When and Where does our Working Memory Take Place?

A multi-level sub-graph analysis of brain functional connectivities.

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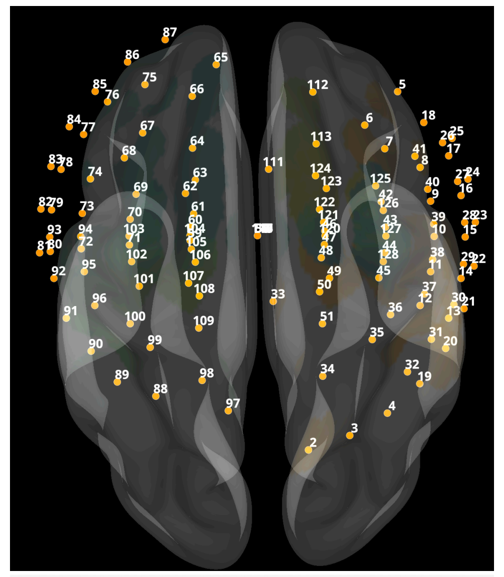
EEG: Measuring our Working Memory

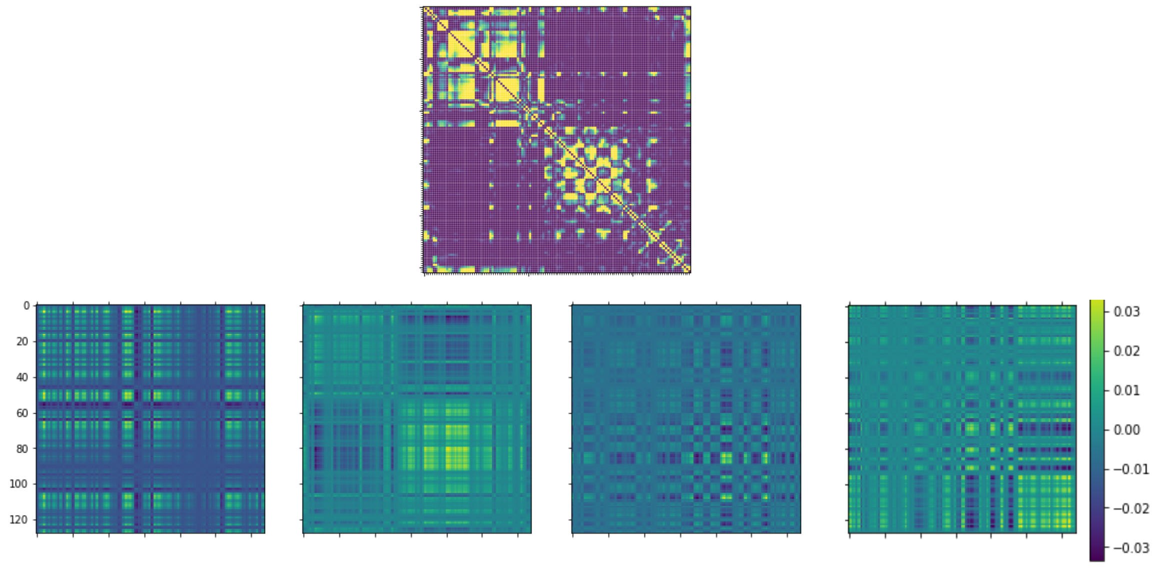
Dataset

- The data was collected in a study of human brain functional connectivities in working memory.
- 20 healthy subjects were asked to do the Sternberg verbal working memory task with three levels of memory load (load 2, 4 and 6). The subjects were wearing EEG head caps with 128 channels of electrodes.



