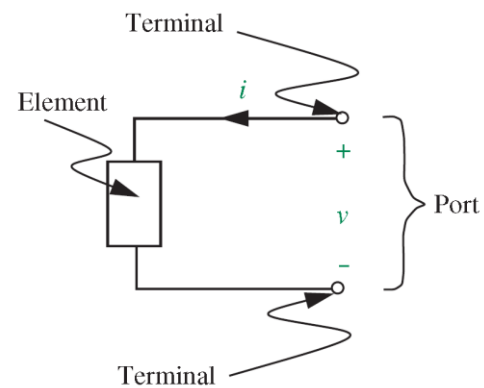
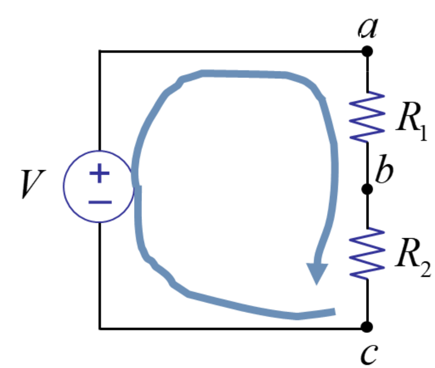
# Electrical and Electronics Circuits (4190.206A 002)

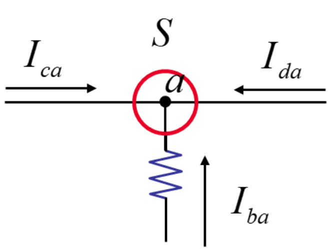
- Lecturer: Taehyun Kim ( taehyun@snu.ac.kr, 301-407)
- Class hour: Mon. and Wed. 2:00~3:15pm / 301-203
- Reservation for make-up class hour
  - 9/30 (Mon) 7pm-8:15pm
- There will be a class on 10/9 (Wed) Hangul Day, but the class attendance won't be checked, and video recording of the class will become available at ETL.
- Office hour: Mon. 7:00~8:00pm, Wed. 3:30~4:30pm / 301-407 (Please check my schedule before you come to my office)
- TA: Chaewon Kim ( kcwchae@gmail.com , 301-416 )
- Textbook : "Foundations of Analog and Digital Electronic Circuits"
- Course homepage: ETL
- Grades: 3 exams 30, 30, 30%   homework + attendance: 10% (If you cannot attend the class for official reason, please let me or TA know in advance.)
- Self-attendance check

# Review

- Associated variable convention
  - Define current to flow in at the device terminal assigned to be positive voltage
  - Never forget to define the polarity of the voltage and the direction of current at the beginning of the circuit analysis.
- I-V characteristics
  - Resistor & diode
  - Voltage source & current source
  - Open and short circuit
- Circuit analysis method
  - Kirchhoff's voltage law (KVL): $\sum_{loop} v_j = 0$ around each loop
  - Kirchhoff's current law (KCL): $\sum_{node} i_j = 0$ at each node



# Chap. 2 Resistive Networks
# Chap. 3 Network Theorems

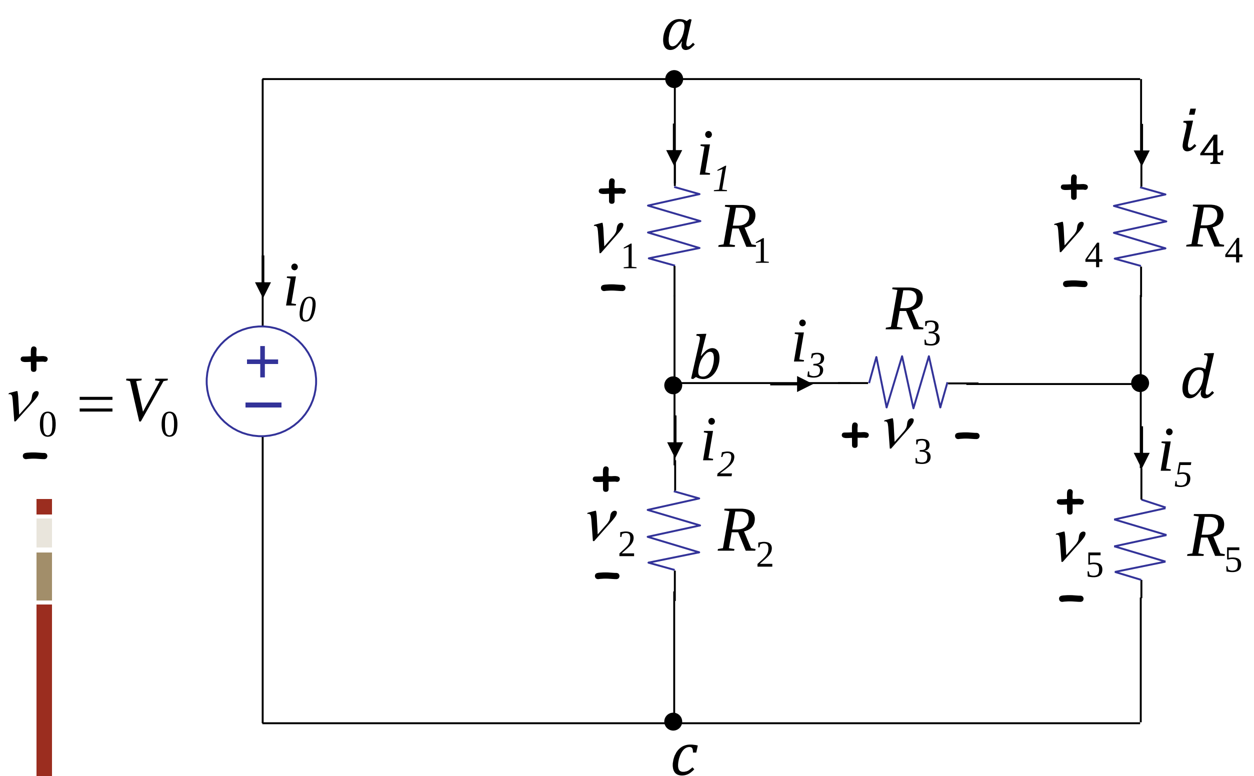
- Kirchhoff's Current Law (KCL)
- Kirchhoff's Voltage Law (KVL)
- Circuit analysis methods
  - Basic KVL, KCL method (2.3)
  - Node analysis method (3.2 ~ 3.3)
  - Element combination method (2.4)
  - Superposition method (3.5)
  - Thevenin Method (3.6)
  - Norton Method (3.6)
- 2.6 Dependent Sources and the Control Concept will be discussed later with amplifier

