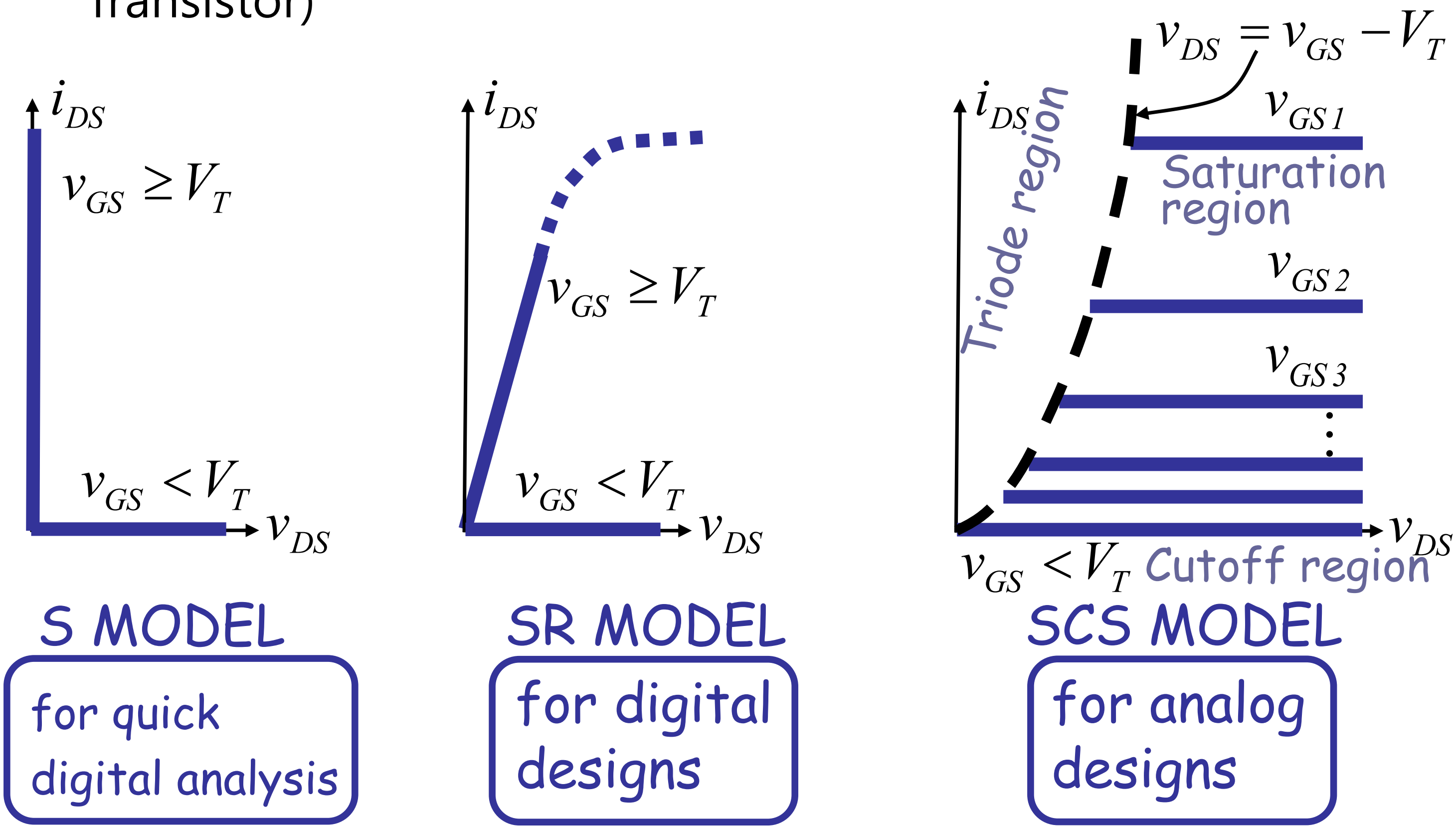
Electrical and Electronics Circuits (4190.206A 002)

- HW #2 is out and due on Oct. 23, 3:15pm
- 1st mid-term exam will be in the next week. I will finalize the date by the next class.

- Self-attendance check

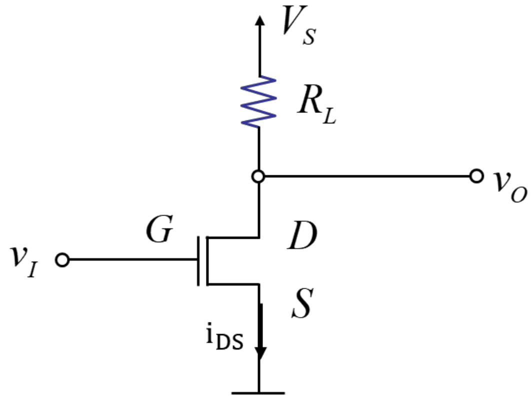
Review I

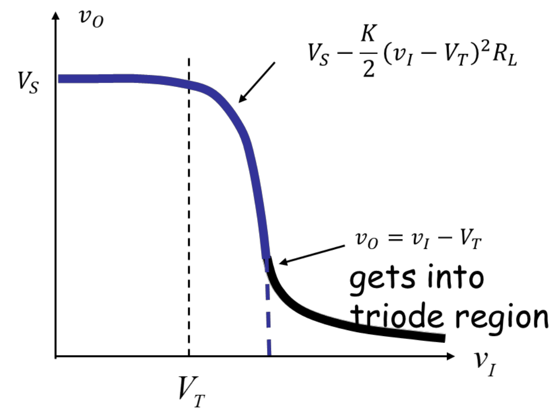
- MOSFET (Metal-Oxide-Semiconductor Field-Effect Transistor)



