A Weighted Spectrum Metric for Comparison of Internet Topologies

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ABSTRACT

Comparison of graph structures is a frequently encountered problem across a number of problem domains. Comparing graphs requires a metric to discriminate which features of the graphs are considered important. The spectrum of a graph is often claimed to contain all the information within a graph, but the raw spectrum contains too much information to be directly used as a useful metric. In this paper we introduce a metric, the *weighted spectral distribution*, that improves on the raw spectrum by discounting those eigenvalues believed to be unimportant and emphasizing the contribution of those believed to be important.

We use this metric to optimize the selection of parameter values for generating Internet topologies. Our metric leads to parameter choices that appear sensible given prior knowledge of the problem domain: the resulting choices are close to the default values of the topology generators and, in the case of some generators, fall within the expected region. This metric provides a means for meaningfully optimizing parameter selection when generating topologies intended to share structure with, but not match exactly, measured graphs.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Network topology; I.6.4 [**Simulation and Modeling**]: Model Validation and Analysis

General Terms

Topology, Performance, Measurement

Keywords

Internet topology, Topology generation, Degree-based generators, graph metrics

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1. INTRODUCTION

In this paper we present a metric, *weighted spectral distribution*, for comparing graphs based on the distribution of their internal structure. Graph comparison is a problem that occurs in many branches of computing, from vision to speech processing to systems. The metric we present differs from existing graph-matching techniques which seek to identify graphs which share common *clusters*, i.e., are similar. Instead, our metric is designed specifically for situations where the graphs being compared are in general dis-similar but can be expected to share, in some sense, a common *structure*. For example, when generating synthetic workloads from trace data described as a graph, the generated graphs should *not* match the original trace data exactly but *should* share some common structure with them. Situations where this is encountered include workload generation, e.g., as in Magpie [2], and Internet topology generation.

We specifically focus on the latter problem domain in this paper, addressing the problem of generating synthetic topologies designed to mimic the structure of the Internet. The Internet topology's structure is not easy to characterize. In the core there is a full mesh formed between various tier-1 Internet Service Providers (ISPs). However, at the edges there are a huge number of smaller ISPs and customer networks which connect through upstream providers. These smaller ISPs and customer networks may have only one upstream provider, or may have many for resilience and performance reasons. This rich and varied structure makes it difficult for researchers to provide a single model, and hence a single metric, that captures all the characteristics of various topologies. Many attempts to capture one or even several characteristics have been made, resulting in several topology generators which each synthesize Internet-like topologies using different models and parameters. Unfortunately, little or no guidance is available on how to set these parameters, with the default values subjectively chosen by the original authors usually being used.

Using our metric, we compare five different topology generators and a measurement of the existing Internet's AS (Autonomous System) topology. Empirical evidence from this comparison shows that the weighted spectral distribution is consistent with expected results. Using our metric we

are also able to give optimum parameter settings for these topology generators with respect to the measured AS data and the weighted spectral distribution.

In summary, we present three contributions in this paper: (*i*) a metric for comparing the structure of graphs that reveals important characteristics such as in what way two graphs differ; (*ii*) a comparison of the outputs of five major Internet topology generators and a measured dataset; (*iii*) optimal parameterizations under our metric of these topology generators with respect to the measured dataset.

2. BACKGROUND

Graph analysis is typically concerned with determining the relationships among the vertices of a graph, and its various applications can be broadly classified as topology tuning, graph matching and cluster determination, examples of which will now be discussed.

Topology tuning is the problem domain we are concerned with here, namely adjusting parameters used to generate topologies with the aim of generating topologies "close enough" to some representative. Graph spectra have not been used for this purpose before, although Hanna [7] uses graph spectra for numerical comparison of architectural spaces in large building plans. By defining space as a graph, he shows that the spectra of two plan types can be effectively used to judge the effects of global *vs.* local changes, and hence the edit distances, to the plans. Hanna believes spectra are a reliable metric for capturing the local relationships and can be used to guide optimization algorithms for reproducing plans.

