KON613E – Probabilistic Methods in Robotics 11 November 2020
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2020-2021 Fall Term
Homework II Return Due: 22 Nov 2020 at 23:55

Question: For the given problem at Appendix –A, solve the problem using EKF method,

Appendix-A:

4. We noted in the text that the EKF linearization is an approximation. To see how bad this approximation is, we ask you to work out an example. Suppose we have a mobile robot operating in a planar environment. Its state is its x-y-location and its global heading direction θ. Suppose we know x and y with high certainty, but the orientation θ is unknown. This is reflected by our initial estimate

$$\mu = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 10000 \end{pmatrix}$$

   (a) Draw, graphically, your best model of the posterior over the robot pose after the robot moves $d = 1$ units forward. For this exercise, we assume the robot moves flawlessly without any noise. Thus, the expected location of the robot after motion will be

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x + \cos\theta \\ y + \sin\theta \\ \theta \end{pmatrix}$$

   For your drawing, you can ignore θ and only draw the posterior in x-y-coordinates.

   (b) Now develop this motion into a prediction step for the EKF. For that, you have to define a state transition function and linearize it. You then have to generate a new Gaussian estimate of the robot pose using the linearized model. You should give the exact mathematical equations for each of these steps, and state the Gaussian that results.

   (c) Draw the uncertainty ellipse of the Gaussian and compare it with your intuitive solution.

   (d) Now incorporate a measurement. Our measurement shall be a noisy projection of the x-coordinate of the robot, with covariance $Q = 0.01$. Specify the measurement model. Now apply the measurement both to your intuitive posterior, and formally to the EKF estimate using the standard EKF machinery. Give the exact result of the EKF, and compare it with the result of your intuitive analysis.

   (e) Discuss the difference between your estimate of the posterior, and the Gaussian produced by the EKF. How significant are those differences? What can be changed to make the approximation more accurate? What would have happened if the initial orientation had been known, but not the robot's y-coordinate?