

# Multiplexing

G54ACC

Lecture 4

[richard.mortier@nottingham.ac.uk](mailto:richard.mortier@nottingham.ac.uk)

# Resource Allocation

- Many resources are expensive
  - Your time, waiting
  - Installation of network cables
  - People to operate, manage, install, repair
- How should we share network?
  - Who gets what, when, and for how long?
  - Greater efficiency == greater profit

# Contents

- Telephone Network
- Multiplexing
- Statistical Multiplexing
- Code Division Multiple Access

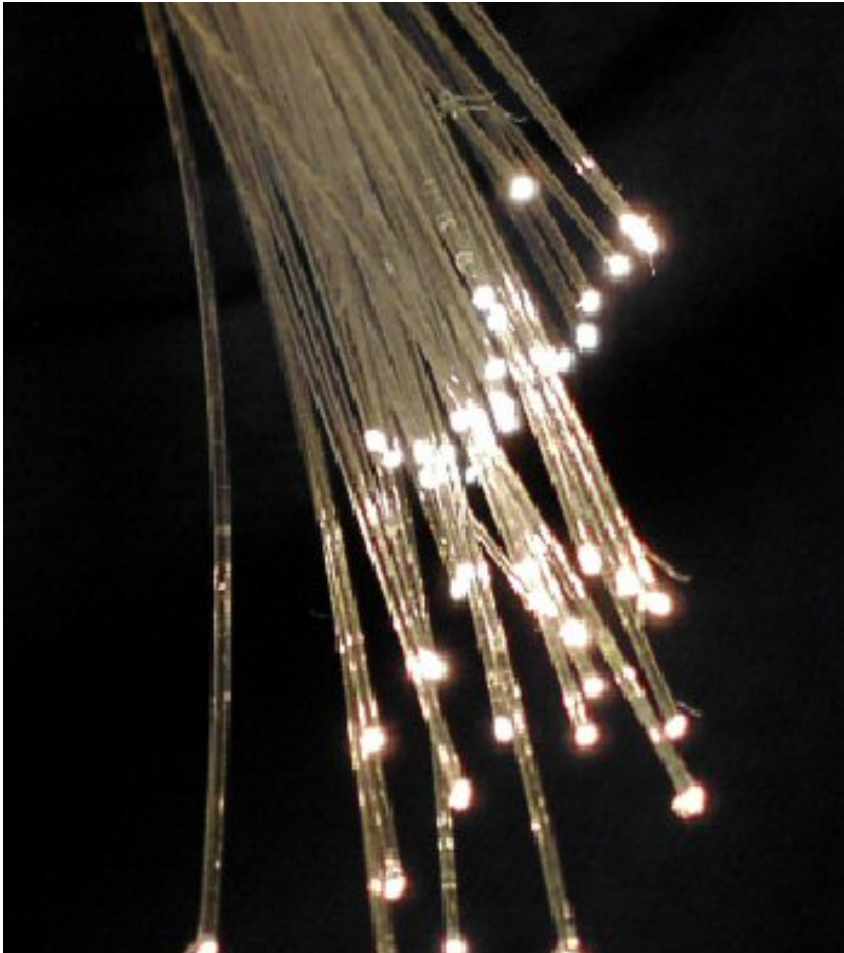
# Contents

- Telephone Network
  - Copper to fibre
  - Simplex vs. Duplex
- Multiplexing
- Statistical Multiplexing
- Code Division Multiple Access

# The Telephone Network, old

- Traditionally, copper cables
  - When you place a call there is an unbroken line of copper from your telephone to theirs
  - Operators at the exchanges plug cables together
  - Signal carried by varying voltage (analogue)
- Simplex vs. half-duplex vs. full-duplex
  - Signal might only travel one-way
  - Signal can go both ways, but separately
  - Signals go both ways, simultaneously

# The Fibre Revolution



- Better encodings mean  
    >1 call per cable
  - Even on copper
  - Thousands on fibre
- Far more efficient
- Far more cost-effective