Alternatively, graph matching is concerned with the comparison of two or more graphs to determine which clusters in the graphs are related. For example, Luo and Hancock [11] compare several images of an object taken at different angles and determine the correspondence between them by representing the images as graphs. Their technique seeks to maximize the likelihood that groups of vertices in one graph correspond to those in another. However, the aim of this work differs from ours as it seeks to identify the edges between specific nodes and clusters.

Techniques for cluster identification differ mainly in the choice of matrix associated with a graph. Ng *et al.* [13] present an algorithm for determining the dominant clusters in a graph by examining the eigenpairs of the normalized graph Laplacian L. They suggest that a graph with n-vertices may be represented in $k < n$-dimensions by choosing the first k eigenvectors as ordered by the k smallest eigenvalues. The value of k is determined by examining the eigenvalues of L, denoted by $\lambda_i, i = 0, \ldots, n-1$. The eigenvector associated with the first (non-zero) eigenvalue determines the largest clusters in the graph with subsequent eigenvectors determining finer subclusters. If there exists an eigenvalues λ_i, for which $\text{abs}(1 - \lambda_{i+1}) >> \text{abs}(1 - \lambda_i)$ then λ_{i+1} to λ_{n-1} are clustered around 1, and this is known as the spectral gap. The spectral gap represents eigenvalues which have little power and can be thought of as representing noise or small variations in the graph structure. Typically, the value of k is determined by examination of a plot of the eigenvalues and set by the spectral gap. The $n \times k$ matrix of eigenvectors is then clustered using the k-means clustering algorithm, although variations using other clustering algorithms exist.

Graph spectra have also been used for characterization of Internet topologies. Gkantsidis *et al.* [5] perform a comparison of clustering coefficients by using the eigenvectors of the k largest eigenvalues of adjacency matrices on BGP [14] topology graphs. However, the choice of k is arbitrary and those chosen are given equal importance. They consider the rest of the spectrum as noise, despite works that have shown that the eigenvalues of the adjacency matrix or the normalized Laplacian matrix can be used to accurately represent a topology and some specific eigenvalues indicate properties such as robustness of a network to failures [16, 10].

Vukadinovic *et al.* [18] use the normalized Laplacian spectrum for analysis of AS graphs, proposing it as a fingerprint for Internet-like graphs. Using the Inet [19] generator and AS graphs extracted from BGP data, they obtain eigenvalues of the normalized Laplacian matrix. They believe that the graph spectrum should be considered as an essential metric when comparing graphs. We expand on this work by demonstrating how an appropriate weighting of the eigenvalues can be used to reveal the structural differences between two topologies.

We now present our metric, the weighted spectral distribution, before using it to compare synthetic and measured topologies, and to optimize parameter selection for the topology generators with respect to the measured topology.

3. METHODOLOGY

We define an undirected graph $G = (V, E)$ where V is the set of vertices (nodes) and E is the set of edges (links). The adjacency matrix of G, $A(G)$ is then defined as:

$$A(G)(u,v) = \begin{cases} 1, & \text{if } u, v \text{ are connected} \\ 0, & \text{if } u, v \text{ are not connected} \end{cases}$$

Several matrices associated with the adjacency matrix exist, such as the computational Laplacian $L(G)$, the normalized Laplacian $\Im(G)$, and the walk Laplacian $\Im_{rw}(G)$:

$$L(G) = D - A(G)$$

$$\Im(G) = D^{-1/2} L(G) D^{-1/2}$$

$$\Im_{rw}(G) = D^{-1} L(G)$$

where D is a diagonal matrix of the row sums of $A(G)$. From a graph theoretic point of view the matrices $A(G)$, $L(G)$, $\Im(G)$, and $\Im_{rw}(G)$ have some very interesting properties associated with the eigenpairs of the matrices, see e.g., [12, 15]. Although the choice of which matrix to use is often application dependent [8], the normalized Laplacian is thought to be superior as it is size independent and so allows comparison of graphs of different sizes. Also, von Luxburg and Bousquet [17] show that spectral clustering based on the normalized Laplacian converges to a nice partition of the data space, but for the computational Laplacian this is only true in the limit and can only be guaranteed for finite samples under assumptions which often cannot be verified by the data. Therefore it would appear that the normalized Laplacian, $\Im(G)$, is the best choice in our case.

