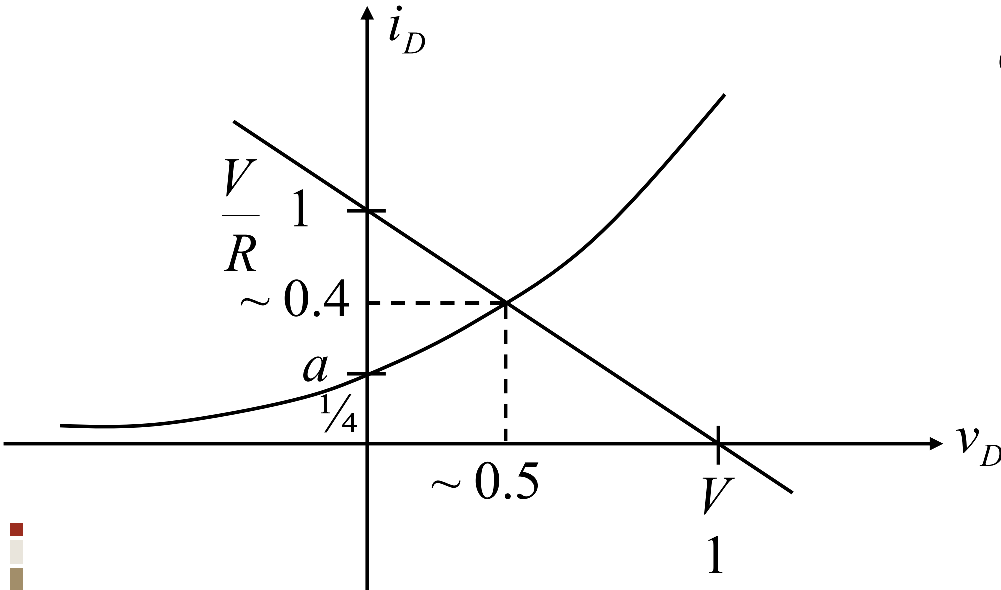
Numerical analysis

- In graphical method, we plotted the two constraints simultaneously.



e.g. $V = 1 \quad v_D = 0.5V$

$R = 1 \quad i_D = 0.4A$

$a = \frac{1}{4}$

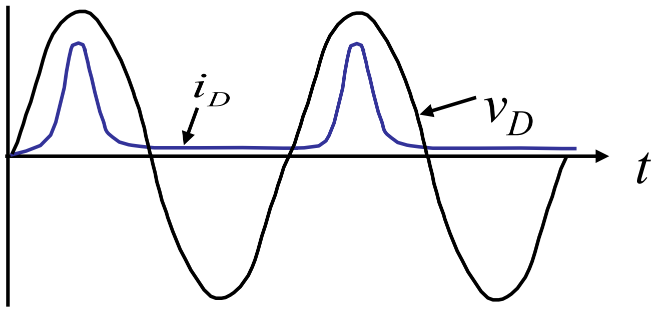
$b = 1$

- In numerical analysis, we should provide the two constraints in functional forms

Numerical analysis

- In numerical analysis, we should provide the two constraints in functional forms
  - Residuals of the eq1 ($i_D = \frac{V}{R} - \frac{v_D}{R}$): $i_D - (V - v_D)/R$
  - Residuals of the eq2 ($i_D = ae^{bv_D}$): $i_D - ae^{bv_D}$
  - We need to define a new function with the following input parameters and return values
    - Input: (v_D, i_D)
    - Output: Residues of $(\mathrm{eq1}, \mathrm{eq2}) = \left(i_D - \frac{V - v_D}{R}, i_D - ae^{bv_D}\right)$
  - Call scipy.optimize.root("new function name", "list of initial guess for (v_D, i_D)")
  - By using the numerical analysis, we can find the correct value should be $v_D = 0.562V$ and $i_D = 0.438A$.
  - Check "Finding numerical solution.ipynb"

