Kerim MORAL
518191043

HW1

$bel(x_t) = \eta \underbrace{p(z_t|x_t)}_{\sim N(z_t; C_t x_t, Q_t)} \overline{bel}(x_t)$

$\hookrightarrow \sim N(x_t; \bar{\mu}_t; \bar{\Sigma}_t)$

$bel(x_t) = \eta \exp\{-J_t\}$

$J_t = \frac{1}{2}(z_t - C_t x_t)^T Q_t^{-1}(z_t - C_t x_t) + \frac{1}{2}(x_t - \bar{\mu}_t)^T \bar{\Sigma}_t^{-1}(x$

① $\frac{\partial J}{\partial x_t} = -C_t^T Q_t^{-1}(z_t - C_t x_t) + \bar{\Sigma}_t^{-1}(x_t - \bar{\mu}_t)$

② $\frac{\partial^2 J}{\partial x^2} = C_t^T Q_t^{-1} C_t + \bar{\Sigma}_t^{-1}$

④ $\Rightarrow \Sigma_t = (C_t^T Q_t^{-1} C_t + \bar{\Sigma}_t^{-1})^{-1}$

① equation is set to 0;

③ $\bar{\Sigma}_t^{-1}(\mu_t - \bar{\mu}_t) = C_t^T Q_t^{-1}(z_t - C_t \mu_t)$

Right side of the eq. can be arranged as:

$C_t^T Q_t^{-1}(z_t - C_t \mu_t) = C_t^T Q_t^{-1}(z_t - C_t \mu_t + C_t \bar{\mu}_t - C_t \bar{\mu}_t)$

$= C_t^T Q_t^{-1}(z_t - C_t \bar{\mu}_t) - C_t^T Q_t^{-1} C_t(\mu_t - \bar{\mu}_t)$

Substitute to ③;

$C_t^T Q_t^{-1}(z_t - C_t \bar{\mu}_t) = \underbrace{(C_t^T Q_t^{-1} C_t + \bar{\Sigma}_t^{-1})}_{\Sigma_t^{-1}}(\mu_t - \bar{\mu}_t)$

$\Sigma_t C_t^T Q_t^{-1}(z_t - C_t \bar{\mu}_t) = \mu_t - \bar{\mu}_t$

define Kalman gain as:

$K_t = \Sigma_t C_t^T Q_t^{-1}$ so;

$\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$

$$K_t = \Sigma_t C_t^T Q_t^{-1} = \Sigma_t C_t^T Q_t^{-1} \underbrace{(C_t \bar{\Sigma}_t C_t^T + Q_t)(C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}}_{I}$$

$$= \Sigma_t (C_t^T Q_t^{-1} C_t \bar{\Sigma}_t C_t^T + C_t^T \underbrace{Q_t^{-1} Q_t}_{I})(C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$

$$= \Sigma_t (C_t^T Q_t^{-1} C_t \bar{\Sigma}_t C_t^T + C_t^T)(C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$

$$= \Sigma_t (C_t^T Q_t^{-1} C_t \bar{\Sigma}_t C_t^T + \underbrace{\bar{\Sigma}_t^{-1} \bar{\Sigma}_t}_{I} C_t^T)(C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$

$$= \Sigma_t (C_t^T Q_t^{-1} C_t + \bar{\Sigma}_t^{-1}) \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$

$$= \underbrace{\Sigma_t \Sigma_t^{-1}}_{I} \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$

$$= \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1} \text{, from } (4)$$

$$(\bar{\Sigma}_t^{-1} + C_t^T Q_t^{-1} C_t)^{-1} = \bar{\Sigma}_t - \bar{\Sigma}_t C_t^T (Q_t + C_t \bar{\Sigma}_t C_t^T)^{-1} C_t \bar{\Sigma}_t$$

$$\Sigma_t = (C_t^T Q_t^{-1} C_t + \bar{\Sigma}_t^{-1})^{-1} = \bar{\Sigma}_t - \bar{\Sigma}_t C_t^T (Q_t + C_t \bar{\Sigma}_t C_t^T)^{-1} C_t \bar{\Sigma}_t$$

$$= [I - \underbrace{\bar{\Sigma}_t C_t^T (Q_t + C_t \bar{\Sigma}_t C_t^T)^{-1}}_{K_t} C_t] \bar{\Sigma}_t$$

$$= (I - K_t C_t) \bar{\Sigma}_t$$