Data Mining for Business Intelligence (IBM 312)

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Department of Management Studies

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Recap and Today

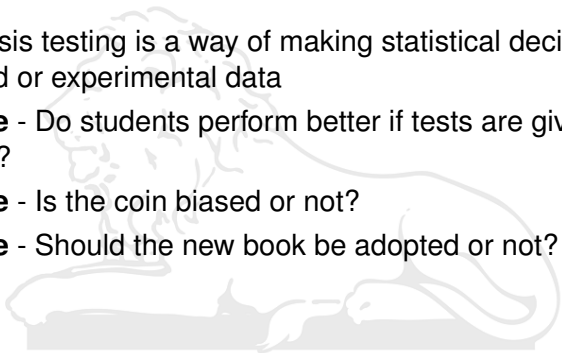**Recap -**
Hypothesis Testing - Introduction
Today -
Examples
p-Value

Hypothesis Testing

❏ Hypothesis testing is a way of making statistical decisions using observed or experimental data

❏ **Example** - Do students perform better if tests are given in the morning?

❏ **Example** - Is the coin biased or not?

❏ **Example** - Should the new book be adopted or not?

Key Terms in Hypothesis Testing

❑ Null Hypothesis, usually denoted by H_0

❑ Alternative Hypothesis, usually denoted by H_1 or H_a

❑ Hypothesis testing is performed after data is available for a sample - $X_1, X_2, \ldots, X_n$

❑ Decision of testing the hypothesis is whether to reject the null hypothesis or not to reject the null hypothesis

❑ Basis - from the value of a **statistic** T, which is calculated from the sample, called the **test statistic**

❑ Whenever the **null hypothesis is rejected, alternative hypothesis is accepted**

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Errors, Level and Power

	Null Rejected	Null not rejected
Null is true	**Type 1 error**	
Null is false		**Type 2 error**

Table: Type 1 and Type 2 error

❑ Probability of type 1 error - level of the test or significance level of the test, denoted by α

❑ Probability of type 2 error usually denoted by β

❑ 1-β is called the power of the test

Remarks on α and β

- [ ] We control the type 1 error by our choice of level of significance α
- [ ] We then try to maximize power $1 - \beta$
- [ ] We cannot reduce the chances of both type 1 and type 2 error simultaneously
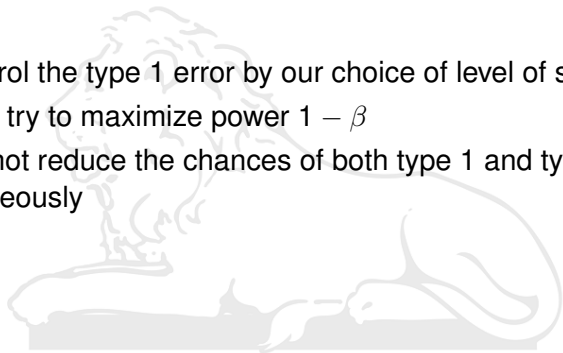
Example

BCCI was accused of using a biased coin in the toss for cricket match. A test was thus performed to check whether the given coin is biased towards heads. The coin was tossed 10 times.

1. Observed result - HHTHHHHTHH
2. Null Hypothesis $p = 0.5$
3. Alternative Hypothesis is $p > 0.5$

It was decided that the null will be rejected if the number of heads is greater than 7?
What is the level of the test?
What is the power of the test?

Hypothesis Testing

❑ Null Hypothesis and Alternative Hypothesis

❑ Null - typically the status quo or the one that people put more faith on, or the one that is easier to describe

❑ Alternative - Everything else comes into this hypothesis

❑ There should be no common possibility in null and alternative hypothesis

❑ There should be no possibility outside of H_0 and H_a

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Building Hypothesis - Examples

$$H_0: p_H \leq 0.5$$
$$H_a: p_H > 0.5$$

$$H_0: p_H = 0.5$$
$$H_a: p_H \neq 0.5$$

- ❑ Coin is biased towards heads
- ❑ Coin is not an unbiased coin
- ❑ Gravity Fitness Gym, Roorkee claims that if you join the gym, you will lose more than 5 kgs in one month on an average.
- ❑ BJP IT team claims that with the marketing strategy that they have adopted, more than 40% people who were voters of other parties will vote for BJP in the coming elections
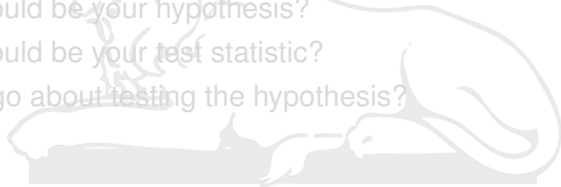
$$H_0: w \leq 5$$
$$H_a: w > 5$$