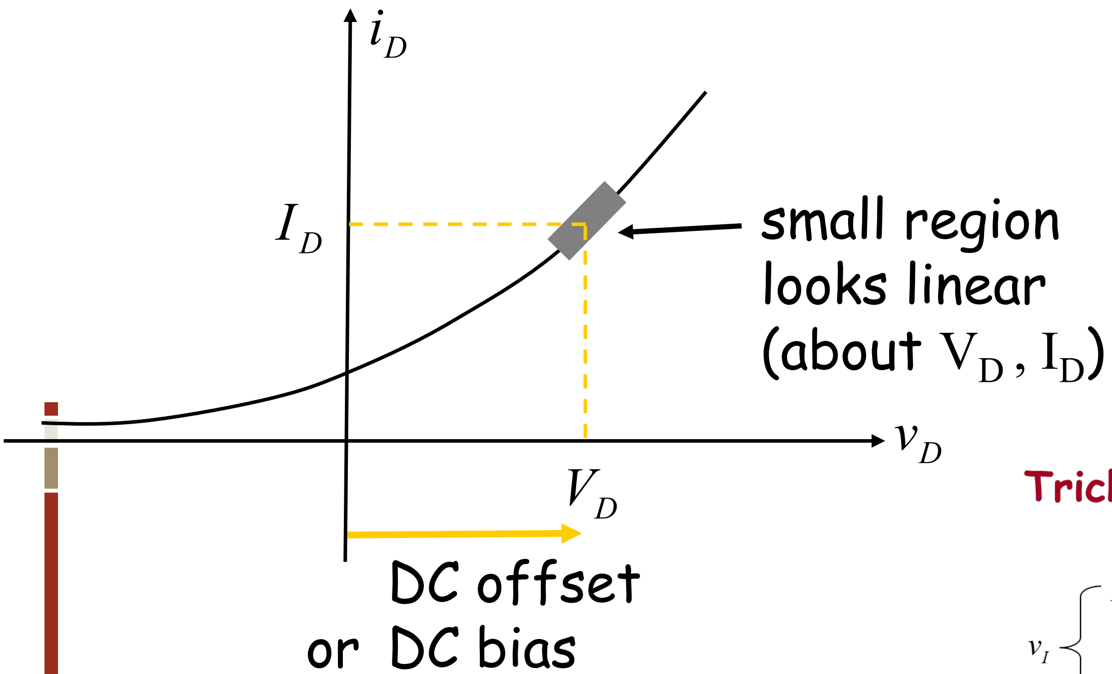
Incremental Analysis



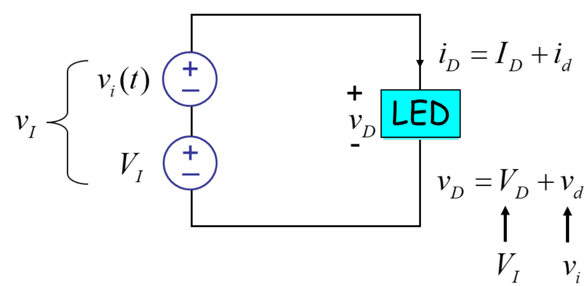
- Demonstration of sound and signal distortion
  - Monotone.ipynb
  - Record sound.ipynb

