Moran_HW4

October 5, 2017

Shannon Moran Phys 514 HW 4 Due: Oct 5

0.1 Import required packages

```
In [1]: %matplotlib inline
        import numpy as np
        from math import *
        import matplotlib.pyplot as plt
        import time
        from scipy import optimize
```

Here, I implement helper functions I use below.

```
In [2]: # Times the run time of a given function for a set number of reps
        def time_test(func,N,reps):
            t_timed = []
            for i in range(reps):
                t_rep = []
                for n in N:
                    x = np.random.random(n)
                    t0 = time.time()
                    func(x)
                    t_rep.append(time.time()-t0)
                t_timed.append(t_rep)
            t = np.average(np.asarray(t_timed),axis=0)
            return t

        # Generates a range of vector sizes to test, output in vector N
        def N_range(start_value,n_test):
            N = [start_value]
            for i in np.arange(1,n_test+1):
                N.append(int(N[i-1]*2))
            return np.asarray(N)

        # Finds the nearest power of 2 to a given value x
        def nearest_p2(x):
            p = 2
            c = 1
            while (x>p):
                p = p*2
                c += 1
            return c-1
```

0.2 01: DFT

Write a function that implements the discrete Fourier transform.

```
In [3]: def DFT(y):
            N = len(y)
            # columns
            k = np.arange(N)
            # rows
            n = np.arange(N).reshape((N, 1))
            C = np.exp(-2j*np.pi*k*n/N)
            c = np.dot(C,y)
            return c
```

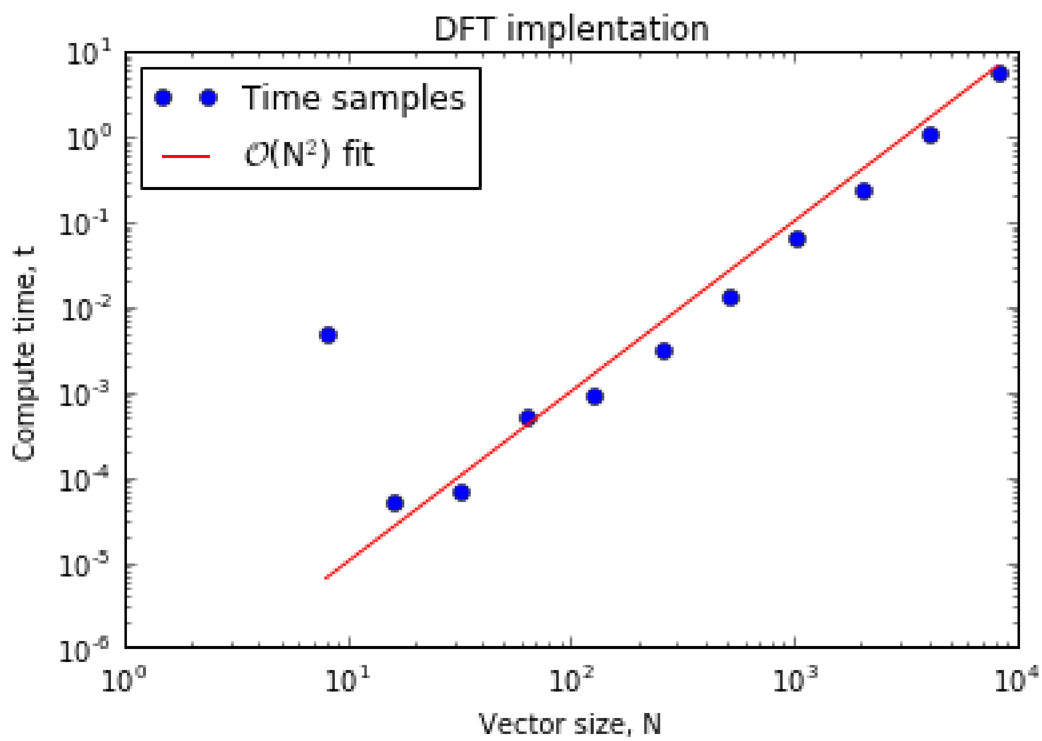
```
In [4]: # Test time for DFT to run.
        N = N_range(8,10)
        t = time_test(DFT,N,10)
```

Show that, as a function of vector length, the computation scales as $\mathcal{O}(N^2)$ on a log-log plot.