# Circuit Analysis Method 1: Basic KVL, KCL method

- Goal: Find all element $V$'s and $I$'s
  1. Write element $I - V$ relationships from lumped circuit abstraction
  2. Write KVL for all loops
  3. Write KCL for all nodes
  4. Solve the set of linear equations

# Example

- $v_0 \cdots v_5, i_0 \cdots i_5$ => 12 unknowns



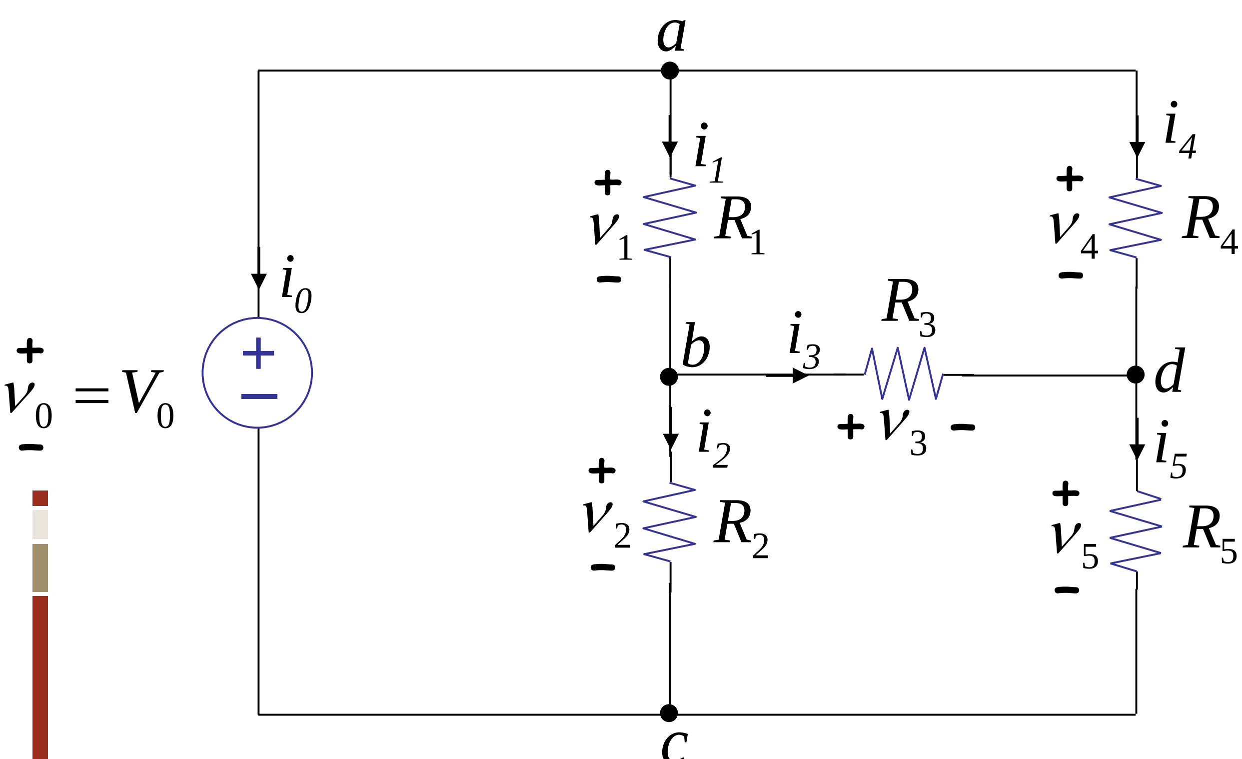
## Example

- Element relations (v, i)

$$v_0 = V_0 \longleftarrow \text{given} \qquad v_3 = i_3 R_3$$

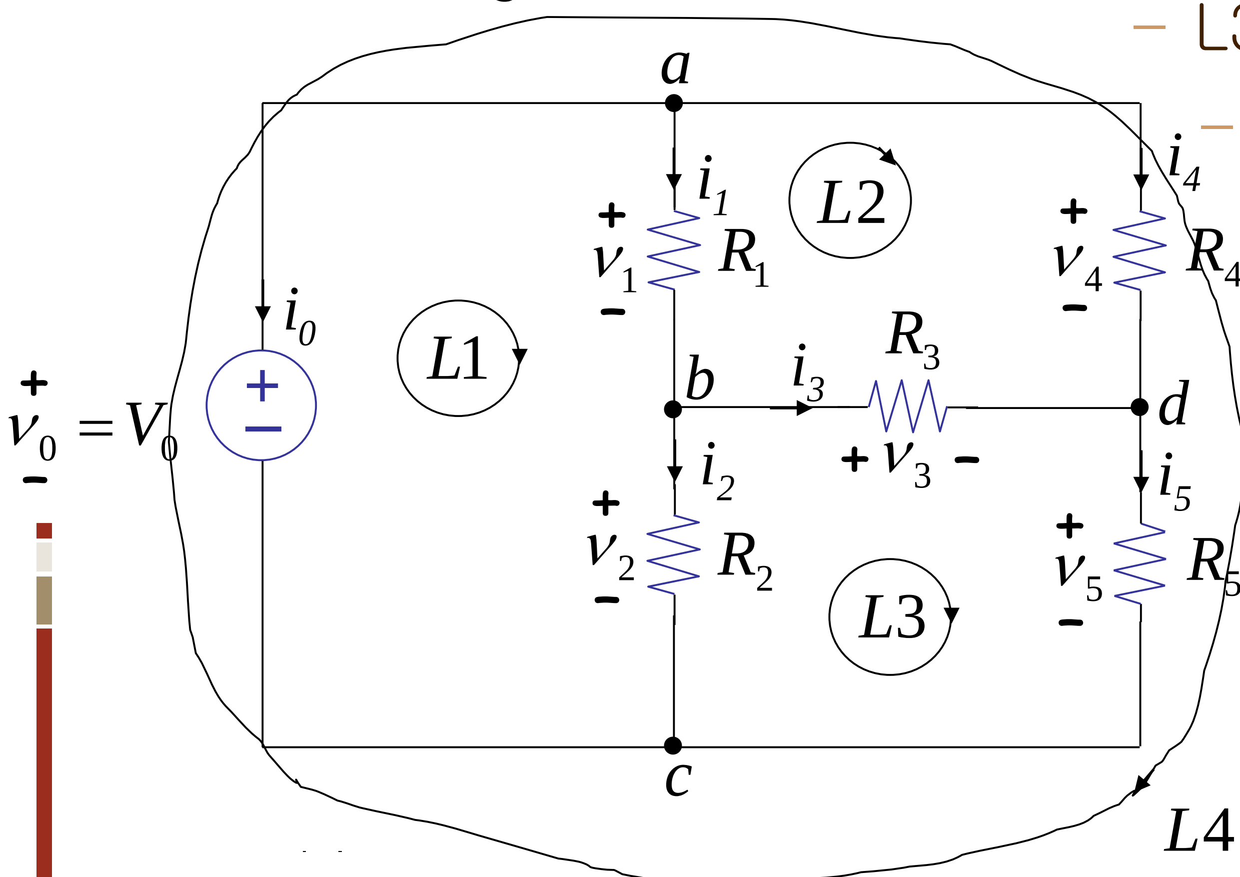
$$v_1 = i_1 R_1 \qquad v_4 = i_4 R_4$$

$$v_2 = i_2 R_2 \qquad v_5 = i_5 R_5$$



## Example

- KVL: as we traverse the loop, if we meet minus terminal, then subtract the voltage, and if we meet plus terminal, add the voltage.




