Hypothesis Testing

$(100,100)$ A: $\overline{X_M} = 72$, $\overline{X_E} = 73$

B: $\overline{X_m} = 72$, $\overline{X_E} = 71.8$

C: $\overline{X_m} = 72$, $\overline{X_E} = 65$

❑ Hypothesis testing is a way of making statistical decisions using observed or experimental data

❑ **Example** - Do students perform better if tests are given in the morning?

❑ **Example** - Is the coin biased or not?

❑ **Example** - Should the new book be adopted or not?

$(10,10)$ D: $\overline{X_m} = 72$, $\overline{X_E} = 65$

Key Terms in Hypothesis Testing

❑ Null Hypothesis, usually denoted by H_0

❑ Alternative Hypothesis, usually denoted by H_1 or H_a

❑ Hypothesis testing is performed after data is available for a sample - $X_1, X_2, \ldots, X_n$

❑ Decision of testing the hypothesis is whether to reject the null hypothesis or not to reject the null hypothesis

❑ Basis - from the value of a **statistic** T, which is calculated from the sample, called the **test statistic**

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Errors, Level and Power

	Null Rejected	Null not rejected
Null is true	**Type 1 error**	
Null is false		**Type 2 error**

Table: Type 1 and Type 2 error

❑ Probability of type 1 error - level of the test or significance level of the test, denoted by α

❑ Probability of type 2 error usually denoted by β

❑ 1-β is called the power of the test

Remarks on α and β

- [ ] We control the type 1 error by our choice of level of significance α
- [ ] We then try to maximize power $1 - \beta$
- [ ] We cannot reduce the chances of both type 1 and type 2 error simultaneously
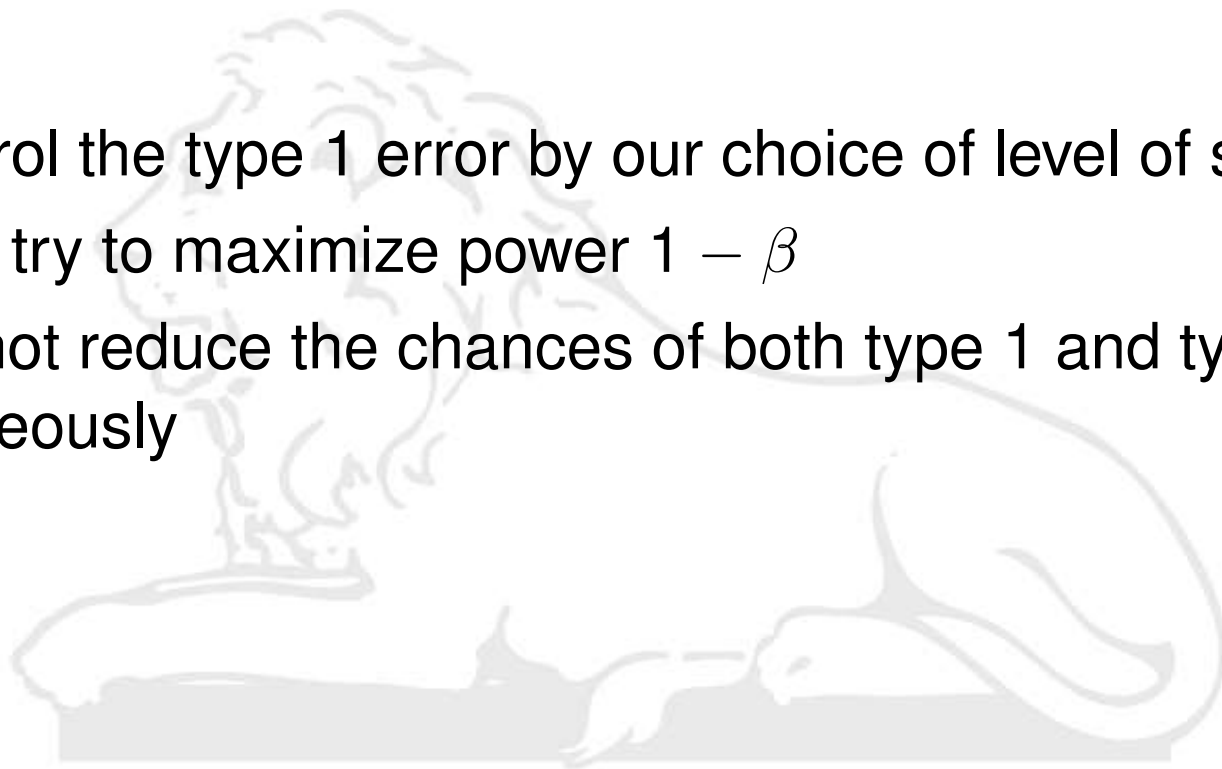
Example

