[SAD2] Gibbs sampler

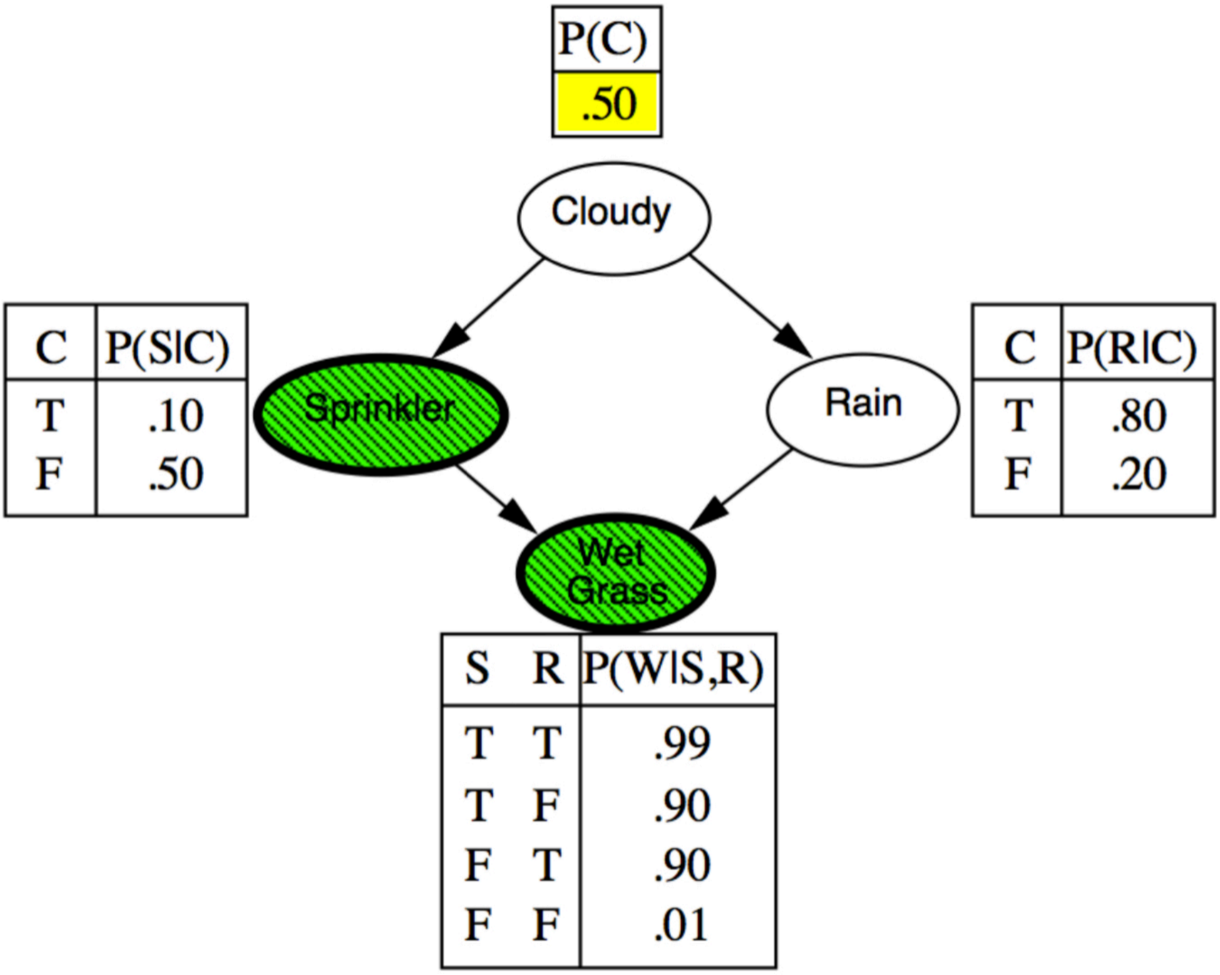

Figure 1. Bayesian Rain network

1.

