When and Where does our Working Memory Take Place?

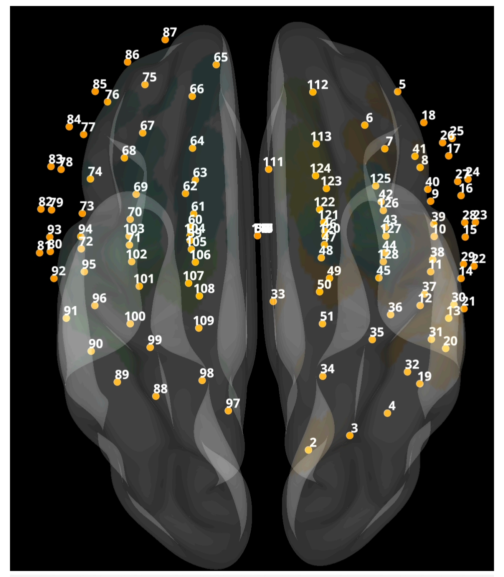
A multi-level sub-graph analysis of brain functional connectivities.

Maoran Xu, Leo Duan
Department of Statistics, University of Florida

EEG: Measuring our Working Memory

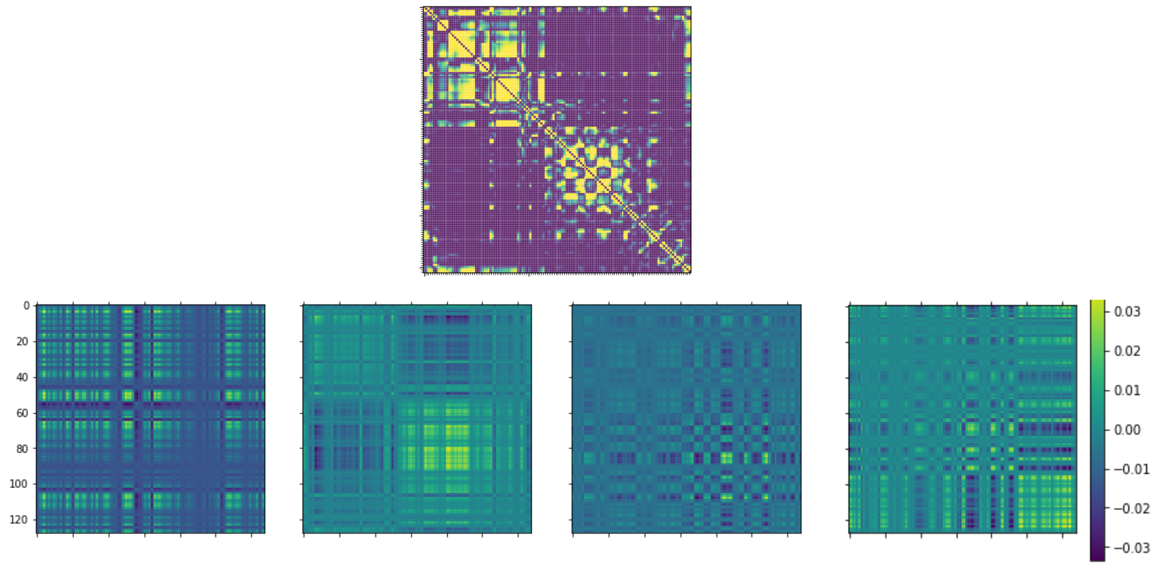
Dataset

- The data was collected in a study of human brain functional connectivities in working memory.
- 20 healthy subjects were asked to do the Sternberg verbal working memory task with three levels of memory load (load 2, 4 and 6). The subjects were wearing EEG head caps with 128 channels of electrodes.