Linear amplifier from nonlinear element

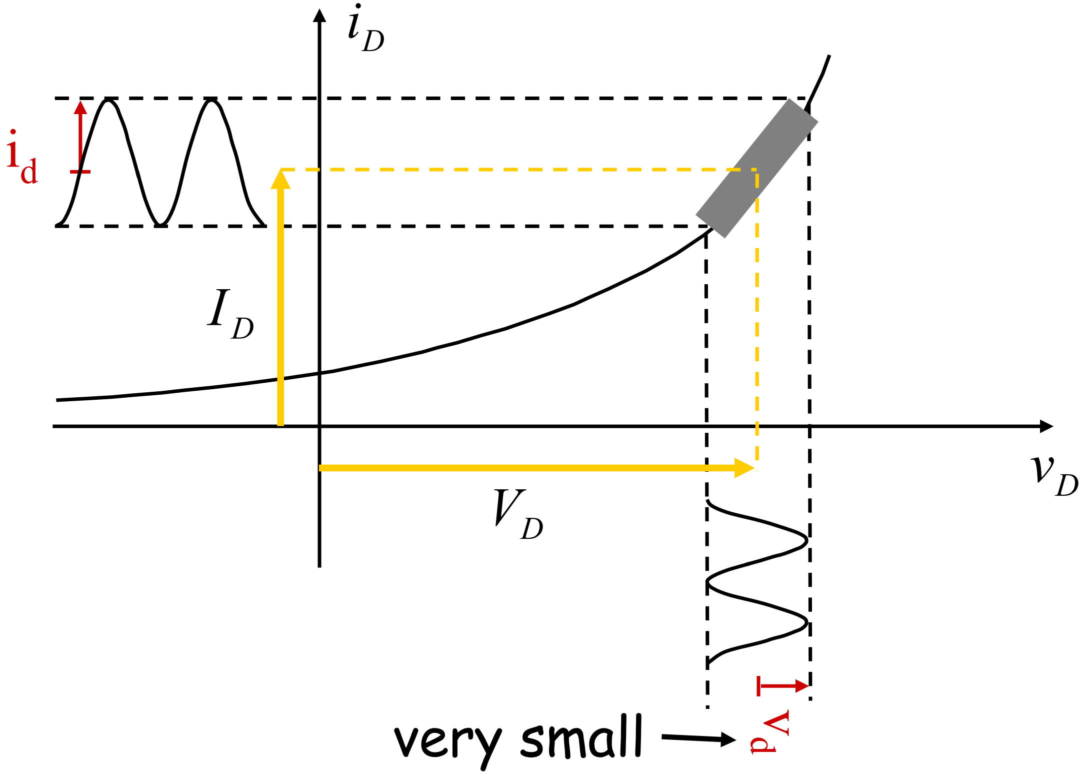
- How can we implement linear amplifier from nonlinear components?



Trick:



Result



Taylor expansion

- Assume that we are given some arbitrary function $f(x)$ of x.
-
- Question: can we approximate this function as a some of polynomials of x? In other words,
$$f(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$
- Answer: it depends on the types of function, but for many of the functions, it is possible.

- Question: the how can we find out $c_0, c_1, c_2, c_3, \ldots$?
- Answer: by matching the values of the derivatives at $x = x_0$.

- In general, $f(x) = f(x_0) + \left.\frac{df}{dx}\right|_{x=x_0} (x - x_0) + \frac{1}{2!}\left.\frac{d^2 f}{dx^2}\right|_{x=x_0} (x - x_0)^2 + \frac{1}{3!}\left.\frac{d^3 f}{dx^3}\right|_{x=x_0} (x - x_0)^3 + \cdots$

- Example 1) at $x = 0$, $f(x) = 1 + 2x + 3x^3 = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$
- Example 2) at $x = 1$, $f(x) = 1 + 2x + 3x^3 = c_0 + c_1(x - 1) + c_2(x - 1)^2 + c_3(x - 1)^3 + \cdots$

Why is the small signal response linear?

- Can we guarantee that the response with respect to the small signal input always linear?

$$i_D = f(v_D)$$

nonlinear

large DC

We replaced

$$v_D = V_D + \Delta v_D$$

v_d

increment about V_D

using Taylor's Expansion to expand $f(v_D)$ near $v_D = V_D$:

$$i_D = f(V_D) + \left.\frac{df(v_D)}{dv_D}\right|_{v_D=V_D} \cdot \Delta v_D$$

$$\cancel{+ \frac{1}{2!}\left.\frac{d^2 f(v_D)}{dv_D^2}\right|_{v_D=V_D} \cdot \Delta v_D^2} + \cdots$$

neglect higher order terms because Δv_D is small

$$i_D \approx \underbrace{f(V_D)}_{\substack{\text{constant} \\ \text{w.r.t. } \Delta v_D}} + \underbrace{\left.\frac{d\,f(v_D)}{dv_D}\right|_{v_D=V_D}}_{\substack{\text{constant w.r.t. } \Delta v_D \\ \text{slope at } V_D,\ I_D}} \cdot \Delta v_D$$