$$P(C=T \mid R=T, S=T, W=T) = P(C=T \mid R=T, S=T) =$$

$$= \frac{P(C=T)P(R=T \mid C=T)P(S=T \mid C=T)}{P(C=T)P(R=T \mid C=T)P(S=T \mid C=T) + P(C=F)P(R=T \mid C=F)P(S=T \mid C=F)} =$$

$$= \frac{0{,}5 \cdot 0{,}8 \cdot 0{,}1}{0{,}5 \cdot 0{,}8 \cdot 0{,}1 + 0{,}5 \cdot 0{,}2 \cdot 0{,}5} = \frac{0{,}04}{0{,}04 + 0{,}05} = \frac{4}{9} \approx 0{,}444$$

$$P(C=T \mid R=F, S=T, W=T) = P(C=T \mid R=F, S=T) =$$

$$= \frac{P(C=T)P(R=F \mid C=T)P(S=T \mid C=T)}{P(C=T)P(R=F \mid C=T)P(S=T \mid C=T) + P(C=F)P(R=F \mid C=F)P(S=T \mid C=F)} =$$

$$= \frac{0{,}5 \cdot 0{,}2 \cdot 0{,}1}{0{,}5 \cdot 0{,}2 \cdot 0{,}1 + 0{,}5 \cdot 0{,}8 \cdot 0{,}5} = \frac{0{,}01}{0{,}01 + 0{,}2} = \frac{1}{21} \approx 0{,}048$$

Cloudy given Rain

Cloudy	Rain	Probability
T	T	0,444
T	F	0,048
F	T	0,556
F	F	0,952

$$P(R=T \mid C=T, S=T, W=T) = \frac{P(R=T \mid C=T)P(W=T \mid R=T, S=T)}{P(R=T \mid C=T)P(W=T \mid R=T, S=T) + P(R=F \mid C=T)P(W=T \mid R=F, S=T)} =$$

$$= \frac{0{,}8 \cdot 0{,}99}{0{,}8 \cdot 0{,}99 + 0{,}2 \cdot 0{,}9} = \frac{0{,}792}{0{,}972} \approx 0{,}815$$

$$P(R=T \mid C=F, S=T, W=T) = \frac{P(R=T \mid C=F)P(W=T \mid R=T, S=T)}{P(R=T \mid C=F)P(W=T \mid R=T, S=T) + P(R=F \mid C=F)P(W=T \mid R=F, S=T)} =$$

$$= \frac{0{,}2 \cdot 0{,}99}{0{,}2 \cdot 0{,}99 + 0{,}8 \cdot 0{,}9} = \frac{0{,}198}{0{,}918} \approx 0{,}216$$

Rain given Cloudy

Rain	Cloudy	Probability
T	T	0,815
T	F	0,216
F	T	0,185
F	F	0,784

2.

Code blocks commented with **# 2.**

3.

Code blocks commented with **# 3.**

P(R = T | S = T, W = T) = 0.39

4.

Code blocks commented with **# 4.**

P(R = T | S = T, W = T) = 0.32448

5.

