Seoul National University

M1522.000900 Data Structure

Fall 2019, Kang

Homework 2: Algorithm Analysis (Chapter 3)

Due: September 25, 11:59 PM

Reminders

- The points of this homework add up to 100.
- Like all homework, this has to be done individually.
- Lead T.A.: Seungcheol Park (ant6si@snu.ac.kr)
- Please type your answers in **English**. Homework written in Korean may get no points.
- **All the homework should be uploaded to the eTL in PDF format.** No handwritten homework (including PDF files made with photos of handwritten papers) will be accepted.
- Your homework should be named as “(studentid)-(yourname)-HW2.pdf” For example, 201912345-GildongHong-HW2.pdf.
- If you have any question about assignments, please upload your question in eTL.
- If you want to use slip-days or consider late submission with penalties, please note that you are allowed one week to submit your assignment after the due date.

Remember that:

- Whenever you are making an assumption, please state it clearly.

Question 1

Rank the following expressions by growth rate from fastest to slowest. [20 points]

$$n! \qquad \log_2 n \qquad 2^{10} \qquad 6\mathrm{n}^2 \qquad \log_2 \log_2 n \qquad 3 \cdot 2^{\mathrm{n}} \qquad 20\mathrm{n}$$

Question 2

For each of the following pairs of functions, determine which relationship of the following relationships is correct: (a) $f(n)$ is in $O\big(g(n)\big)$. (b) $f(n)$ is in $\Omega\big(g(n)\big)$. (c) $f(n)$ is in $\Theta\big(g(n)\big)$ [20 points].

(1) $f(n) = \log n^2$ and $g(n) = \log n + 5$ [5 points]

(2) $f(n) = \sqrt{n}$ and $g(n) = \log n^2$ [5 points]

(3) $f(n) = n$ and $g(n) = \log^2 n$ [5 points]

(4) $f(n) = \log n^2$ and $g(n) = \log^2 n$ [5 points]

Question 3

Answer the following questions [20 points].

(1) What are the time and space complexities of following code [8 points]?

```
Random rand = new Random();

int a=0, b=0;
for (int i=0; i<N; i++) {
    a = a + rand.nextInt();
}
for (int j=0; j<M; j++) {
    b = b + rand.nextInt();
}
```

(2) What is the time complexity of the following code [8 points]?

```
int i, j, k = 0;
for (i = n/2; i <= n; i++) {
    for (j = 2; j <= n; j = j * 2) {
        k = k + n / 2;
    }
}
```

(3) What does it mean when we say that an algorithm X is asymptotically more efficient than Y [4 points]?

(a) X will always be a better choice for small inputs.

(b) X will always be a better choice for large inputs.

(c) Y will always be a better choice for small inputs.

(d) X will always be a better choice for all inputs.

Question 4

Suppose that a particular algorithm has time complexity $\boldsymbol{T}(n) = 3\boldsymbol{T}(n-1) + 2$ in all cases, and that executing an implementation of the algorithm on a particular machine X takes t seconds for n inputs. Answer the following questions.

(1) What is time complexity of this algorithm in asymptotic terms? [10 points]

(2) Given a new machine Y 27 times faster than X, how many inputs could we process in t seconds in Y? [10 points]

Question 5

A power function is a function of the form $f(x) = x^n$. The figures below shows an implementation of the algorithm in Java. Answer the following questions. [20 points]

```
public static long powerN(long x, int n) {

    if (n==0) return 1;
    return x*powerN(x, n-1);
}
```

Figure 1. A power function implementation in Java.

(1) In Figure 1, what will the time complexity of the power function be in asymptotic terms? Express your answer using n. [5 points]

```
public static long powerN(long x, int n) {
    if(n==0) return 1;
    if (n % 2 == 0) {
        long a = powerN(x, n/2);
        return a*a;
    }
    else {
        long a= powerN(x, (n-1)/2);
        return x*a*a;
    }
}
```

Figure 2. A modified power function implementation in Java.

(2) If we change the implementation of the power function from Figure 1 to Figure 2, what will the time complexity be in asymptotic terms? Express your answer using n. [15 points]