- L1: $-v_0 + v_1 + v_2 = 0$
- L2: $-v_1 + v_4 - v_3 = 0$
- L3: $-v_2 + v_3 + v_5 = 0$
- L4: $-v_0 + v_4 + v_5 = 0$

➔ redundant

L1 + L2 + L3 => L4

If we draw all the loops in clockwise direction, each internal element such as $R_1$, $R_2$, $R_3$ appears twice with opposite sign in two elementary loop calculations. Therefore, if we add all the small loops, all the internal elements will cancel out, and the calculation for the outer loop will be obtained.

Also, the calculation for any other shape of loops can be also obtained by combining several elementary loops.

# Example

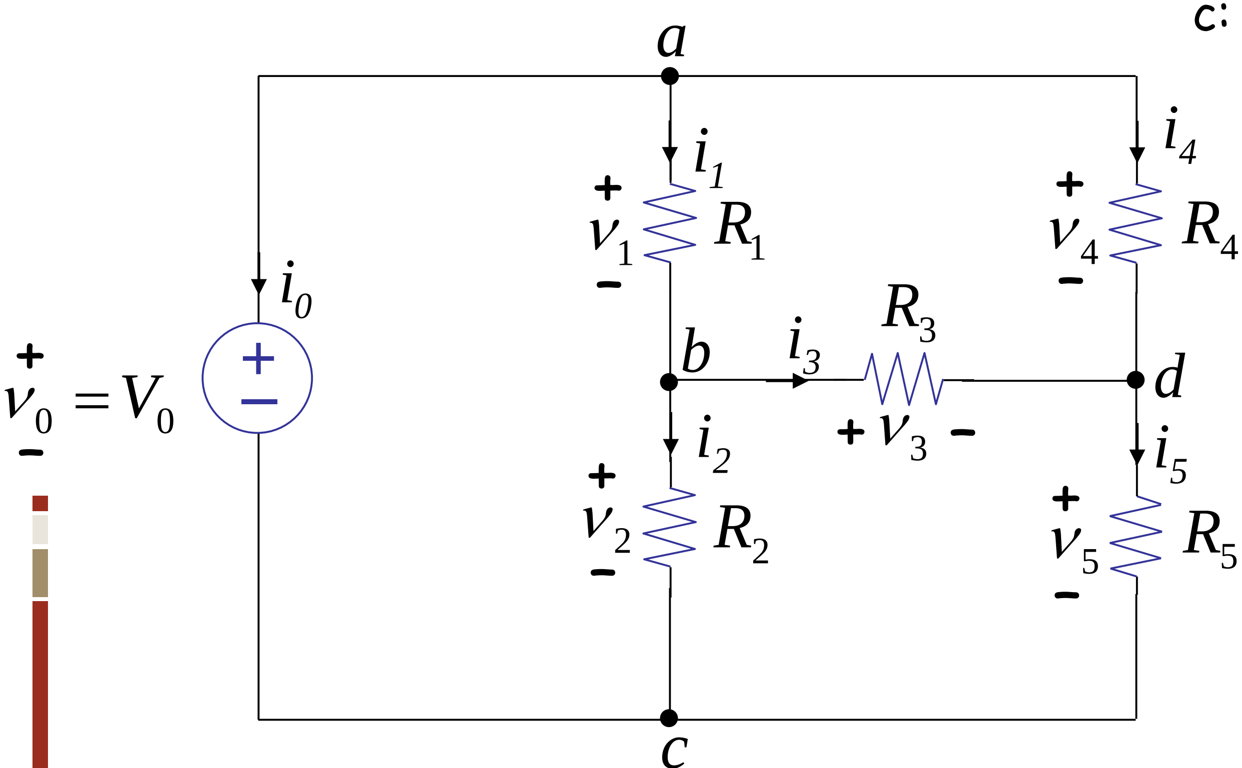
- KCL

a: $i_0 + i_1 + i_4 = 0$

b: $i_2 + i_3 - i_1 = 0$

d: $i_5 - i_3 - i_4 = 0$

c: $-i_0 - i_2 - i_5 = 0$ redundant




If we add all the outgoing currents at each node except one specific node (e.g. node $c$), then the same current will appear in two summations with opposite signs except those currents exiting the specific node. Therefore, the summation of all the equations will cancel out all other currents except those currents flowing into the specific node.

In general, there will be always a single node whose KCL equation is redundant.

# Example

$v_0, v_1, \ldots, v_5, i_0, i_1, \ldots, i_5$

12 unknowns

1. Element relations $(v, i)$

$v_0 = V_0 \longleftarrow$ given

$v_1 = i_1 R_1$

$v_2 = i_2 R_2$

$v_3 = i_3 R_3$

$v_4 = i_4 R_4$

$v_5 = i_5 R_5$

6 equations

2. KVL for loops

L1: $-v_0 + v_1 + v_2 = 0$

L2: $-v_1 + v_4 - v_3 = 0$

L3: $-v_2 + v_3 + v_5 = 0$

L4: $-v_0 + v_4 + v_5 = 0$ ← redundant

3 independent equations

3. KCL at nodes

a: $i_0 + i_1 + i_4 = 0$

b: $-i_1 + i_2 + i_3 = 0$

d: $-i_3 - i_4 + i_5 = 0$

c: $-i_0 - i_2 - i_5 = 0$ ← redundant

3 independent equations

12 equations

# Derivation of node analysis method

- How to solve 12 equations?
  - Unknowns: $v_0, v_1, \dots, v_5, i_0, i_1, \dots, i_5$
  - Remove all currents with elements relations