# Contents

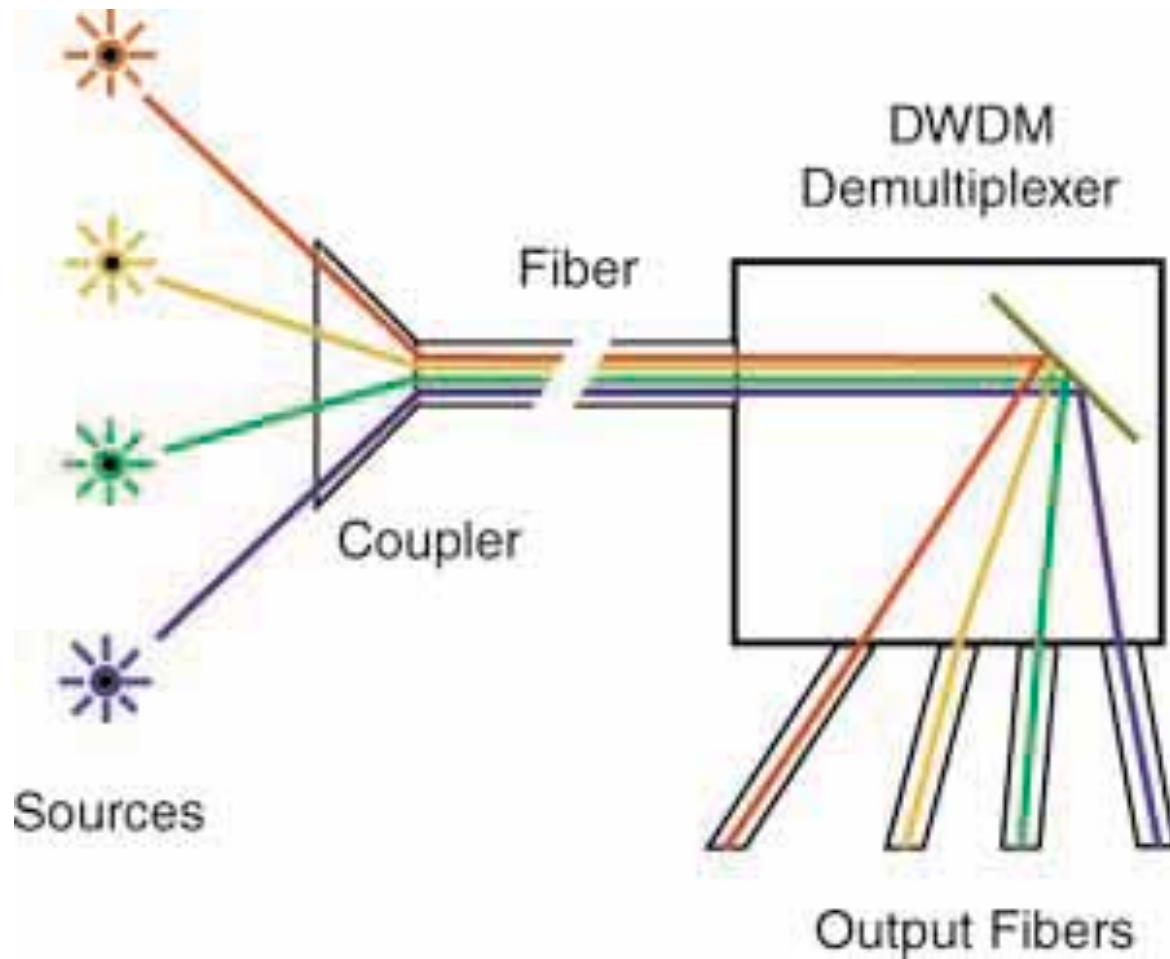
- Telephone Network
- Multiplexing
  - SDM, WDM
  - TDM, STM
  - PTM, ATM
  - Trade-offs
- Statistical Multiplexing
- Code Division Multiple Access

# Multiplexing

- Space Division Multiplexing
  - Reuse channel in different places
- Frequency/Wavelength Division Multiplexing
  - Use of different frequencies within a channel
  - Radio vs. optical
- Time Division Multiplexing
  - Coming next
- Code Division Multiplexing
  - Coming later