As noted by Zhu and Wilson [20] the eigenvalues of two isomorphic graphs will be equal but the converse is not true. However, studies have shown [20] that the number of cospectral graphs, i.e., graphs that share the same eigenvalues of $\Im(G)$ but are not isomorphic, fall drastically with n

and so for large n two graphs that share the same eigenvalues can be considered "almost certainly" (in the statistical sense) isomorphic. The question next arises with regards to two graphs which have similar but different eigenvalues. Although the eigenvalues may be close, the eigenvectors may be different and thus represent different clusters of data. Again, Zhu and Wilson [20] empirically show that the edit distance between graphs may be linearly related to the sum squared difference in the eigenvalues. However, their simulations were carried out on graphs for small values of n and so the relationship may not hold at higher values. Further evidence relating the importance of the eigenvalues in a graph can be seen from the following relationship due to Chung [3], relating graph subsets and the eigenvalues:

$$\min_{i \neq j} \text{dist}(X_i, X_j) = \max_{i \neq j} \left\lceil \frac{\ln \sqrt{\frac{\text{vol}(\bar{X}_i)\text{vol}(\bar{X}_j)}{\text{vol}(X_i)\text{vol}(X_j)}}}{\ln \frac{\lambda_{n-1}+\lambda_k}{\lambda_{n-1}-\lambda_k}} \right\rceil \quad (1)$$

where $X \subset G$, $\bar{X}$ is the complement of that subset, $\text{vol}(X)$ is the total number of edges in X and $\text{dist}(X_i, X_j)$ is the distance between subset i and j. Equation 1 may be interpreted as representing the distance between subsets for k subsets k = 1,...,$n-1$. In other words the eigenvalues may be used to estimate the number of subsets in a network without forcing the distances to be too short [15]. The interesting point to note about Equation 1 is the way in which λ_k is expressed; as the distance between the k^{th} eigenvalue and the largest eigenvalue, λ_{n-1}. This implies that the *distribution*[†] of the eigenvalues is important in the structure of a graph.

Next, we examine the interpretation of the eigenpairs as a spectral decomposition of a graph:

$$\Im(G) = \sum_i \lambda_i e_i e_i^T$$

where e_i are the eigenvectors of $\Im(G)$. As the graphs examined here are undirected, $\Im(G)$ is symmetrical and thus $\{e_1, e_2, ..., e_{n-1}\}$ form a basis for $\Im(G)$ with $\{\lambda_1, \lambda_2, ..., \lambda_{n-1}\}$ representing the strength of the projection of the matrix onto each of these bases. This may be viewed from a statistical point of view [15] where each $\lambda_i e_i e_i^T$ may be used to approximate $A(G)$ with approximation error inversely proportional to $1 - \lambda_i$. Thus, again, the distribution of eigenvalues can be seen to be linked to the structure of a graph.

As mentioned in the introduction, we propose a metric for comparing the structure of two graphs. We are not interested in the particulars of the structure, i.e., the actual clusters, but rather in their number, size and distribution. From the discussion above the distribution of the eigenvalues would appear ideal for this task. If we define the eigenvalue distribution as $f_\lambda(\lambda)$, we construct a metric based on $f_\lambda(\lambda)$ as:

$$J(G_1, G_2) = \int_\lambda \mu(\lambda)(F_\lambda^1(\lambda) - F_\lambda^2(\lambda))^p d\lambda \quad (2)$$

