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Problem Set 02

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Contents

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Part I. The model

Partial equilibrium problem of a representative agent firm with two technologies, tangible and intangible capital.

$$(states_{i,t}) = (k^T_{i,t}, k^I_{i,t}, b_{i,t}, z_{1,i,t}, z_{2,i,t})$$

$$(controls_{i,t}) = (k^T_{1,i,t}, k^I_{1,i,t}, k^T_{i,t+1}, l_{1,i,t}, l_{2,i,t}, b_{i,t+1})$$

The producer faces the following recursive problem:

$$J(states_{i,t}) = \max_{controls_{i,t}} \left(d_{i,t} + \frac{1}{1+r_{i,t}} E_{zz}\left[J(states_{i,t+1})\right] \right)$$

$$
\begin{aligned}
\text{S.T.}\quad d_{i,t} &= \left(\theta_1(k_{1,i,t}^T)^{\frac{\lambda_1-1}{\lambda_1}} + (1-\theta_1)(k_{1,i,t}^I)^{\frac{\lambda_1-1}{\lambda_1}}\right)^{\frac{\lambda_1}{\lambda_1-1}\alpha_1} (e^{z_{1,i,t}} l_{1,i,t})^{\gamma_1} + (1-\delta_T)k_{i,t}^T - k_{i,t+1}^T - \\
&\quad w(l_{1,i,t}+l_{2,i,t}) + b_{i,t+1} - (1+r_t)b_{i,t} - 0.02(b_{i,t+1}-b_{i,t})^2 - 0.01(b_{i,t}-0.2)^2 \\
k_{i,t+1}^I &= \left(\theta_2(k_{2,i,t}^T)^{\frac{\lambda_2-1}{\lambda_2}} + (1-\theta_2)(k_{2,i,t}^I)^{\frac{\lambda_2-1}{\lambda_2}}\right)^{\frac{\lambda_2}{\lambda_2-1}\alpha_2} (e^{z_{2,i,t}} l_{2,i,t})^{\gamma_2} + (1-\delta_I)k_{i,t}^I \\
k_{2,i,t}^T &= k_{i,t}^T - k_{1,i,t}^T \\
k_{2,i,t}^I &= k_{i,t}^I - k_{1,i,t}^I
\end{aligned}
$$

which gives the following equilibrium conditions (I drop subndices i,t and use recursive formulation notation to make the equations easier to read):

$$
\begin{aligned}
[k_1^T]: &\quad \theta_1\alpha_1(k_1^T)^{\frac{-1}{\lambda_1}} \left(\theta_1(k_1^T)^{\frac{\lambda_1-1}{\lambda_1}} + (1-\theta_1)(k_1^I)^{\frac{\lambda_1-1}{\lambda_1}}\right)^{\frac{\lambda_1}{\lambda_1-1}\alpha_1-1} (e^{z_1}l_1)^{\gamma_1} \\
&= E_z\left[\frac{1}{1+r_i} J_{k_1^T}(k'^T, k'^I, b\prime, z_1\prime, z_2\prime)\right] = E_z\left[\frac{1}{1+r_i} J_{k_1^T}(states')\right] \\
&= E_z\left[\frac{1}{1+r_i} J_{k^I}(k'^T, k'^I, b\prime, z_1\prime, z_2\prime) k'^I_{k_1^T}(k^T, k^I, b, z_1, z_2)\right] \\
&= E_z\left[\frac{1}{1+r_i}\{(1-\theta_1)\alpha_1(k_1^I(states'))^{\frac{-1}{\lambda_1}} \left(\theta_1(k_1^T(states'))^{\frac{\lambda_1-1}{\lambda_1}} + (1-\theta_1)(k_1^I(states'))^{\frac{\lambda_1-1}{\lambda_1}}\right)^{\frac{\lambda_1}{\lambda_1-1}\alpha_1-1}\right. \\
&\quad \left(e^{z_1'}l_1(states')\right)^{\gamma_1}\Big\} \\
&\quad \left.\left\{\theta_2\alpha_2(k^T-k_1^T)^{\frac{-1}{\lambda_2}} \left(\theta_2(k^T-k_1^T)^{\frac{\lambda_2-1}{\lambda_2}} + (1-\theta_2)(k^I-k_1^I)^{\frac{\lambda_2-1}{\lambda_2}}\right)^{\frac{\lambda_2}{\lambda_2-1}\alpha_2-1} (e^{z_2}l_2)^{\gamma_2}\right\}\right] \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
[k_1^I]: &\quad (1-\theta_1)\alpha_1(k_1^I)^{\frac{-1}{\lambda_1}} \left(\theta_1(k_1^T)^{\frac{\lambda_1-1}{\lambda_1}} + (1-\theta_1)(k_1^I)^{\frac{\lambda_1-1}{\lambda_1}}\right)^{\frac{\lambda_1}{\lambda_1-1}\alpha_1-1} (e^{z_1}l_1)^{\gamma_1} \\
&= E_z\left[\frac{1}{1+r_i} J_{k_1^I}(k'^T, k'^I, b\prime, z_1\prime, z_2\prime)\right] = E_z\left[\frac{1}{1+r_i} J_{k_1^I}(states')\right] \\
&= E_z\left[\frac{1}{1+r_i} J_{k^I}(k'^T, k'^I, b\prime, z_1\prime, z_2\prime) k'^I_{k_1^I}(k^T, k^I, b\prime, z_1, z_2)\right] \\
&= E_z[\frac{1}{1+r_i}\{(1-\theta_1)\alpha_1(k_1^I(states'))^{\frac{-1}{\lambda_1}} \left(\theta_1(k_1^T(states'))^{\frac{\lambda_1-1}{\lambda_1}} + (1-\theta_1)(k_1^I(states'))^{\frac{\lambda_1-1}{\lambda_1}}\right)^{\frac{\lambda_1}{\lambda_1-1}\alpha_1-1} \\
&\quad \left(e^{z_1'}l_1(states')\right)^{\gamma_1}\} \\
&\quad \left\{(1-\theta_2)\alpha_2(k^I-k_1^I)^{\frac{-1}{\lambda_2}} \left(\theta_2(k_2^T)^{\frac{\lambda_2-1}{\lambda_2}} + (1-\theta_2)(k^I-k_1^I)^{\frac{\lambda_2-1}{\lambda_2}}\right)^{\frac{\lambda_2}{\lambda_2-1}\alpha_2-1} (e^{z_2}l_2)^{\gamma_2} + (1-\delta_I)\right\}\Big] (2)
\end{aligned}
$$