$$H_0: p_n \leq 0.4$$
$$H_a: p_n > 0.4$$

Building Hypothesis - Example

There is a perception that IITR students are made to work much harder than students from other similar programs. A recent study states that on average a student in such a program works for 18 hours per week with an SD of 4 hours per week

- ☐ What would be your hypothesis?
- ☐ What would be your test statistic?
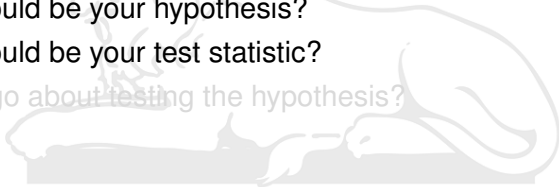- ☐ How to go about testing the hypothesis?

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Building Hypothesis - Example

$$X_1, X_2, \text{_____} \qquad , X_{100} \qquad \boxed{\bar{X} = 18.2}$$

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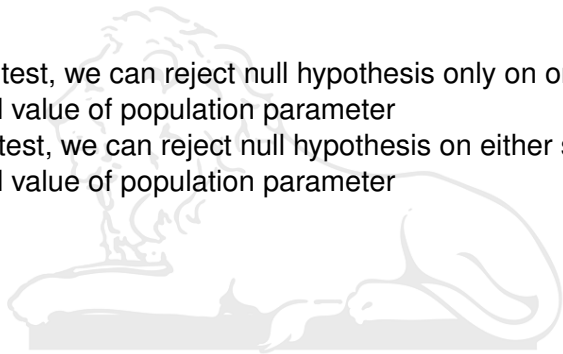- ❑ How to go about testing the hypothesis?

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One tail versus two tail test

In one-tailed test, we can reject null hypothesis only on one side of the hypothesized value of population parameter
In two-tailed test, we can reject null hypothesis on either side of the hypothesized value of population parameter

P-value

□ p-value is the probability of obtaining a result as extreme as the one that was actually observed, under the assumption that null hypothesis is true

□ If the p-value is low, the null hypothesis should go

□ In the toss experiment, observed number of heads is 8. What is the p-value of this test statistic??

□ *p*-value for the shop-sales problem? How to make a decision??

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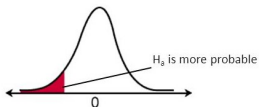❑ *p*-value for the shop-sales problem? How to make a decision??

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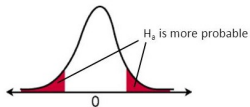
One Tail and Two Tail Test
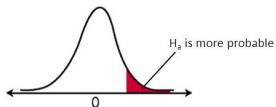
Right-tail test

$H_a: \mu > \text{value}$

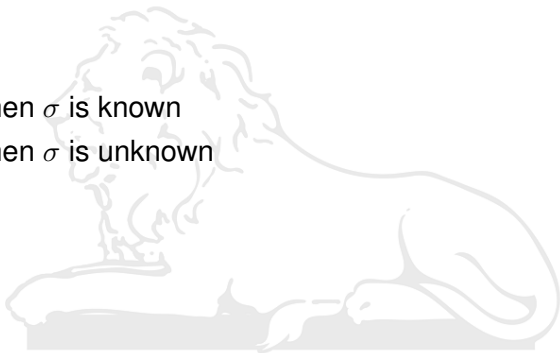**Left-tail test**

$H_a: \mu < \text{value}$

Two-tail test

$H_a: \mu \neq \text{value}$

Source - https://www.fromthegenesis.com/difference-between-one-tail-test-and-two-tail-test/

Hypothesis testing for population mean

❏ Case when σ is known
❏ Case when σ is unknown

Steps in Hypothesis Testing

1. Formulate the problem by clearly writing null hypothesis and alternative hypothesis
2. Decide on the level, α, for the test
3. Decide on the test statistic T. Calculate the value of the test statistic based on the sample
4. Find the *p*-value of the test statistic (or find the rejection criteria)
5. Give a verdict