Note that I do not plot the $\mathcal{O}(N)$ or lower terms to more easily demonstrate the $\mathcal{O}(N^2)$ fit; however, I do use these terms for predicting the largest vector I can transform in a second.

```
In [5]: # First, we test the fit to a second order function in N
        z = np.polyfit(N,t,2)
        fit = [z[0]*np.power(n,2) for n in N]

        plt.loglog(N,t,'bo',label="Time samples")
        plt.loglog(N,fit,'r-',label=r"$\mathcal{O}$(N$^2$) fit")
        plt.title('DFT implentation')
        plt.xlabel('Vector size, N')
        plt.ylabel('Compute time, t')
        plt.legend(loc=0)
        plt.show()
```



What is the largest vector that you can transform within a second?

```
In [6]: # Use the scipy implementation of fsolve to solve for N when t=1s
        def func(N):
            global z
            [a,b,c] = z
            t1 = 1
            return a*np.power(N,2)+b*N+c-t1
        N_t1 = optimize.fsolve(func,1e4)

        print('Longest vector we can transform in a second is: %s elements' % int(N_t1))
        print('We can confirm this by testing a DFT on vector of length %s' % int(N_t1))
        x = np.random.random(int(N_t1))
        %timeit DFT(x)
```

```
Longest vector we can transform in a second is: 3804 elements
We can confirm this by testing a DFT on vector of length 3804
1 loops, best of 3: 897 ms per loop
```

The actual time to run a DFT of size N_t1 may be smaller than 1 due to changes in programs running in the background at various points. I typically found that the longest vector I could transform in a second was on the order of 4k elements.

0.3 02: FFT

Write a function that implements the fast Fourier transform algorithm. Show that, as a function of vector length, the computation scales as O(N log N) (You can also show it in log-log plot, but remember to plot a reference curve as N log N). What is the largest vector that you can transform within a second?

```
In [7]: def FFT(y):
            N = len(y)

            # Allows us to catch the lowest-level recursion
            # Choose this based on the value of N at which DFT run time starts increasing linearly agai
            if (N<=32):
                return DFT(y)
            # Runs the recursion over all other sizes N
            else:
                E = FFT(y[::2]) # get even-indexed values
                O = FFT(y[1::2]) # get odd-indexed values
                k = np.arange(N)
                C = np.exp(-2j*np.pi*k/N)
                # solve first and second halves
                Y = np.concatenate([E+C[:N/2]*O, E-C[N/2:]*O])
                return Y
```

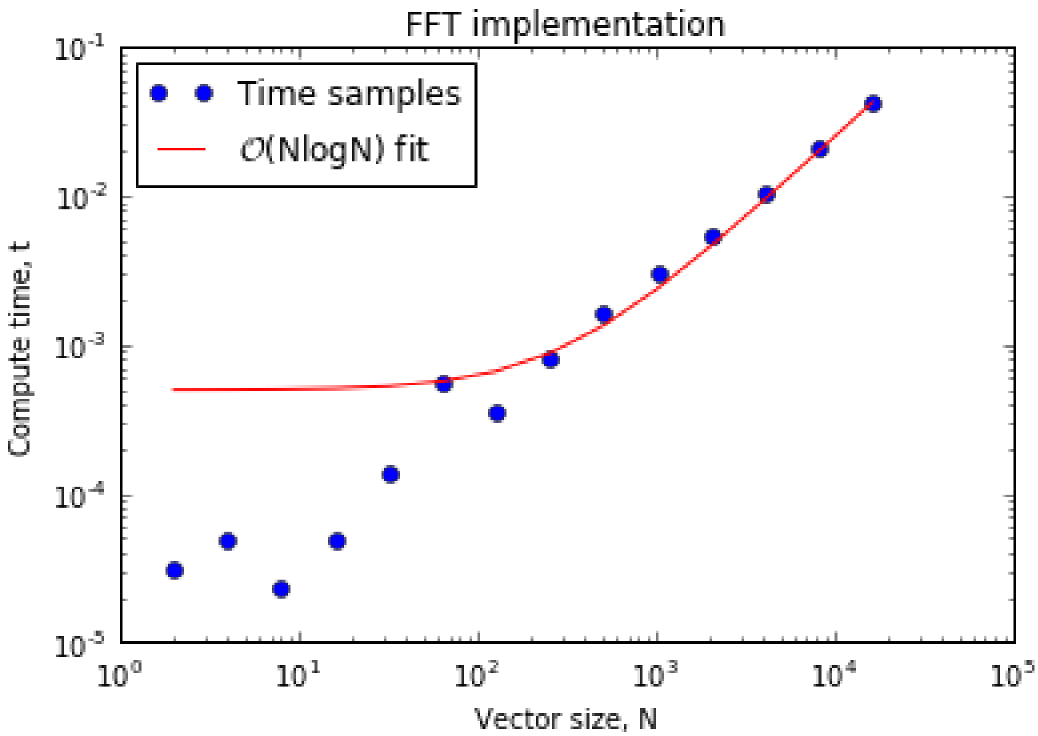
```
In [8]: N = N_range(2,13)
        t = time_test(FFT,N,10)
```

```
/Users/shannonmoran/miniconda3/lib/python3.5/site-packages/ipykernel/__main__.py:15: DeprecationWarning:
```