We can write

$$I_D + \Delta i_D \approx f(V_D) + \left.\frac{d\,f(v_D)}{dv_D}\right|_{v_D=V_D} \cdot \Delta v_D$$

equating DC and time-varying parts,

so, $\Delta i_D \propto \Delta v_D$

$$I_D = f(V_D) \longrightarrow \text{operating point}$$

$$\Delta i_D = \underbrace{\left.\frac{d\,f(v_D)}{dv_D}\right|_{v_D=V_D}}_{\text{constant w.r.t. } \Delta v_D} \cdot \Delta v_D$$

By notation,

$\Delta i_D = i_d$

$\Delta v_D = v_d$

Incremental Method (Small Signal Method)

1. Operate at some DC offset or bias point V_D, I_D.
2. Superimpose small signal v_d (music) on top of V_D.
3. Response i_d to small signal v_d is approximately linear.

Notation: $i_D = I_D + i_d$

total variable — DC offset — small superimposed signal

Example

$$i_D = a\, e^{b v_D}$$

$$I_D + i_d \approx a\, e^{bV_D} + a\, e^{bV_D} \cdot b \cdot v_d$$

Equate DC and incremental terms,

$$\boxed{I_D = a\, e^{bV_D}}$$ → operating point

aka bias pt.

aka DC offset

$$i_d = a\, e^{bV_D}\, b \cdot v_d$$

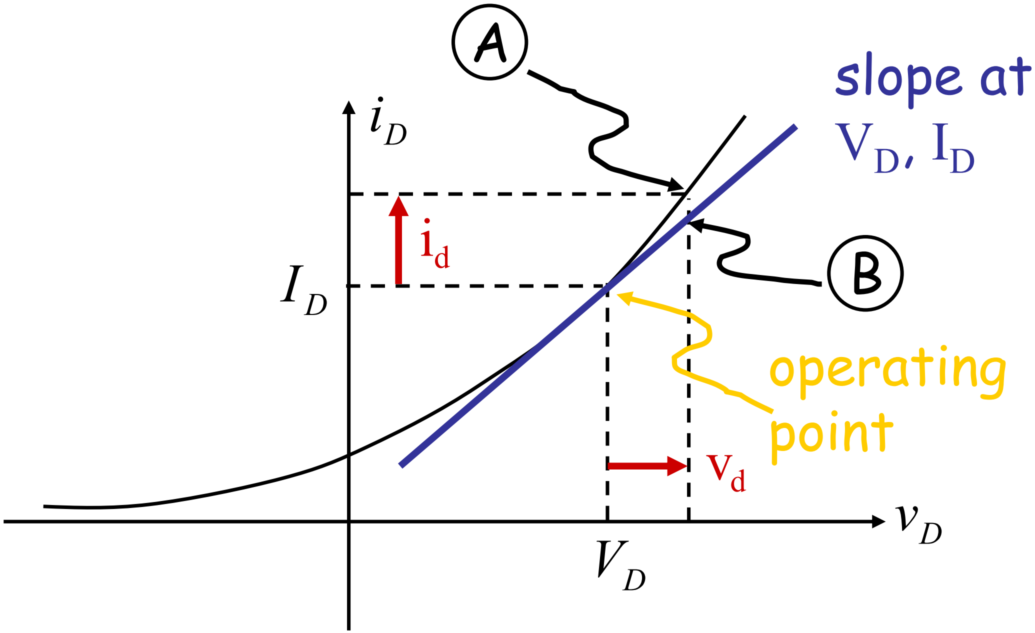
$$i_d = \underbrace{I_D \cdot b}_{\text{constant}} \cdot v_d$$ → small signal behavior

constant → linear!

