2019 Fall, Electrical and Electronic Circuits (4190.206A 002)

Final

Dec. 11, 2019 2:00 pm

Name : ____________________

Student ID Number : ____________________

1								2		3		4					
a	b	c	d	e	f	G	h	a	b	a	b	a	b	c	d		
5					6				7				8				
a	b	c	d	e	a	b	C	d	a	b	c	d	a	b			

Unit prefix

$m: 10^{-3}$ $k: 10^{3}$
$\mu: 10^{-6}$ $M: 10^{6}$
$n: 10^{-9}$ $G: 10^{9}$
$p: 10^{-12}$ $T: 10^{12}$
$f: 10^{-15}$ $P: 10^{15}$

Problem 1 - 25 Points

Consider the NMOS inverter in figure 1. As we learned in class, we'll use SR (Switch-Resistor) model for the MOSFET and calculate the average power dissipation in the circuit under the input voltage shown in figure 2.

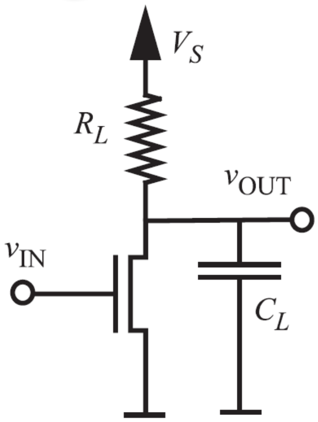


Figure 1

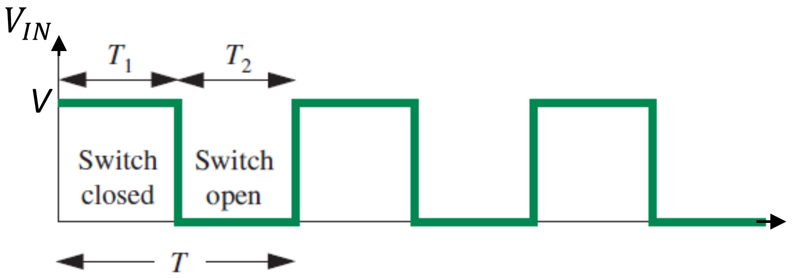


Figure 2

(A) Figure 3 is the equivalent circuit when the switch is closed. Express V_{TH} *and* R_{TH} in the Thevenin equivalent circuit shown in figure 4 in terms of V_S, R_L *and* R_{ON}. (3 points)

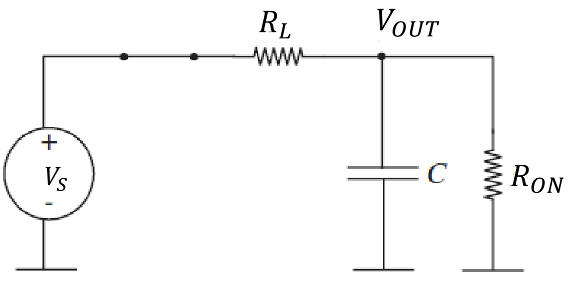


Figure 3

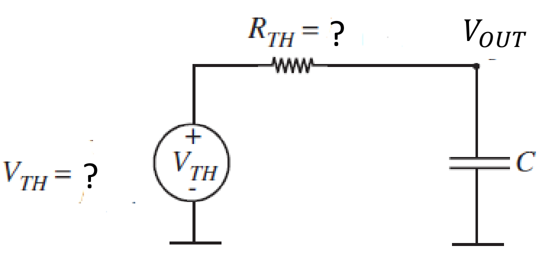


Figure 4

(B) Assuming that V_{out} is initially V_S right after the switch is closed (V_{out}(0) = V_S). Calculate V_{out}(t) in terms of *C*, V_S , V_{TH} *and* R_{TH}. (4 points)

(C) Find the expression for the power dissipated in the resistors of this circuit in terms of *C*, V_{TH}, R_{TH}, V_S, R_L *and* R_{ON} during T_1 period. Note that we are asking for the instantaneous power, not the integrated energy, so you don't need to calculate the integration. (4 points)

Answer : p(t) =

(D) Figure 5 is the equivalent circuit when the switch is open. Assuming that V_{out} is initially V_{TH}, right after the switch is closed (V_{out}(0) = V_{TH}). Calculate V_{out}(t) in terms of *C*, V_S, V_{TH} *and* R_L. (3 points)