We see that $\mathcal{O}$(NlogN) scaling is a good fit for the data.

```
In [9]: z = np.polyfit(N*np.log2(N),t,1)
        fit = [z[0]*n*log2(n)+z[1] for n in N]

        # Should be linear
        plt.loglog(N,t,'bo',label="Time samples")
        plt.loglog(N,fit,'r-',label=r"$\mathcal{O}$(NlogN) fit")
        plt.title('FFT implementation')
        plt.xlabel('Vector size, N')
        plt.ylabel('Compute time, t')
        plt.legend(loc=0)
        plt.show()
```




```
In [10]: # Use the scipy implementation of fsolve to solve for N when t=1s
         def func(N):
             global z
             [a,b] = z
             t1 = 1
             return N*np.log2(N) - (t1-b)/a
         N_t1 = optimize.fsolve(func,1e4)
         p = nearest_p2(N_t1)

         print('Longest vector we can transform in a second is: %s elements' % int(N_t1))
         print('We can confirm this by testing my FFT implementation on vector of length 2^%s' % int(p))
         x = np.random.random(int(2**(p)))
         %timeit FFT(x)
```

```
Longest vector we can transform in a second is: 298669 elements
We can confirm this by testing my FFT implementation on vector of length 2^18
1 loops, best of 3: 694 ms per loop
```

```
/Users/shannonmoran/miniconda3/lib/python3.5/site-packages/ipykernel/__main__.py:15: DeprecationWarning:
```

Because my FFT implementation only works for vectors that are of 2^m elements, I test the nearest power of 2 that is less than the calculated size of the longest vector I can transform in a second. If I tested my FFT implementation on a vector of size 2^{19}, the time to run would be over 1s.

0.4 03: FFTW

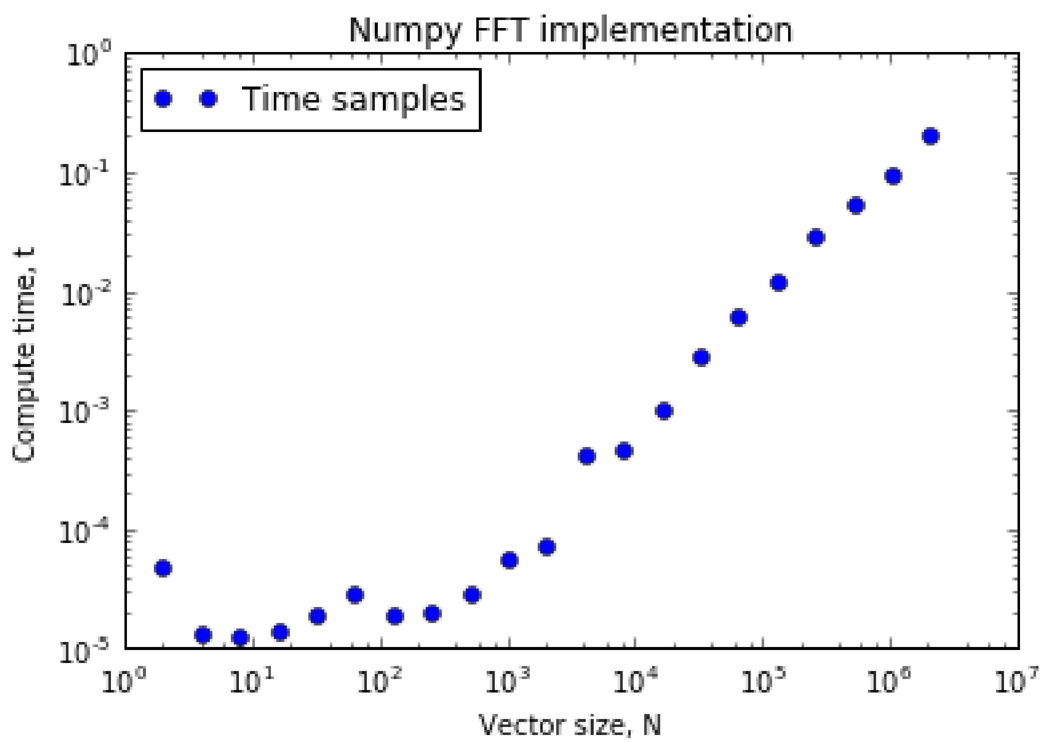
Write a function that calls the built-in FFT interface (matlab), the scipy/numpy fftpack (Python), or the FFTW library for fast Fourier transforms (http://www.fftw.org/, C++ or Fortran).

