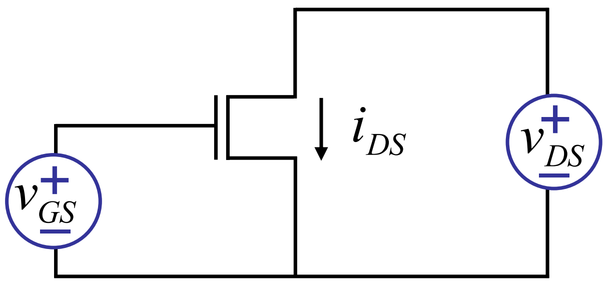
Electrical and Electronics Circuits (4190.206A 002)

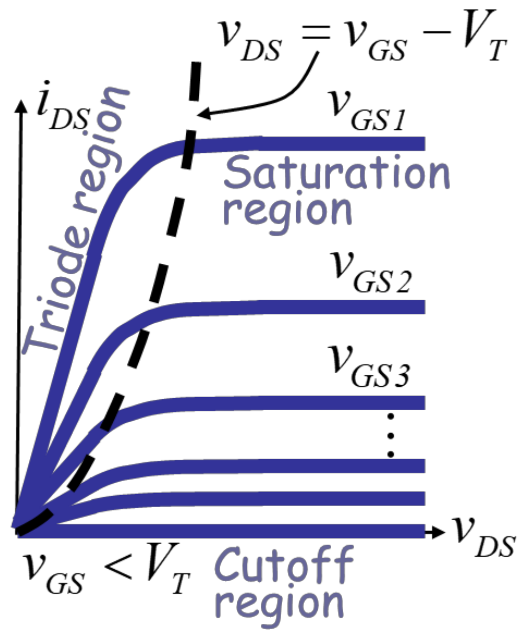
- HW #1 is due today 3:15pm
- There will be no class on 10/9 (Wed) Hangul Day and new make-up class will be scheduled later.
- The late homework will be accepted only until 10/10 (Thu) 10am. The late submitted homework will receive only 80% of the full credit.

