University of Pennsylvania
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Professor Gomes — Problem Set 2
Finance 937

Part I: Liquidity/Covenant Default

Consider the following dynamic problem for the equity holders in the firm,

$$\begin{aligned} e(z,k,b) &= \max_{k',b'} \left[d(z,k,k',b,b') + \mathbb{E}_z M e(z',k',b') \right], \\ s.t. \quad & d(z,k,k',b,b') = \pi(z,k) - i(k,k') + b' - R^b b - \tau(z,k,b), \end{aligned}$$

where $d(\cdot)$ is the value of gross distributions

Operating profits obey,

$$\pi = z k^{\alpha_k} l^{\alpha_l} - Wl$$

Investment is:

$$i(k,k') = k' - (1-\delta)k,$$

Suppose that taxes payments are:

$$\tau(z,k,b) = \tau_c \left[\pi(z,k) - \delta k - b\left(R^b - 1\right) \right]$$

where $\tau_c = 0.15$.
Suppose firms will default whenever

$$(1-\tau_c)\pi(z,k) + (1-\delta)k \leq b$$

The stochastic process for z is given by:

$$\log(z') = (1-\rho)\log \bar{z} + \rho \log(z) + \sigma\epsilon'.$$

and the innovations ϵ are follow a truncated standard normal distribution.

To solve the model use the following initial parameter values: $M = 0.99$, $\delta = 0.1$, $\alpha_k = 0.3$, $\alpha_l = 0.6$, $W = 2$, $\rho = 0.7$, $\sigma = 0.05$. There is also a (proportional) cost of issuing equity, $\lambda = 0.025$.

a) Compute and plot the probability of default next period, conditional on the value of the shocks today $p(z,b',k')$.

b) Use the conditional default probability to compute the required rate of return by bondholders, $R^b(b',k';z)$ that ensures they make 0 profits. Assume for simplicity bondholders get paid 0 upon default.

c) Solve the Bellman equation for the equity holders taking as given the function for the required rate of return by bondholders, $R^b(\cdot)$.

Consider now a world with many such firms and no entry or exit. Specifically, suppose that upon hitting the default threshold debt claims are settled so $b = 0$. The restructured firm continues to operate but with capital, $k = 0$ and the previous productivity shock, z.

d) Compute the stationary distribution of firms. Use this distribution to construct a table reporting the cross-sectional average values of:

(1) probability of default, $p(\cdot)$;
(2) required return on risky bonds, $R^b(\cdot)$;
(3) leverage ratio, b/k;
(4) investment to capital ratio, i/k.
(5) fraction of firms issuing equity;

Part II: Method of Simulated Moments

Consider the stationary distribution of the model above.

a) Estimate the following leverage and investment regressions on the steady-state distribution of firms and report the coefficients:

$$\begin{aligned} i/k &= \beta_0 + \beta_1 Q + \beta_2 \pi/k \\ b/k &= \gamma_0 + \gamma_1 Q + \gamma_2 \pi/k + \gamma_3 \log(k) \end{aligned}$$

In what follows assume all structural parameters are known except ρ and λ. Suppose we have the following empirical estimates of the regression coefficients: $\beta_1 = 0.004$, $\beta_2 = 0.2$; $\gamma_1 = -0.15$, $\gamma_2 = -0.4$, $\gamma_3 = 0.05$.

b) Estimate the values of ρ and λ using indirect inference and the above five moments. To do this construct a grid with only 5 values for each of these two parameters, centered around the choice value above.

Part III: Optimal Default

Now consider the following dynamic problem for the equity holders in the firm,

$$e(z, k, b) = \max_{k', b'} \left[d(z, k, k', b, b') + \mathbb{E}_z M \max\{e(z', k', b'), 0\} \right],$$

where $d(\cdot)$ is the value of gross distributions and the innovations ϵ are follow a truncated standard normal distribution.

a) Assume $R^b = 1.01R^f$. Solve the Bellman equation for equity and plot the optimal investment and default decision for the equity holders.

b) Compute and plot the probability of default next period, conditional on the value of the shocks today $p(z, b', k')$.

c) Now use the conditional default probability to compute the required rate of return by bondholders, $R^b(b', k'; z)$ that ensures they make 0 profits. Assume for simplicity bondholders get paid 0 upon default.

d) Solve the Bellman equation for the equity holders taking as given this new function for the required rate of return by bondholders, $R^b(\cdot)$. Plot this new value function against the one found in section a).