Here, I use numpy's fft implementation to test the scaling of implemented packages.

```
In [11]: N = N_range(2,20)
         t = time_test(np.fft.fft,N,10)
```

Plotting crudely, it appears that there is some power law scaling at play, emergent at larger vector sizes.

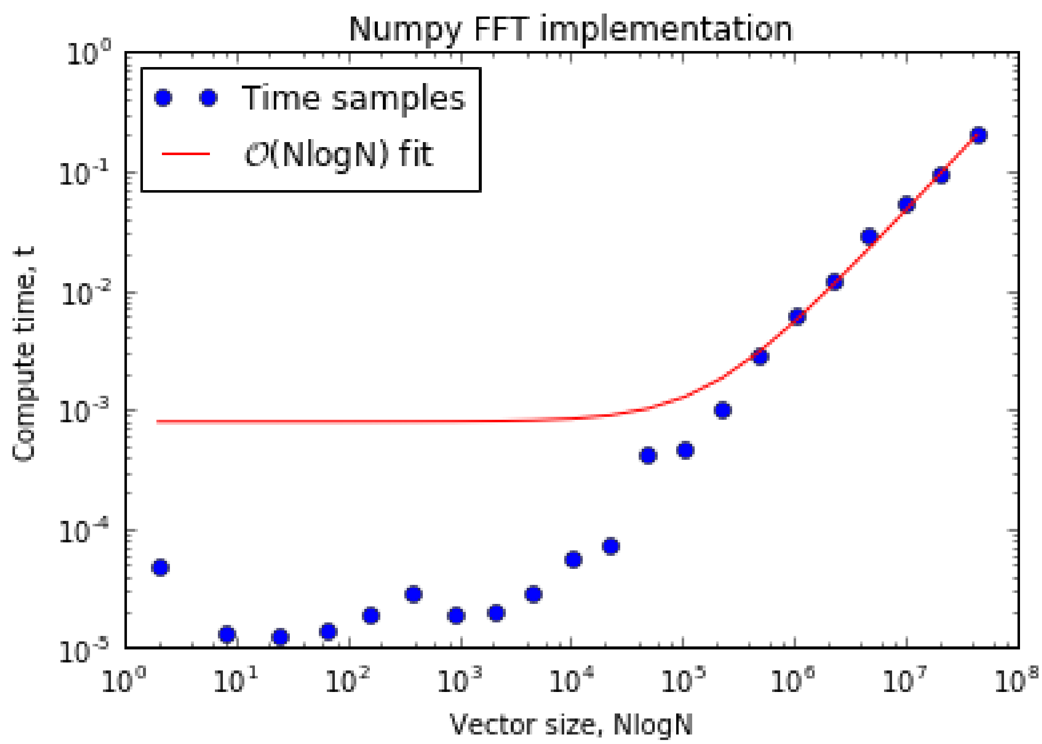
```
In [12]: plt.loglog(N,t,'bo',label="Time samples")
         plt.title('Numpy FFT implementation')
         plt.xlabel('Vector size, N')
         plt.ylabel('Compute time, t')
         plt.legend(loc=2)
         plt.show()
```



We can confirm this with a polyfit for $\mathcal{O}$(NlogN), seeing that the time points follow this fit particularly at longer time.

```
In [13]: z = np.polyfit(N*np.log2(N),t,1)
         fit = [z[0]*n*np.log2(n)+z[1] for n in N]

         # Should be linear
         plt.loglog(N*np.log2(N),t,'bo',label="Time samples")
         plt.loglog(N*np.log2(N),fit,'r-',label=r'$\mathcal{O}$(NlogN) fit')
         plt.title('Numpy FFT implementation')
         plt.xlabel('Vector size, NlogN')
         plt.ylabel('Compute time, t')
         plt.legend(loc=0)
         plt.show()
```




```
In [14]: # Use the scipy implementation of fsolve to solve for N when t=1s
         def func(N):
             global z
             [a,b] = z
             t1 = 1
             return N*np.log2(N) - (t1-b)/a
         N_t1 = optimize.fsolve(func,1e7)
         p = nearest_p2(N_t1)

         print('Longest vector we can transform in a second is: %s elements' % int(N_t1))
         print('We can confirm this by testing np.fft on vector of length 2^%s' % int(p))
         x = np.random.random(int(2**(p)))
         %timeit np.fft.fft(x)
```

```
Longest vector we can transform in a second is: 9411452 elements
We can confirm this by testing np.fft on vector of length 2^23
1 loops, best of 3: 724 ms per loop
```