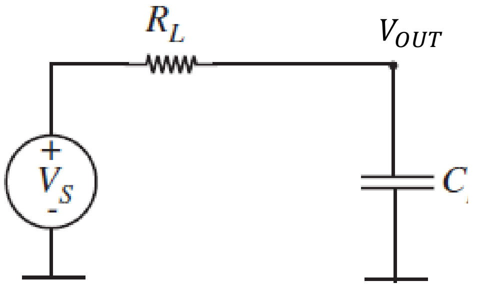


Figure 5

(E) Find the expression for the power dissipated in the resistors of this circuit in terms of *C*, V_{TH}, R_{TH}, V_S, R_L *and* R_{ON} during T_2 period. Note that we are asking for the instantaneous power, not the integrated energy, so you don't need to calculate the integration. (3 points)

Answer : p(t) =

(F) Under the assumption that T_1 and T_2 are long enough, by integrating each dissipated power we solved in question (C) and (E),

$$w_1 = \frac{V_S^2}{R_L + R_{ON}} T_1 + \frac{V_S^2 R_L^2 C_L}{2(R_L + R_{ON})^2}.$$

$$w_2 = \frac{V_S^2 R_L^2 C_L}{2(R_L + R_{ON})^2}.$$

Assuming that $T_1 = T_2$, $T = T_1 + T_2$ and $w = w_1 + w_2$, calculate the average power dissipation $\bar{p} = w / T$. (You can use the fact that frequency $f = 1 / T$)
(4 points)

(G) Consider a chip with 10^6 NMOS inverters with C = 1fF, R_L = 10kΩ, V_S = 5V and the clock frequency f = 100MHz. Assuming $R_L \gg R_{ON}$, calculate the average power dissipation. (2 points)
(f : 10^{-15}, k : 10^3, M : 10^6)

(H) Figure 6 is a CMOS inverter circuit. As we learned in the class CMOS inverter has no static power, and the average power dissipation $\bar{p} = C * V_S{}^2 * f$. Calculate the average power dissipation when we use CMOS inverters instead of NMOS inverters in (G). (2 points)

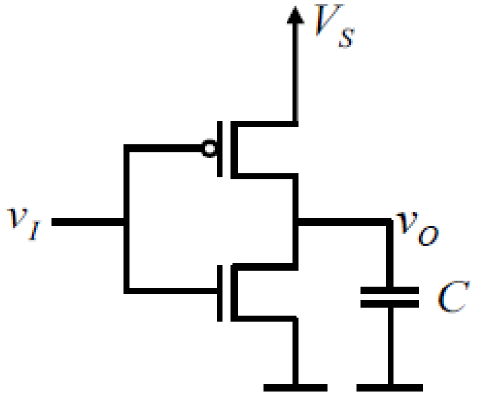
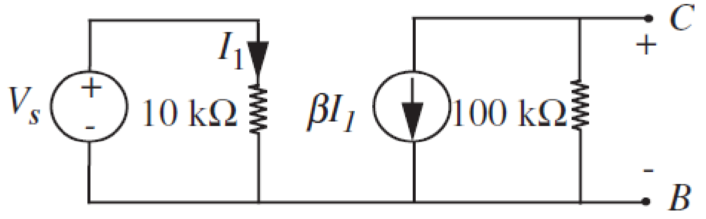


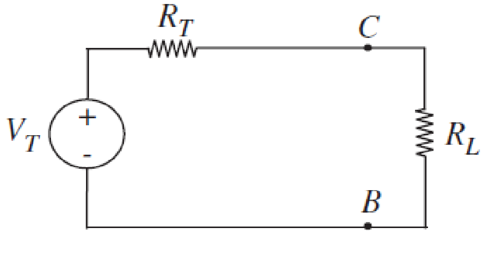
Figure 6

Problem 2 - 10 Points

(A) Find the Thevenin equivalent for the following network at terminals CB. The current source is a controlled source. (5 points)



(B) Now suppose you connect a load resistor across the output of your equivalent circuit as shown in the following figure. Find R_L which will provide the maximum power transfer to the load. (5 points)



Problem 3 – 10 Points

Figure 1 and figure 2 is a capacitor and an inductor each. We learned in class that the energy stored in a capacitor is $\frac{1}{2}CV^2$ and the energy stored in a inductor is $\frac{1}{2}LI^2$.