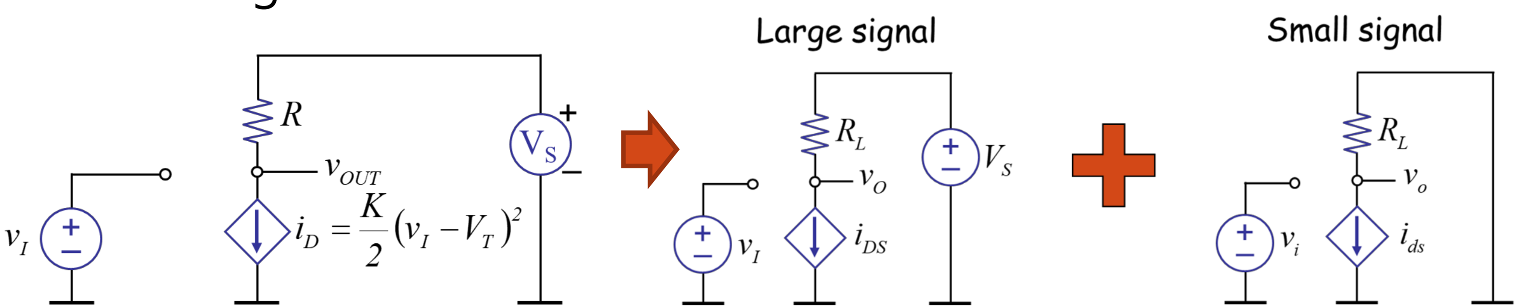
Review II

- Typical configuration for amplifier



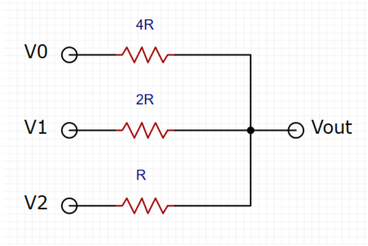


- Small signal model

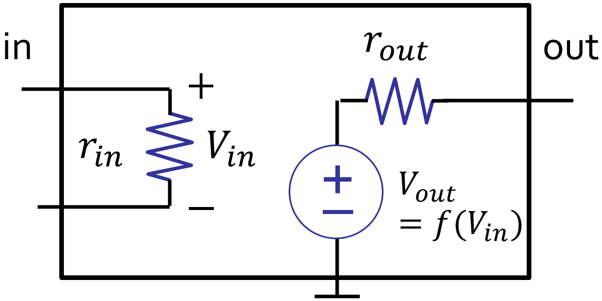


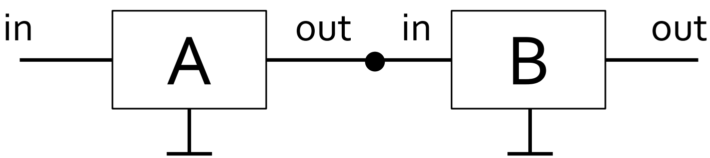
Connecting Two Circuits

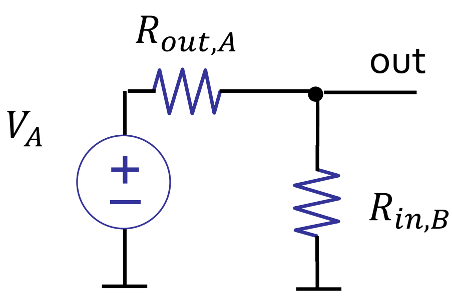
- What happens if we connect a load to the output of DAC?



- Design rule of thumb
  - Assume that circuit A drives circuit B
  - $R_{out,A}$: output resistance of circuit A
  - $R_{in,B}$: input resistance of circuit B
  - Make sure that $R_{out,A} \le R_{in,B}$