$$(1000, 700) \to$$

$$(1000000, 700000) \to$$

BCCI was accused of using a biased coin in the toss for cricket match. A test was thus performed to check whether the given coin is biased towards heads. The coin was tossed 10 times.

1. Observed result - HHTHHHHTHH
2. Null Hypothesis $p = 0.5$
3. Alternative Hypothesis is $p > 0.5$

It was decided that the null will be rejected if the number of heads is greater than 7?

What is the level of the test?

What is the power of the test?

$$P\left(\text{Heads} \geqslant 8\right)$$

$$= {}^{10}C_8 (0.5)^8 (0.5)^2 + {}^{10}C_9 (0.5)^9 0.5^1 + 0.5^{10}$$

Hypothesis Testing

❑ Null Hypothesis and Alternative Hypothesis

❑ Null - typically the status quo or the one that people put more faith on, or the one that is easier to describe

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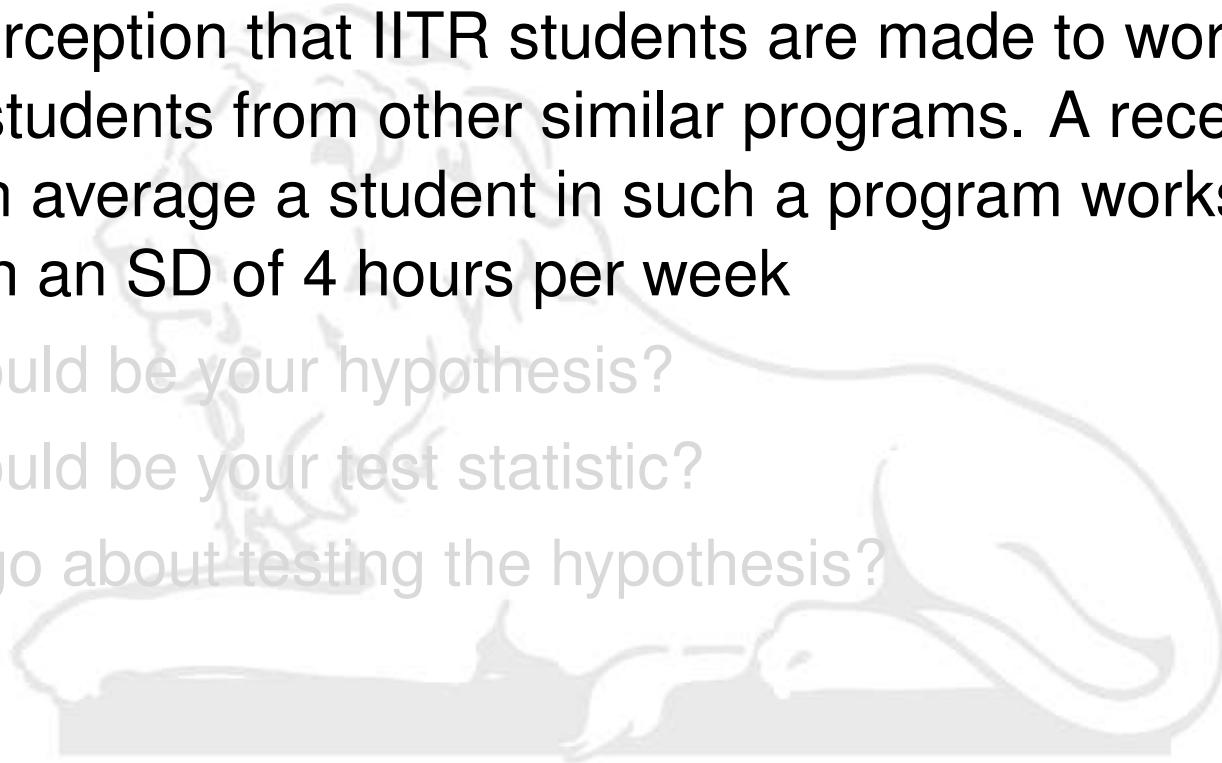
Building Hypothesis - Examples

- ❏ Coin is biased towards heads

- ❏ Coin is not an unbiased coin

- ❏ Gravity Fitness Gym, Roorkee claims that if you join the gym, you will lose more than 5 kgs in one month on an average.