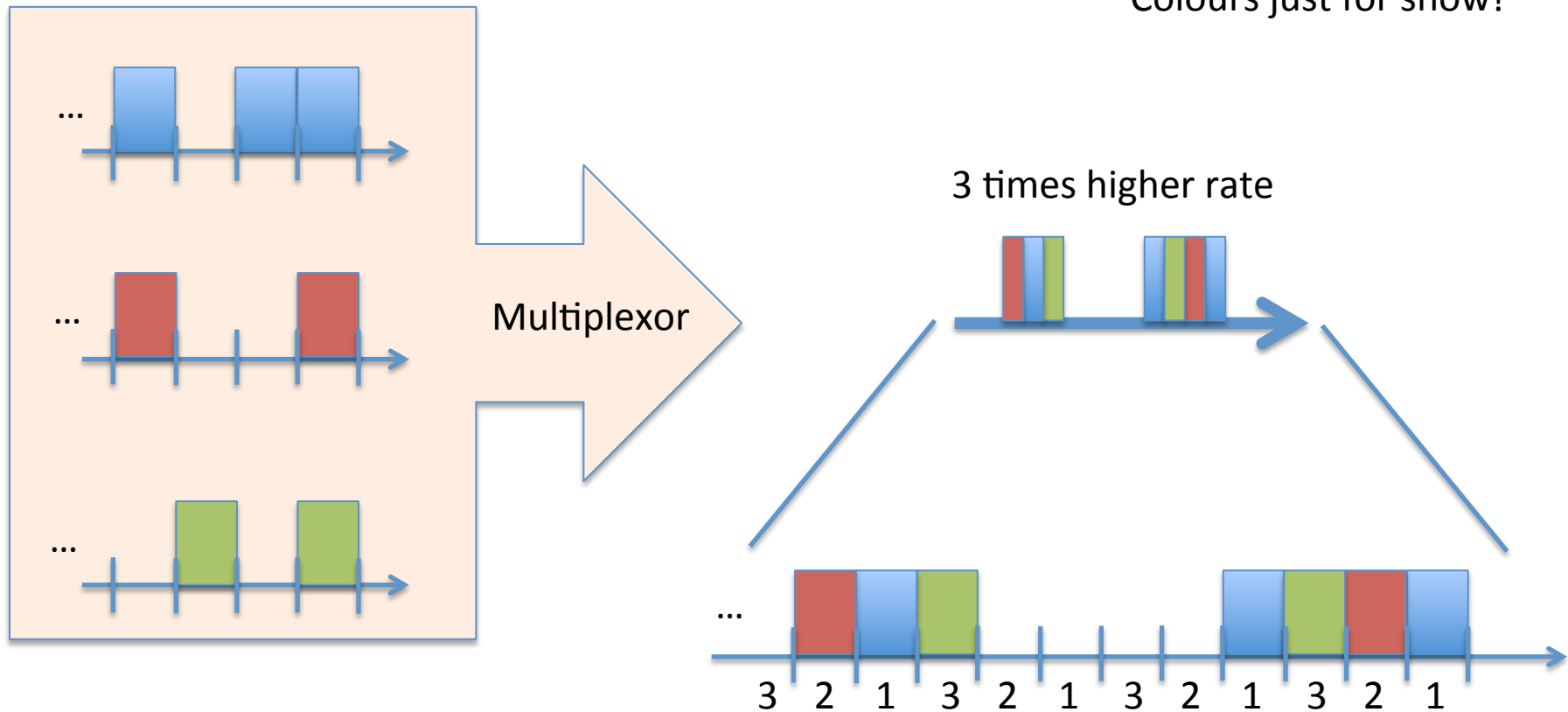


# Wavelength Division Multiplexing



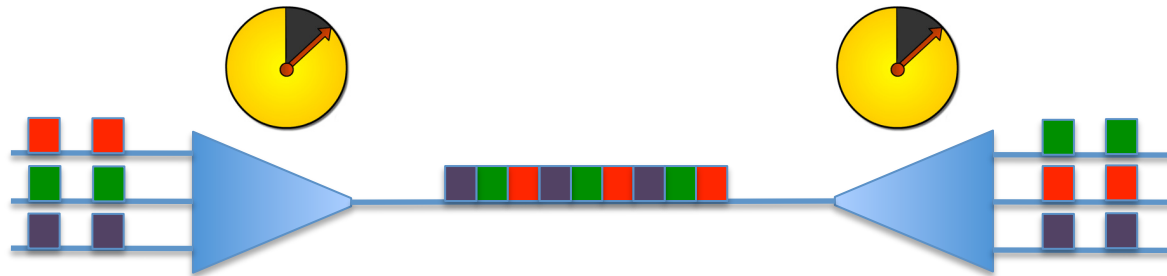
# Time Division Multiplexing

Single wavelength:  
Colours just for show!