Graphical Interpretation

$I_D = a\, e^{bV_D}$ ➡ operating point

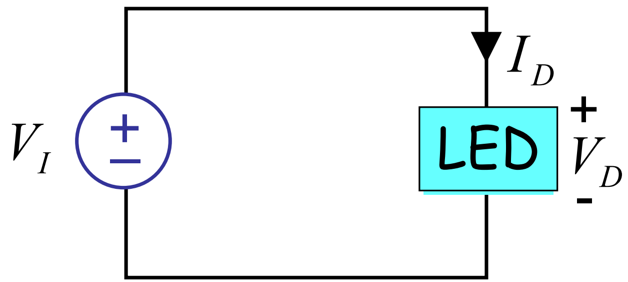
$i_d = I_D \cdot b \cdot v_d$



we are approximating A with B

Combined Together

Large signal circuit:



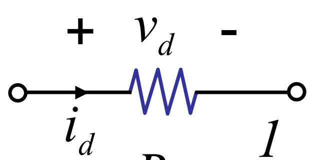
$$I_D = a\, e^{bV_D}$$

By using graphical or analytical solution, find bias point.

Small signal reponse: $i_d = I_D\, b\, v_d$

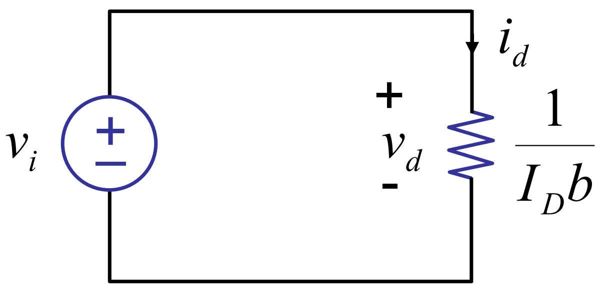
Linearization

behaves like:



$$R = \frac{1}{I_D\, b}$$

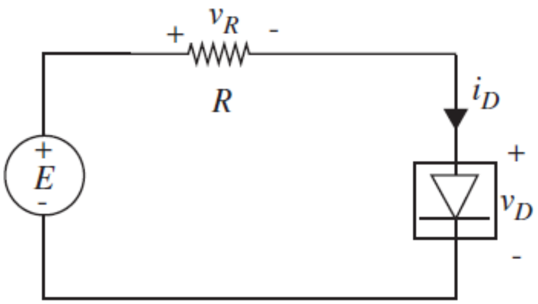
small signal circuit:

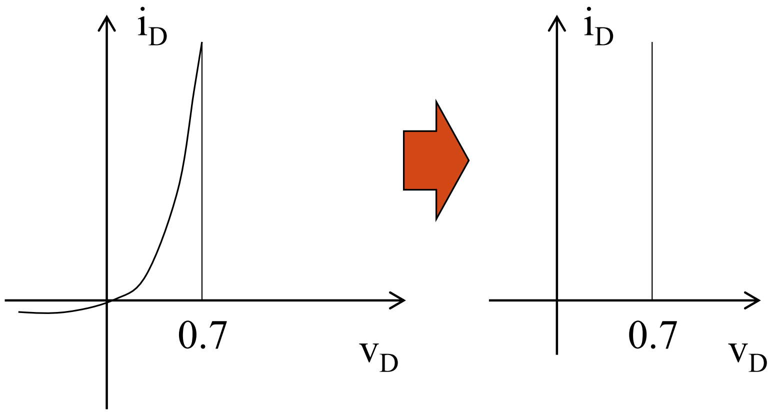


Linear!