1. Element relations $(v, i)$

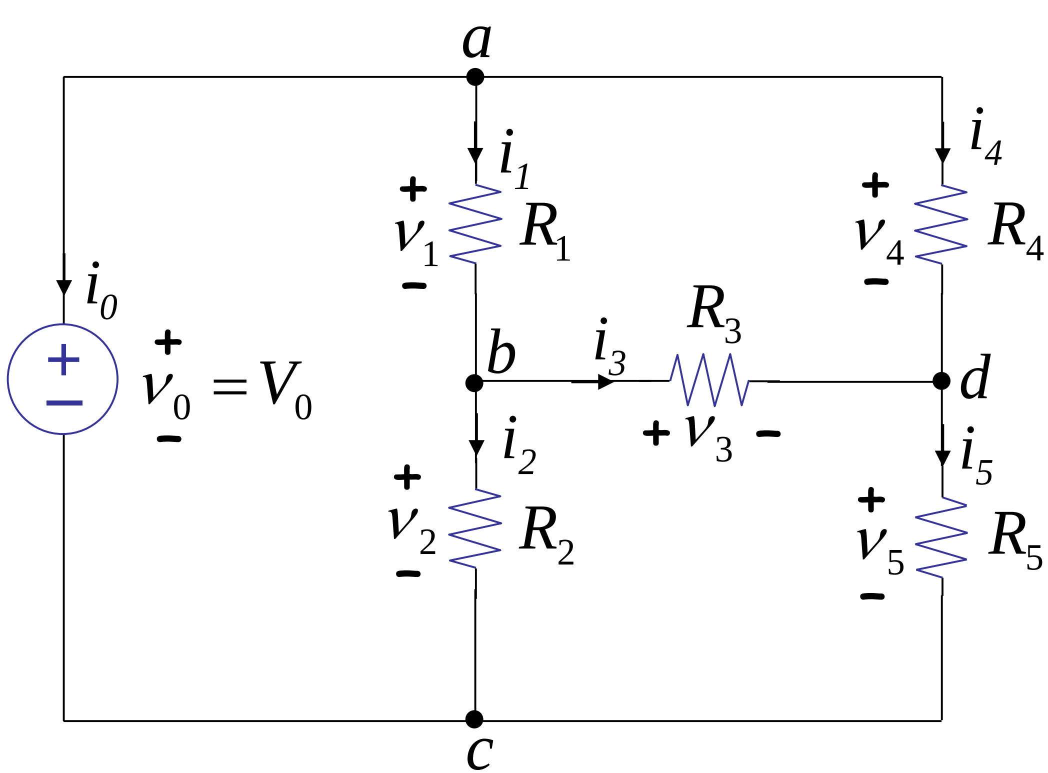
$$v_0 = V_0 \qquad v_3 = i_3 R_3$$
$$v_1 = i_1 R_1 \qquad v_4 = i_4 R_4$$
$$v_2 = i_2 R_2 \qquad v_5 = i_5 R_5$$

2. KCL at nodes

a: $i_0 + i_1 + i_4 = 0$
b: $-i_1 + i_2 + i_3 = 0$
d: $-i_3 - i_4 + i_5 = 0$
c: $-i_0 - i_2 - i_5 = 0$ ← redundant

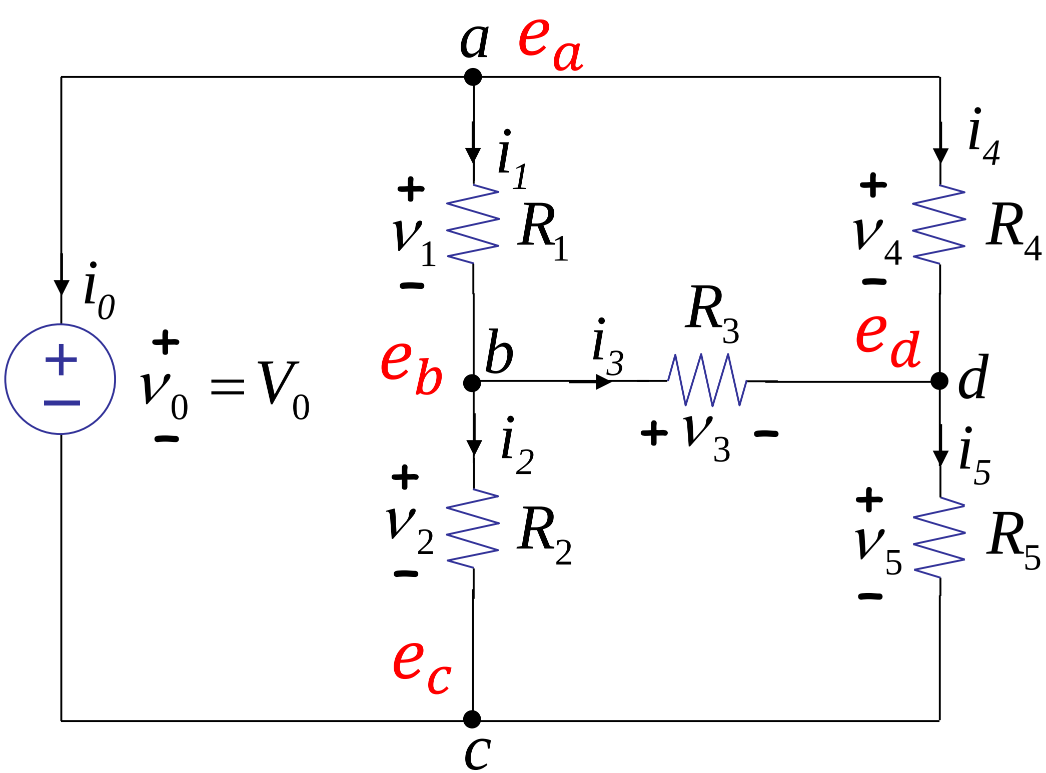
3. KVL for loops

L1: $-v_0 + v_1 + v_2 = 0$
L2: $-v_1 + v_4 - v_3 = 0$
L3: $-v_2 + v_3 + v_5 = 0$
L4: $-v_0 + v_4 + v_5 = 0$ ← redundant



# Derivation of node analysis method

- How to solve 12 equations?
  - Unknowns: $v_0, v_1, \ldots, v_5, i_0, i_1, \ldots, i_5$
  - Remove all currents with elements relations ➔ 5 voltages + 1 current
  - Define node voltage rather than voltage across each element. ➔ Automatically satisfy the KVL eqs.