# Synchronous Transfer Mode

- The destination of data in the network depends on *where* it comes from, and *when* it came
  - Telephone network is referred to as STM
- ✗ Idle users consume bandwidth
- ✗ Links are shared on a fixed cyclical schedule
  - Quantization of link capacity
  - Can't adjust bandwidth

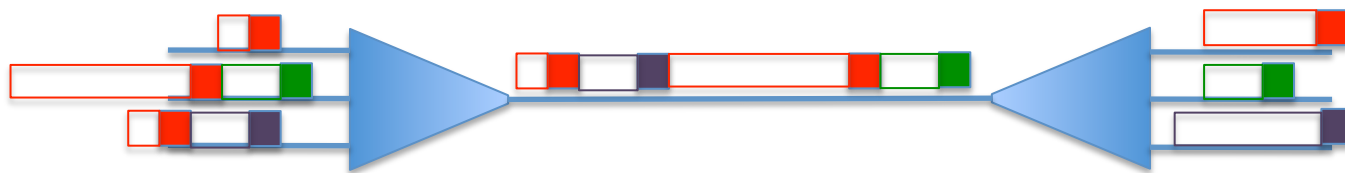


# The Internet

- No connection at all! Physical or virtual!
  - A radical departure: they said it couldn't be done
- Rely on computers running software
  - Subdivide data into variable sized packets
  - Label with the destination
  - Send them on their way
- Packet/Data Multiplexing

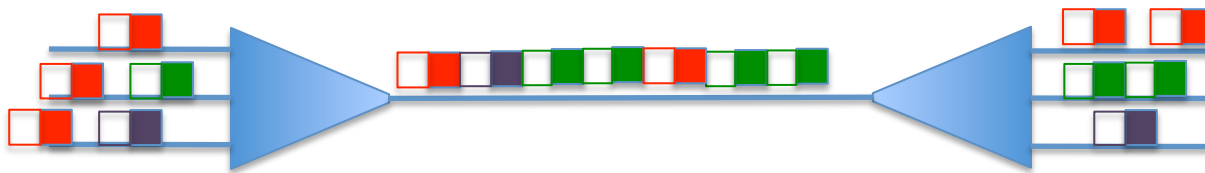
# Packet Transfer Mode

- The destination of data in the network depends on a *label* in the front of the *packet*
  - The Internet uses PTM
- Label can be *global*
  - Connectionless – e.g., IP
- ...or *local* (context sensitive)
  - Connection oriented or virtual circuit – e.g., MPLS
- Packets can be an arbitrary size



# Asynchronous Transfer Mode

- Mix the benefits of STM and PTM
- The destination of data in the network is based on a label
- All packets are the same size



# Trade-offs

- 😊 Far more efficient
  - Share based on dynamically varying need
- 😊 Far more robust
  - If a packet is lost or link fails, can recover
- 😞 Far less predictable
  - Variability *vs.* failure
- 😞 Requires more on-going work
  - No connection to setup (control)
  - *vs.* Need to think about every packet (data)

# Contents

- Telephone Network
- Multiplexing
- Statistical Multiplexing
  - Randomness
  - A Simple Stochastic Process
  - Poisson
- Code Division Multiple Access



# Statistical Multiplexing

*What is the benefit of PTM/ATM?*

- Consider a 10Mb/s link and an application that uses 1Mb/s for 10% of the time
- How many can you support with STM vs. PTM?
  - STM,  $N = 10$ 
    - Must allocate 1Mb/s of bandwidth to each application
  - PTM,  $10 \leq N \leq 100$ 
    - Depending on how lucky you are,  
and how much delay and loss you can tolerate

# Randomness

- No network buffering makes packet loss intolerable
  - If everyone talks at once,  $N = 10$
  - If by magic only 10 of 100 talk together,  $N = 100$
- So – what is the probability that we lose packets?
  - I.e., that more than 10 talk at once?
- Means understanding the random transmission of packets from the application
  - The *stochastic process* of the packets
  - A research topic for the last 30 years...
    - ...and as new applications arise we find new behaviours

# An Old Problem

- Erlang 1909

*How many phone lines does a small village need to the outside world to ensure that the probability of a call blocking is  $< 5\%$ ?*