where μ is a weighting function, F_λ^i is the distribution of eigenvalues of graph i, p is an integer, and $J(G_1, G_2)$ is the distance between two graphs G_1 and G_2. The next question arises as to the appropriate weighting function. In this paper we choose the weighting by noting a result from [3]:

$$dev(G) = \sum_i (1 - \lambda_i)^4 + 20\sqrt{Irr(G)}$$

where $Irr(G)$ is the irregularity of the graph [3]. The deviation of a graph may be used as a measure of the structure in a graph or its distance away from randomness. In addition we choose $p = 2$ to give the standard sum-squared-error although other values may be of interest, resulting in:

$$J(G_1, G_2) = \int_\lambda (1 - \lambda)^4 (F_\lambda^1(\lambda) - F_\lambda^2(\lambda))^2 d\lambda$$

However, calculating the eigenvalues of a large (even sparse) matrix is computationally expensive. In addition, the aim here is to represent the *global* structure of a graph and so precise estimates of *all* the eigenvalues are not required. Thus, the distribution of eigenvalues is sufficient. In this paper the distribution of eigenvalues $f(\lambda = k)$ is estimated using pivoting and Sylvester's Law of Inertia to compute the number of eigenvalues that fall in a given interval. To estimate the distribution we use K equally spaced bins.[1] A measure of the graph can then be constructed by considering the distribution of the eigenvalues as

$$\omega(G, N) = \sum_{k \in K} (1 - k)^N f(\lambda = k) \quad (3)$$

where the elements of $\omega(G, N)$ form the *weighted spectral distribution*:

$$WSD : G \rightarrow \Re^{|K|} \{k \in K : ((1 - k)^N f(\lambda = k))\} \quad (4)$$

The corresponding metric is then constructed from $\omega(G)$ for comparing two graphs, G_1 and G_2, as

$$J(G_1, G_2, N) = \sum_{k \in K} (1 - k)^N (f_1(\lambda = k) - f_2(\lambda = k))^2 \quad (5)$$

where f_1 and f_2 are the eigenvalue distributions of G_1 and G_2 and the distribution of eigenvalues is estimated in the set K of bins $\in [0, 2]$.

For each type of topology generator a *family* of WSDs may be generated by varying the parameters of the generator. The aim at this point is to show that these WSD families map onto different curves for different topology generators, i.e., the WSDs generated by an AB model should not correspond to any of those of the GLP model or the Waxman model, etc. It is not possible to plot these families side by side as in Figure 3 as the plot becomes too cluttered: the key problem is that the data is essentially of dimension K and so cannot be distinguished in a 2-d plot. First the dimension of the data must be reduced from K to two.

We begin by sampling from the family of WSDs for each topology generator. Specifically we generate 100 topologies of each using random parameters. 70 bins are used in this experiment, resulting in a data matrix of 400 WSDs (4 topology generators) of size 400×70. The next stage is to reduce the dimension of this data to 400×2. As we require *any* projection that separates the data classes (generators), *not* specifically an optimal projection, random projection is used [4]. Random projection is a technique often used in compressed sensing in which a high dimensional matrix is reduced to a low dimensional matrix by multiplying the

[†]This is not a distribution in the strict statistical sense as the eigenvalues are deterministic quantities for a given graph.

[1] K can be increased depending on the granularity required.