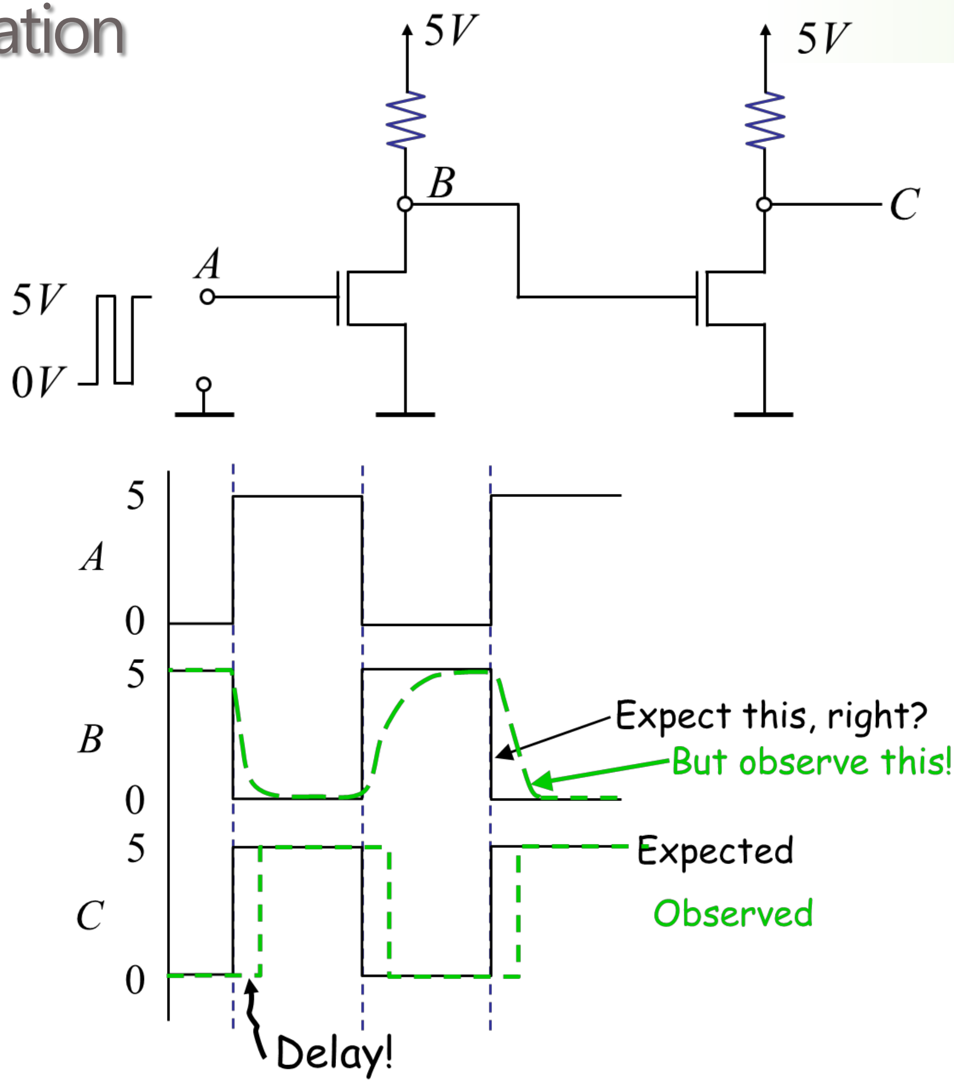




Where we are…

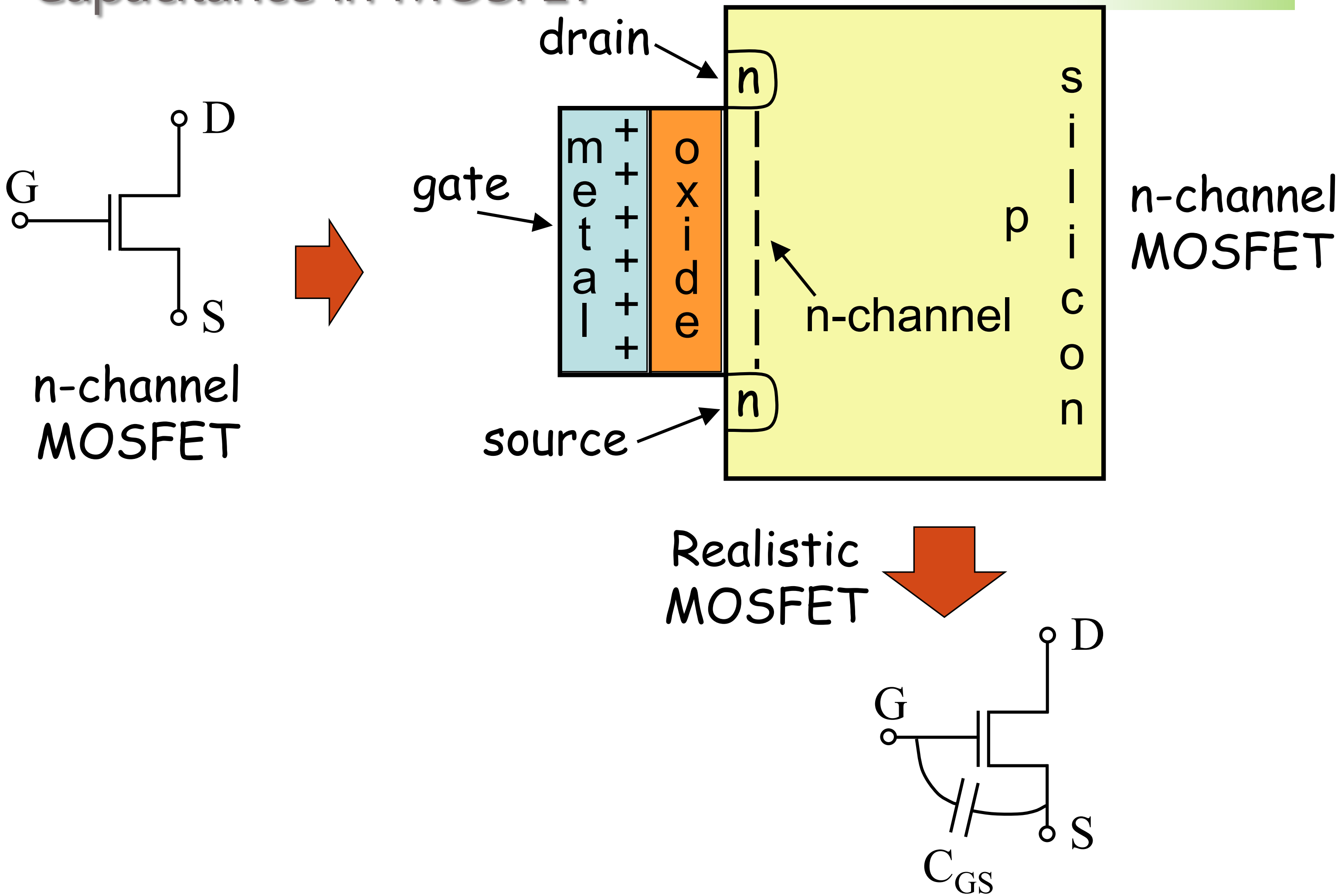
- Chap. 9 Energy Storage Elements
  - 9.1 Constitutive Laws
    - 9.1.1 Capacitors
    - 9.1.2 Inductors
  - 9.2 Series and Parallel Connections
  - 9.3 Special Examples
  - 9.5 Energy, Charge, and Flux Conservation
- Chap. 10 First-order Transients in Linear Electrical Networks
  - 10.1 Analysis of RC Circuits
  - 10.2 Analysis of RL Circuits
  - 10.4 Propagation Delay and the Digital Abstraction
  - 10.7 Digital Memory