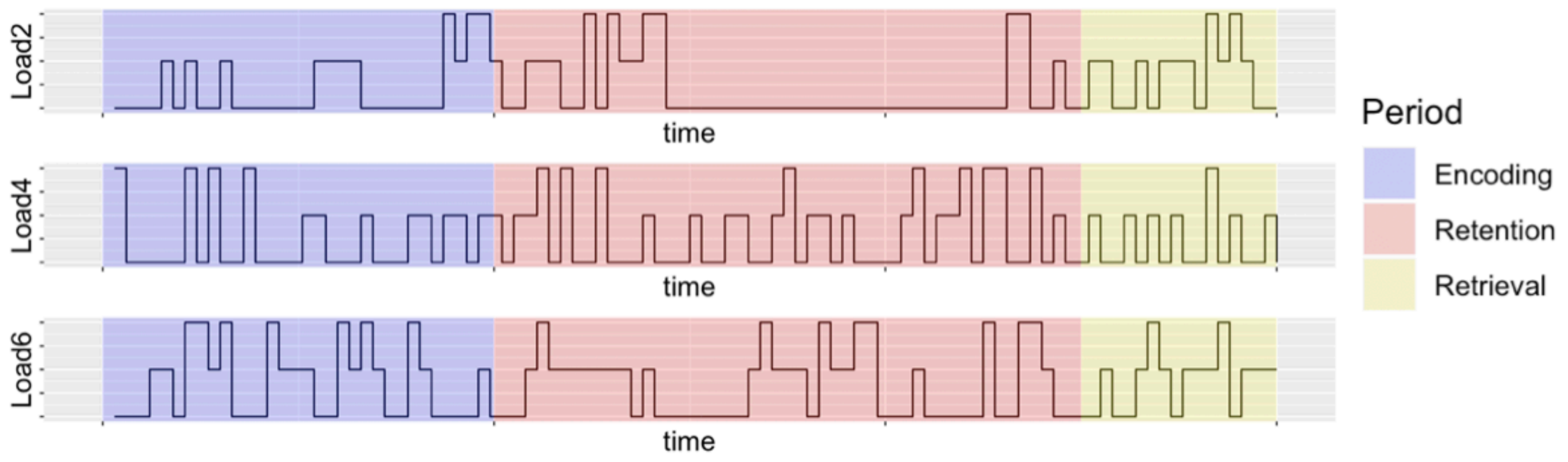
Motivation

- We extract the correlation between each pair of the channels. We study the adjacency matrices of measurement over time period.
- The brain network is large. However, the brain connectivity displays coarse-grain structures that may relate to latent low-dimensional estimation.
- During different period of memory, and loads, our brains display different connectivity.



A Case Study: Change Point Detection

We model the change point detection task as a combination of Hidden Markov Model and clustering problem.



Treed-SVD: Partition the Brian Network

The Tree-Spanning Stiefel Space

For a given graph $G=(V, E, A)$ with adjacency matrix A, we view it as a common signal-plus-noise graph

$$\boldsymbol{A} = \boldsymbol{P}\boldsymbol{\Lambda}\boldsymbol{P}^{\intercal} + \mathcal{E},$$

where P is an orthonormal matrix and every column of P has a maximum of 2^{l-1} unique values. Λ is a diagonal matrix indicating the weight of each level.

We call such matrix space a Tree-Spanning Stiefel Space

$$\mathcal{V}_{\mathcal{T}}^{d\times n} = \mathcal{T}_d \cap \mathcal{V}^{d\times n}.$$

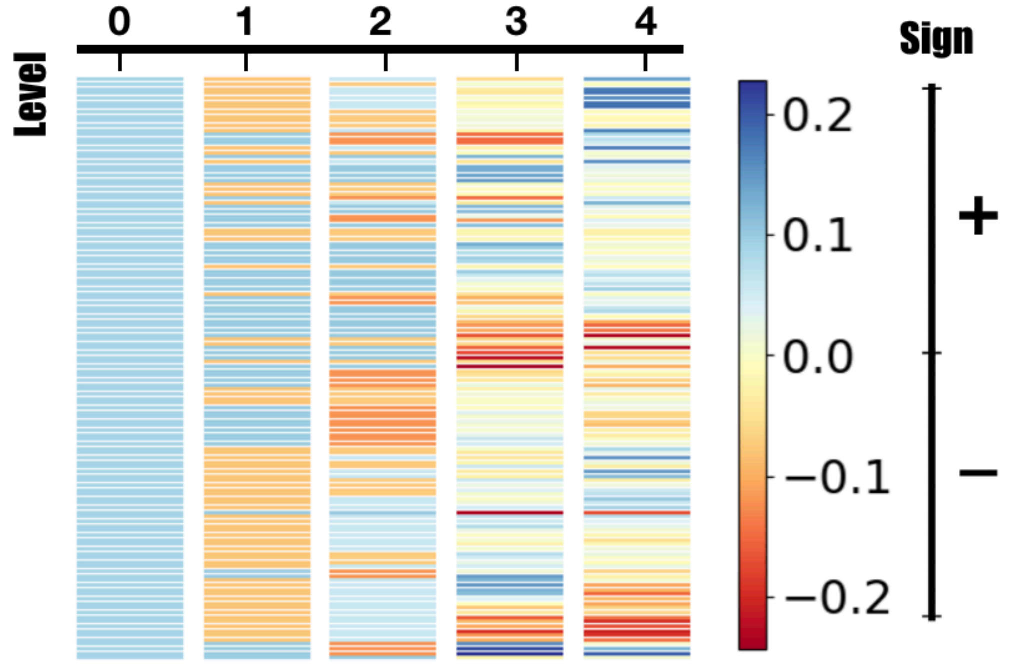
where

$$\mathcal{V}^{d\times n} = \{\boldsymbol{P} \in \mathbb{R}^{n\times d} : \boldsymbol{P}^{\intercal}\boldsymbol{P} = \boldsymbol{I}_d\}$$