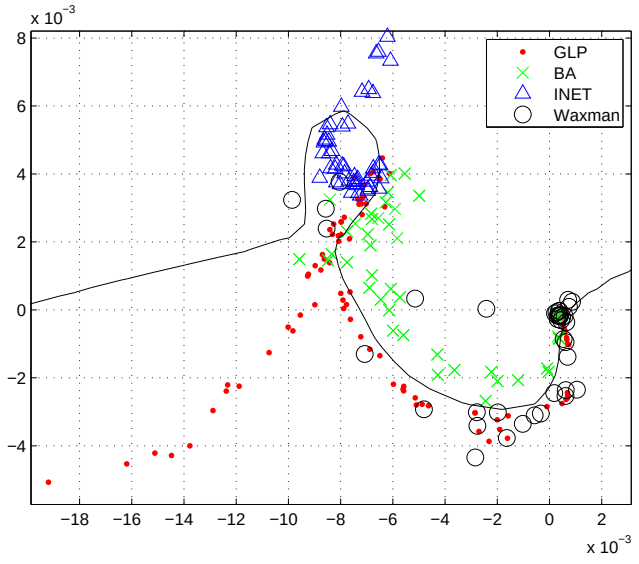


Figure 1: 2-d random projection of WSD families corresponding to four different topology generators. Axis labels are not relevant: the crucial information is in the separation of points. See text for details.

data by a random matrix as:

$$Z = XT \tag{6}$$

where $Z \in R^{M,k}$ is the projected data matrix, $X \in R^{M,K}$ is the WSD sample data, $T \in R^{M,k}$ is the random projection matrix where each of the elements of T are drawn from a Gaussian distribution $T \sim N(0,1)$,[‡] M is the number of samples in the data (400 in this case) K is the original dimension (70 in this case) and $k << K$ is the reduced dimension (2 in this case).

Figure 1 shows the projection of the sampled families onto two dimensions using random projection. The first thing to note is that most of the Waxman WSDs lie well outside the range of the figure. This is to be expected as Waxman topologies differ significantly from the others. Second, the actual units of the graph are of no importance; only the separation of the points is significant. At the right of the graph (around 0,0) there is a clustering in which the GLP, AB and Waxman models all overlap. This occurs at low parameter values when the graphs contain few links and so are difficult to discern from each other. The GLP and AB graphs are very close for a large section of the families. This occurs as GLP is similar in structure to AB but not equal. In order to demonstrate this, a Support Vector Machine (SVM) [9] was used to determine the boundary between the the AB class (topologies) and the GLP class. The decision boundary is shown in Figure 1 as a solid black line. Note the boundary value is irrelevant outside of the training range. As can be seen the boundary separates the two classes efficiently with an 11% false classification rate (out-of sample). The Inet models generate a different cluster of projections which is shown quite clearly in Figure 1. Thus the WSD has been shown empirically to generate unique WSDs for different topologies.

[‡]As the rows of T are normally distributed independent variables their correlation is zero in expectation and so they are (near) orthogonal vectors.

4. RESULTS AND DISCUSSIONS

In this section we use the *weighted spectral distribution*, Equation 5 to obtain parameter estimates for four Internet topology generators [6]: the Waxman model, the 2nd Barabasi and Albert Model (BA2), the Generalized Linear Preference model (GLP) and the Inet model. These are compared with the Skitter dataset [1] and the Positive Feedback Preference model (PFP), which has no parameters. In addition, we show that the spectrum on its own, equivalent to $p = 1$, $\mu(\lambda) = 1$, is not sufficient to obtain parameter estimates.

Figure 2 shows a plot of the weighted spectral distribution distance between the four topology generators and the Skitter dataset, as a function of values of the topology generator parameters. Each grid (Figure 2(d) is a curve as Inet has only one parameter) shows a quantile contour plot of the surface of the distances at different parameter values. It is encouraging to note that the minima in each case lie close to the default values (see Table 1). In addition, it is known that the behavior of the BA2 model splits into two regions: exponential behavior and scale free behavior. The Internet is known to exhibit scale free behavior, and the area of minimum distance lies in this area.