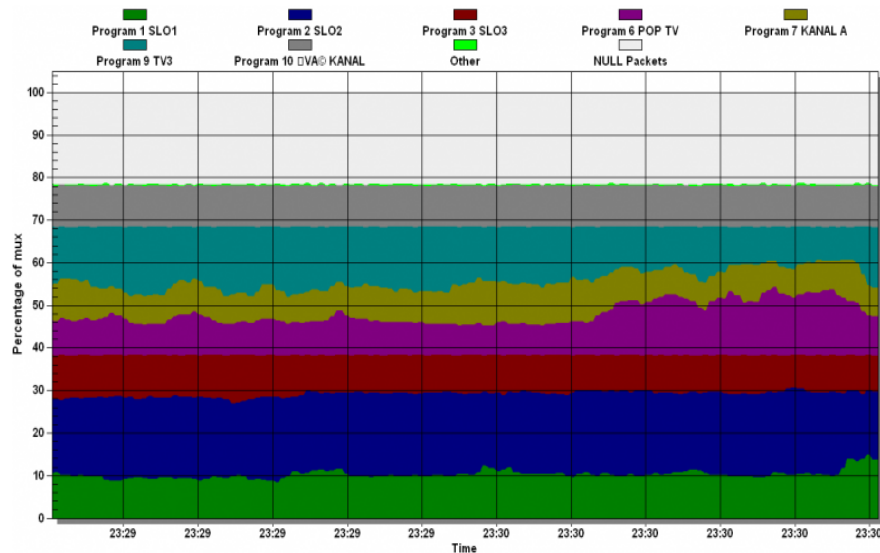
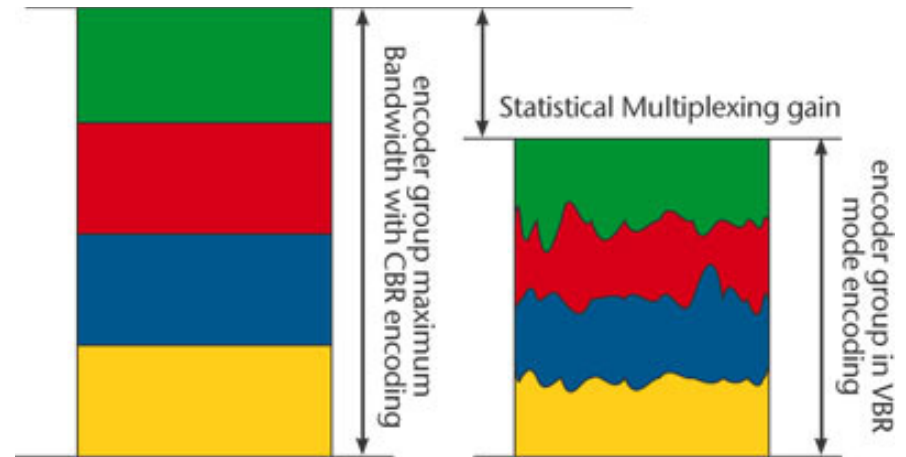
## 1. THE THEORY OF PROBABILITIES AND TELEPHONE CONVERSATIONS

*First published in "Nyt Tidsskrift for Matematik" B, Vol. 20 (1909), p. 33.*

Although several points within the field of Telephony give rise to problems, the solution of which belongs under the Theory of Probabilities, the latter has not been utilized much in this domain, so far as can be seen. In this respect the Telephone Company of Copenhagen constitutes an exception as its managing director, Mr. *F. Johannsen*, through several years has applied the methods of the theory of probabilities to the solution of various problems of practical importance; also, he has incited others to work on investigations of similar character. As it is my belief that some point or other from this work may be of interest, and as a special knowledge of telephonic problems is not at all necessary for the understanding thereof, I shall give an account of it below.

# ...And A Very Modern One

- Digital TV uses MPEG coding of video which naturally leads to a variable data rate