Piecewise Linear Analysis

- Diode: $i_D = I_s(e^{v_D/V_{TH}} - 1)$
- Analyze the circuit in two regimes





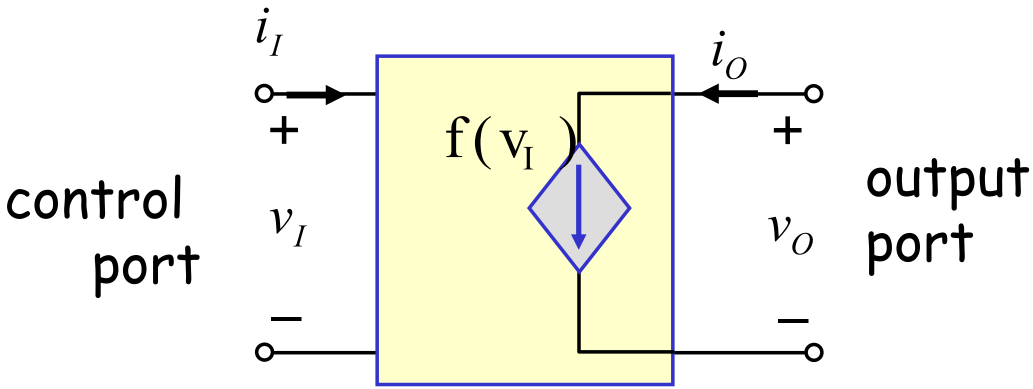
Open circuit case
- When $v_D < 0.7$ V

Close circuit case
- When $v_D > 0.7$ V

Dependent sources

- Section 2.6 in the textbook

New type of device: Dependent source



2-port device

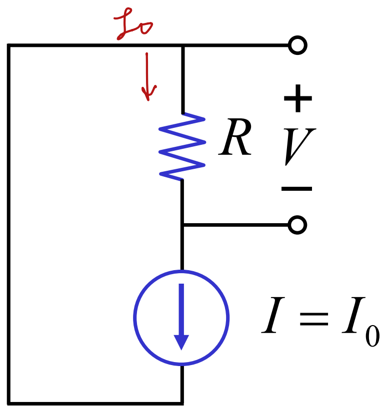
E.g., Voltage Controlled Current Source

Current at output port is a function of voltage at the input port

Independent Source Example

Example 1: Find V

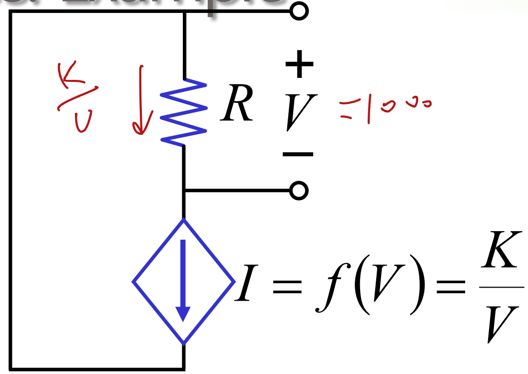
independent
current
source

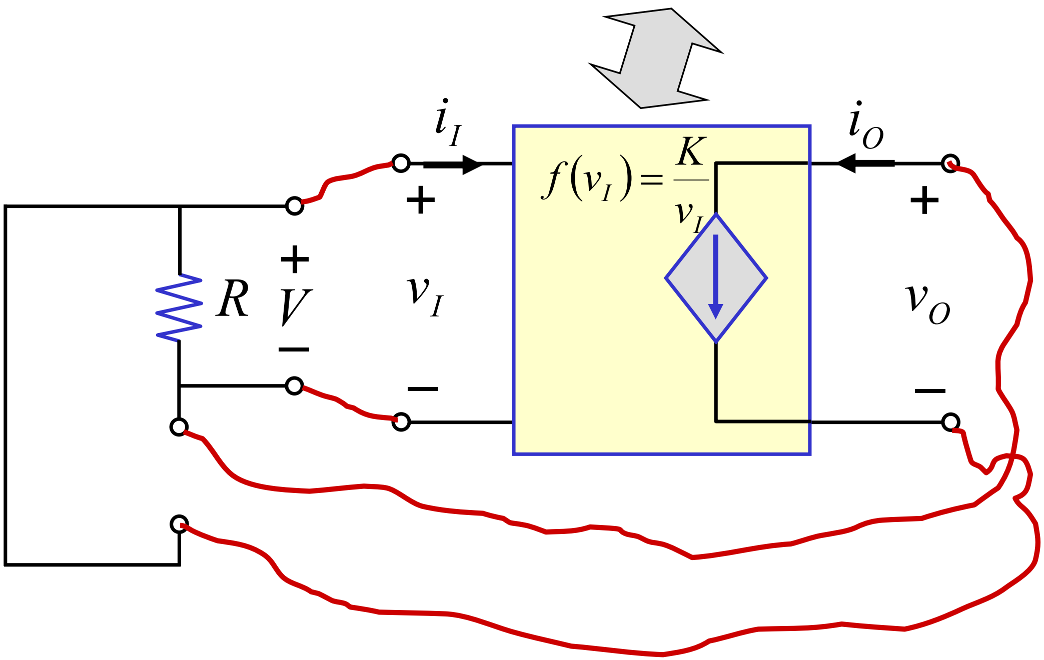


$$V = I_0 R$$

Dependent sources: Example

voltage