- Self-attendance check

Review I

- Full Characteristics of MOSFET Device



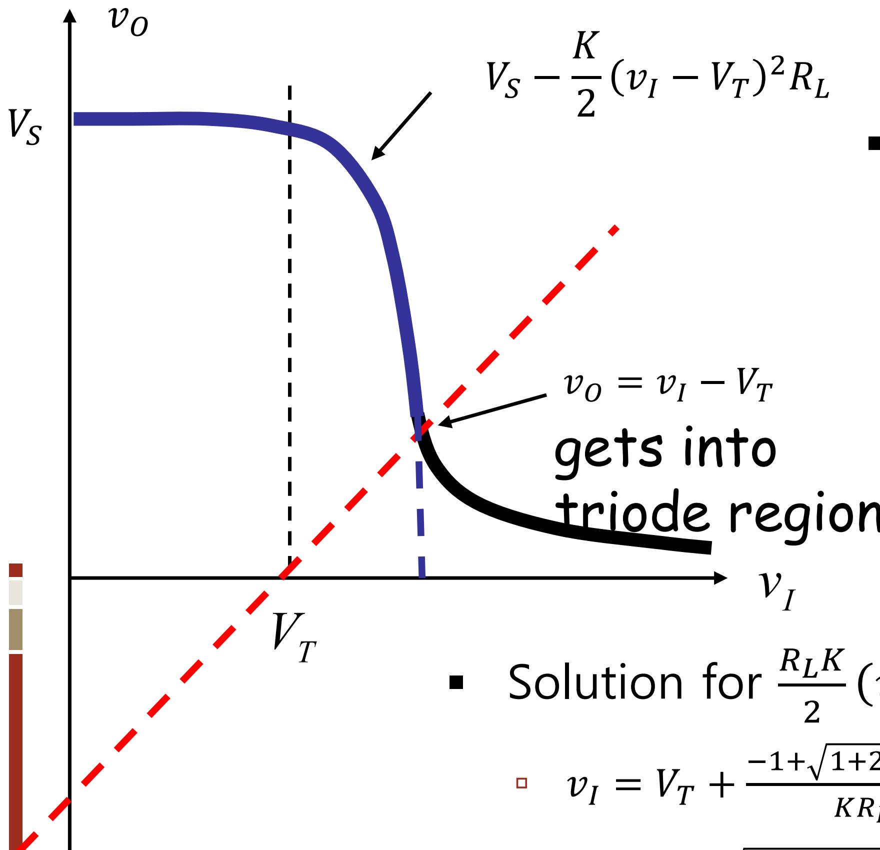


$$
i_{DS} = \begin{cases} K\left[(v_{GS} - V_T)v_{DS} - \dfrac{v_{DS}^2}{2}\right] & \text{for } v_{GS} \geq V_T \text{ and } v_{DS} < v_{GS} - V_T \\ \dfrac{K(v_{GS} - V_T)^2}{2} & \text{for } v_{GS} \geq V_T \text{ and } v_{DS} \geq v_{GS} - V_T \\ 0 & \text{for } v_{GS} < V_T \end{cases}
$$

- Saturation region ➔ voltage-controlled current source
  - Control voltage: v_{GS}
  - In ideal situation, v_{DS} should NOT influence any current

$\frac{K}{2}(v_{GS}-V_T)^2 - \frac{K}{2}(v_{DS}-(v_{GS}-V_T))^2$

Review II: Large Signal Analysis



- Where the two curves meet
  - $v_O = V_S - \frac{K}{2}(v_I - V_T)^2 R_L$
  - $v_O = v_I - V_T$

  ➔ $V_S - \frac{K}{2}(v_I - V_T)^2 R_L = (v_I - V_T)$

- Solution for $\frac{R_L K}{2}(v_I - V_T)^2 R_L + (v_I - V_T) - V_S = 0$
  - $v_I = V_T + \frac{-1+\sqrt{1+2KR_LV_S}}{KR_L}$
  - $v_O = \frac{-1+\sqrt{1+2KR_LV_S}}{KR_L}$

