eigenvector / characteristic vector

a vector that has its direction unchanged by a given linear transformation.

$$\underbrace{T}_{\text{linear transformation}}\ \underbrace{v}_{\text{vector}} = \lambda v$$

λ → scaled value / eigenvalue

$$Tv = \lambda I v \rightarrow Tv - \lambda I v = 0 \Rightarrow \underline{(T - \lambda I) v = \vec{0}}$$

- $\underline{A \vec{x} = \vec{0}}$ one solution: $\vec{x}$ is $\vec{0}$

  or A must be singular matrix

  example:

  $$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{cases} x_1 + 2x_2 = 0 \\ 2x_1 + 4x_2 = 0 \end{cases} \rightarrow \dots\ x_1 = -2x_2$$

$(T - \lambda I)\vec{v} = \vec{0}$, if exists non-zero $\vec{v}$

$$\det(T - \lambda I) = 0$$

~~Hypothesis~~

Theorem: for singular $T - \lambda I$, we have n eigenvalues where T is a $n \times n$ matrix. And we have infinite numbers of eigenvectors.

proof: $\det(T - \lambda I)$ is a polynomial of degree n.

it has n roots (roots can be complex and can be equal with each other)

for any root λ_i, we have

$$\underbrace{(T - \lambda_i I)}_{\text{singularity}} v = 0$$

So v does not has only one solution, but infinite

or we can do this: $\underbrace{c}_{\text{constant}} (T - \lambda_i I) v = 0$

→ $v \rightarrow cv$