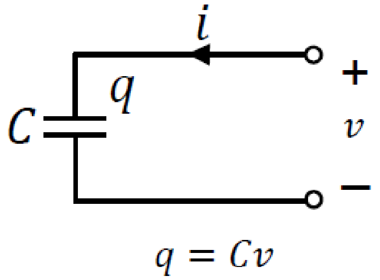


Figure 1

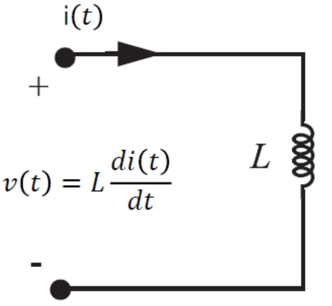


Figure 2

Using these formulas, you'll calculate the energy stored in the undriven circuit in figure 3.

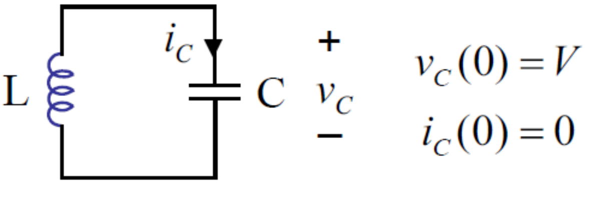


Figure 3

(A) Calculate $v_c(t)$ and $i_c(t)$. (5 points)

(B) Show that energy stored in the circuit is a constant. (5 points)

Problem 4 - 12 Points

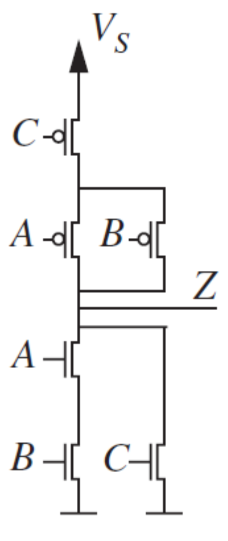
Draw a CMOS implementation for the given Boolean expressions.

(A) C = $\overline{A \cdot B}$ (3 points)

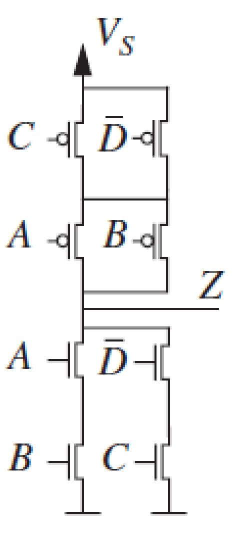
(B) C = $\overline{A + B}$ (3 points)

Write Boolean expressions for the given CMOS circuits.

(C) (3 points) Z =



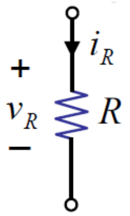
(D) (3 points) Z =



Problem 5 - 12 Points

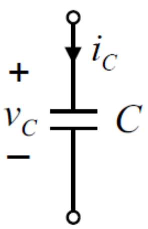
When we analyze sinusoidal steady state, we can assume that all components are oscillating with the same frequency. Using complex exponential drive, we can make the analysis with complex amplitude. For example, when $v_i = V_i e^{i\omega t}, v_o = V_o e^{i\omega t}$, we think about the magnitude $\left|\frac{V_o}{V_i}\right|$ and the phase $\varphi = \angle\frac{V_o}{V_i}$.

(A) Prove that $V_r = RI_r$ in the following circuit. (2 points)



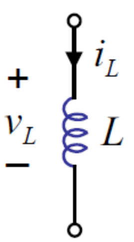
$$i_R = I_r e^{j\omega t}$$
$$v_R = V_r e^{j\omega t}$$

(B) Prove that $V_c = \frac{1}{j\omega C} I_c$ in the following circuit. (2 points)



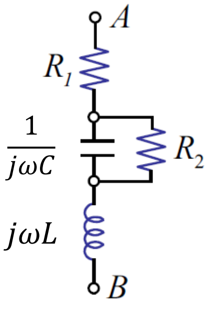
$$i_C = I_C e^{j\omega t}$$
$$v_C = V_C e^{j\omega t}$$

(C) Prove that $V_l = j\omega L\ I_l$ in the following circuit. (2 points)

