Strategic Solving

1

Equations with Variables on Both Sides

WARM UP

Solve each equation.

1. $2.5x + 100 = 600$

2. $10 = 2x - 4$

3. $\frac{1}{4}x + 5 = 30$

LEARNING GOALS

- Use strategies to solve linear equations with variables on both sides of the equals sign.
- Solve linear equations with rational number coefficients.
- Combine like terms and use the Distributive Property to solve linear equations.

You have solved equations by combining like terms and using inverse operations. How can you solve equations when there are variables on both sides of the equation?

Getting Started

Build It Up and Break It Down

The Properties of Equality allow you to solve equations.

Properties of Equality	For all numbers *a, b,* and *c*
Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $ac = bc$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

These properties also allow you to create more complex equations. For example, given the equation $x = 2$, you can use the Addition Property of Equality to create $x + 1 = 2 + 1$, which is the same as $x + 1 = 3$. Since you used the Properties of Equality, the two equations have the same solution.

To solve a two-step equation, isolate the variable term on one side of the equation and the constant on the other side of the equation. Then multiply or divide both sides of the equation by the numeric coefficient to determine the value of the variable.

1. Consider each given equation. Use the Properties of Equality to create an equivalent equation in the form $ax + b = c$, where *a, b,* and *c* can be any number. Record the Properties of Equality you used to create your new equation.

a. $x = 5$

b. $x = -1$

2. Give each of your equations to a partner to verify that each equation has the correct solution.

ACTIVITY 1.1

Factoring to Solve Equations

You have previously solved two-step equations using a variety of strategies. In this activity you will learn different strategies to solve equations with variables on both sides. Remember, to solve an equation means to determine the value of the unknown that makes the equation true.

> To begin solving an equation with variables on both sides of the equation, move all the variable terms to one side of the equation and all the constants to the other side of the equation.

Consider the equation $5x + 3 = 2x + 5$.

Teddy and Topher each solved it in a different way. Analyze their solution strategies.

Teddy

$$\begin{array}{rcl} 5x + 3 & = & 2x + 5 \\ -5x & & -5x \\ \hline 3 & = & -3x + 5 \\ -5 & & -5 \\ \hline \frac{-2}{-3} & = & \frac{-3x}{-3} \\ \frac{2}{3} & = & x \\ x & = & \frac{2}{3} \end{array}$$

Topher

$$\begin{array}{rcl} 5x + 3 & = & 2x + 5 \\ -2x & & -2x \\ \hline 3x + 3 & = & 5 \\ -3 & & -3 \\ \hline 3x & = & 2 \\ x & = & \frac{2}{3} \end{array}$$

1. Compare the two solution strategies.

a. How were their solution strategies the same? How were they different?

b. Which strategy do you prefer? Explain your choice.

2. Solve each equation. Describe why you chose your solution strategy.

a. $x - 6 = 5x + 10$

b. $2x - 7 = -5x + 14$

NOTES

Consider the two different equations that Sandy and Sara solved.

Sandy

$$3x + 9 = 6x - 30$$
$$\frac{3x + 9}{3} = \frac{6x - 30}{3}$$
$$x + 3 = 2x - 10$$
$$-x \qquad -x$$
$$3 = x - 10$$
$$+10 \qquad +10$$
$$13 = x$$
$$x = 13$$

Sara

$$-x - 2 = -4x - 1$$
$$\frac{-x - 2}{-1} = \frac{-4x - 1}{-1}$$
$$x + 2 = 4x + 1$$
$$-x \qquad -x$$
$$2 = 3x + 1$$
$$-1 \qquad -1$$
$$1 = 3x$$
$$\frac{1}{3} = \frac{3x}{3}$$
$$\frac{1}{3} = x$$
$$x = \frac{1}{3}$$

3. Sandy and Sara each divided both sides of their equations by a factor and then solved.

   a. Explain the reasoning used by each.

   b. Do you think this solution strategy will work for any equation? Explain your reasoning.

4. Solve each equation using the strategy similar to Sandy and Sara.

   a. $-4x + 8 = 2x + 10$

   b. $-42x = -4x - 1$

ACTIVITY 1.2

Solving Equations with Efficiency

As you saw in the last activity, there can be more than one way to solve an equation. Sometimes an efficient strategy involves changing the numbers in the equation—in mathematically appropriate ways.

A savvy mathematician (you!) can look at an equation, see the structure of the equation, and look for the most efficient solution strategy.

WORKED EXAMPLE

Consider the equation $\frac{1}{3}(2x + 7) + \frac{5}{6} = \frac{5}{3}x$.

You can multiply both sides of the equation by the least common denominator (LCD) of the fractions to convert the fractions to whole numbers.