Alternative Graphical Method

- Constraint for saturation region

$$i_{DS} = \frac{K}{2}(v_I - V_T)^2 \ ,$$

for

$$v_O \geq v_I - V_T$$

$$\Downarrow$$

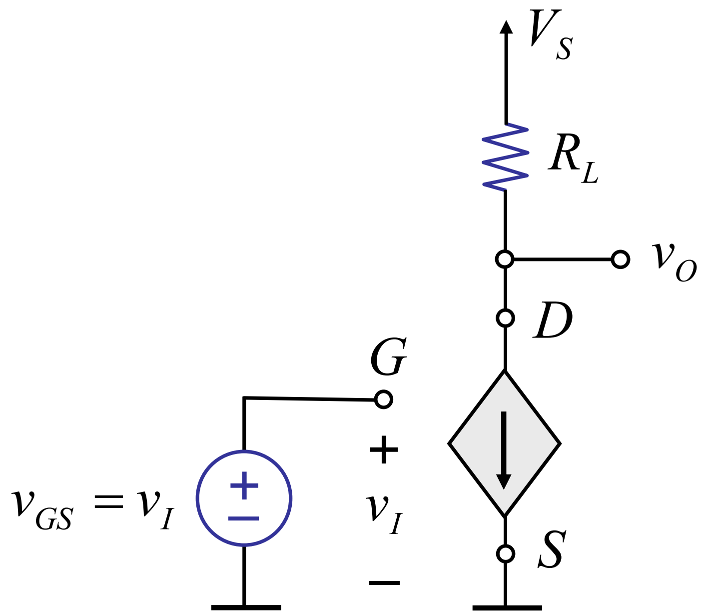
$$v_O \geq \sqrt{\frac{2i_{DS}}{K}}$$

$$\Downarrow$$

$$i_{DS} \leq \frac{K}{2} {v_O}^2$$

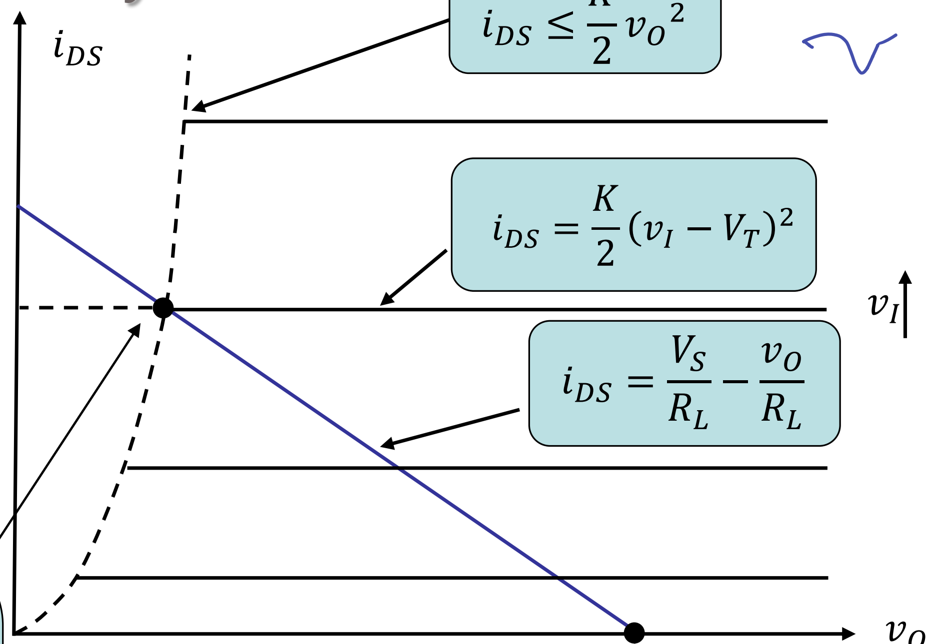
- Load relation

$$i_{DS} = \frac{V_S}{R_L} - \frac{v_O}{R_L}$$



Alternative Graphical Analysis

- Only interested in the saturation region
- What are the required ranges for v_I, v_O, i_{DS}?