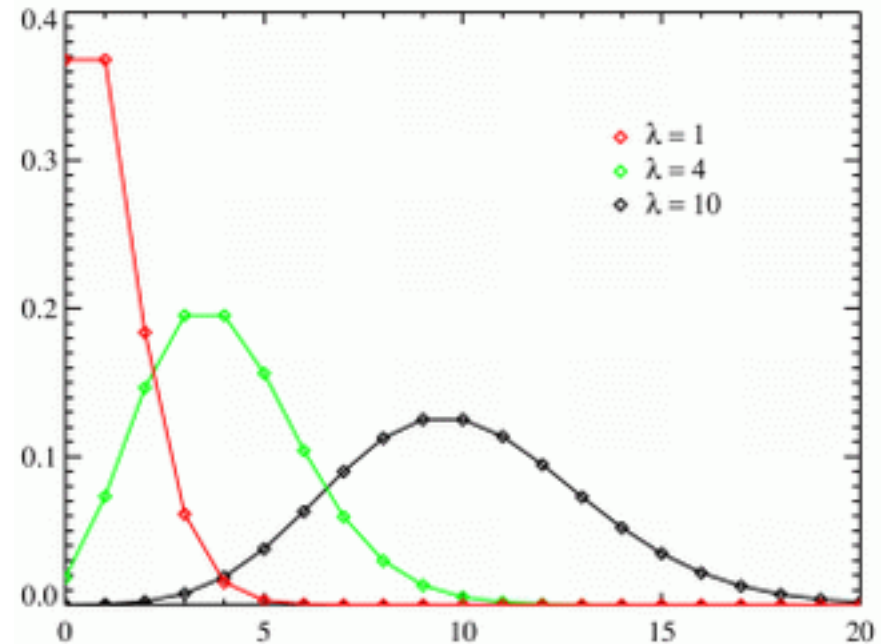


# A Simple Stochastic Process

**Poisson** process:

If the expected number of occurrences in an interval is  $\lambda$ , then the probability that there are exactly  $k$  occurrences ( $k$  being a non-negative integer,  $k = 0, 1, 2, \dots$ ) is equal to:

$$f(k; \lambda) = \lambda^k e^{-\lambda} / k!$$



# Poisson Appears Everywhere



## Do Outbreaks of War Follow a Poisson-Process?

H. W. Houweling

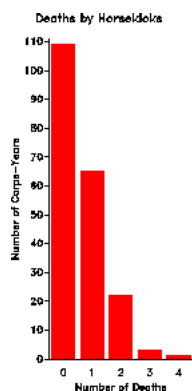
J. B. Kuné

University of Amsterdam

### Abstract

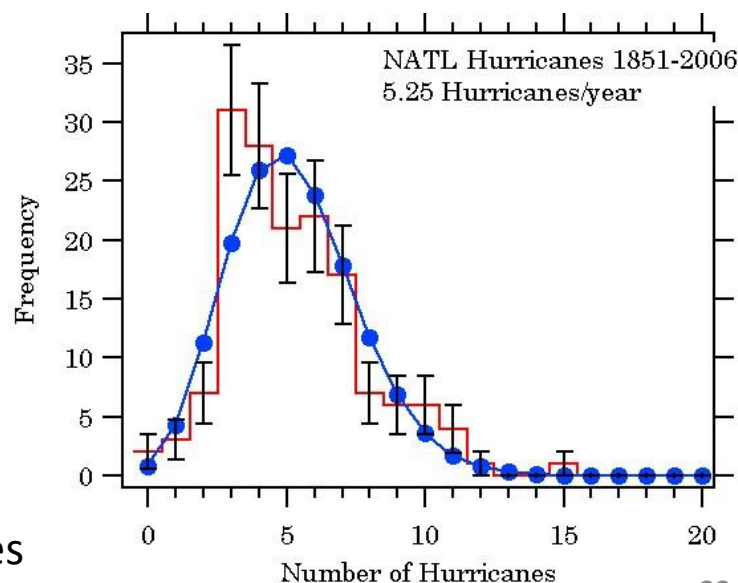
Richardson's finding that the distribution of war outbreaks in time conforms to the Poisson distribution has been repeated over and over again. In this article, we argue that the close correspondence between the two distributions does not imply that the mechanism generating the data is the Poisson process. Because the time sequence is broken down in Richardson's analysis, his finding does not imply that war outbreaks follow a distinct pattern in time. The Parzen test on arrival times revealed that war outbreaks are not generated by a Poisson random process.

*Journal of Conflict Resolution*, March 1984; vol. 28, 1: pp. 51-61.



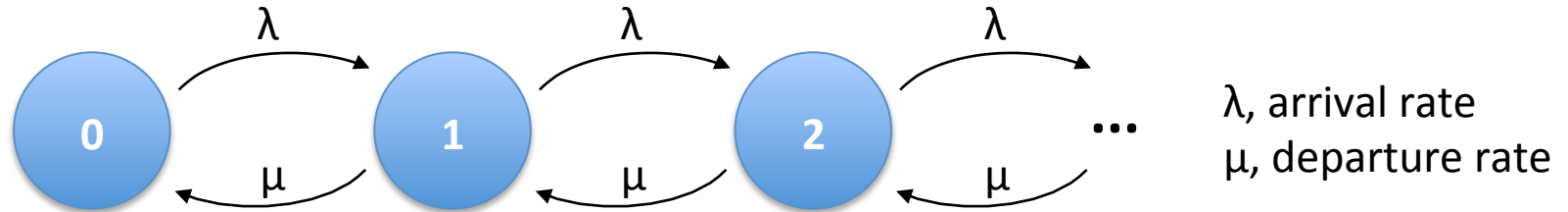
von Bortkiewicz (1898) –  
horse kick fatalities in  
the Prussian Cavalry