controled
current
source

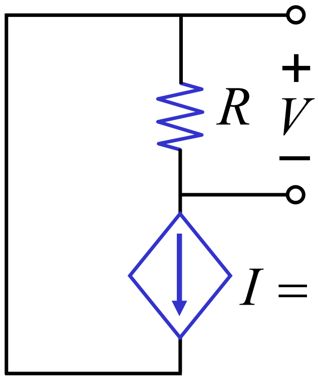




Dependent sources: Example

- Find V

voltage
controlled
current
source



$$I = f(V) = \frac{K}{V}$$

e.g. $K = 10^{-3}$ Amp·Volt
$R = 1\text{k}\Omega$

$$V = IR = \frac{K}{V}R$$

or $V^2 = KR$

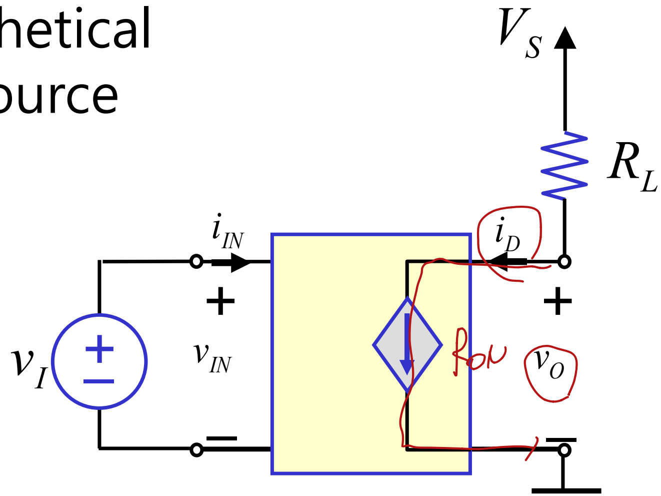
or $V = \sqrt{KR}$

$$= \sqrt{10^{-3} \cdot 10^{3}}$$

$$= 1\,Volt$$

Another Dependent Source Example

- Assume there exists a hypothetical voltage-controlled current source



$$i_D = f(v_{IN})$$

e.g. $i_D = f(v_{IN})$

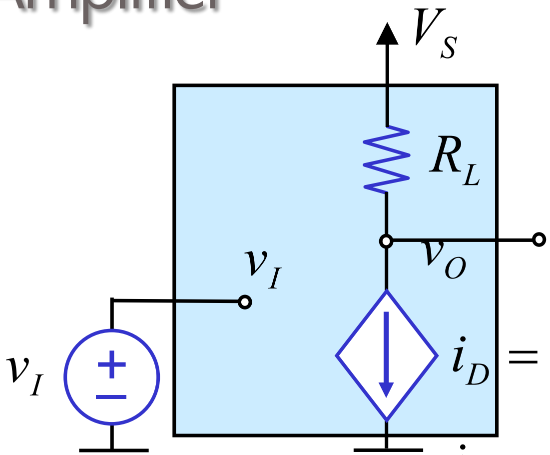
$$= \frac{K}{2}(v_{IN} - 1)^2 \quad \text{for } v_{IN} \geq 1$$

$$i_D = 0 \qquad \text{otherwise}$$

Find v_O as a function of v_I.

