Homework 4

M1522.000900 Data Structure (2019 Fall)

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**Question 1.**

For a binary tree that has n leaves, the number of 2-degree nodes be .

In the tree that has k nodes, let the number of leaves be and of 2-degree nodes be .

Let the proposition for the binary tree that has nodes, .

Proof.

[Base case] At and .

, thus, holds for .

[Inductive Steps] Assume that holds for .

When tree grows by 1 node, there are 3 cases by the degree.

(Case 1) The new node cannot be attached to existing node if it is 2-degree node.

(Case 2) The new node can be attached to existing node if it is 1-degree node.

In this case, this node becomes 2-degree node by new node.

(Case 3) The new node can be attached to existing node if it is leaf.

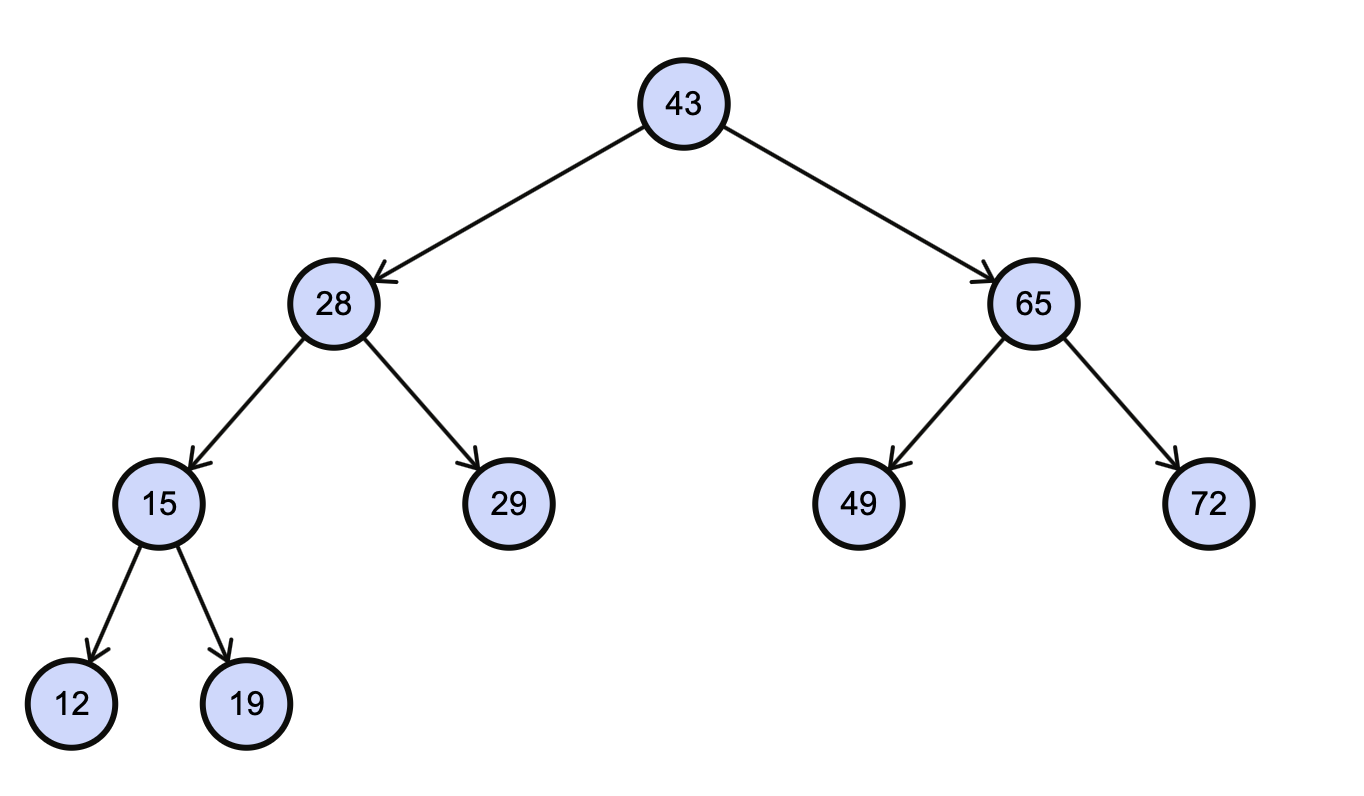
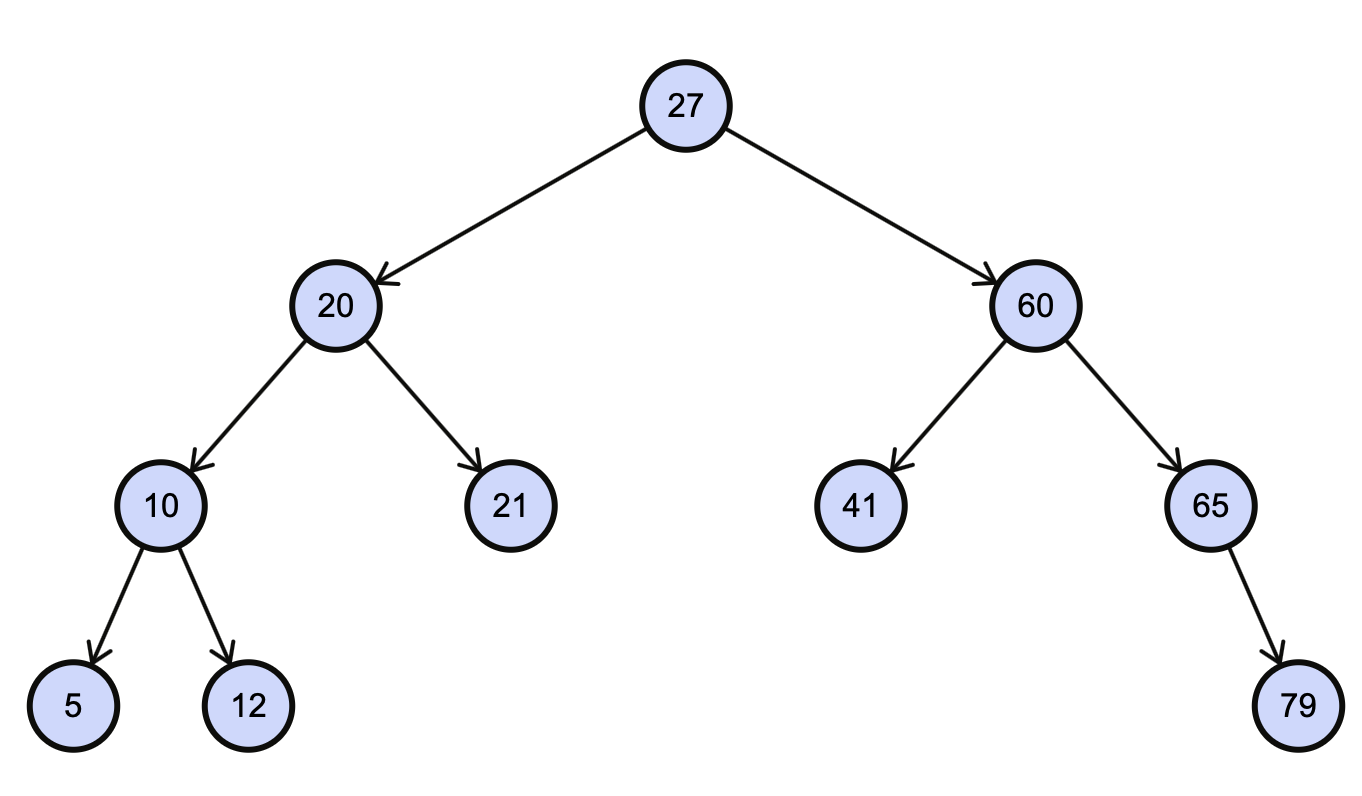
In this case, there is no change in and from .

In all cases, .

∴ holds for .