Motivation

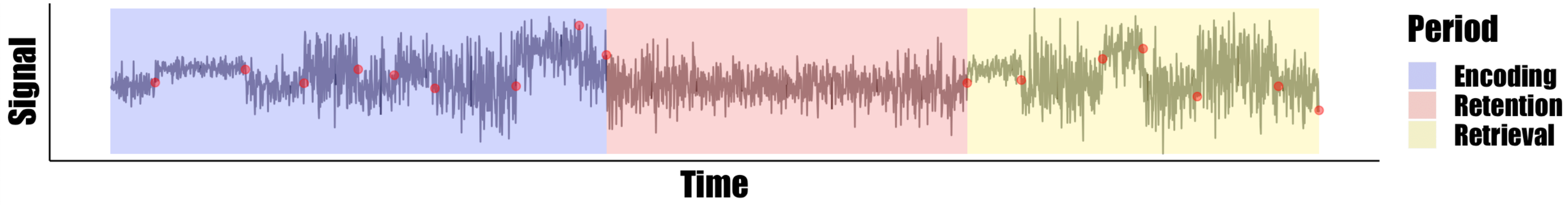
- We extract the correlation between each pair of the channels. We study the adjacency matrices of measurement over time period.
- The brain network is large. However, the brain connectivity displays coarse-grain structures that may relate to latent low-dimensional estimation.
- During different period of memory, and loads, our brains display different connectivity.



A Case Study: Change Point Detection

We model the change point detection task as a combination of Hidden Markov Model and clustering problem. Suppose there are K hidden states, where the center for the state is $\mu^{(k)}$. A_t belongs to state z_t. Denote the transition probability from k to l by $p_{k,l}$.
The likelihood is therefore

$$L(\boldsymbol{A}_t, z_t, \boldsymbol{\mu}^{(z_t)}) = \prod_{t=1}^{T} p_{z_{t-1}, z_t} \exp\{-\|\boldsymbol{A}_t - \boldsymbol{\mu}^{(z_t)}\|\},$$



Treed-SVD: Partition the Brian Network

Framework:

- For a given adjacency matrix A, we want to find out the Treed-SVD decomposition as

$$A = P\Lambda P^T,$$

where P is an orthonormal matrix and every column of P displays a level partition.

- The structure is generated level-by-level:
  1. Compute the eigenvector of A corresponding to the largest eigenvalue, and partition by the sign as $\mathcal{T}_{l,s}$.
  2. Solve the quadratic programming

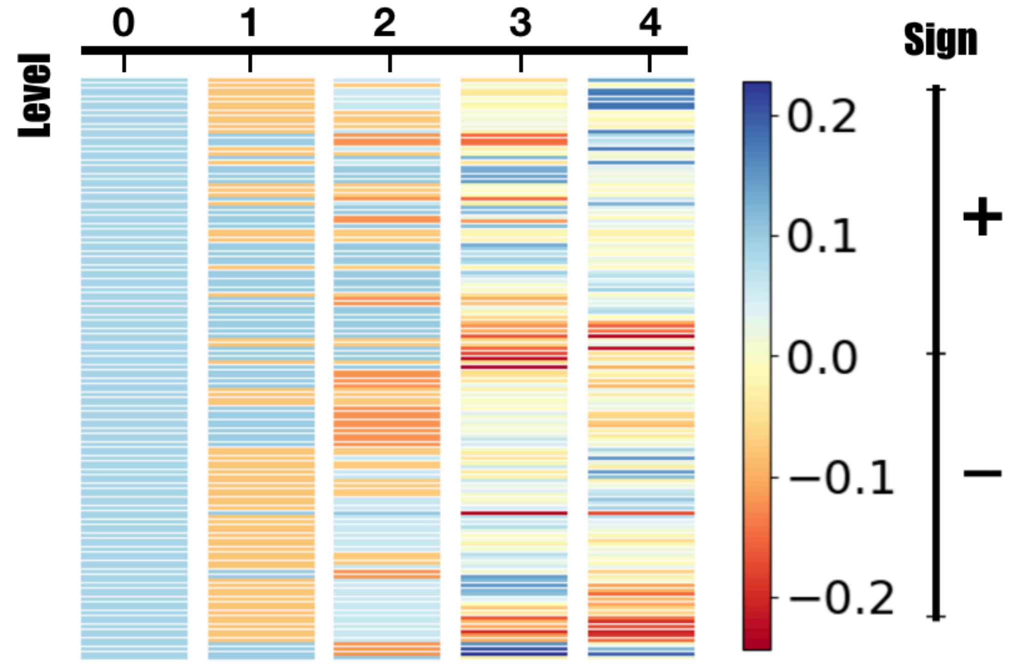
$$\arg\min_{\lambda_k, \boldsymbol{p}_k} \left|\left|\boldsymbol{R}_{k-1} - \lambda_k \boldsymbol{p}_k \boldsymbol{p}_k^{\intercal}\right|\right|_F$$