1. Element relations $(v, i)$

$v_0 = V_0 \quad v_3 = i_3R_3$

$v_1 = i_1R_1 \quad v_4 = i_4R_4$

$v_2 = i_2R_2 \quad v_5 = i_5R_5$

2. KCL at nodes

a: $i_0 + v_1/R_1 + v_4/R_4 = 0$

b: $-v_1/R_1 + v_2/R_2 + v_3/R_3 = 0$

d: $-v_3/R_3 - v_4/R_4 + v_5/R_5 = 0$

c: $-i_0 - i_2 - i_5 = 0$ ← redundant

3. KVL for loops

L1: $-v_0 + v_1 + v_2 = 0$

L2: $-v_1 + v_4 - v_3 = 0$

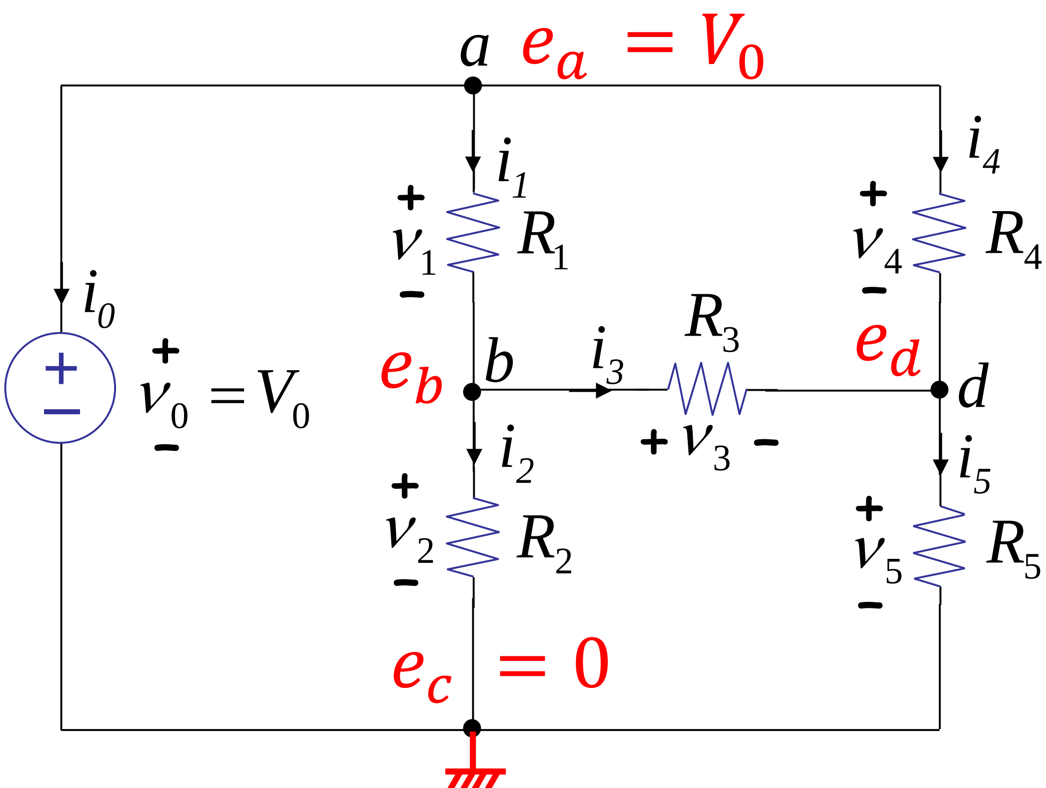
L3: $-v_2 + v_3 + v_5 = 0$

L4: $-v_0 + v_4 + v_5 = 0$ ← redundant

E.g. L1 loop: $-v_0 + v_1 + v_2$
$= -(e_a - e_c) + (e_a - e_b) + (e_b - e_c)$
$= 0$

# Derivation of node analysis method

- Unknown 4 voltages and one current
- We have a freedom to choose one of the voltages as ground. E.g. $v_c = 0$.
- When a node is connected to the ground through a voltage source, voltage of that node will be fixed, e.g. $v_a = V_0$ and we don't need to solve the KCL at that node.
- Problem is reduced to find two unknown voltages $e_b$ and $e_d$.




2. KCL at nodes

a: $\cancel{i_0 + v_1/R_1 + v_4/R_4 = 0}$

b: $-v_1/R_1 + v_2/R_2 + v_3/R_3 = 0$

d: $-v_3/R_3 - v_4/R_4 + v_5/R_5 = 0$

By using

- $v_1 = e_a - e_b = V_0 - e_b$
- $v_2 = e_b - e_c = e_b$
- $v_3 = e_b - e_d$
- $v_4 = e_a - e_d = V_0 - e_d$
- $v_5 = e_d - e_c = e_d$

KCL at nodes turns into

b: $-\frac{(V_0 - e_b)}{R_1} + \frac{e_b}{R_2} + \frac{(e_b - e_d)}{R_3} = 0$

d: $-\frac{(e_b - e_d)}{R_3} - \frac{(V_0 - e_d)}{R_4} + \frac{e_d}{R_5} = 0$

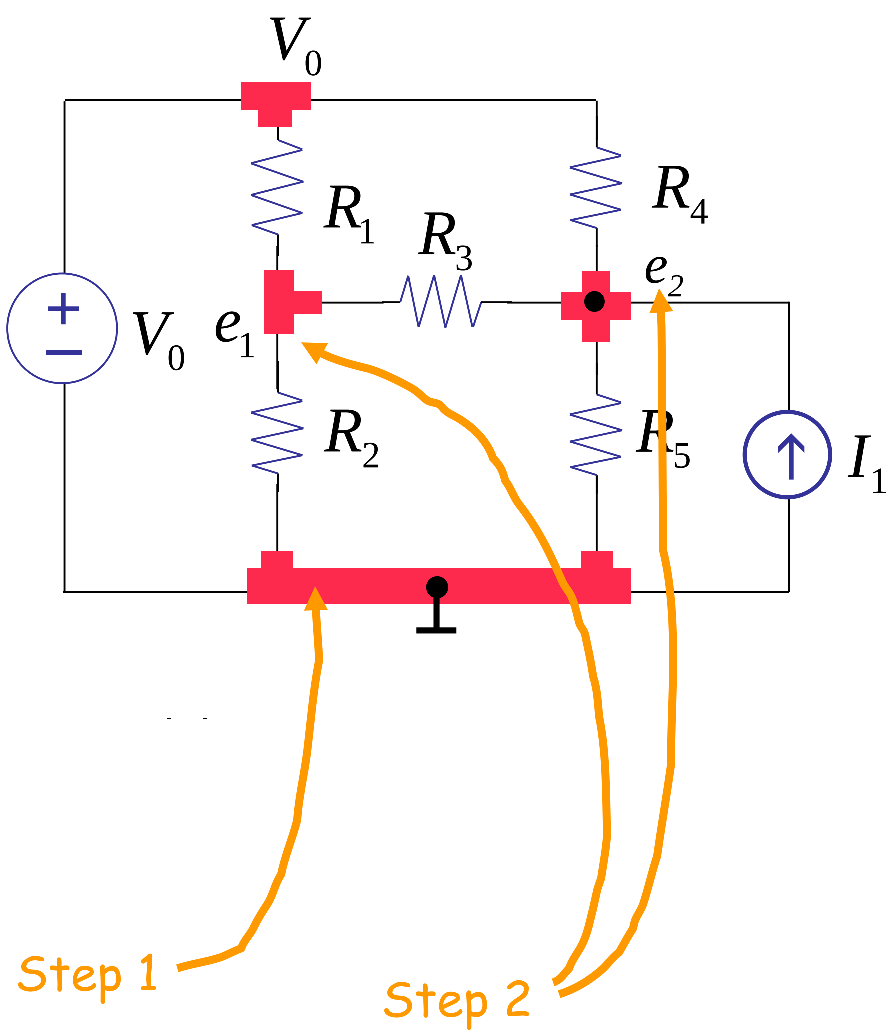
# Circuit Analysis Method 3: Node analysis