where equation (2) uses: $k_2^T = k^T - k_1^T$.

$$
\begin{aligned}
[k'^T]: \quad & 1 \\
= \quad & E_z\left[\frac{1}{1+r_i}\left\{J_{k'^T}(k'^T, k'^I, b\prime, z_1\prime, z_2\prime)\right\}\right] = E_z\left[\frac{1}{1+r_i}\left\{J_{k'^T}(states')\right\}\right] \\
= \quad & E_z\left[\frac{1}{1+r_i}\left\{1-\delta_T+\theta_1\alpha_1 k_1^I(states')^{\frac{-1}{\lambda_1}}\right.\right. \\
& \left.\left.\left(\theta_1 k_1^T(states')^{\frac{\lambda_1-1}{\lambda_1}}+(1-\theta_1)k_1^I(states')^{\frac{\lambda_1-1}{\lambda_1}}\right)^{\frac{\lambda_1}{\lambda_1-1}\alpha_1-1}\left(e^{z_1}l_1(states')\right)^{\gamma_1}\right\}\right] \quad (3)
\end{aligned}
$$

$$
\begin{aligned}
[b']: \quad & 1 \\
= \quad & 0.04(b'-b)+E_z\left[\frac{1}{1+r_i}\{J_{b'}(k'^T, k'^I, b\prime, z_1\prime, z_2\prime)\}\right] \\
= \quad & 0.04(b'-b)+E_z\left[\frac{1}{1+r_i}\{J_{b'}(states')\}\right] \\
= \quad & 0.04(b'-b)+E_z\left[\frac{1}{1+r_i}\{(1+r)+0.02(b'-0.2)-0.04(b''-b\prime)\}\right] \quad (4)
\end{aligned}
$$

$$
\begin{aligned}
[l_1]: \quad & \gamma_1(e^{z_1})^{\gamma_1}l_1^{\gamma_1-1}\left(\theta_1(k_1^T)^{\frac{\lambda_1-1}{\lambda_1}}+(1-\theta_1)(k_1^I)^{\frac{\lambda_1-1}{\lambda_1}}\right)^{\frac{\lambda_1}{\lambda_1-1}\alpha_1} \\
= \quad & w \quad (5)
\end{aligned}
$$