Motivation

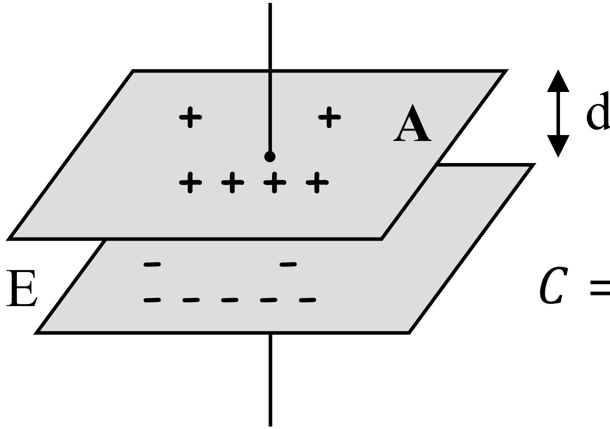


5V
5V
B
C
A
5V
0V
5
A
0
5
B
0
5
C
0
Expect this, right?
But observe this!
Expected
Observed
Delay!

Capacitance in MOSFET

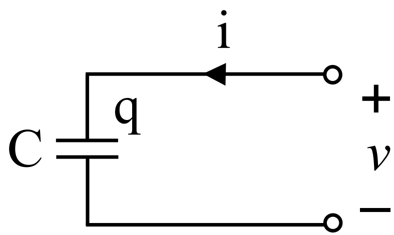


Ideal Linear Capacitor



$$C = \epsilon \frac{A}{d}$$

obeys Lumped Matter Discipline!
total charge on capacitor
$= +q - q = 0$

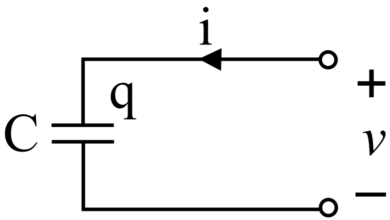


$$q = C \; v$$

coulombs farads volts

1′50″

Ideal Linear Capacitor



$$q = C\,v$$

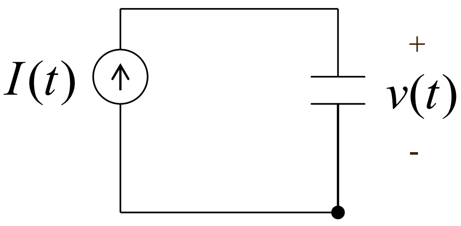
$$\boxed{i = \frac{dq}{dt}} = \frac{d(Cv)}{dt} = C\frac{dv}{dt}$$

$$\underbrace{\begin{aligned} \frac{d\omega_E(t)}{dt} &= i(t)v(t) \\ d\omega_E(t) &= v(t)\big(i(t)dt\big) = v(t)dq(t) \\ \omega_E &= \int_0^q v\,dx = \frac{q^2(t)}{2C} = \frac{Cv(t)^2}{2} \end{aligned}}$$

A capacitor is an energy storage device

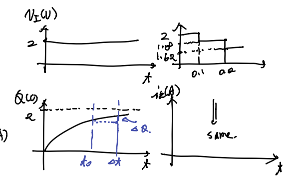
→ memory device → history matters!

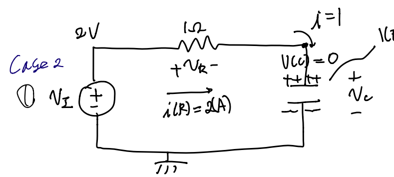
Current Source and Capacitor



What is the voltage across the capacitor?

Case 1) $Q = 0\,(C)$ $V_I = 0\,(V)$
$V_C = 0\,(V)$ $V_R = 0,\ i_R = 0$





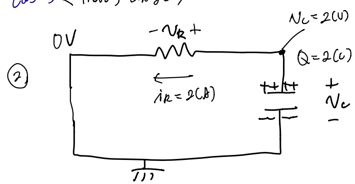
Case 2) $Q = 0\,(C)$ $V_I = 2\,(V)$
$V_C = 0$ $V_R = 2V,\ i_R = -2(A)$

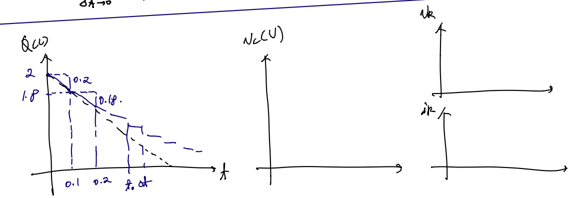
$\Delta Q = \frac{V_I - V_C}{R} \cdot \Delta t$

$\lim_{\Delta t \to 0} C \frac{\Delta V_C}{\Delta t} = \frac{\Delta Q}{\Delta t} = \frac{V_I - V_C}{R} \Rightarrow RC \cdot \frac{dV_C}{dt} = V_I - V_C.$

what if.

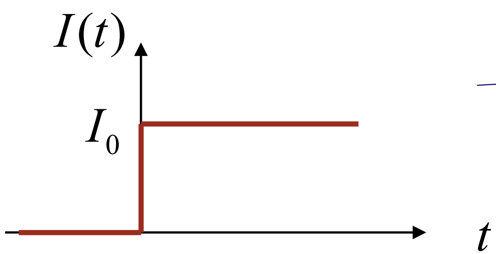
Case 3 < There's charge >





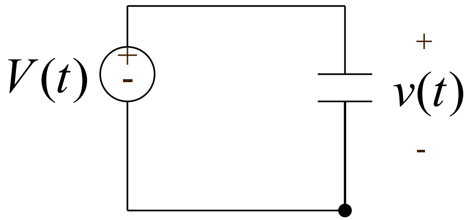
in, Case 3

$\Delta Q = C \cdot \Delta V_C$ $\quad \frac{C \Delta V_C}{\Delta t} = -\frac{V_C - 0}{R} \cdots \Rightarrow RC \cdot \frac{dV_C}{dt} + V_C = 0.$

