_T \\ \frac{K}{3}(v_{GS} - V_T)^3 & \text{when } v_{GS} > V_T \end{cases}$$

Note that the current flowing from drain to source only depends on the voltage between gate and source, and it is independent of drain voltage. Also no current is flowing in the gate terminal.

Let's analyze the following amplifier circuit with this new transistor. In this problem, we will only consider the case when $v_I > V_T$.



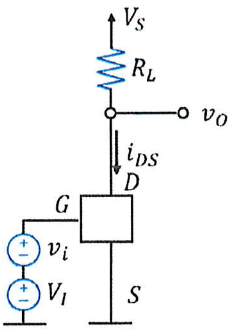
(a) (point 2) Draw the equivalent circuit for the amplifier using voltage-controlled current source (VCCS) model when $v_I > V_T$.



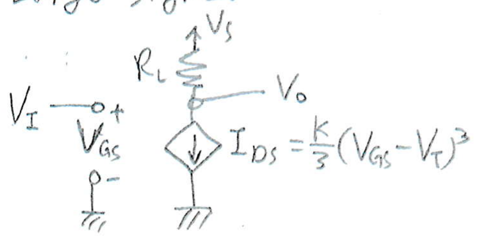
(b) (point 3) Write an expression for v_O in terms of v_I and the circuit parameters given above.

Answer : $V_o = V_s - \frac{R_L K}{3}(V_I - V_T)^3$

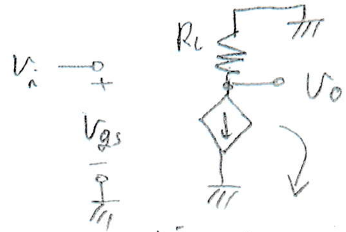
(c) (point 6) Assume that the input voltage v_I is given as a sum of DC bias voltage V_I and small signal v_i. Please draw a large signal circuit driven by V_I. Also draw a small signal circuit driven by the small signal v_i. Finally, find out an expression for small signal response v_o to the small signal input v_i.



Large signal



Small signal



$$i_{ds} = \left.\frac{d\, i_{DS}}{d V_{GS}}\right|_{v_{GS}=V_I} v_{gs} = K(V_I - V_T)^2 v_i$$

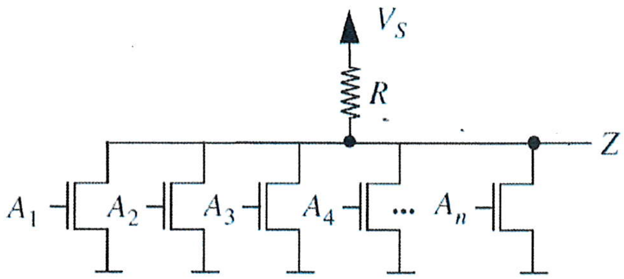
Answer : $v_o = -R_L K (V_I - V_T)^2 v_i$

(d) (point 4) We want the voltage amplification ratio of the small signal is -8. When $V_S = 10\ \mathrm{V}$, $V_T = 1\ \mathrm{V}$, $K = 2mA/V^3$ and $R = 1\ \mathrm{k\Omega}$, please find out the proper input bias voltage V_I and the resulting output bias voltage V_O.

Answer : $V_I = 3\,(V),\ V_o = \frac{14}{3}\,(V)$,

$$A = \frac{v_o}{v_i} = -R_L K (V_I - V_T)^2$$
$$= -1\,(k\Omega)\cdot 2\,(mA/V^3)\,(V_I - 1\,(V))^2 = -8$$
$$V_I = 3\,(V) \rightarrow I_{DS} = \frac{2}{3}(3-1)^3 = \frac{16}{3}\,(mA)$$
$$V_o = 10 - 1\,(k\Omega)\cdot\frac{16}{3}(mA) = \frac{14}{3}\,(V)$$

2.9. (point 6) Consider the N-input NOR gate shown in the figure below. Assume that the on-state resistance of each of the MOSFETs is R_{ON}. For what set of inputs does this gate consume the maximum amount of power? Compute this worst-case power.

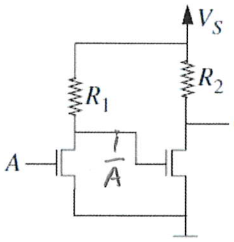


Answer : $\dfrac{V_S^2}{R + \frac{R_{ON}}{n}}$

For fixed voltage, the total power is determined by the resistance by $\frac{V_s^2}{R_{total}}$. When R_{total} is minimum, power consumption is maximum. For R_{total} to be minimum, all N-MOS needs to be on $\Rightarrow R_{total} = R + \frac{R_{ON}}{n}$ $\therefore P_{max} = \frac{V_S^2}{R + R_{ON}/n}$

2.10. (point 14) Write a Boolean expression that describes the function of each of the circuits in the figure.