- KCL at nodes using V's referenced w.r.t. ground

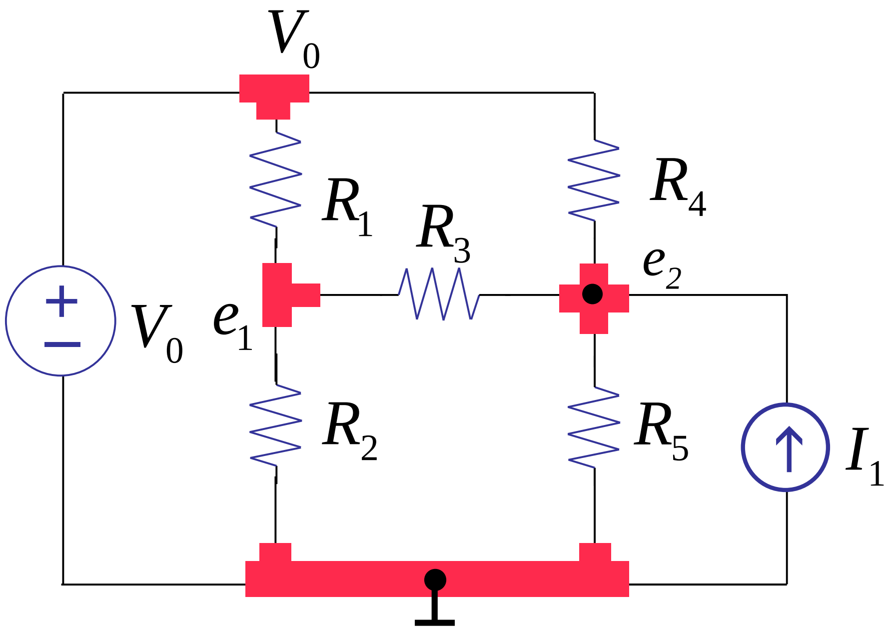
1. Select reference node (⏚ ground) from which voltages are measured.
2. Label voltages of remaining nodes with respect to ground. These are the primary unknowns.
3. Write KCL for all but the ground node, substituting device laws.
4. For nodes directly connected to terminals of voltage sources, write down the voltage drop relation (KVL).
5. Solve for node voltages.
6. Back solve for branch voltages and currents (i.e., the secondary unknowns)

- Exceptional case happens with a floating independent voltage source at step 3. Check section 3.3.2 of the text for the solution.

# Circuit Analysis Method 3: Node analysis



# Circuit Analysis Method 3: Node analysis




for convenience, write

$$G_i = \frac{1}{R_i}$$

Step 3

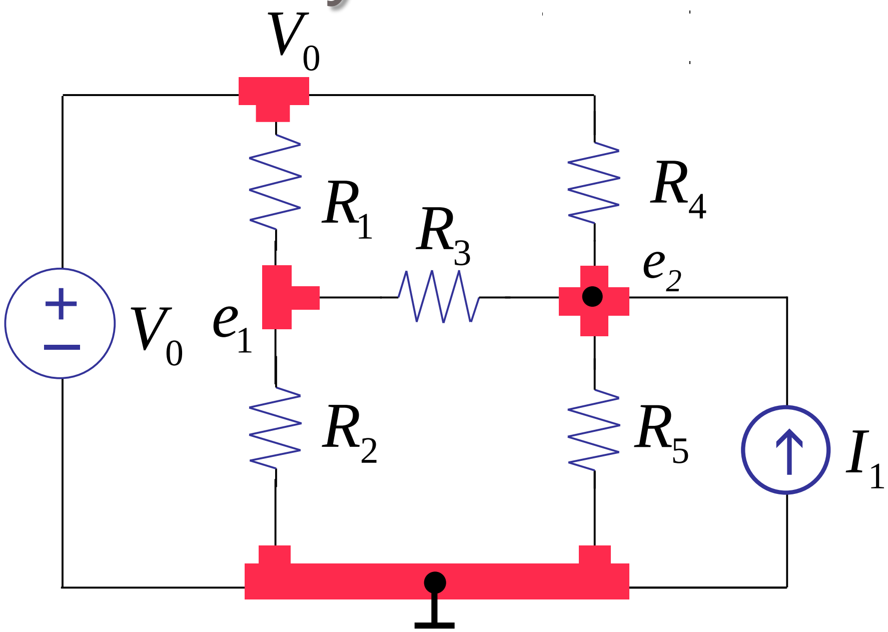
KCL at $e_1$

$$(e_1 - V_0)G_1 + (e_1 - e_2)G_3 + (e_1)G_2 = 0$$

KCL at $e_2$

$$(e_2 - e_1)G_3 + (e_2 - V_0)G_4 + (e_2)G_5 - I_1 = 0$$

# Circuit Analysis Method 3: Node analysis




$$G_i = \frac{1}{R_i}$$

KCL at $e_1$ $\quad (e_1 - V_0)G_1 + (e_1 - e_2)G_3 + (e_1)G_2 = 0$

KCL at $e_2$ $\quad (e_2 - e_1)G_3 + (e_2 - V_0)G_4 + (e_2)G_5 - I_1 = 0$

move constant terms to RHS & collect unknowns

$$e_1(G_1 + G_2 + G_3) + e_2(-G_3) = V_0(G_1)$$

$$e_1(-G_3) + e_2(G_3 + G_4 + G_5) = V_0(G_4) + I_1$$

2 equations, 2 unknowns $\longrightarrow$ Solve for e's

Step 4

# Circuit Analysis Method 3: Node analysis

## In matrix form:

$$\left[\begin{array}{c|c} G_1+G_2+G_3 & -G_3 \\ \hline -G_3 & G_3+G_4+G_5 \end{array}\right] \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} G_1V_0 \\ G_4V_0+I_1 \end{bmatrix}$$

conductance matrix

unknown node voltages

sources

## Solve

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \frac{\left[\begin{array}{c|c} G_3+G_4+G_5 & G_3 \\ \hline G_3 & G_1+G_2+G_3 \end{array}\right] \begin{bmatrix} G_1V_0 \\ G_4V_0+I_1 \end{bmatrix}}{(G_1+G_2+G_3)(G_3+G_4+G_5)-G_3^{\,2}}$$