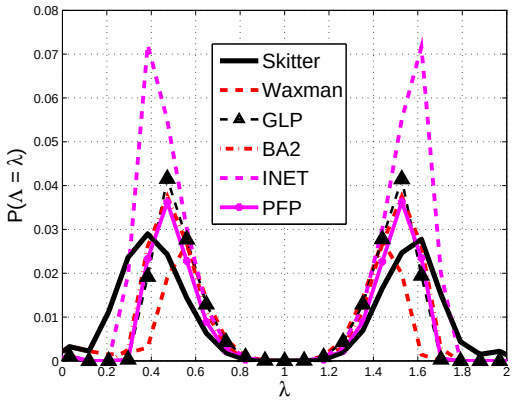
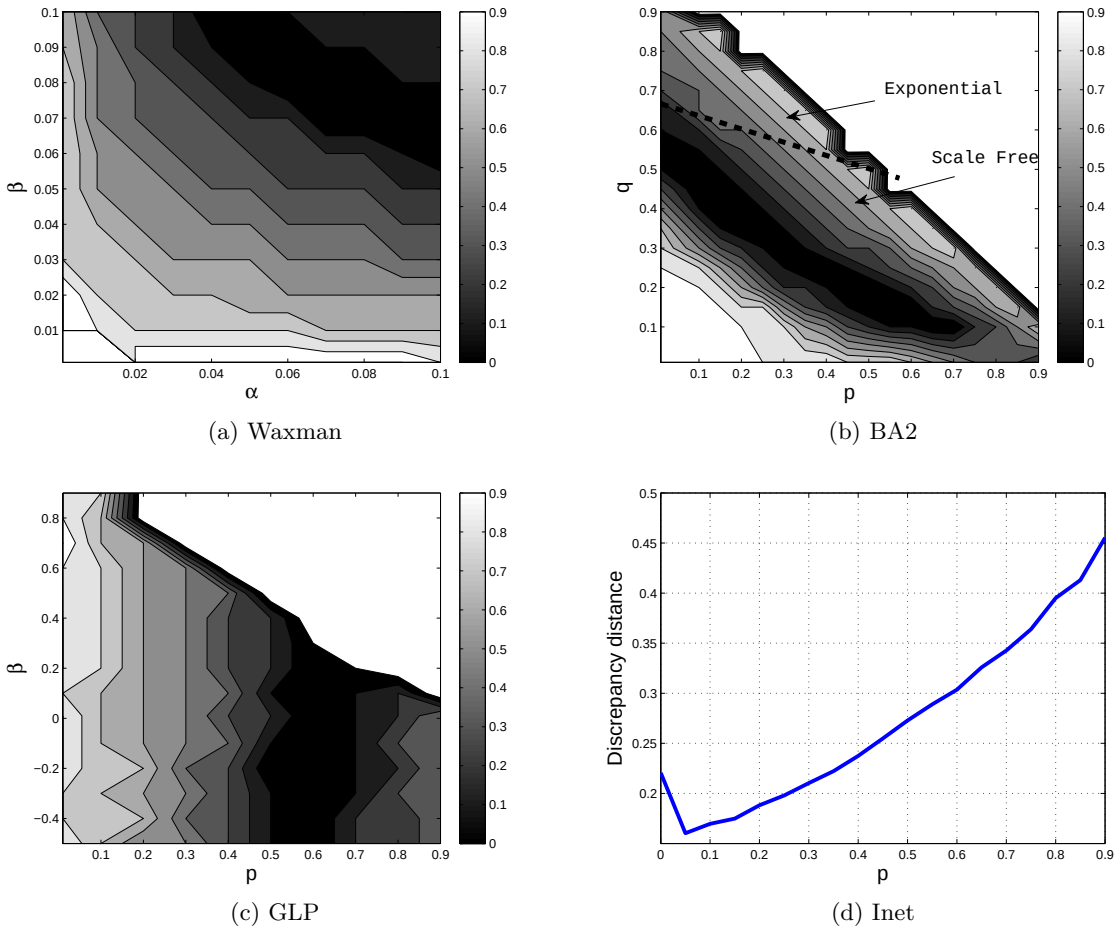


Figure 3: Comparison of the weighted spectra.