$$i_{DS} \le \frac{K}{2} {v_O}^2$$

$$i_{DS} = \frac{K}{2}(v_I - V_T)^2$$

$$i_{DS} = \frac{V_S}{R_L} - \frac{v_O}{R_L}$$

$v_I = V_T$ threshold.
$i_{DS} = 0$ and $v_O = V_S$

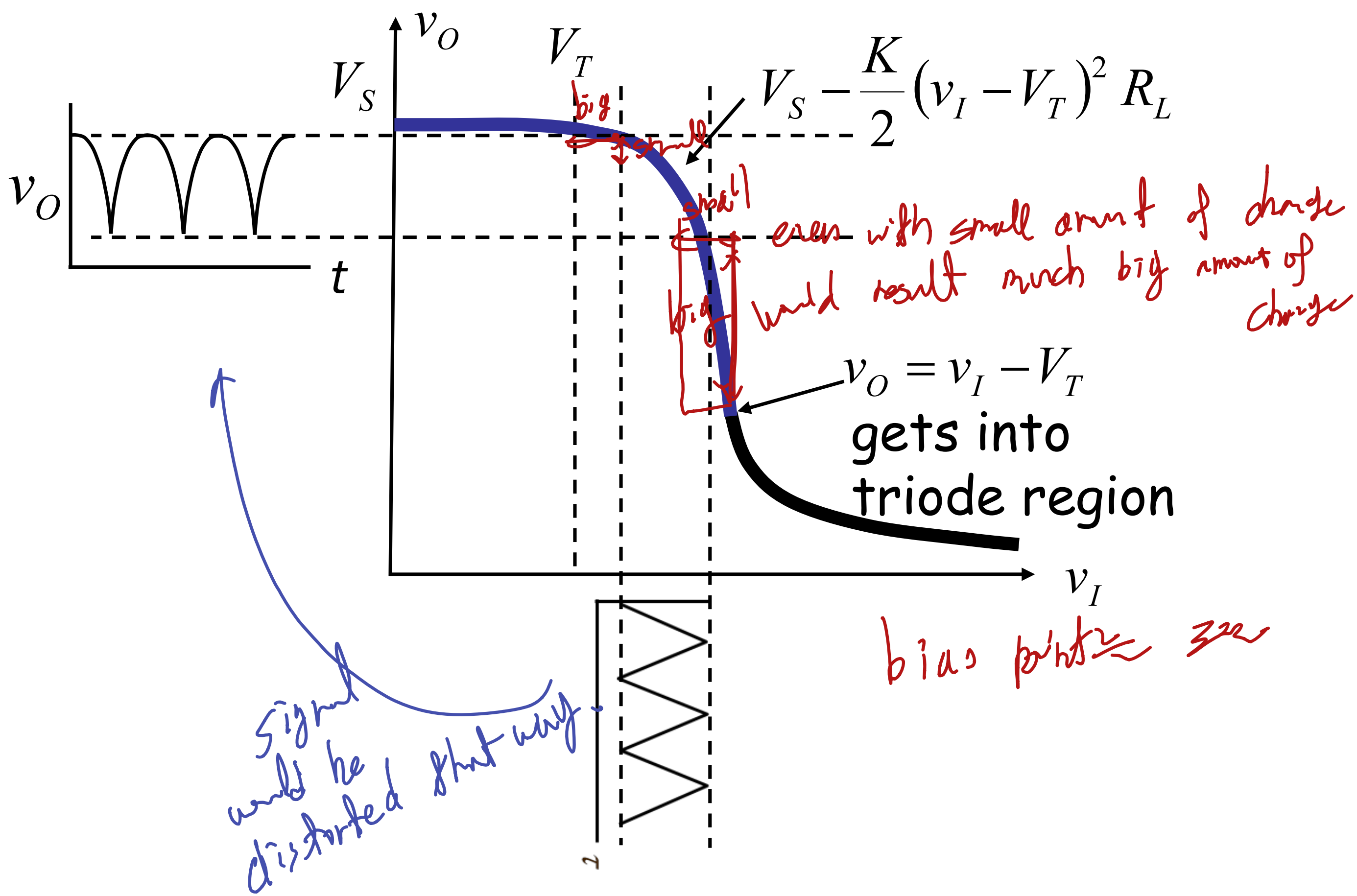
$$v_O = \frac{-1 + \sqrt{1 + 2KR_LV_S}}{KR_L}$$