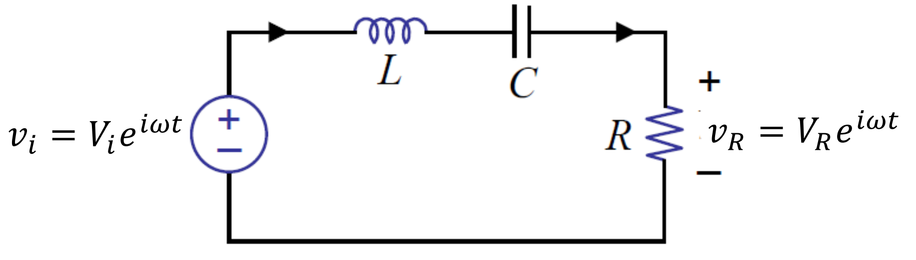


$$i_L = I_l e^{j\omega t}$$
$$v_L = V_l e^{j\omega t}$$

(D) Calculate the impedance between A, B. (3 points)



(E) Express V_R in terms of V_i, R, L, C. (3 points)



Problem 6 - 10 Points

In the following questions, when you sketch the plots, you can use either log-log scale or linear scale.

(A) Sketch the transfer function of the following circuit. (Use impedance model.) (3 points)

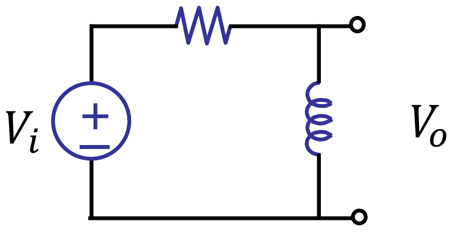


Figure 1

$$|H(\omega)| = \left|\frac{V_o}{V_i}\right|$$

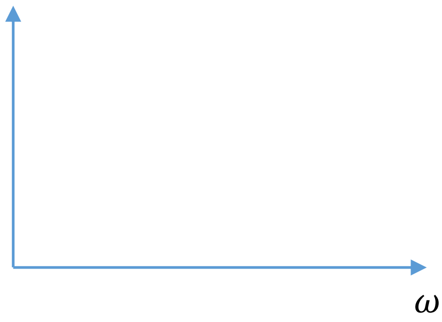


Figure 2

(B) Is it high pass filter or low pass filter? (2 points)

(C) Sketch the transfer function of the following circuit. (Use impedance model.) (3 points)

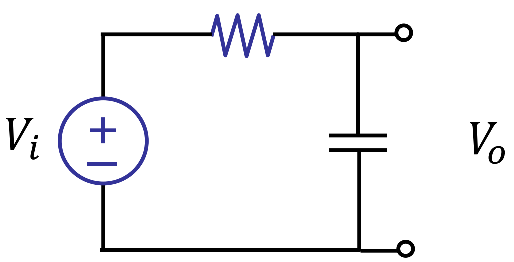


Figure 3

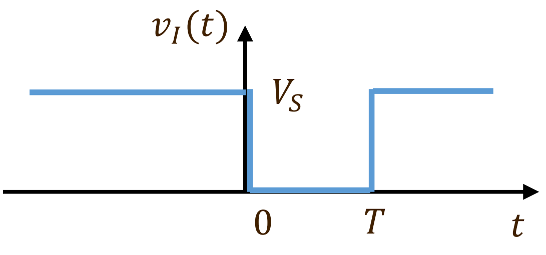
$$|H(\omega)| = \left|\frac{V_o}{V_i}\right|$$

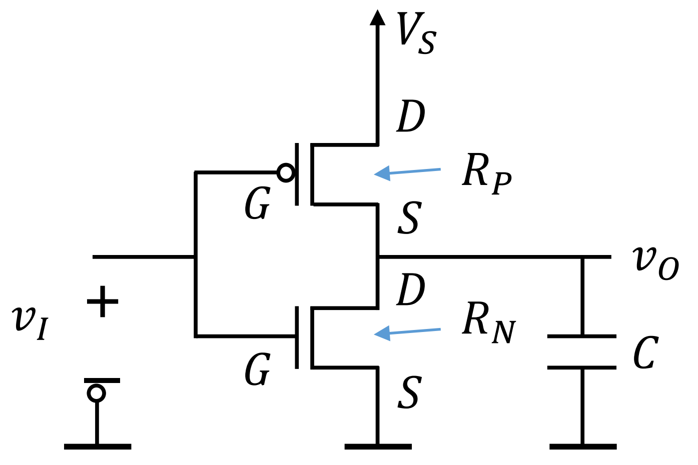
Figure 4

(D) Is it high pass filter or low pass filter? (2 points)