$\frac{1}{3}(2x + 7) + \frac{5}{6} = \frac{5}{3}x$ ← The LCD of the fractions is 6. Multiply both sides by 6.

$2(2x + 7) + 5 = 10x$ ← Rewrite using the Distributive Property.

$4x + 14 + 5 = 10x$

1. Explain how both sides of the equation were multiplied by 6 in the first step.

2. What is the solution to the equation $\frac{1}{3}(2x + 7) + \frac{5}{6} = \frac{5}{3}x$? Check your solution.

3. **Explain why Cody's reasoning is incorrect.**

Cody

$$-\frac{3}{4}x = \frac{1}{2}x + \frac{5}{4}$$

$$4\left(-\frac{3}{4}x\right) = 4\left(\frac{1}{2}x + \frac{5}{4}\right)$$

$$-3x = 2x + 5$$

$$-5x = 5$$

$$x = -1$$

Since I multiplied both sides by 4 to get the solution, I have to divide the solution by 4:

$$x = -\frac{1}{4}$$

4. **Solve each equation by first multiplying both sides of the equation by the LCD. Check your solutions.**

   a. $\frac{1}{4}(x - 5) + 9 = \frac{1}{2}x$

   b. $\frac{5}{4}\left(x + \frac{1}{2}\right) + 8 = \frac{1}{8}x$

5. **Mindy and David multiplied both sides of the equation $2.5x + 1.4 = 0.5x + 2$ by 10 before solving the equation. The first step of each strategy is shown. Who's correct? What is the error in the other strategy?**

Mindy

$$25x + 14 = 5x + 2$$

David

$$25x + 14 = 5x + 20$$

You can multiply both sides of an equation by powers of 10 to convert all numbers to whole numbers.

ACTIVITY 1.3

Practice Solving Equations

NOTES

Solve each equation.

1. $12.6 + 4x = 9.6 + 8x$

2. $-12.11x - 10.5 = 75.6 - 3.5x$

3. $\frac{10x + 2}{2} = 4x + \frac{1}{4}$

4. $\frac{3}{8}(x + 8) = \frac{1}{2}(x + 5) + \frac{1}{4}$

5. $\frac{-2(5x + 4)}{3} = -3(3x + 2) - \frac{7}{3}$

NOTES

TALK the TALK

Building Strategically

Use each starting equation to build an equation with variables on both sides that can be solved using the given strategy. Then, give your equations to a partner to solve.

1. $h = 1.6$, factor out a number from both sides

2. $j = 5$, multiply both sides by the LCD to rewrite fractions as whole numbers

3. $k = \frac{1}{3}$, multiply both sides by a power of 10 to rewrite decimals as whole numbers

Assignment

Write

Explain the process of solving an equation with variables on both sides.

Remember

You can use Properties of Equality to rewrite equations and increase your efficiency with solving equations.

- Factor out a number from both sides.
- Multiply both sides of an equation by the least common denominator of the fractions to rewrite fractions as whole numbers.
- Multiply both sides of an equation by a power of 10 to rewrite decimals as whole numbers.
- Use the Distributive Property to rewrite expressions.

Practice

Solve each equation.

1. $5x + 15 = 75 - 25x$
2. $\frac{1}{4}x - 3 = \frac{1}{2}x + 12$
3. $4x = 20x - 24$
4. $11.3x + 12.8 = 7.5x + 35.6$
5. $9.6x - 15.4 = -4.3x + 26.3$
6. $-2x - 1.4 = 6 + 3x$

Stretch

You can solve an equation with two variables by trying different values. What is the solution to the equation $2x + 3y = 13$?

Review

1. Rodell took a survey of his classmates. The data from the survey are shown in the two-way table.

Student's Lunch Preference

		Lunch Options				
Gender		Chicken Nuggets	Peanut Butter & Jelly	Pizza	Salad	Total
	Male	2	3	4	0	9
	Female	3	1	3	4	11
	Total	5	4	7	4	20

a. Which lunch option is the most favorite of the males?

b. Which lunch option is the most favorite of the females?

2. Isabel surveyed three classes about their favorite season. The data from the survey are shown in the two-way table. Complete the relative frequencies for each row. If necessary, round decimals to the nearest thousandth.

Student's Season Preference

		Seasons				
		Winter	Spring	Summer	Fall	Total
Classes	Class A	9	2	7	6	24
	Class B	2	5	9	4	20
	Class C	8	6	10	4	28
	Total	19	13	26	14	72

Student's Season Preference

		Seasons				
		Winter	Spring	Summer	Fall	Total
Classes	Class A	$\frac{9}{24} = 0.375$				
	Class B					
	Class C					

3. Calculate the slope of the line represented by each table.

a.

X	*Y*
4	8
10	11
16	14
20	16

b.

X	*Y*
2	5
4	3
5	2
8	–1