$$v_I = V_T + \frac{-1 + \sqrt{1 + 2KR_LV_S}}{KR_L}$$

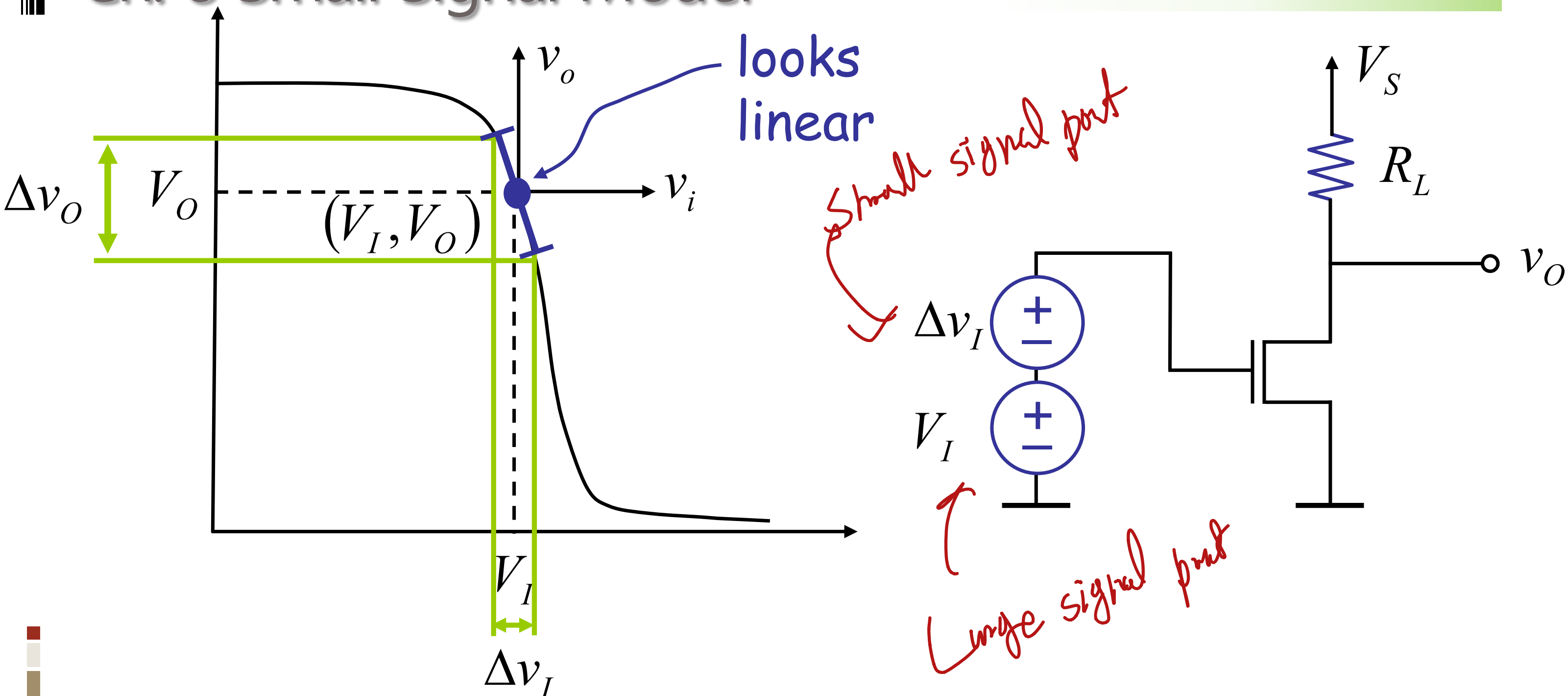
$$i_{DS} = \frac{V_S}{R_L} - \frac{v_O}{R_L}$$

Large Signal Analysis

- Piecewise analysis for v_O vs v_I



Ch. 8 Small Signal Model



- Operate amp at V_I, V_O ➔ DC bias
- Superimpose small signal on top of V_I
- Notation: $v_{\mathrm{I}} = V_I + v_i$, $v_{\mathrm{O}} = V_O + v_o$

Small Signal Analysis

$$v_O = V_S - \frac{R_L K}{2}\left(v_I - V_T\right)^2 \qquad \Big| \qquad V_O = V_S - \frac{R_L K}{2}\left(V_I - V_T\right)^2$$

substituting $v_I = V_I + v_i \qquad v_i << V_I$

$$v_O = V_S - \frac{R_L K}{2}\left(\left[V_I + v_i\right] - v_T\right)^2$$

$$= V_S - \frac{R_L K}{2}\left(\left[V_I - V_T\right] + v_i\right)^2$$

$$= V_S - \frac{R_L K}{2}\left(\left[V_I - V_T\right]^2 + 2\left[V_I - v_T\right]v_i + v_i\right)$$

$$V_O + v_o = V_S - \frac{R_L K}{2}\left(V_I - V_T\right)^2 - R_L K\left(V_I - V_T\right)v_i$$