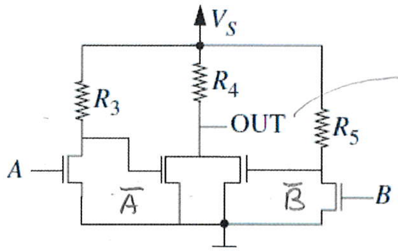
(a) (point 2)



$= (\bar{\bar{A}}) = A$

Answer : A

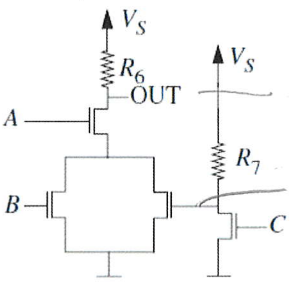
(b) (point 3)



$\overline{(\bar{A}+\bar{B})} = A \circ B$

Answer : $A \circ B$

(c) (point 4)

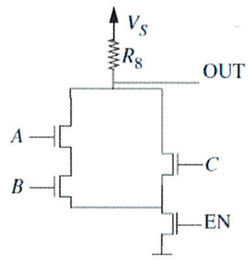


$\overline{(A \cdot (B + \bar{C}))}$

$\bar{C}$

Answer : $\bar{A} + \bar{B} \circ C$

(d) (point 4)



$= \overline{(A \circ B + C) \circ EN}$

Answer : $(\bar{A}+\bar{B}) \cdot \bar{C} + \overline{EN}$

2.11. (point 6) Answer the following questions using the actual sine and cosine values for $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \ldots$. For example, use $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$.

$\sin\frac{\pi}{6} = \frac{1}{2}, \; \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

(a) (point 1) $\sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right) = \sin\frac{\pi}{6} \cdot \cos\frac{\pi}{4} + \cos\frac{\pi}{6} \cdot \sin\frac{\pi}{4} = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4}$

(b) (point 1) $\sin^2\left(\frac{\pi}{8}\right) = \frac{1}{2}\left(1 - \cos\frac{\pi}{4}\right) = \frac{1}{2}\left(1 - \frac{\sqrt{2}}{2}\right) = \frac{2-\sqrt{2}}{4}$

$\cos 2\theta = 1 - 2\sin^2\theta$

(c) (point 1) $e^{i\pi} = \cos\pi + i\sin\pi = -1$

(d) (point 1) $e^{\frac{i\pi}{2}} =$ $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$

(e) (point 1) $e^{i\left(\frac{\pi}{6}+\frac{\pi}{4}\right)} = \cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right) =$ $\cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4} + i\frac{\sqrt{2}+\sqrt{6}}{4}$

$= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}+\sqrt{6}}{4} = \frac{1}{4}\left(\sqrt{6}-\sqrt{2}+i(\sqrt{2}+\sqrt{6})\right)$

(f) (point 1) Confirm that $e^{i\frac{\pi}{6}} \cdot e^{i\frac{\pi}{4}} = \underbrace{\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)}_{=\frac{1}{2}(\sqrt{3}+i)} \cdot \underbrace{\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)}_{=\frac{\sqrt{2}}{2}(1+i)}$ produces the same result as (e).

$\frac{\sqrt{2}}{4}(\sqrt{3}+i\sqrt{3}+i-1) = \frac{1}{4}\left(\sqrt{6}-\sqrt{2}+i(\sqrt{6}+\sqrt{2})\right)$

2.12 (point 2) Derive $\cos(\theta_1+\theta_2) = \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$ by using Euler relation. (Hint: Expand the equality $e^{i(\theta_1+\theta_2)} = e^{i\theta_1} \cdot e^{i\theta_2}$ using Euler relation and equate the real terms and imaginary terms separately.)

$e^{i(\theta_1+\theta_2)} = \underbrace{\cos(\theta_1+\theta_2)}_{\text{real term}} + i\sin(\theta_1+\theta_2)$

$e^{i\theta_1} \cdot e^{i\theta_2} = (\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)$

$= \underbrace{\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2}_{\text{real term}} + i(\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2)$

2.13 (point 4) Answer the following questions. In the following question, a is a positive constant.

(a) (point 1) $\frac{d}{dx}a^x =$ $\frac{d}{dx}\left(e^{\ln a}\right)^x = \frac{d}{dx}\left(e^{(\ln a)\cdot x}\right) = e^{(\ln a)\cdot x}\cdot(\ln a)$

$= a^x(\ln a)$

(b) (point 1) $\frac{d}{dx}e^{x^2} =$ $e^{x^2}\cdot 2x$

(c) (point 1) $\frac{d}{d\theta}e^{a*\sin(\theta)} =$ $e^{a*\sin(\theta)} \cdot a\cos\theta.$

(d) (point 1) When $f(x) = 3*x^3 + 2*x^2 + x$, find the Taylor series of f(x) around x = 1. Note that if you expand the result of the Taylor series, you should get the original function.

$f(x) = f(x)\Big|_{x=1} + \frac{df}{dx}\Big|_{x=1}(x-1) + \frac{1}{2!}\frac{d^2f}{dx^2}\Big|_{x=1}(x-1)^2 + \cdots$

$= 6 + (9\cdot1+4\cdot1+1)(x-1) + \frac{1}{2}(18\cdot1+4+0)(x-1)^2 + \frac{1}{6}(18)(x-1)^3 + 0\cdots$

$= 6 + 14(x-1) + 11\cdot(x-1)^2 + 3(x-1)^3$

If I expand $f(x)$, $6 + 14x - 14 + 11x^2 - 22x + 11 + 3x^3 - 9x^2 + 9x - 3$

$= 3x^3 + (11-9)x^2 + (14-22+9)x + 6 - 14 + 11 - 3$

$= 3x^3 + 2x^2 + x$