2019 Fall, Electrical and Electronic Circuits (4190.206A 002)

Homework #3

Due date: Nov. 11 (Mon), 2019 3:15pm

If you hand-in after the due date, your score will be deducted by 20%. No more submission will be accepted after Nov. 13, 2019. 3:15pm.

Name : ______________________

Student ID Number : ____________________

1	2	3	4	5	6	7	8	Total

To reduce the grading burden of TA, the homework will be graded all-or-nothing style. Within each problem, there are several sub-problems, but we will grade only one sub-problem within each problem, and the score of each problem will be determined by the graded problems. For example, if problem 1 is composed of 5 sub-problems, we will decide which sub-problem will be graded later, and if you solved that sub-problem correctly, you will get the full credit of problem 1. In the worst case, you might have solved all other sub-problems correctly, and made a mistake only in the graded sub-problem. That is an unfortunate situation, but the score for that entire problem will become 0. Without this policy, we cannot grade so many homework efficiently. We cannot return the graded homework, so before you submit your homework, please make your own copy or scan it. We will post solutions for all the problems and announce which sub-problem will be graded.

One nice thing about the differential equation problem is that at least you can easily verify whether your solution is correct or not by putting your solution back into the equation. Please confirm your solution if you want to avoid mistake.

3-1. (point 15) Find the homogeneous solutions for the following differential equations. If you have a difficulty in factoring the characteristic equation, try integers between -3 and +3 for the root.

(example) $\frac{d^2}{dt^2}y + 3\frac{d}{dt}y + 2y = 0$

(example answer) $s^2 + 3s + 2 = 0 = (s+2)(s+1)$ ➔ $y_h(t) = Ae^{-2t} + Be^{-t}$

(a) $\frac{d}{dt}y + 4y = 0$

$C_1 \cdot e^{-4t}$

(b) $2\frac{d^2}{dt^2}y + 5\frac{d}{dt}y + 2y = 0$

$C_1 e^{-\frac{1}{2}t} + C_2 e^{-2t}$

(c) $\frac{d^3}{dt^3}y + 2\frac{d^2}{dt^2}y - \frac{d}{dt}y - 2y = 0$

$C_1 e^{t} + C_2 e^{-t} + C_3 e^{-2t}$

(d) $\frac{d^2}{dt^2}y + 4y = 0$

$C_1 e^{2it} + C_2 e^{-2it}$

(e) $\frac{d^3}{dt^3}y + y = 0$

$C_1 e^{-t} + C_2 e^{\frac{1+\sqrt{3}i}{2}t} + C_3 e^{\frac{1-\sqrt{3}i}{2}t}$

3-2. (point 15) Find the homogeneous solutions for the following differential equations. If you have a difficulty in factoring the characteristic equation, try integers between -3 and +3 for the root.

(a) $\frac{d^2}{dt^2}y - 2\frac{d}{dt}y = 0$

$$C_1 + C_2 e^{2t}$$

(b) $\frac{d^2}{dt^2}y + 6\frac{d}{dt}y + 9y = 0$

$$C_1 e^{-3t} + C_2 t e^{-3t}$$

(c) $\frac{d^3}{dt^3}y + 2\frac{d^2}{dt^2}y - 4\frac{d}{dt}y - 8y = 0$

$$C_1 e^{2t} + C_2 e^{-2t} + C_3 t e^{-2t}$$

(d) $2\frac{d^2}{dt^2}y + 4\frac{d}{dt}y + 12y = 0$

$$C_1 e^{(-1+\sqrt{5}i)t} + C_2 e^{(-1-\sqrt{5}i)t}$$

(e) $\frac{d^3}{dt^3}y - y = 0$

$$C_1 e^{t} + C_2 e^{\frac{-1+\sqrt{3}i}{2}t} + C_3 e^{\frac{-1-\sqrt{3}i}{2}t}$$

3-3. (point 15) Find the particular solutions for the following differential equations.