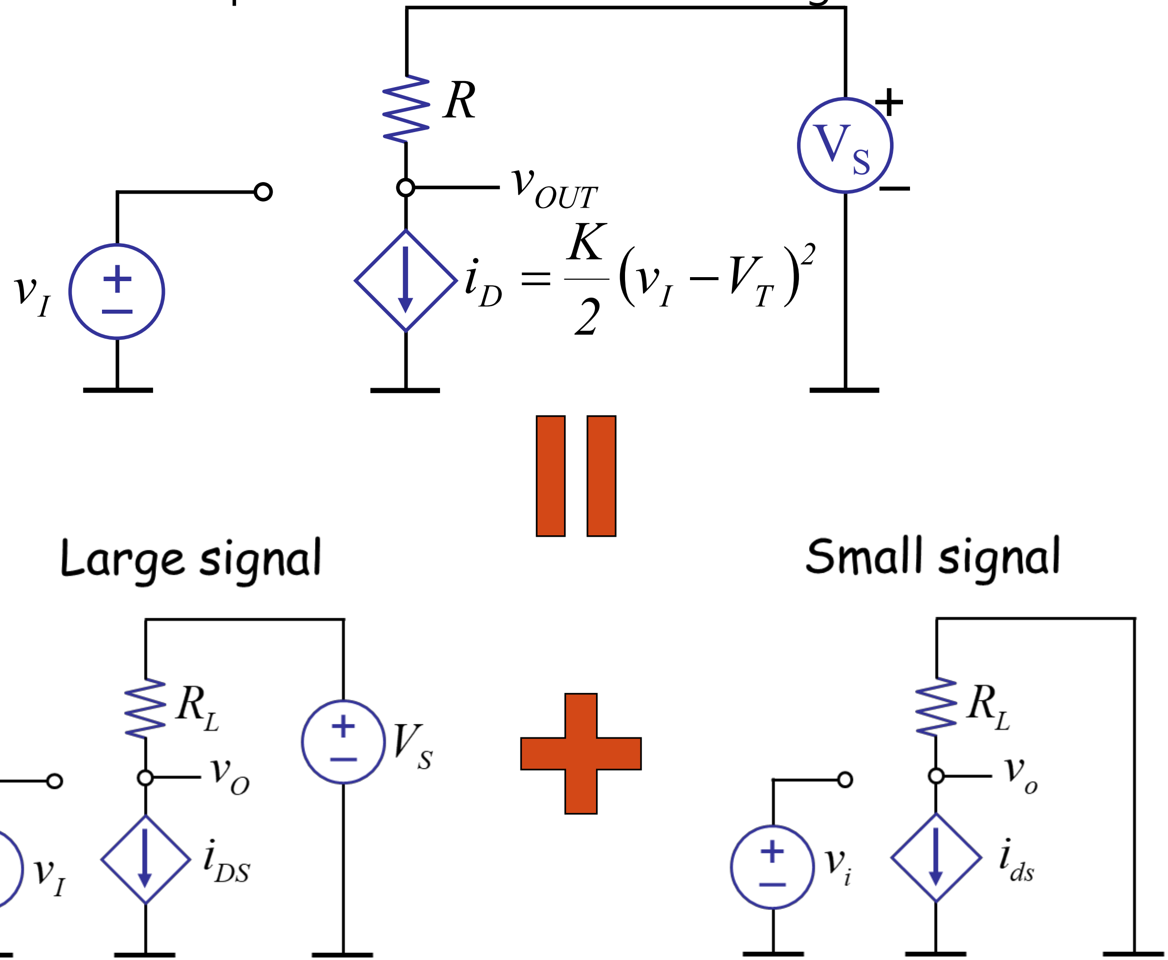
$$v_o = -R_L \underbrace{K\left(V_I - V_T\right)}_{g_m} v_i$$

g_m related to V_I

➔ Same result can be also obtained by differentiation.