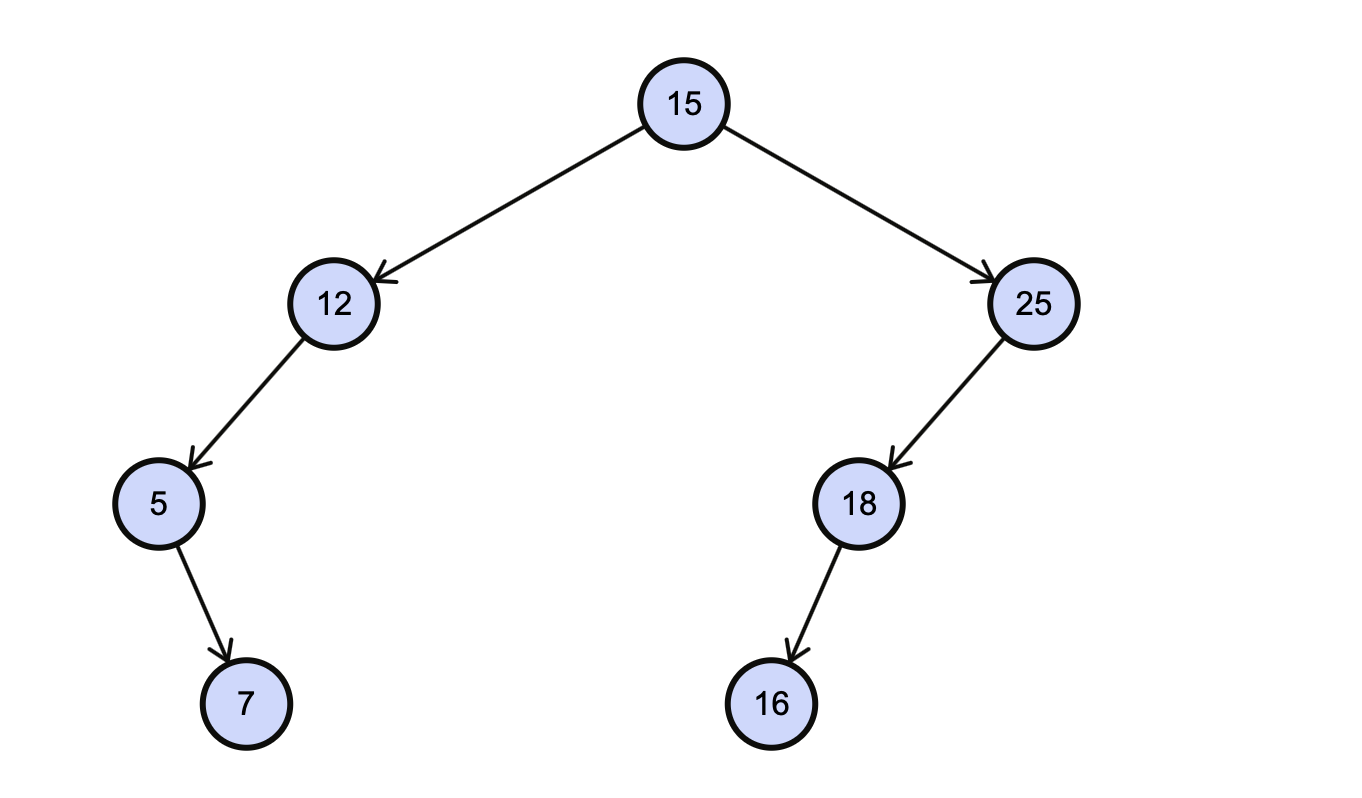
**QED**.

**Question 2.**

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**(a) (b)**

**Question 3.**

(a) 

(b) A. 15-12-5-7-25-18-16

B. 5-7-12-15-16-18-25

C. 7-5-12-16-18-25-15

**Question 4.**

(1) (c), (d)

(2)

**Question 5.**

Let be .

And let a perfect binary tree of height h be .

Proof. The sum of depth of each node in is .

[Base case] In , sum of depth of each node is .

holds for .

[Inductive Steps] Assume that holds for, then, sum of depth of each node is .

When grows to , new leaves are attached at existing leaves.  
This procedure is like below,

1) has leaves from its definition.  
2) Each leaves in became ’s nodes that have 2 children.

3) ’s leaves are 2 times of ’s leaves.

From these, sum of depth of each node in is

**∴ S(h) also holds for h=(n+1).**

**QED.**