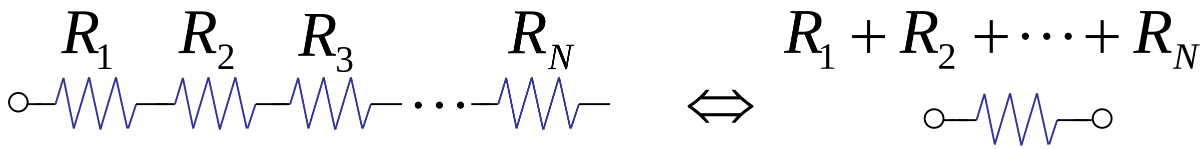
$$e_1 = \frac{(G_3+G_4+G_5)(G_1V_0)+(G_3)(G_4V_0+I_1)}{G_1G_3+G_1G_4+G_1G_5+G_2G_3+G_2G_4+G_2G_5+G_3^{\,2}+G_3G_4+G_3G_5}$$

$$e_2 = \frac{(G_3)(G_1V_0)+(G_1+G_2+G_3)(G_4V_0+I_1)}{G_1G_3+G_1G_4+G_1G_5+G_2G_3+G_2G_4+G_2G_5+G_3^{\,2}+G_3G_4+G_3G_5}$$

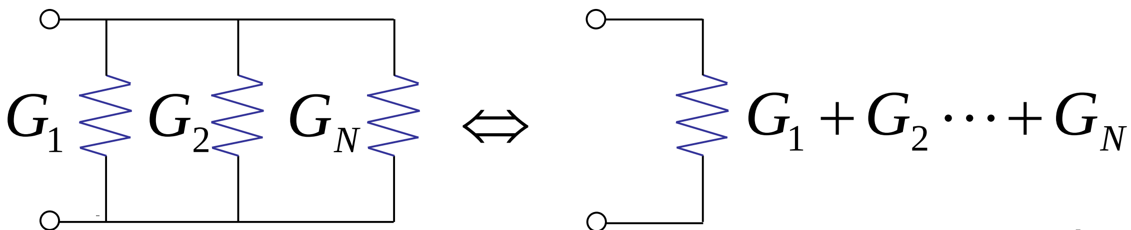
(same denominator)

# Circuit Analysis Method 2: Apply element combination rules

A

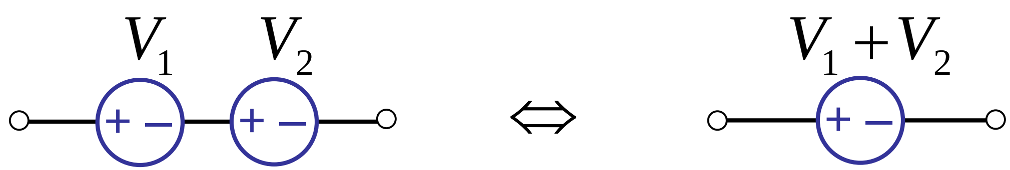



B

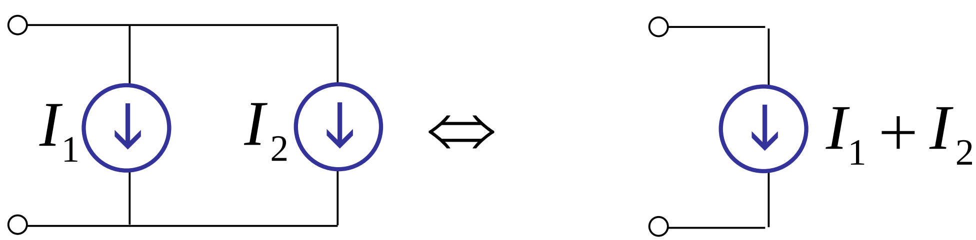



Conductance $G_i \equiv \frac{1}{R_i}$

C

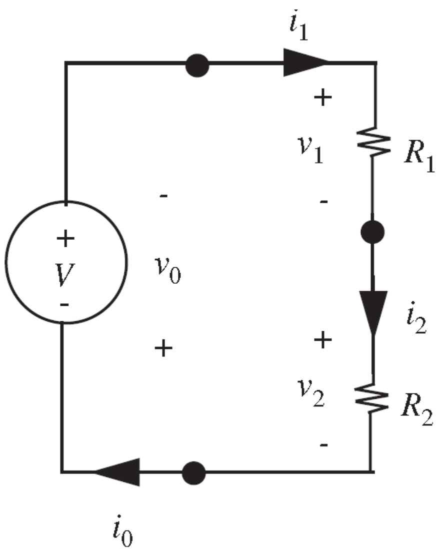



D



# Voltage divider

- The only constraint: current should be the same




1. Element laws

$$v_0 = -V$$
$$v_1 = R_1 i_1$$
$$v_2 = R_2 i_2$$

2. KCL

$$-i_0 + i_1 = 0$$
$$-i_1 + i_2 = 0$$

3. KVL

$$v_0 + v_1 + v_2 = 0$$

4. Solve the equations

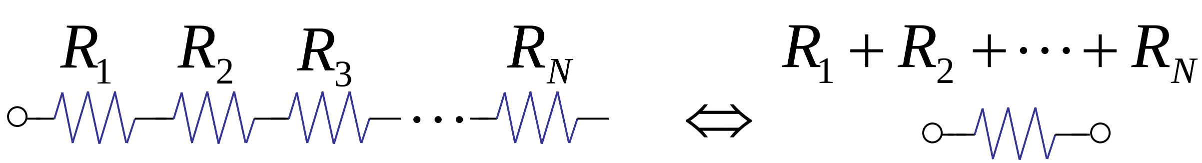
$$i_0 = i_1 = i_2 = \frac{V}{R_1 + R_2}$$

From the relation between the voltage $V$ and current $i_0$ from the voltage source, we can conclude that the equivalent resistance $R_{total}$ of two resistors connected in series is $R_1 + R_2$.

What is the voltage $v_2$ across $R_2$?