(example) $\frac{d}{dt}y + 4y = 2$

(example answer) trial solution: $y_p(t) = A$. $\frac{d}{dt}y_p + 4y_p = 0 + 4A = 2$ ➔ $A = \frac{1}{2}$ ➔ $y_p(t) = \frac{1}{2}$

(a) $\frac{d^2}{dt^2}y + 3\frac{d}{dt}y + 2y = 2t$

$t - \frac{3}{2}$

(b) $\frac{d^2}{dt^2}y + 3\frac{d}{dt}y + 2y = 4t^2$

$2t^2 - 6t + 7$

(c) $\frac{d^2}{dt^2}y + 3\frac{d}{dt}y + 2y = e^t$

$\frac{1}{6} e^t$

(d) $\frac{d^2}{dt^2}y + 3\frac{d}{dt}y + 2y = e^t + e^{3t}$

$\frac{1}{6} e^t + \frac{1}{20} e^{3t}$

(e) $\frac{d}{dt}y + 2y = te^t$

$\left(\frac{1}{3}t - \frac{1}{9}\right)e^t$

3-4. (point 15) Find the particular solutions for the following differential equations.

(a) $\frac{d}{dt}y - 2y = e^{2t}$

te^{2t}

(b) $\frac{d^2}{dt^2}y + 6\frac{d}{dt}y + 9y = 2e^{-3t}$

$t^2 e^{-3t}$

(c) $\frac{d}{dt}y + 2y = \sin t$

$-\frac{1}{5}\cos t + \frac{2}{5}\sin t$

3-5. (point 15) Find the particular solutions for the following differential equations.

(a) $\frac{d^2}{dt^2}y + \frac{d}{dt}y + 2y = 2\cos t$

$\cos t + \sin t$

(b) $\frac{d}{dt}y + y = 2t\cos t$

$t\cos t + t\sin t - \sin t$

(c) $\frac{d^2}{dt^2}y + \frac{d}{dt}y + y = e^{-t}\cos t$

$-e^{-t}\sin t$

3-6. (point 20) Find the solutions for the following differential equations with the given constraints.

(example) $\frac{d}{dt}y - 2y = 0$ with $y'(0) = \left.\frac{dy}{dt}\right|_{t=0} = -4$

(example answer) homogeneous solution: $y_h(t) = Ae^{2t}$, particular solution: $y_p(t) = 0$, total solution: $y_t(t) = y_h(t) + y_p(t) = Ae^{2t}$. ➔ $\frac{dy}{dt} = 2Ae^{2t}$ ➔ $\left.\frac{dy}{dt}\right|_{t=0} = 2A = -4$ ➔ $A = -2$ ➔ $y_t(t) = -2e^{2t}$

(a) $\frac{d^2}{dt^2}y + 3\frac{d}{dt}y + 2y = 0$ with $y(0) = 1,\ y'(0) = 0$

$2e^{-t} - e^{-2t}$

(b) $\frac{d^2}{dt^2}y + 4y = 0$ with $y(0) = 0,\ y'(0) = 2$

$\sin 2t$

(c) $\frac{d^3}{dt^3}y + \frac{d^2}{dt^2}y + \frac{d}{dt}y + y = 0$ with $y(0) = 0,\ y'(0) = 1,\ y''(0) = -2$

$-e^{-t} + \cos t$

(d) $\frac{d^2}{dt^2}y + 2\frac{d}{dt}y + 2y = 0$ with $y(0) = 1,\ y'(0) = -1$

$e^{t} \cos t$

3-7. (point 20) Find the solutions for the following differential equations with the given constraints.

(a) $\frac{d^2}{dt^2}y + 3\frac{d}{dt}y + 2y = 4$ with $y(0) = 2,\ y'(0) = -1$

$$e^{-t} + e^{-2t} + 2$$

(b) $\frac{d^2}{dt^2}y - y = 4t$ with $y(0) = 0,\ y'(0) = -2$

$$e^{t} - e^{-t} - 4t$$

(c) $\frac{d^2}{dt^2}y + \frac{d}{dt}y = 2t$ with $y(0) = 2,\ y'(0) = -3$ (Hint: If you have a trouble in finding a particular solution, note that you can define $z \equiv \frac{d}{dt}y$ and solve the equation for z first, because there is no $c_0 y$ term. Then obtain y by integrating z.)