- ❏ BJP IT team claims that with the marketing strategy that they have adopted, more than 40% people who were voters of other parties will vote for BJP in the coming elections

Building Hypothesis - Example

There is a perception that IITR students are made to work much harder than students from other similar programs. A recent study states that on average a student in such a program works for 18 hours per week with an SD of 4 hours per week
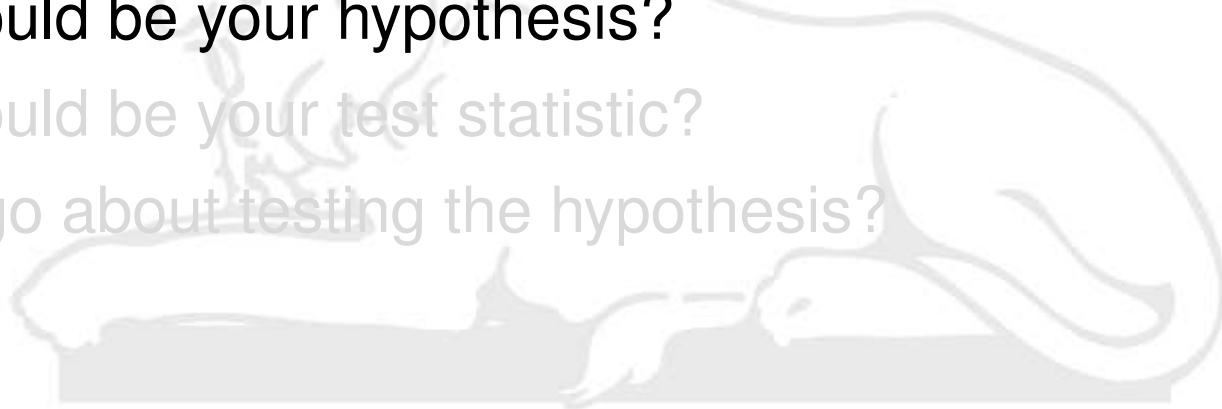
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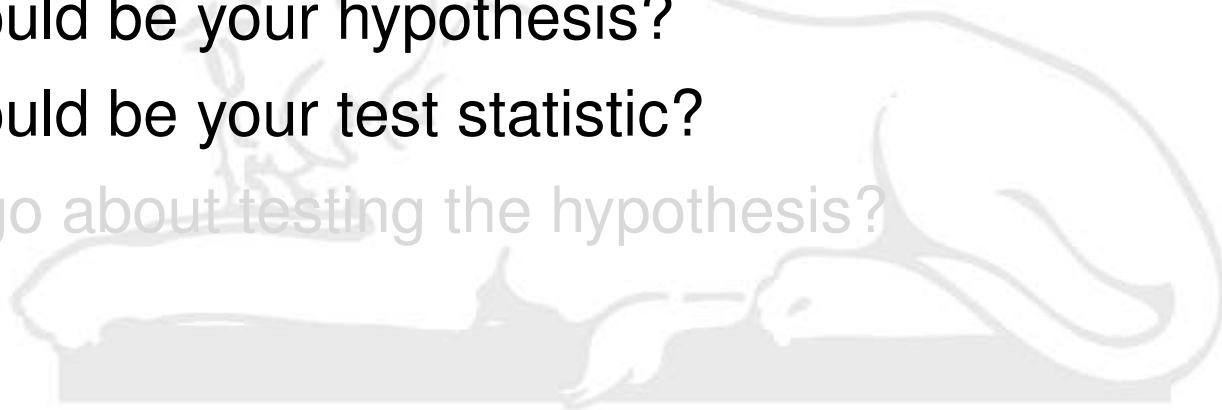- ❑ How to go about testing the hypothesis?

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