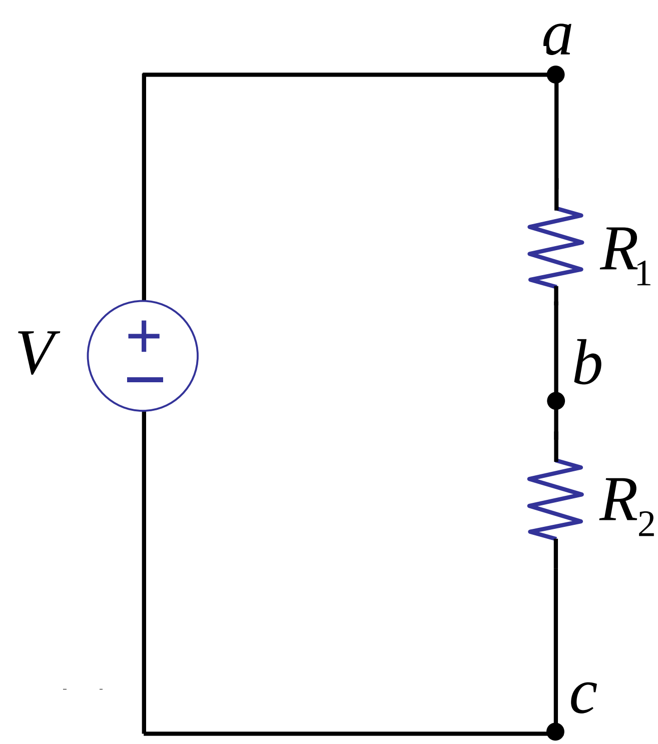
$$v_2 = i_2 R_2 = \frac{V}{R_1+R_2} R_2 = \frac{R_2}{R_1+R_2} V$$

➔ Voltage divider

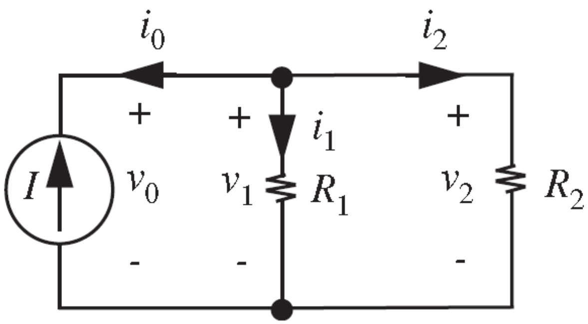


# Voltage drop

- "Voltage drop" is one of the most frequently used terminologies in the class, so get used to it.
- "Voltage drop" across $R_1$
- "Voltage drop" across $R_2$
- "Voltage drop" across voltage source



# Current divider




1. Element laws

$$i_0 = -I$$
$$v_1 = R_1 i_1$$
$$v_2 = R_2 i_2$$

2. KVL

$$-v_0 + v_1 = 0$$
$$-v_1 + v_2 = 0$$

3. KCL

$$i_0 + i_1 + i_2 = 0$$

4. Solve the equations

Conductance
$G_i \equiv 1/R_i$

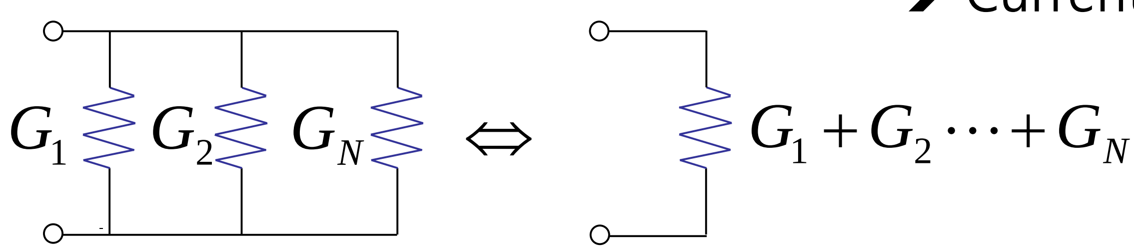
$$v_0 = v_1 = v_2 = \frac{I}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{I}{G_1 + G_2}$$

From the relation between the voltage $v_0$ and current $i_0$ from the current source, we can conclude that the equivalent resistance $R_{total}$ of two resistors connected in parallel is $\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$. Or equivalently, $G_{total} = G_1 + G_2$.

What is the current $i_2$ across $R_2$?

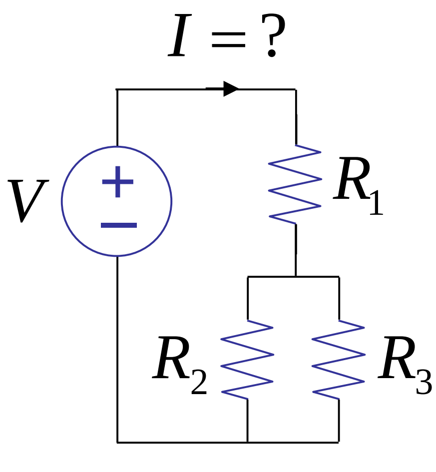
$$i_2 = \frac{v_2}{R_2} = \frac{I}{R_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} = \frac{R_1}{R_2 + R_1} I$$

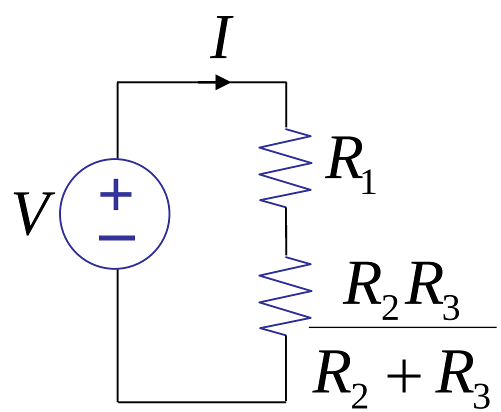
➔ Current divider

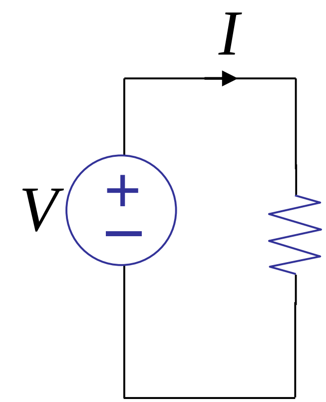


# Circuit Analysis Method 2: Apply element combination rules

- Example




$$R = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

$$I = \frac{V}{R}$$

# Summary

- Various circuit analysis methods
  - Method 1: Basic KVL $\sum_{loop} V_i = 0$ and KCL $\sum_{node} I_i = 0$
  - Method 2: Elementary combination method
    - $R_{total} = \sum_{series} R_i$ for series resistors
    - $G_{total} = \sum_{parallel} G_i$ for parallel resistors where $G_i = 1/R_i$
  - Method 3: Node analysis method
    - Define voltage variables $e_j$ at each node $j$ except ground
    - Apply KCL at each node $j$ while using "$(e_j - e_i)/R_{ij}$" as the current flowing through the branch between node $j$ and node $i$.
- Forthcoming
  - Superposition method and linearity
  - Thevenin equivalent circuits