Seoul National University

M1522.000900 Data Structure

Fall 2019, Kang

Homework 1: Mathematical Preliminaries (Chapter 2)

Due: September 18, 11:59 PM

Reminders

- The points of this homework add up to 100.
- Like all homework, this has to be done individually.
- Lead T.A.: Huiwen Xu (xuhuiwen33@snu.ac.kr)
- Please type your answers in **English**. Homework written in Korean may get no points.
- **All the homework should be uploaded to the eTL in PDF format.** No handwritten homework (including PDF files made with photos of handwritten papers) will be accepted.
- Your homework should be named as "(studentid)-(yourname)-HW1.pdf" For example, 201912345-GildongHong-HW1.pdf.
- If you have any question about assignments, please upload your question in eTL.
- If you want to use slip-days or consider late submission with penalties, please note that you are allowed one week to submit your assignment after the due date.

Remember that:

- Whenever you are making an assumption, please state it clearly.

Question 1

For each of the following relations, either prove that it is an equivalence relation or prove that it is not an equivalence relation. [18 points]

(1) For integers a and b, $a \equiv b$ if and only if $a + b$ is even.

(2) For integers a and b, $a \equiv b$ if and only if $a + b$ is odd.

(3) For nonzero rational numbers a and b, $a \equiv b$ if and only if $a \times b > 0$.

(4) For nonzero rational numbers a and b, $a \equiv b$ if and only if a / b is an integer.

(5) For rational numbers a and b, $a \equiv b$ if and only if $a - b$ is an integer.

(6) For rational numbers a and b, $a \equiv b$ if and only if $|a - b| \leq 2$.

Question 2

Answer whether each of following relations is a partial ordering or not. [18 points]

(1) “is father of” on the set of people

(2) “is ancestor of” on the set of people

(3) “is older than” on the set of people

(4) “is sister of” on the set of people

(5) $\{\langle a, b\rangle, \langle a, a\rangle, \langle b, a\rangle\}$ on the set $\{a, b\}$

(6) $\{\langle 2,1\rangle, \langle 1,3\rangle, \langle 2,3\rangle\}$ on the set $\{1,2,3\}$

Question 3

The Tower of Hanoi is a game to move all the disks from leftmost pole to rightmost pole (see a figure below). You can move a disk on top of the towers to other towers at a time. You cannot place a larger disk onto a smaller disk. Let $T(n)$ be the number of disk movements to finish the game with n disks. Answer the following questions. [25 points]

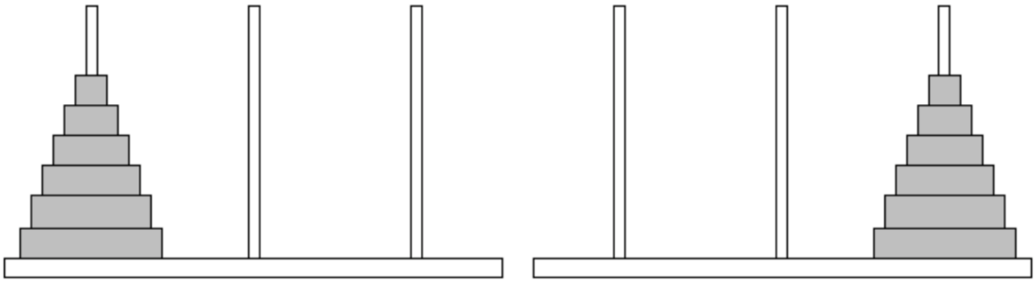

Figure 1. Tower of Hanoi game with 6 disks. The left figure is an initial state of disks and the right figure is the final state of disks.

(1) What is value of $T(10)$?

(2) Write $T(n)$ as a recurrence relation.

(3) What is a closed form solution of $T(n)$?

Question 4

Expand the following recurrence to help you find a closed-form solution, and then use induction to prove your answer is correct. [25 points]

$$S(n) = 2S(n/2) + n \text{ for } n = 2^i, \text{where } i > 0; S(1) = 1$$

(1) Derive closed-form solution.

(2) **Basis:** Show that $S(n)$ holds for $n = 2$.

(3) **Inductive step**: Show that the following recurrence holds: If $S(n)$ holds for $n = 2^k$, then $S(n)$ also holds for $n = 2^{k+1}$.

Question 5

Use mathematical induction to show that when n is an exact power of 2, the solution of the recurrence

$$\mathrm{T(n)} = \begin{cases} 2 & \text{if } \mathrm{n} = 2 \\ 2\mathrm{T}\left(\frac{\mathrm{n}}{2}\right) + \mathrm{n} & \text{if } \mathrm{n} = 2^{\mathrm{k}}, \text{for } \mathrm{k} > 1 \end{cases}$$

is $\mathrm{T(n)} = \mathrm{n} \lg \mathrm{n}$. [14 points]