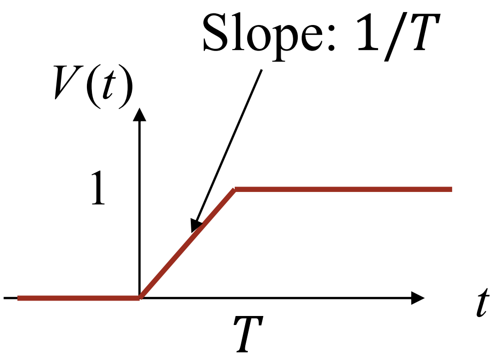


Step input

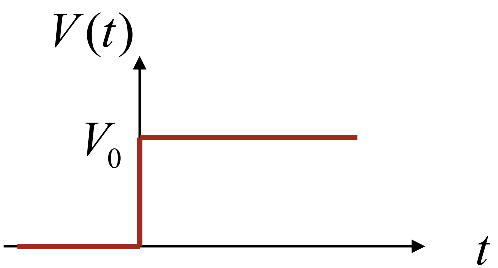
Voltage Source and Capacitor



What is the current through the capacitor?

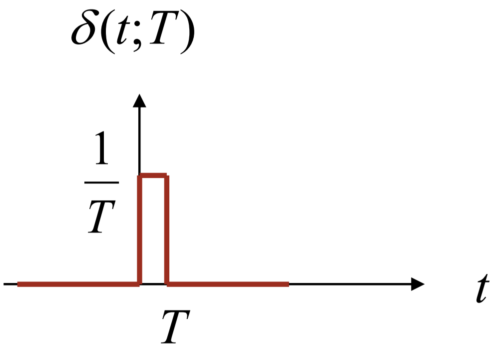


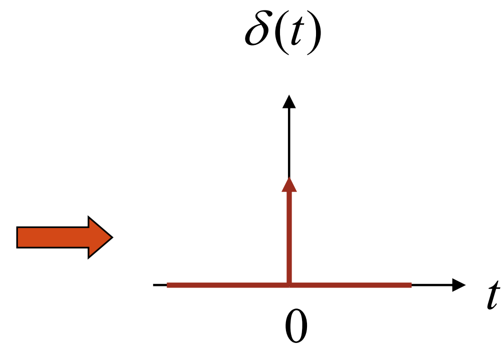
Ramp input



Step input

Unit Impulse Function





$$\delta(t) = 0 \quad for \quad t \neq 0$$

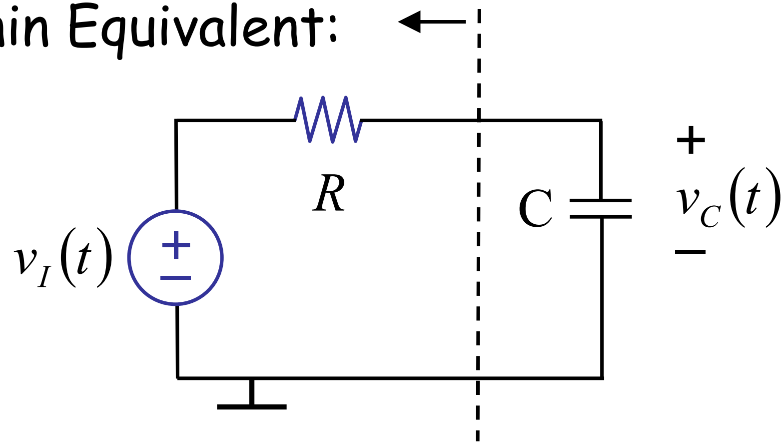
$$\int_{-\infty}^{t} \delta(t)dt = u(t) \Leftrightarrow \delta(t) = \frac{du(t)}{dt}$$

$$\int_{-\infty}^{+\infty} \delta(t)dt = 1$$

Analysis of an RC circuit (Section 10.1)

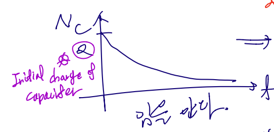
(y using)

Thévenin Equivalent: ←



in Case 3) $RC \cdot \frac{dv_C}{dt} + v_C = 0$.

should satisfy that equation..



$\Longrightarrow \frac{1}{2^t}, \frac{1}{3^t}, \ldots$

finding value : objective

just unknown

$\frac{1}{a^t} = a^{-t} = e^{-(\ln a)t} = e^{-st} \Rightarrow e^{st}$.

$e^{st} \cdot s$.

$\Rightarrow RC \cdot \frac{dv_C}{dt} + v_C = 0 \Rightarrow \cdots \Rightarrow s = -\frac{1}{RC} \Rightarrow v_C = A \times e^{-\frac{1}{RC}t}$

if) $t = 0$, $v_C = 1$.