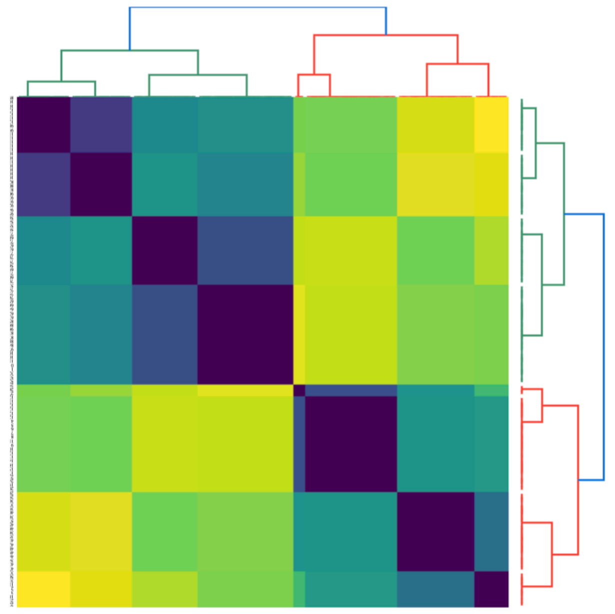
$$\text{subject to } (\boldsymbol{p}_1, \boldsymbol{p}_2, \ldots, \boldsymbol{p}_k) \sim \{\mathcal{T}_{l,s}\}_{l=1,\ldots,}$$

and set

$$\lambda_k = tr\left(\boldsymbol{p}_k^{\intercal} \boldsymbol{R}_{k-1} \boldsymbol{p}_k\right).$$

  3. Compute the residual matrix as $R_k = R_{k-1} - \lambda_k p_k p_k^T$.





The Hidden Markov Clustering

1. Conditioned on $\{z_t\}_t$, update $\{\mu^{(k)}\}_k$ using Treed-SVD algorithm.
2. Conditioned on $\{\mu^{(k)}\}_k$, update the latent assignment one-at-a-time, from the categorical distribution with

$$pr(z_t = k \mid z_{t-1}, z_{t+1}) = \frac{p_{z_{t-1},k} p_{k,z_{t+1}} \exp\{-\|\boldsymbol{A}_t - \boldsymbol{\mu}^{(k)}\|\}}{\sum_{l=1}^{K} p_{z_{t-1},l} p_{l,z_{t+1}} \exp\{-\|\boldsymbol{A}_t - \boldsymbol{\mu}^{(l)}\|\}}$$

3. Conditioned on $\{z_t\}_t$, update the transition probabilities by

$$(p_{k,1}, \ldots, p_{k,K}) \sim \text{Dir}\{\alpha + \sum_{t=1}^{T} \mathbf{1}(z_{t-1} = k, z_t = 1), \ldots, \alpha + \sum_{t=1}^{T} \mathbf{1}(z_{t-1} = k, z_t = K)\}$$

Reference:

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[2] Fornito, A., Zalesky, A., & Breakspear, M. (2013). Graph analysis of the human connectome: promise, progress, and pitfalls. Neuroimage, 80, 426-444.
[3] Yu, Y., Wang, T., & Samworth, R. J. (2014). A useful variant of the Davis – Kahan theorem for statisticians. Biometrika, 102(2), 315-323.
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