$= i_D \cdot R_{oN}$

Amplifier



$$i_D = \frac{K}{2}(v_{IN} - 1)^2 \quad \text{for } v_{IN} \geq 1$$

$$i_D = 0 \quad \text{otherwise}$$

$$v_O = V_S - \frac{K}{2}(v_I - 1)^2 R_L \quad \text{for } v_I \geq 1$$

$$v_O = V_S \quad \text{for } v_I < 1$$

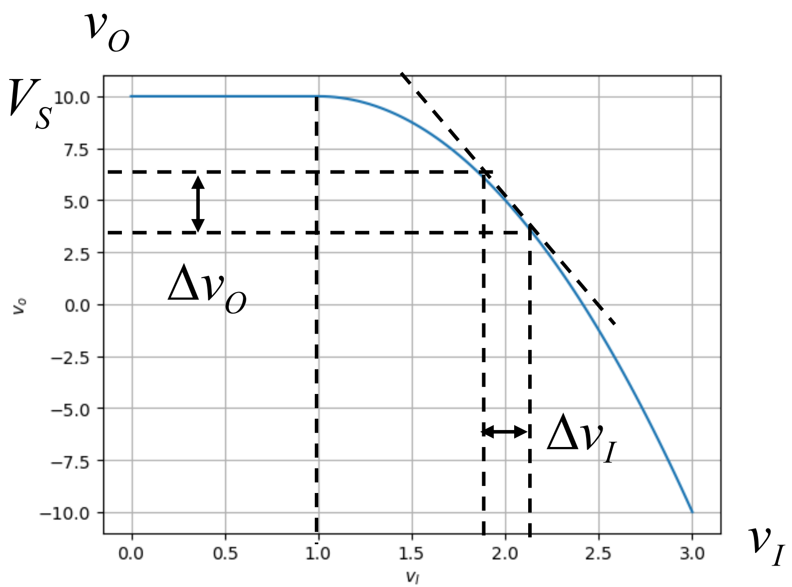
What is the gain of this amplifier?

Let's look at the v_O versus v_I curve.

e.g. $V_S = 10V, \quad K = 2\dfrac{mA}{V^2}, \quad R_L = 5k\Omega$

$$v_O = V_S - \frac{K}{2} R_L \left(v_I - 1\right)^2$$

$$v_O = 10 - 5\left(v_I - 1\right)^2$$

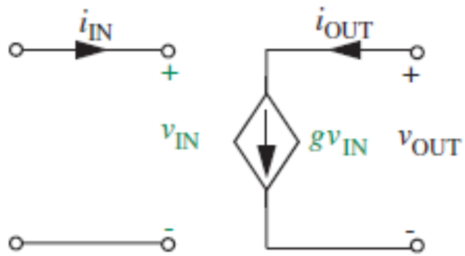


$$\frac{\Delta v_O}{\Delta v_I} > 1 \longrightarrow \text{amplification}$$

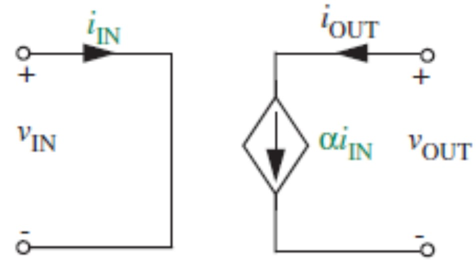
Amplification $\dfrac{\Delta v_O}{\Delta v_I} = -10$

around $v_I = 2V$.

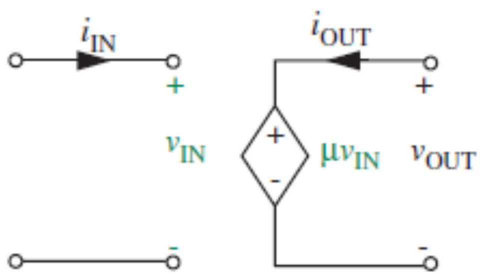
Other Types of Controlled Sources



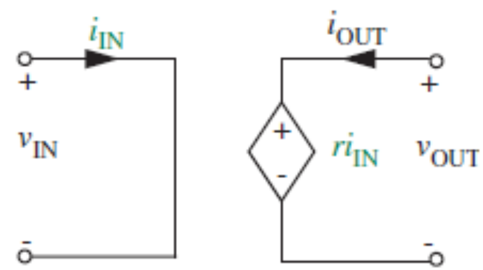
VCCS (voltage-controlled current source)



CCCS (current-controlled current source)



VCVS (voltage-controlled voltage source)



CCVS (current-controlled voltage source)