Code blocks commented with **# 5.**

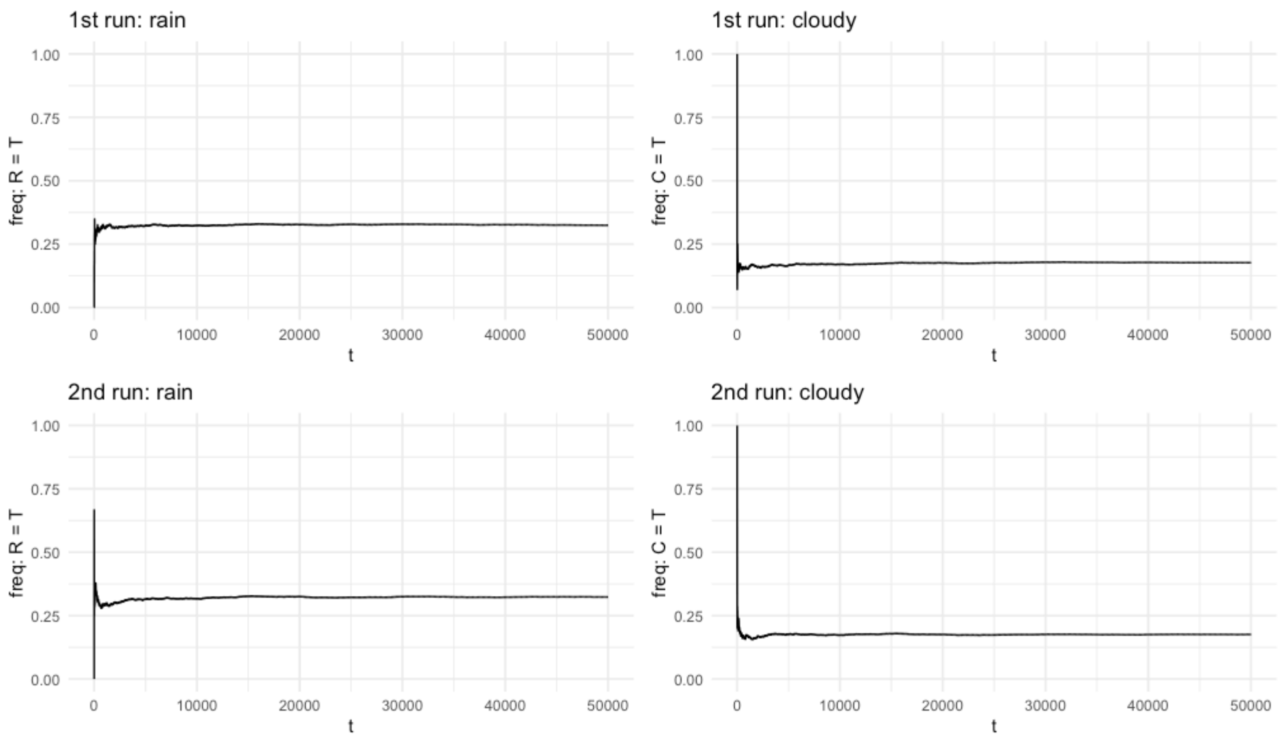

Figure 2. Relative frequencies plots

Based on these plots, I'd suggest *burn-in* time = 7 500

6.

Code blocks commented with **# 6.**

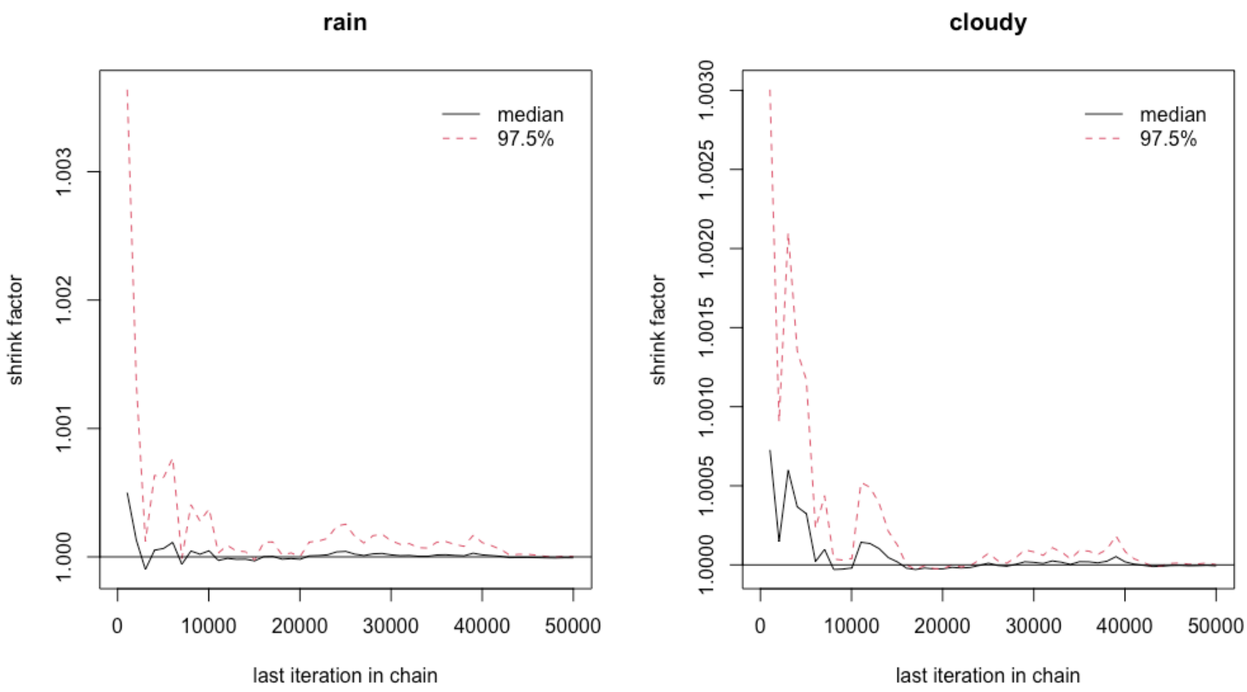

Figure 3. Gelman plots

Based on Gelman plots, I'd suggest *burn-in* time = 7 500

7.