$$
\begin{aligned}
[l_2]: \quad & E_z\left[\frac{1}{1+r_i}J_{k^I}(k'^T, k'^I, b\prime, z_1\prime, z_2\prime)k'^I_{l_2}(k^T, k^I, b\prime, z_1, z_2)\right] \\
= \quad & E_z\left[\frac{1}{1+r_i}\{(1-\theta_1)\alpha_1(k_1^I(states'))^{\frac{-1}{\lambda_1}}\left(\theta_1(k_1^T(states'))^{\frac{\lambda_1-1}{\lambda_1}}+(1-\theta_1)(k_1^I(states'))^{\frac{\lambda_1-1}{\lambda_1}}\right)^{\frac{\lambda_1}{\lambda_1-1}\alpha_1-1}\right. \\
& \left.\left(e^{z_1'}l_1(states')\right)^{\gamma_1}\right\} \\
& \left.\left\{\gamma_2(e^{z_2})^{\gamma_2}l_2^{\gamma_2-1}\left(\theta_2(k_2^T)^{\frac{\lambda_2-1}{\lambda_2}}+(1-\theta_2)(k_2^I)^{\frac{\lambda_2-1}{\lambda_2}}\right)^{\frac{\lambda_2}{\lambda_2-1}\alpha_2}\right\}\right] \\
= \quad & w \quad (6)
\end{aligned}
$$

where again: $k_2^T = k^T - k_1^T$ and $k_2^I = k^I - k_1^I$.
Gatherer:

$$V^G(z,w) = \max_{c,k',b'}\left(u(c)+\beta_G E_z\left[V(z',w')\right]\right)$$

$$\text{S.T.} \qquad c+qk'+b' = G(k)+qk+Rb$$

which gives the following equilibrium conditions (after FOCs):

$$
\begin{aligned}
q &= \beta_G E_z \left[\frac{u'(c')}{u'(c)} \left(G'(k') + q \right) \right] \\
1 &= \beta_G E_z \left[R \frac{u'(c')}{u'(c)} \right]
\end{aligned}
$$

Parameters

```
params.a=1;
```

```
...
grids.theta80grid.kF = { linspace(0.01, 0.99, 10) };
grids.theta80grid.levF = { linspace(0.6,1,20) };
...
expers = {'bench',{}; % bench experiment
    'theta875',{'theta',0.875; 'grid','theta875grid'}; % second
    'alpha90',{'alpha',0.9; 'alower',0.85; 'grid','alpha90grid'}; % third
    'theta80',{'theta',0.80; 'grid','theta80grid'};% fourth
};
```

Part II

$$
b^F_{t+1} \leq \theta \left(a(x_{t+1}) + q(x_{t+1}) \right) k^F_{t+1} \qquad \forall x_{t+1} \in X_{t+1}
$$

The :

$$
V^F(z,w) = \max_{c,k',b'} \left(u(c) + \beta_F E_z \left[V(z',w') \right] \right)
$$

$$
\begin{aligned}
\text{s.t.} \qquad c + qk' + b' &= ak + qk + Rb - \phi(k'/k-1)^2 k/2 \\
b' &\leq \theta \left(a' + q' \right) k'
\end{aligned}
$$

which gives the following equilibrium conditions (after FOCs):

$$
\begin{aligned}
q &= \beta_F E_z \left[\frac{u'(c')}{u'(c)} (a + q + \frac{\phi((k'/k)^2 - 1)}{2}) \right] + \mu\theta(a' + q') \\
1 &= \mu + \beta_F E_z \left[R \frac{u'(c')}{u'(c)} \right] \\
0 &= \mu \left[\theta(a' + q')k' + b' \right]
\end{aligned}
$$

Figure 1: IRFs 1 for unmodified code:

Figure 2: IRFs 2 for unmodified code:

2.a)

The .

2.b) New Steady State.

Again, the steady state is very similar as the previous case, with a few modifications, namely:

$$
\begin{aligned}
\mu_F &= 1 - \beta_F/\beta_G \\
q &= \frac{\beta_F a + \theta a \mu_F}{1 - \beta_F - \theta \mu_F} \\
k_{F,ss} &= \left[\frac{\alpha \underline{a}(1 - \beta_F - \theta\mu_F)\beta_G)}{(1 - \beta_G)a(\beta_F + \theta\mu_F)}\right]^{\frac{1}{1-\alpha}} \\
b_F &= -\theta(q + a)k_{F,ss}
\end{aligned}
$$

2.c) Rewrite constraint.

Using the minimum of (q+a) at any given time is enough, as that guarantees that it will always be binding (if it is binding for the lowest state). The code already has an implementation for such variable, which needs to be indicated in the 'experdef' file, and needs to add $min(q + a))$ in the 'calcEquations' files.

2.f) Main economic effects.