Paul Linsay (2006) –  
Atlantic hurricanes



# A Simple Queue: M/M/1

Markov (Poisson) arrivals, Markov (Poisson) departures, 1 Server



- $\rho = \lambda / \mu$ , the traffic intensity
  - What if  $\rho \geq 1$ ?
- If  $\rho < 1$ ,
  - Probability of  $j$  things in queue,  $P_j = \rho^j (1 - \rho)$  for  $j = 0, 1, \dots$
  - Average queue length,  $L = \sum j P_j = \lambda / (\mu - \lambda)$
- Unfortunately real network traffic doesn't look like Poisson – why?

# Contents

- Telephone Network
- Multiplexing
- Statistical Multiplexing
- Code Division Multiple Access
  - Frequency Hopping
  - Direct Sequence



# Code Division Multiple Access (CDMA)

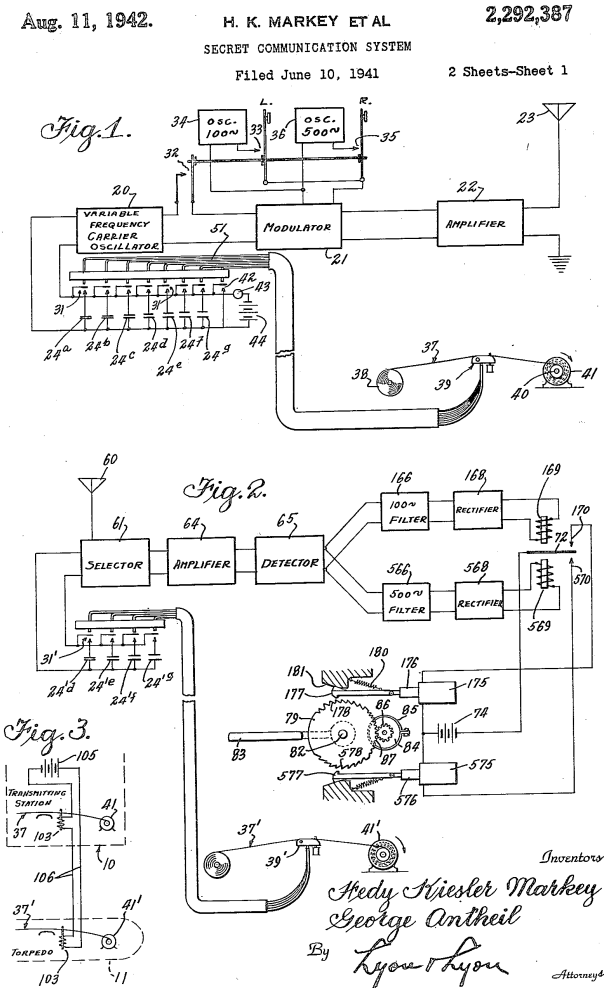
- Share same frequency space using unique per-user code
  - Frequency hopping
  - Time hopping
  - Direct Sequence
- Allows multiple users to “coexist” and transmit simultaneously with minimal interference
  - If codes are “orthogonal”
  - A large code can be selected at random and will probably be orthogonal to any other random one!
- Can’t receive signal unless you know the code
  - In fact hard to interfere if you don’t know the code

# Frequency Hopping

- 1941 patent by Hedy Keiser Markey & George Antheil
  - Code on paper tape selects frequency
  - Synchronise transmitter and receiver tapes
- Designed for radio control torpedoes to prevent radio jamming