① ② two main requisition

cf. w.o. well defined initial condition ⇒ there will be infinite numbers of equations constant

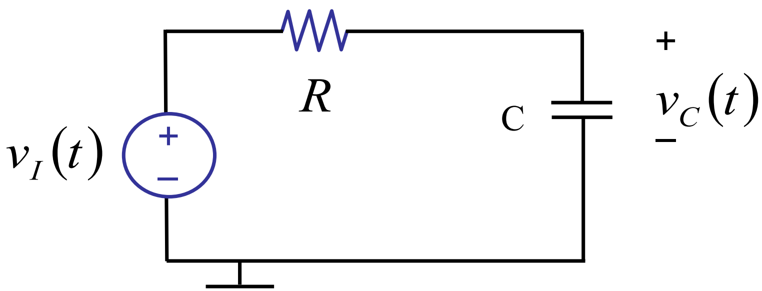
Node method:

$$\frac{v_C - v_I}{R} + C\frac{dv_C}{dt} = 0$$

$$RC\frac{dv_C}{dt} + v_C = v_I \quad \begin{cases} t \geq t_0 \\ v_C(t_0) \text{ given} \end{cases}$$

RC time constant

Example Analysis of an RC circuit



$$v_I(t) = V_I$$

$$v_C(0) = V_0 \quad \text{given}$$

$$RC\,\frac{dv_C}{dt} + v_C = V_I \quad \text{———} \quad \otimes$$

Example Analysis of an RC circuit

$$v_I(t) = V_I$$

$$v_C(0) = V_0 \quad \text{given}$$

$$RC\,\frac{dv_C}{dt} + v_C = V_I \quad \text{———} \quad ⊗$$

$$\underset{\text{total}}{v_C(t)} = \underset{\text{homogeneous}}{v_{CH}(t)} + \underset{\text{particular}}{v_{CP}(t)}$$

Method of homogeneous and particular solutions:

1. Find the homogeneous solution.
2. Find the particular solution.
3. The total solution is the sum of the particular and homogeneous solutions.
4. Use the initial conditions to solve for the remaining constants.

Homogeneous Solution

$$RC\,\frac{dv_{CH}}{dt} + v_{CH} = 0 \quad \text{——— (y)}$$

v_{CH}: solution to the homogeneous (y) equation (set drive to zero)

$v_{CH} = Ae^{st}$ assume solution of this form. A, s ?

$$RC\,\frac{dAe^{st}}{dt} + Ae^{st} = 0 \qquad RCAs\cancel{e^{st}} + A\cancel{e^{st}} = 0$$

Discard trivial $A = 0$ solution,

$$RCs + 1 = 0$$ Characteristic equation

$$\Rightarrow \quad s = -\frac{1}{RC}$$

$$\boxed{v_{CH} = Ae^{\frac{-t}{RC}}}$$

RC called time constant τ

Particular Solution

$$RC\,\frac{dv_{CP}}{dt} + v_{CP} = V_I$$

$\boxed{v_{CP} = V_I}$ works

$$RC\,\underbrace{\cancel{\frac{dV_I}{dt}}}_{0} + V_I = V_I$$

In general, use trial and error.

v_{CP} : any solution that satisfies the original equation ⓧ

Total Solution

$$v_C = v_{CP} + v_{CH}$$

$$v_C = V_I + A\, e^{\frac{-t}{RC}}$$

Find remaining unknown from initial conditions:

Given, $v_C = V_0$ at $t = 0$

so, $V_0 = V_I + A$

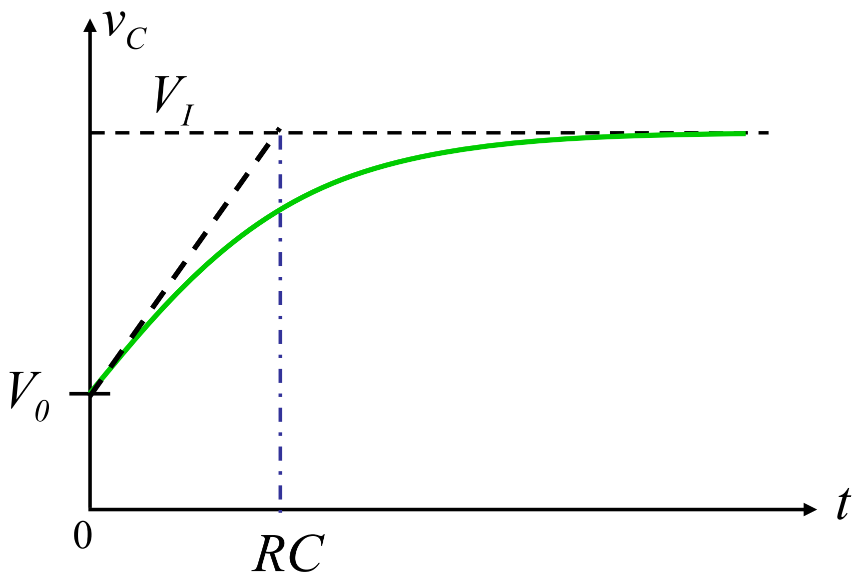
or $A = V_0 - V_I$

thus $$\boxed{v_C = V_I + \left(V_0 - V_I\right) e^{\frac{-t}{RC}}}$$

also $$i_C = C\frac{dv_C}{dt} = -\frac{\left(V_0 - V_I\right)}{R} e^{\frac{-t}{RC}}$$