Code blocks commented with **# 7.**

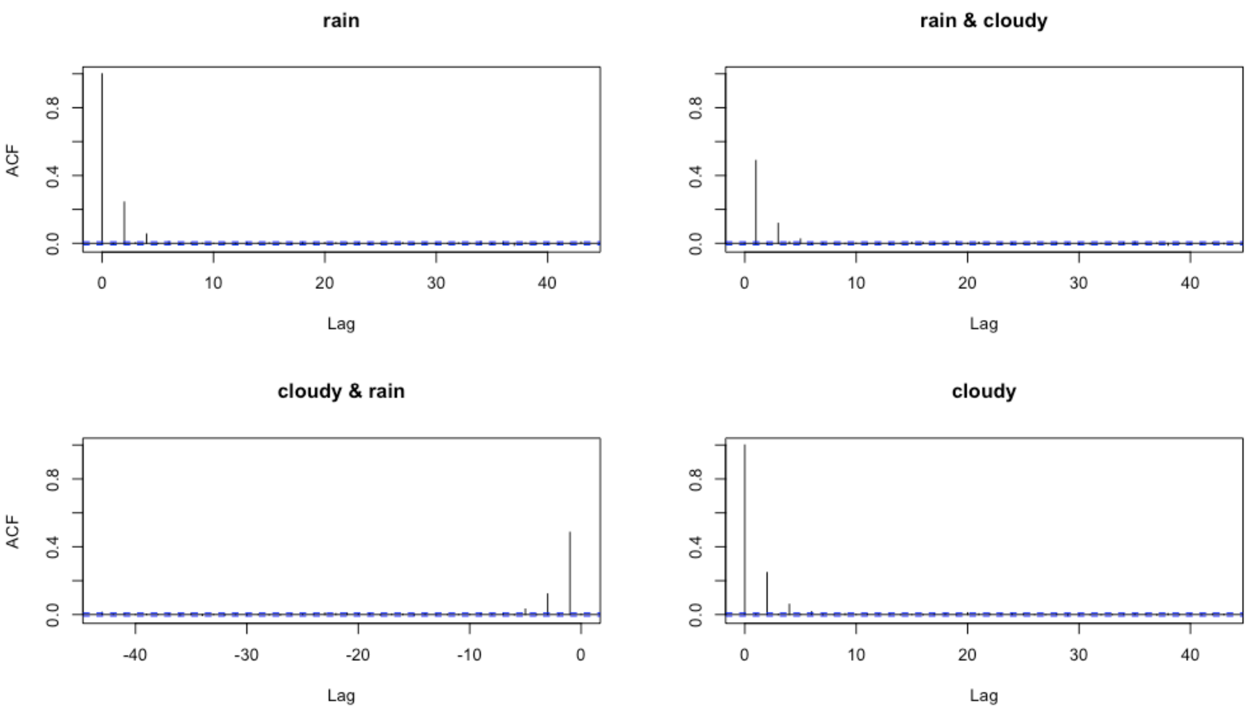

Figure 4. Auto-correlation plots, run #1

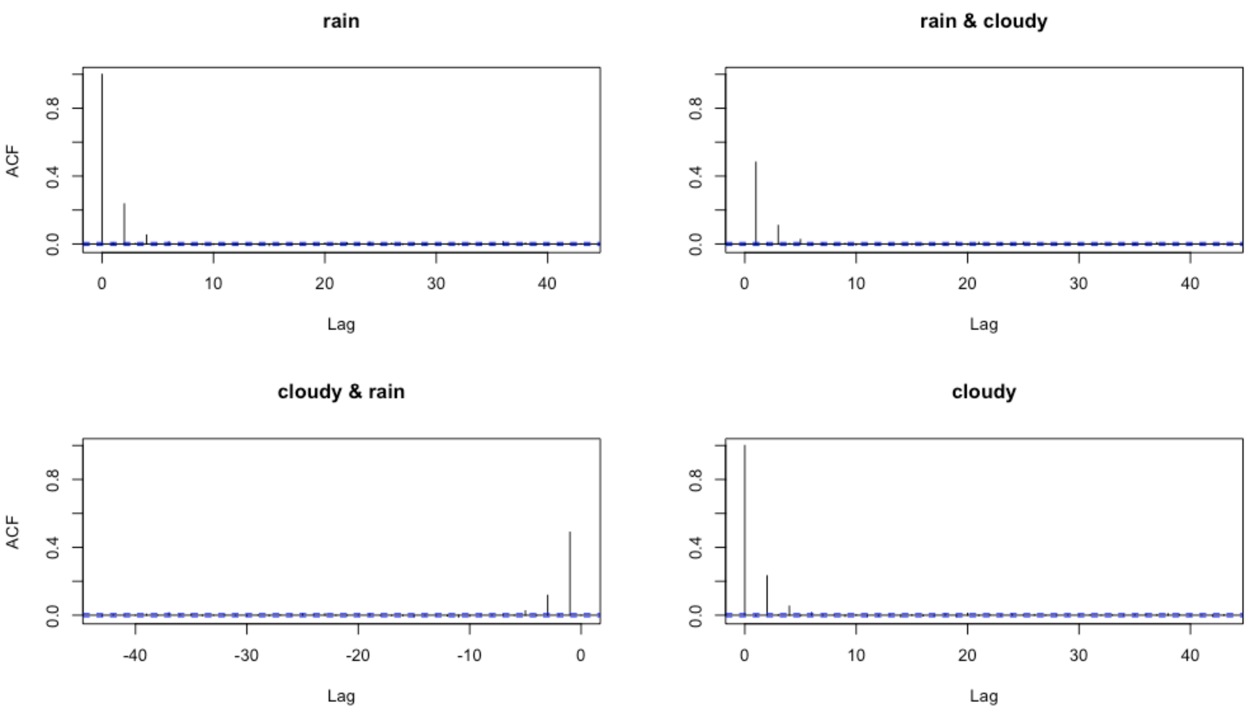

Figure 5. Auto-correlation plots, run #2

Based on auto-correlation plots, I'd suggest *interval* = 3

8.

Code blocks commented with **# 8.**

Sampling with:
burn-in = 7 500
thin-out = 3

P(R = T | S = T, W = T) = 0.32

Comparing to P from **3.** (100 samples), it's much lower, closer to P from **4.** (50 000 samples). But such small sample highly depends on randomness, thus should not be trusted that much. After drawing few hundreds times 100 samples from Gibbs sampler with and without burned-in, thinned-out enhancements, mean probability values were very similar, but with burned-in and thinned-out often closer to 0.32 (which is analytically right answer for this Bayesian network), with also significantly lower variances.