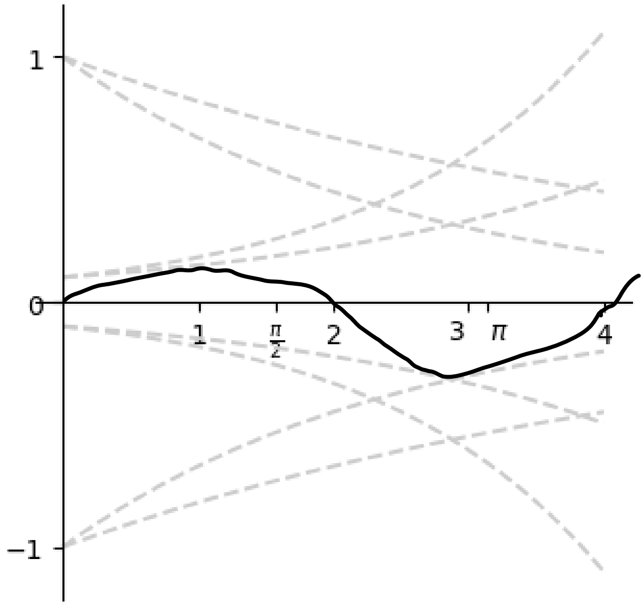
$$e^{-t} + t^2 - 2t + 1$$

(d) $\frac{d^2}{dt^2}y - 3\frac{d}{dt}y + 2y = -e^t$ with $y(0) = 0,\ y'(0) = 2$

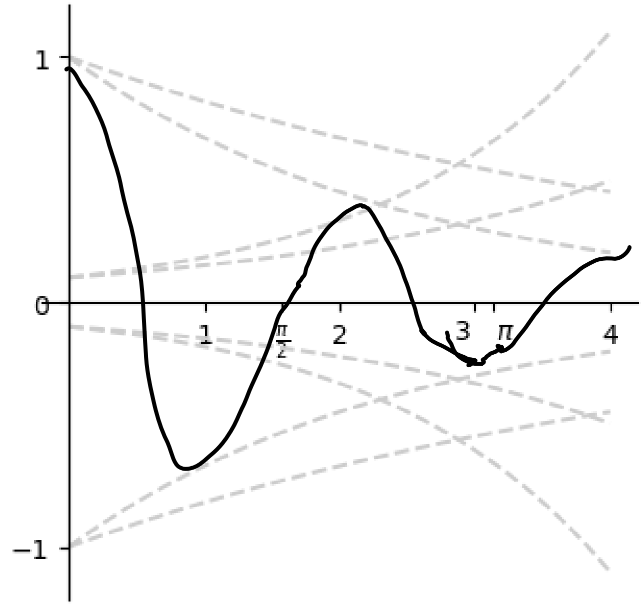
$$-e^{t} + e^{2t} + te^{t}$$

3-8. (point 20) In the following blank graphs, please sketch the functions given in the sub problems. I included plots of $\pm 0.1e^{0.4t}, \pm 0.1e^{0.6t}, \pm e^{-0.2t}, \pm e^{-0.4t}$ functions in grey dashed lines so that you can use them as a guideline for drawing your plots. You should identify which line corresponds to which function by considering the properties of those exponential functions.

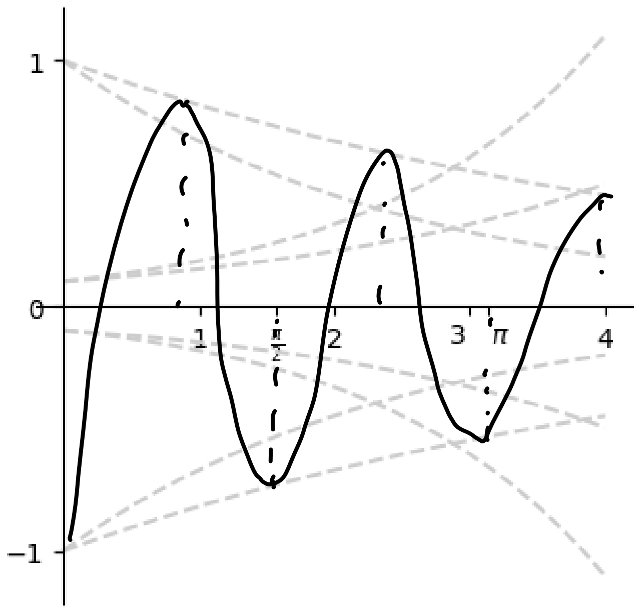
(a) $0.1e^{0.4t}\sin\left(\frac{\pi}{2}t\right)$



(b) $e^{-0.4t}\cos(\pi t)$



(c) $-e^{-0.2t}\cos(4t)$



(d) $\frac{0.1}{\sqrt{2}}[e^{0.6t}\sin(2t) + e^{0.6t}\cos(2t)]$ $0.1\, e^{0.6t} \sin(2t + \frac{\pi}{4})$