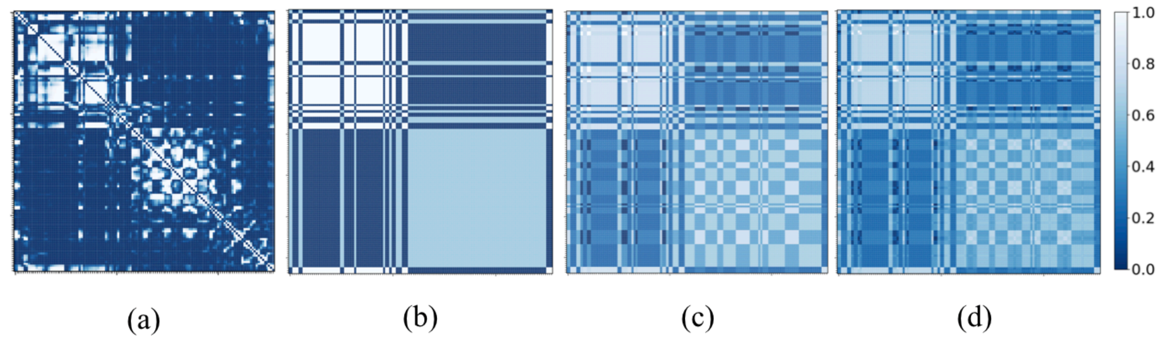
and

$$\mathcal{T}_d = \{\boldsymbol{P} \in \mathbb{R}^{n\times d} : P_{i,l} = P_{i',l} \text{ if } i, i' \in \mathcal{N}_{l,s}\ \forall l, s\}.$$

The Tree-Spanning Stiefel Space reduces the dimensionality while keeping the independence between different scales.



The P matrix for a 4-depth tree decomposition.
Each column shows the bi-partition within each scale while summing to 0.



The sequentially generated estimate for a 3-layer Treed-SVD decomposition.

The Computing Algorithm

- Estimating the structure:

1. Compute the eigenvector of $A - \tilde{A}_{1:k}$ corresponding to the largest eigenvalue, and partition by the sign as $\mathcal{T}_{k+1}$.
2. Maximize the conditional likelihood

$$L(P_{k+1}, \lambda_{k+1} | \tilde{\boldsymbol{A}}_{1:k}, \boldsymbol{A}) = \exp\{-||(\boldsymbol{A} - \tilde{\boldsymbol{A}}_{1:k}) - \lambda_{k+1} P_{k+1} P_{k+1}^{\intercal}||_F\}.$$

by solving the equivalent quadratic programming:

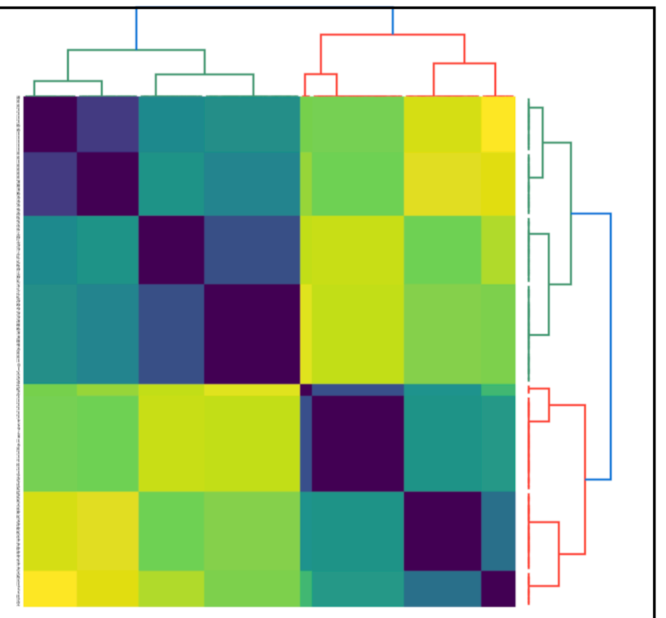
$$\max_{P_{k+1}} P_{k+1}^{\intercal}\left(\boldsymbol{A} - \tilde{\boldsymbol{A}}_{1:k}\right)P_{k+1}$$

$$\text{subject to } \boldsymbol{P} \in \mathcal{V}_{\mathcal{T}}^{(k+1)\times n},$$

and set $\lambda_{k+1} = P_{k+1}^{T} A P_{k+1}$.

Future Applications:

The estimated matrix forms treed structure, which can be used for clustering, Region of Interest detection and change point detection.



Reference:

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