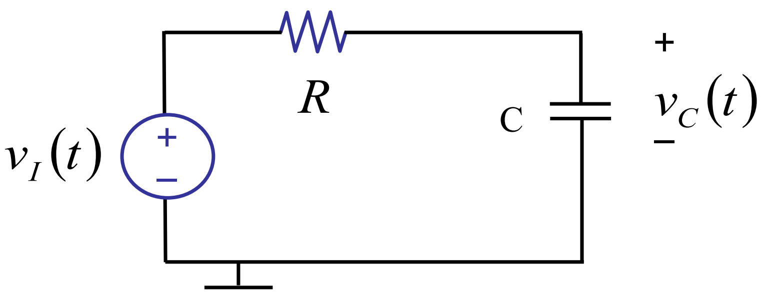
Plot of Total Solution

$$v_C = V_I + \left(V_0 - V_I\right) e^{\frac{-t}{RC}}$$

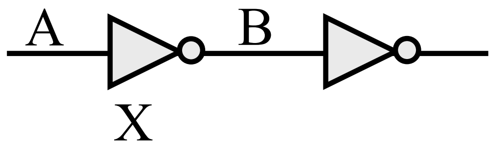


Intuitive Analysis

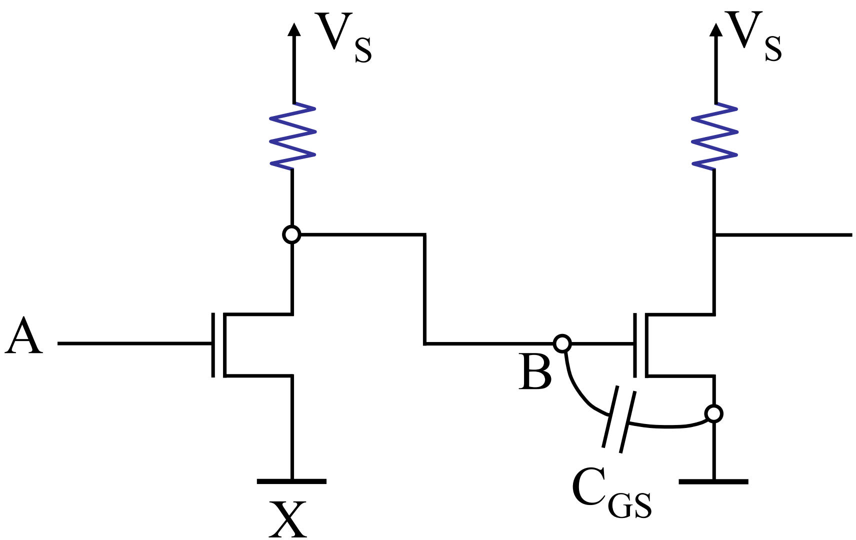
- Response to step input
- $v_C = V_I + (V_0 - V_I)e^{-t/RC} = V_I\left(1 - e^{-t/RC}\right) + V_0 e^{-t/RC}$
- At $t = 0$, $1 - e^{-\frac{t}{RC}} = 0$ and $e^{-t/RC} = 1$
- At $t = \infty$, $1 - e^{-\frac{t}{RC}} = 1$ and $e^{-t/RC} = 0$
- Especially at $t = \infty$, treat capacitor as open circuit

