Seoul National University

M1522.000900 Data Structure

Fall 2019, Kang

Homework 4: Binary Trees (Chapter 5)

Due: October 21, 11:59 PM

Reminders

- The points of this homework add up to 100.
- Like all homeworks, this has to be done individually.
- Lead T.A.: Chaeheum Park (chaeheum@snu.ac.kr)
- Please type your answers in English. Illegible handwriting may get no points, at the discretion of the graders.
- If you have a question about assignments, please upload your question in eTL.
- If you want to use slipdays or consider late submission with penalties, please note that you are allowed four days to submit your assignment after the due date.

Remember that:

1. Whenever you are making an assumption, please state it clearly

Question 1

Define the **x-degree** of a node in a binary tree as the number of its non-empty children. For example, A 2-degree node has two childrens. Let the number of leaves in a binary tree be n. What is the number of 2-degree nodes? Represent the number using n. The answer should be true for any binary tree. (No further explanation needed just represent using n). [10 points]

Question 2

Figure 1 and Figure 2 represent a BST.

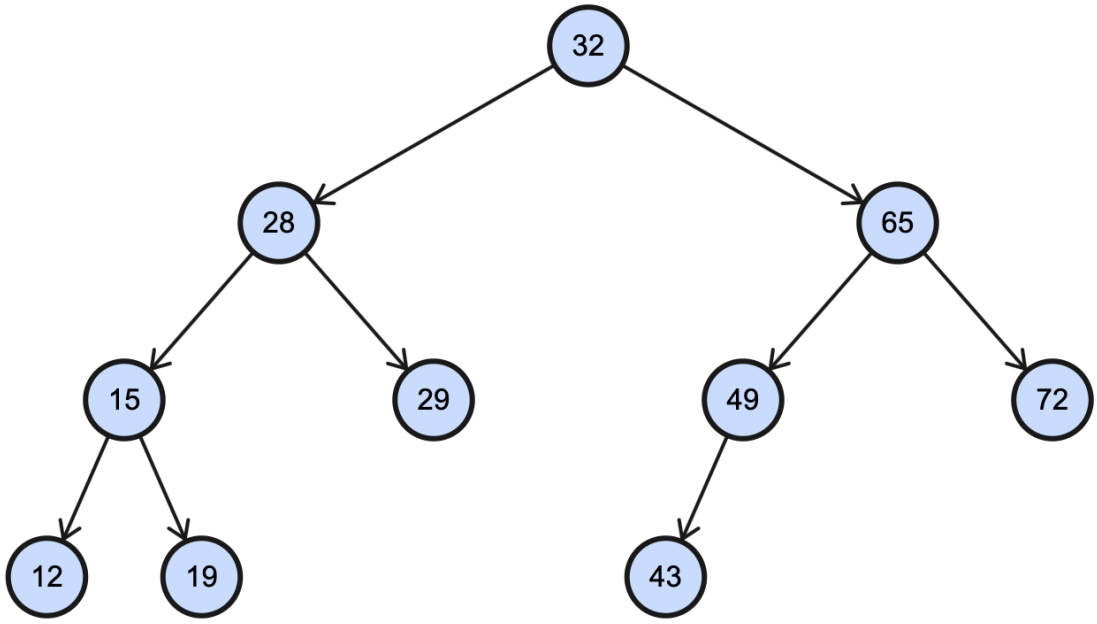

Figure 1. Binary Search Tree.