Small Signal Circuit

- We can derive equivalent models for small signal circuit



Small Signal Circuit

- To find the relationship between the small signal parameters of a circuit, we can replace large signal device models with corresponding small signal device models, and then analyze the resulting small signal circuit.
- KVL, KCL applied to some circuit C yields:

$$\cdots + v_A + \quad \cdots \quad + v_{OUT} + \quad \cdots \quad + v_B + \cdots \qquad (1)$$

topology of the circuit

- Replace total variables with operating point variables plus small signal variables

$$\cdots + V_A + v_a \cdots + V_{OUT} + v_{out} \quad + V_B + v_b + \cdots$$

- Operating point variables themselves satisfy the same KVL, KCL equations

$$\cdots + V_A \qquad \cdots + V_{OUT} \qquad\qquad + V_B \qquad + \cdots$$

bias point only.

- So, we can cancel them out, leaving

$$\cdots \qquad + v_a \cdots \qquad + v_{out} \qquad + v_b + \cdots \qquad (2)$$

final results: changes for the sames

- But (2) is the same equation as (1) with small signal variables replacing total variables, so (2) must reflect same topology as in C, except that small signal models are used.
- Since small signal models are linear, our linear tools will now apply…

Small Signal Circuit Analysis

1. Find operating point using DC bias inputs using large signal model.

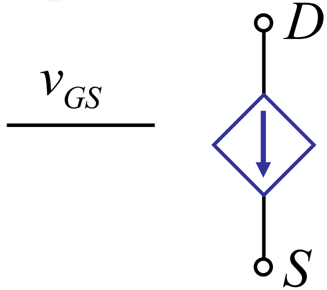
2. Develop small signal (linearized) models for elements.

3. Replace original elements with small signal models.

Once small signal circuit with only linearized elements is obtained, we can use all the linear circuit analysis tools such as superposition and Thevenin equivalent circuits.

Small Signal Model of MOSFET

large signal



$$i_{DS} = \frac{K}{2}\left(v_{GS} - V_T\right)^2$$

Small signal analysis:

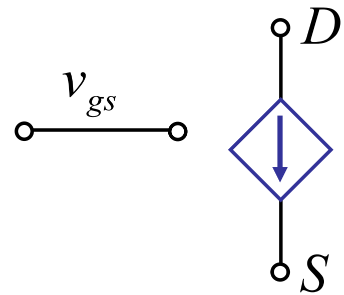
$$i_{DS} = \frac{K}{2}\left(v_{GS} - V_T\right)^2$$

$$i_{ds} = \frac{\partial}{\partial v_{GS}}\left[\frac{K}{2}\left(v_{GS} - V_T\right)^2\right]\Bigg|_{v_{GS}=V_{GS}} \cdot v_{gs}$$

$$i_{ds} = \underbrace{K\left(V_{GS} - V_T\right)}_{g_m} \cdot v_{gs}$$

i_{ds} is linear in v_{gs}

small signal model

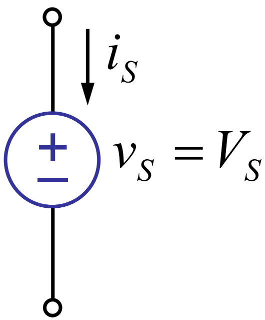


$$i_{ds} = K\left(V_{GS} - V_T\right) v_{gs}$$

$$i_{ds} = g_m v_{gs}$$

Small Signal Model of DC Voltage Source

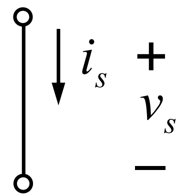
large signal



small signal model

Small signal

$$v_s = \left.\frac{\partial V_S}{\partial i_S}\right|_{i_S = I_S} \cdot i_s$$

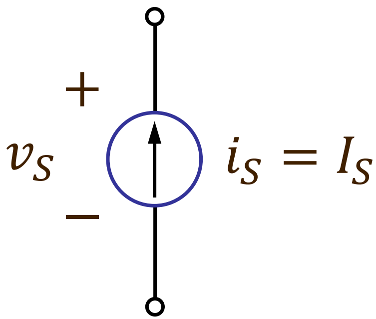


$$v_s = 0$$

DC voltage source behaves as short to small signals.