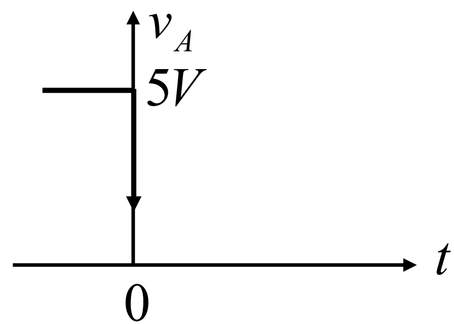


Double Inverter Circuit



First, rising delay t_r at B

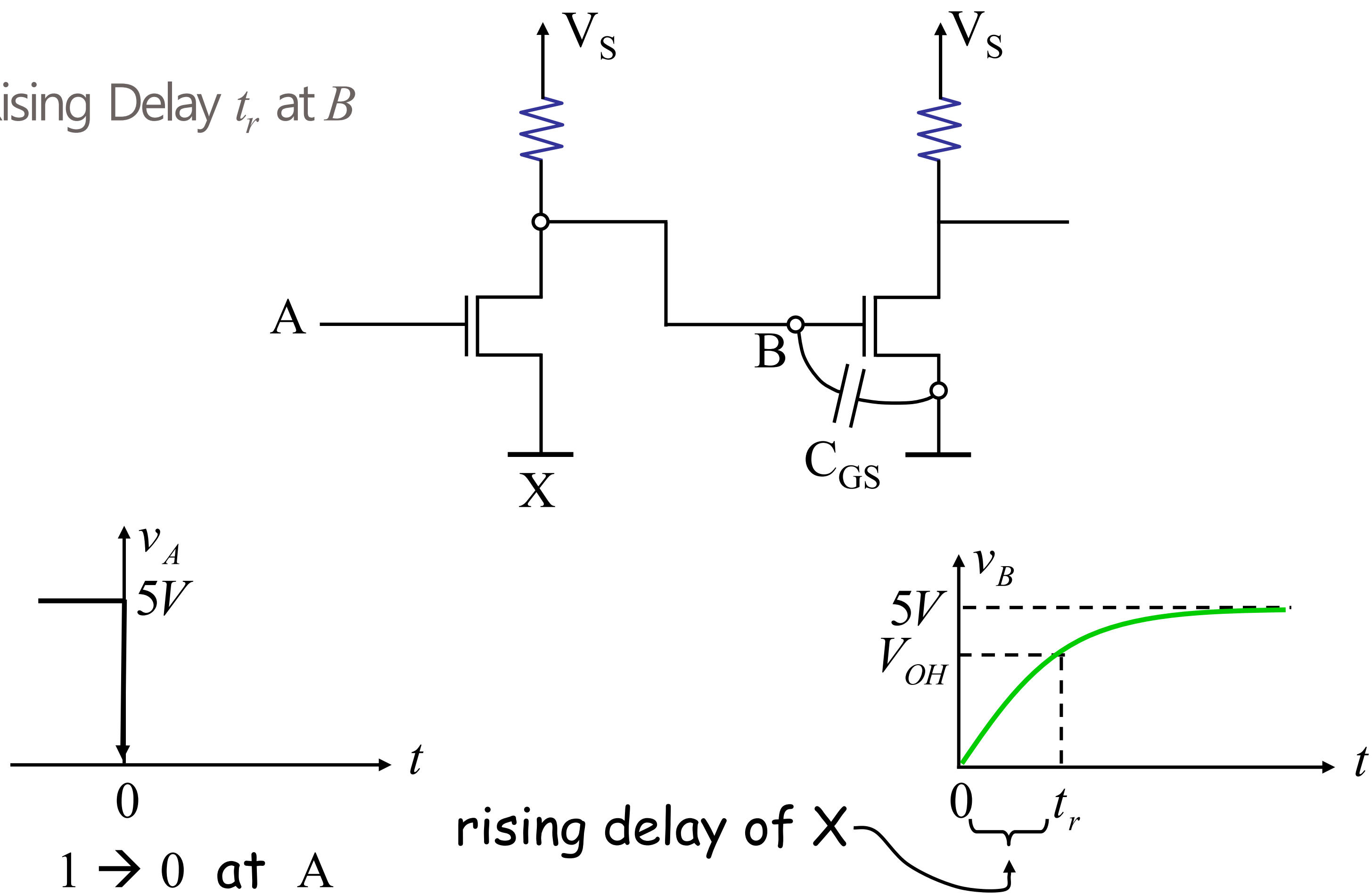




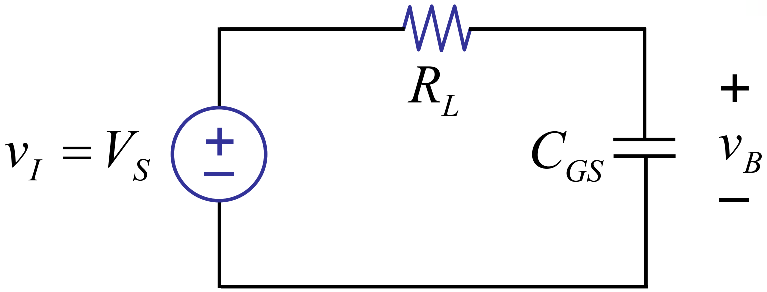
1 → 0 at A

Double Inverter Circuit

Rising Delay t_r at B



Equivalent circuit for 0➔1 at B



$$v_I = V_S$$
$$v_B(0) = 0$$
for $t \geq 0$

$$v_B = V_S + (0 - V_S)\, e^{\frac{-t}{R_L C_{GS}}}$$

Now, we need to find t for which $v_B = V_{OH}$.

$$v_{OH} = V_S - V_S\, e^{\frac{-t}{R_L C_{GS}}}$$

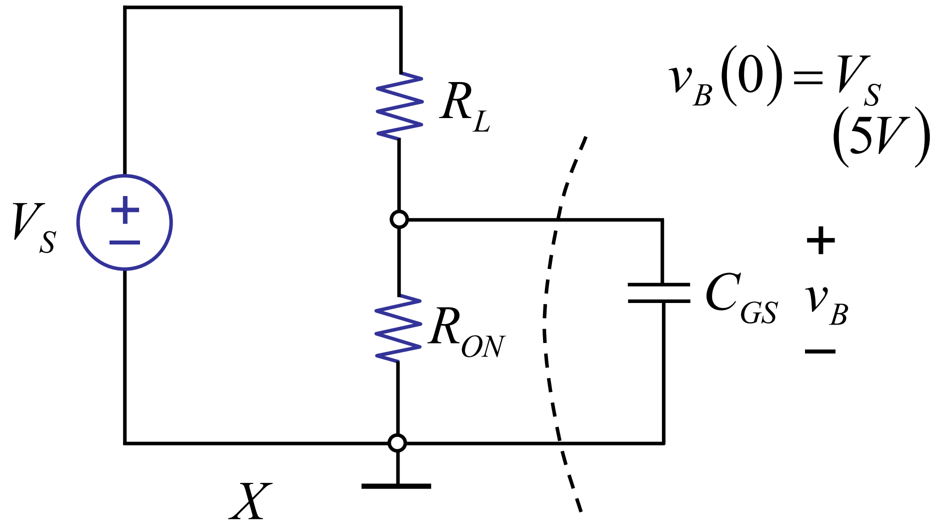
Find t_r :

$$V_S\, e^{\frac{-t_r}{R_L C_{GS}}} = V_S - V_{OH}$$

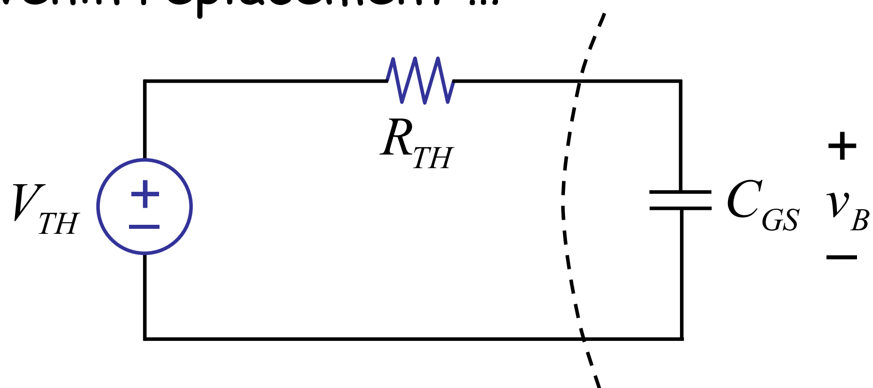
$$t_r = -R_L C_{GS} \ln \frac{V_S - V_{OH}}{V_S}$$

Falling Delay

Equivalent circuit for 1 → 0 at B



Thévenin replacement …



Falling Delay

$$v_B = V_{TH} + \left(V_S - V_{TH}\right) e^{\frac{-t}{R_{TH} C_{GS}}}$$

Falling decay t_f is the t for which v_B falls to V_{OL}

$$V_{OL} = V_{TH} + \left(V_S - V_{TH}\right) e^{\frac{-t_f}{R_{TH} C_{GS}}}$$

or

$$t_f = -R_{TH} C_{GS} \ln \frac{V_{OL} - V_{TH}}{V_S - V_{TH}}$$