Problem 7 - 16 Points

The output of a CMOS (complementary MOS) switch is connected to a capacitor as shown in the right figure and the input voltage $v_I(t)$ of the switch is being driven by the voltage shown in the left figure. For MOSFET devices, we will use SR (Switch-Resistor) model. Assume that NMOS has on-resistance of R_N when it is on, and PMOS has on-resistance of R_P when it is on. If you remember how to analyze intuitively for RC circuit, you don't need to derive differential equation. Of course, you are free to derive differential equation if you want.



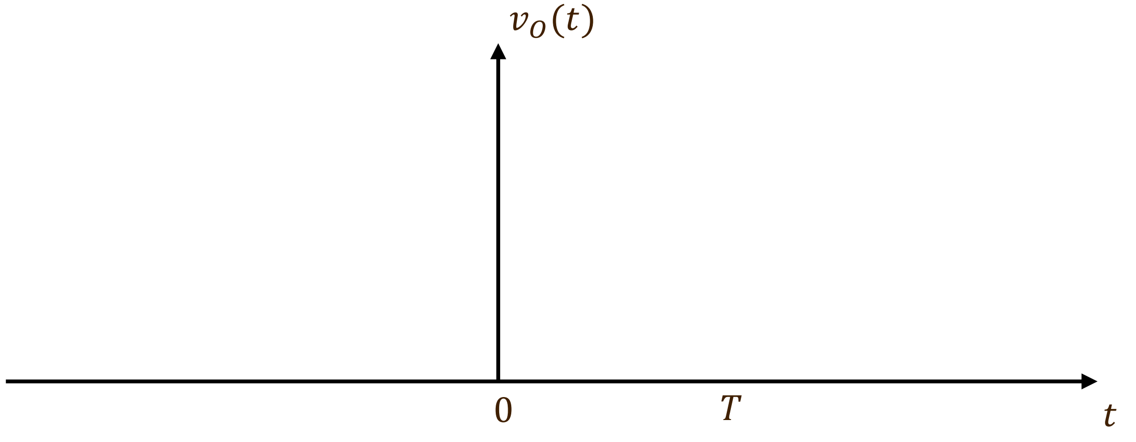


(A) When t<0, what is the output voltage $v_O(t)$? We assume that $v_I(t)$ remained at V_S for a long time, so you can ignore transient behavior. (4 points)

(B) Please plot the shape of output voltage $v_O(t)$ when 0<t<T. Assume that T is long enough compared to the relevant time constant. Indicate the initial voltage, asymptotic final voltage, and specify the slope of the exponential curve at t=0. (4 points)

(C) Please plot the shape of output voltage $v_O(t)$ when t>T. Assume that T is long enough compared to the previous time constant so that $v_O(t)$ already reached its asymptotic final value right before t=T. Indicate the voltage at t=T, asymptotic final voltage, and specify the slope of the exponential curve at t=T. (4 points)

(D) Please combine all the previous result and plot in the following graph. (4 points)

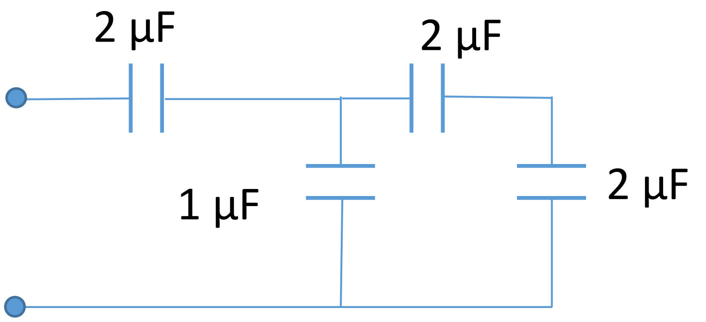


Problem 8 - 6 Points

Find out the equivalent capacitance and inductance between the indicated terminals.

(A) (3 points)

1μF



(B) (3 points)

3 mH