Small Signal Model of DC Current Source

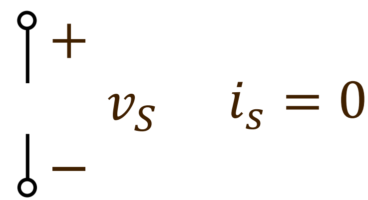
large signal



small signal model

Small signal

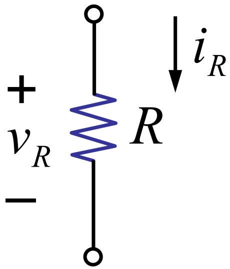
$$i_s = \left.\frac{\partial I_S}{\partial v_S}\right|_{v_S = V_S} \cdot v_s$$



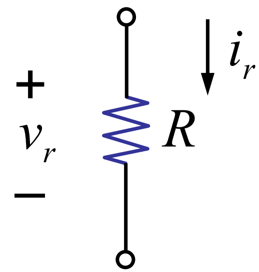
DC current source behaves as open circuit to small signals.

Small Signal Model of Resistor

Large signal



small
signal
model



Small signal analysis:

$$v_R = R\, i_R$$

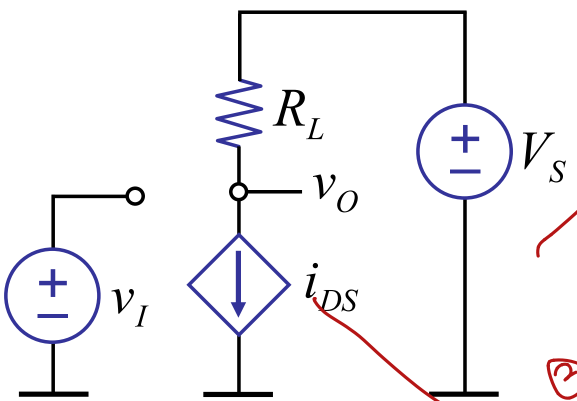
$$v_r = \left. \frac{\partial (R i_R)}{\partial i_R} \right|_{i_R = I_R} \cdot i_r$$

$$v_r = R \cdot i_r$$

Large Signal and Small Signal Circuits

① to get this.

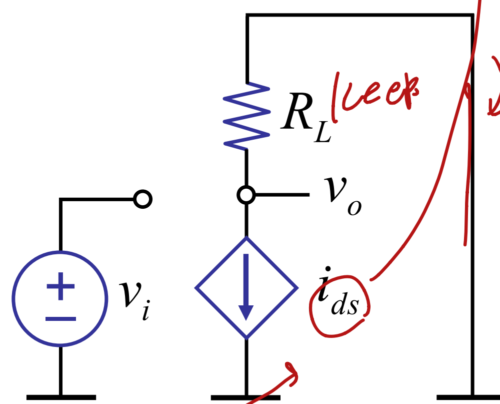
Large signal



Linearize

② analyze is needed

Small signal



$$i_{DS} = \frac{K}{2}(v_I - V_T)^2$$

$$v_O = V_S - \frac{K}{2}(v_I - V_T)^2 R_L$$

- To obtain this value, we need a bias value. Therefore large signal should be done before small signal analysis.

$$i_{ds} = K(V_I - V_T)\cdot v_i$$

$$i_{ds}R_L + v_o = 0$$

$$v_o = -i_{ds}R_L$$

$$v_o = -K(V_I - V_T)R_L \cdot v_i$$

$$= -g_m R_L \cdot v_i$$