	mean	**variance**
Gibbs sampler	0.32622	0.003706525
Gibbs sampler burn-in = 7500, thin-out = 3	0.31912	0.001882591

Table 1. Probability mean and variance after 500x drawing 100 samples from specific sampler

9.

$$P(R=T \mid S=T, W=T) = \frac{P(R=T, S=T, W=T)}{P(S=T, W=T)} = \frac{A}{B} = \frac{0{,}1782}{0{,}5562} \approx 0{,}320$$

$$A = P(R=T \mid C=T)P(S=T \mid C=T)P(W=T \mid S=T, R=T) + P(R=T \mid C=F)P(S=T \mid C=F)P(W=T \mid S=T, R=T) = \begin{bmatrix} 0{,}8 \cdot 0{,}1 \cdot 0{,}99 \\ 0{,}2 \cdot 0{,}5 \cdot 0{,}99 \end{bmatrix}$$

$$= 0{,}1782$$

$$B = P(R=T, S=T, W=T) + P(R=F \mid C=T)P(S=T \mid C=T)P(W=T \mid S=T, R=F) + P(R=F \mid C=F)P(S=T \mid C=F)P(W=T \mid S=T, R=F) = \begin{bmatrix} 0{,}1782 \\ 0{,}2 \cdot 0{,}1 \cdot 0{,}9 \\ 0{,}8 \cdot 0{,}5 \cdot 0{,}9 \end{bmatrix}$$

$$= 0{,}5562$$