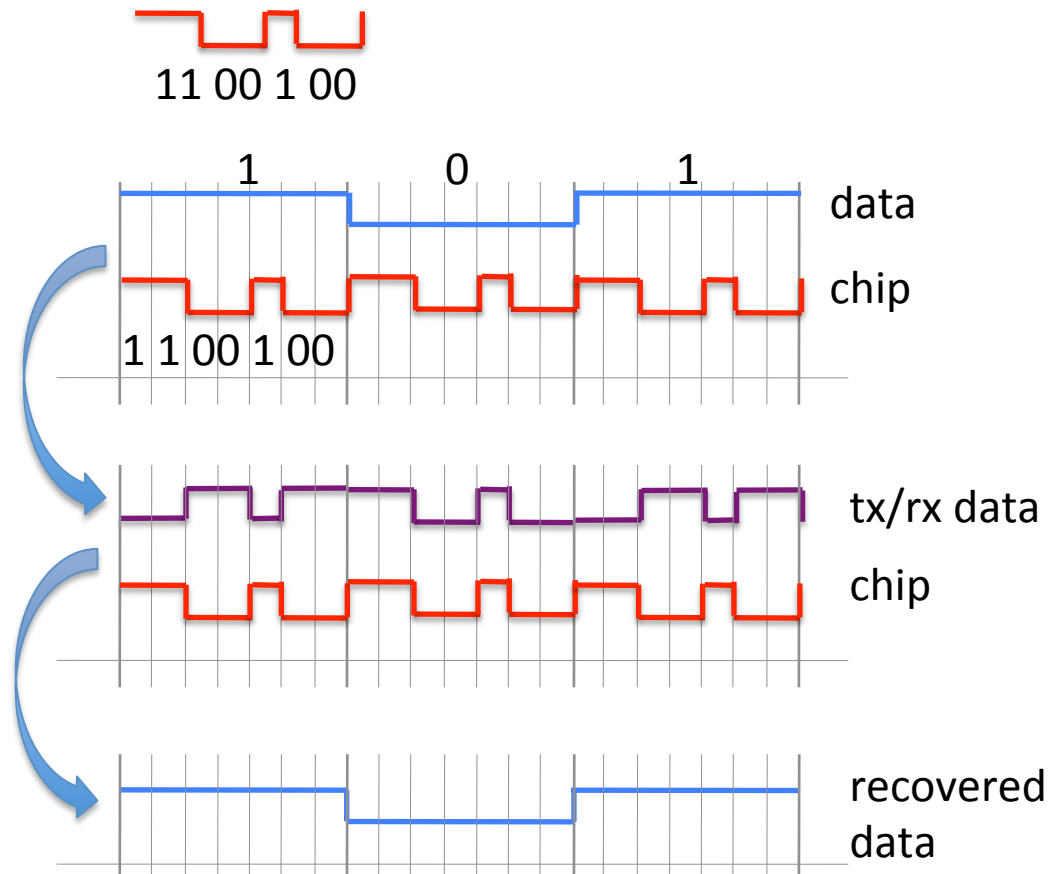


Hedy had a professional name  
 – Hedy Lamarr, star of stage and screen



# Direct Sequence CDMA

- Code is called *chip sequence*
- Transmitter, Tx:  
XOR data and chip sequence
- Receiver, Rx:  
XOR received data and chip sequence
- CDMA is basis of:
  - 3G mobile telephony
  - GNSS/GPS



# Contents

- Telephone Network
  - Copper to fibre
  - Simplex vs. Duplex
- Multiplexing
  - SDM, WDM
  - TDM, STM
  - PTM, ATM
  - Trade-offs
- Statistical Multiplexing
  - Randomness
  - A Simple Stochastic Process
  - Poisson
- Code Division Multiple Access
  - Frequency Hopping
  - Direct Sequence

# Summary

- Multiplexing is the sharing of a resource by signals
  - ...in such a way that the original signals can be recovered
  - Applies to other resources than the network
- Several different techniques for doing this
  - Different trade-offs arise
  - Cost, performance, complexity
- Description of STM, PTM, ATM
  - Role of statistical multiplexing
  - Problems then in queuing theory
- CDMA

# Quiz

1. Name three different multiplexing techniques, describe how they operate, where they are might be used, and why.
2. What are two different issues posed by STM?
3. What type of multiplexing are ATM and PTM both examples of, and how do they differ?
4. Why is statistical multiplexing often beneficial, particularly for computer networks? What problem does it cause?
5. Describe the M/M/1 queue model of a network buffer, and show the average queue length and conditions under which it applies.
6. Why is the M/M/1 model typically a poor model of a network buffer in an IP network element?
7. Give three different types of CDMA and say how they differ.
8. Illustrate Direct Sequence CDMA when transmitting and then receiving the ASCII string "G54ACC" using the chip sequence 1001101.