Figure 3 shows the weighted spectrum (each element that makes up the summation in Equation 5) for the optimal values of the parameters calculated using the Nelder Meade optimization algorithm. As can be seen the results are quite distinct, showing that no single topology is capable of generating the same distribution of structure as the Skitter data (it is in any case questionable that Skitter represents the actual Internet due to inherent measurement difficulties). However, it is the way in which each topology differs that is revealing. The Waxman model has peaks at 0.6, the closest peaks to 1 of all the topologies examined. This implies that the Waxman topologies exhibit more random behavior than desired, as expected. Alternatively the Inet topology correctly peaks at 0.4, but exhibits too strong structure at this point. The best model is the PFP model which is a non-linear model considered to be a good approximation to the actual Internet. Table 1 confirms these results.



(a) Waxman (b) BA2

(c) GLP (d) Inet

Figure 2: Grid of sum squared error of weighted spectra for topology generators.

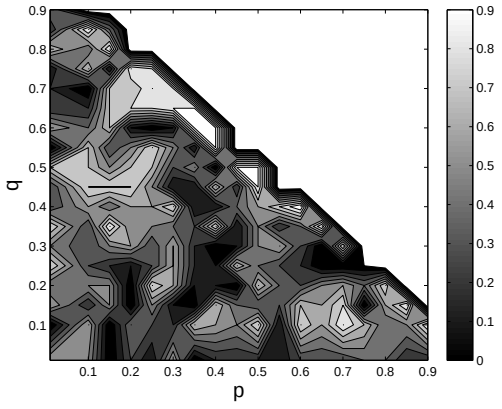


Figure 4: Parameter grid for sum of absolute differences of BA2 spectrum CDF.

As an example of using the unweighted spectrum, the grid for the BA2 model is shown in Figure 4. As can be seen there exists no minimum. This is because the unweighted spectrum weights each eigenvalue equally and so random structure, mainly in the spectral gap, makes the surface noisy. We show only the BA2 result here due to space constraints, but a similar situation was found for all of the topology generators. We thus conclude that the spectrum on its own is not sufficient to reveal the structure of the topologies.

5. CONCLUSIONS

Comparison of graph structures is a frequently encountered problem across a number of problem domains. To perform a useful comparison requires definition of a metric that encodes which features of the graphs are considered important. Although the spectrum of a graph is often claimed to be a way to encode a graph's features, the raw spectrum contains too much information to be useful on its own. In this paper we have introduced a new metric, the weighted spectral distribution, that improves on the graph spectrum by discounting those eigenvalues that are believed to be unimportant and emphasizing the contribution of those believed to be important.

We use this metric to optimize the selection of parameter values of Internet topology generation. The weighted spectral distribution was shown to be a useful metric in that it leads to parameter choices that appear sensible given prior knowledge of the problem domain: the resulting choices are close to the default values and, in the case of the BA2 generator, fall within the expected region. In addition, as the met-

Table 1: Optimum parameter values for matching Skitter topology.

Waxman	$\alpha = 0.08$ (default=0.15)	$\beta = 0.08$ (default=0.2)	$J(\theta) = 0.0026$	$\overline{J}(\theta) = 0.0797$
BA2	$p = 0.2865$ (default=0.6)	$q = 0.3145$ (default=0.3)	$J(\theta) = 0.0014$	$\overline{J}(\theta) = 0.0300$
GLP	$p = 0.5972$ (default=0.45)	$\beta = 0.1004$ (default=0.64)	$J(\theta) = 0.0021$	$\overline{J}(\theta) = 0.0446$
Inet	$\alpha = 0.1013$ (default=0.3)	–	$J(\theta) = 0.0064$	$\overline{J}(\theta) = 0.0150$
PFP	–	–	$J(\theta) = 0.0014$	$\overline{J}(\theta) = 0.0371$

ric is formed from a summation it is possible to go further and identify which particular eigenvalues are responsible for significant differences. Although it is currently difficult to assign specific features to specific eigenvalues, it is hoped that this feature of our metric will be useful in the future.

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