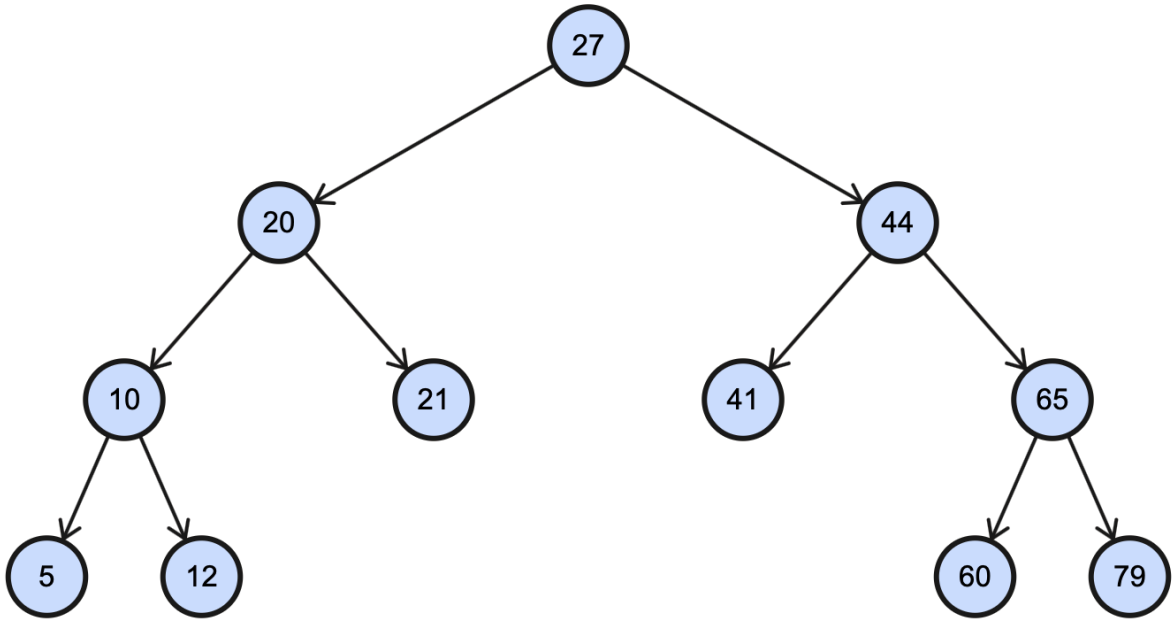

Figure 2. Binary Search Tree.

(a) Draw a BST that results from deleting value 32 from **Figure 1**. [10 points]

(b) Draw a BST that results from deleting value 44 from **Figure 2**. [10 points]

Question 3

(a) Show the BST that results from inserting the values 15, 12, 25, 18, 16, 5, and 7 (in that order). [6 points]

(b) Show the enumerations (ex. 1-2-3-4) for the tree of (a) that result from doing a preorder traversal, an inorder traversal, and a postorder traversal. [24 points, 8 points each]

A. Preorder

B. Inorder

C. Postorder

Question 4

A max-heap is a complete binary tree where the value of each parent is greater than or equal to the values of its children. Answer the following questions.

(1) Which of the following binary trees is a max-heap? [10 points]

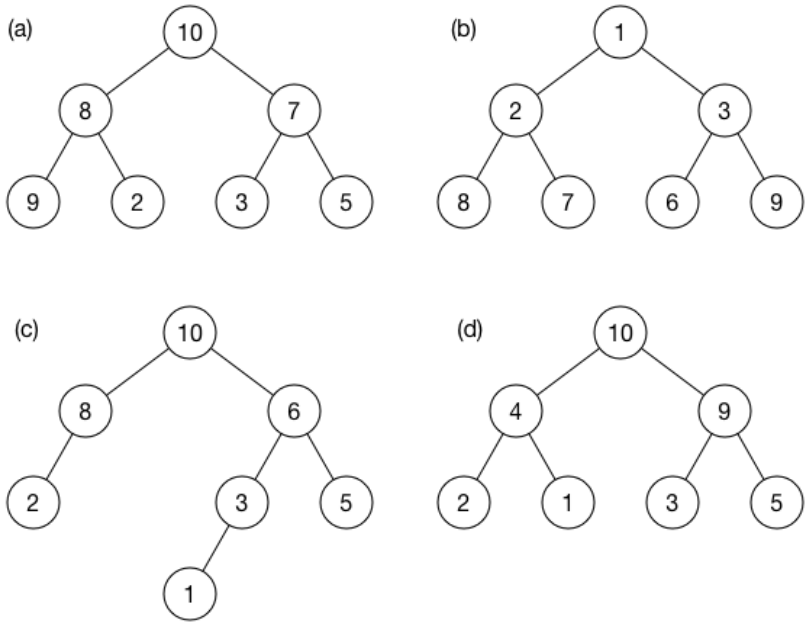

Figure 3. Binary trees.

(2) The following tree is not a max-heap. But we can make the tree a max-heap by moving some nodes. What is the minimum number of nodes that should be moved to make the tree a max-heap? Show your answer by drawing a tree for each node movement. (Assume that you can move a node from any position to any position at once.) [10 points]

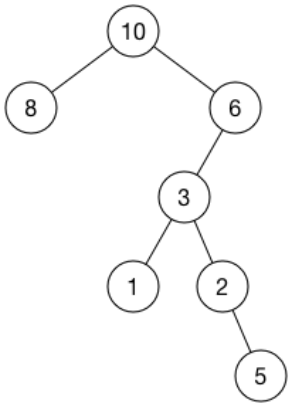

Figure 4. A binary tree which is not a max-heap.

Question 5

A perfect binary tree of height h is a binary tree where:

1. All leaves have the same depth $h - 1$.
2. All other nodes have two children nodes.

Prove that the sum of depth of each node in a perfect binary tree of height h is $(h - 2)(2^h - 1) + h$. [20 points]