A Student Guide to the Master of Pure and Applied Logic

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Contents

1 Introduction

This document has to provide the coming student with an idea what this master is about and how it is organised. Please, play due attention to the date of the document since we will update as minor changes occur.

Our master is called *Pure and Applied Logic*. In a sense, most people will consider our master as a pure and theoretical logic master. Indeed, we have a strong theoretical tradition and as a matter of fact, within Europe it may be one of the more mathematically oriented masters.

However, the applications can be found within other fields of logic. In the same sense as the *Journal of Pure and Applied Logic* refers to applications of logic within branches of mathematics or even within other fields of logic themselves.

Apart from this reading of 'applied', we do have a couple of courses that are applied in the sense that they are very much related to Computer Science which may be the more classical connotations of the word *applied*.

Be it as it is, one can take a more pragmatic approach and ask

> What's in a name? That which we call a rose; By any other name would smell as sweet.

As such, the best description of our master is not given by its name but it is simply its extension, the courses being taught, the research activities in which the master is embedded, and in general, the thriving academic fabric that one finds in the larger Barcelona area and which surrounds this master.

Another clear indication of what our master is, is also provided by the repository of a selection of master theses that were written in our program under the guidance of our professors and often in collaboration with foreign supervisors too. As a rule, new students are expected to proudly add their thesis to the repository so that it may grow with many high quality master theses to come. We kindly invite you to have a look at

`http://diposit.ub.edu/dspace/handle/2445/133559`

and be inspired to add your future contribution.

2 General structure of the master

The master is in its current set up biennial which means, we start every other year and the duration is two years in total. Each new edition starts at the even year and the next edition is 2022 - 2024.

Once you have started the master, you will have to obtain a total of **90 credits** within the *European Credit Transfer System*. There are 30 credits of mandatory courses and 40 credits of optional courses to choose from. Your final and mandatory assignment consists of writing a master thesis (*treball final de* *màster*) for a total of 20 credits. The standard of these theses is quite high and often they lead to publications or conference presentations.

As mentioned before, your thesis will in principle be published in the theses repository of our master:

`http://diposit.ub.edu/dspace/handle/2445/133559`

Thus, the structure of the master is as follows:

Activity type	Number of credits
Compulsory courses	30
Optional courses	40
Master thesis	20

The courses are taught both at the *University of Barcelona* (UB) and at the *Poly-technical University of Catalonia* (UPC).

The master is coordinated through the Faculty of Philosophy of the University of Barcelona where also various courses are being taught. At the University of Barcelona you will also follow courses in the Faculty of Mathematics and Computer Science.

At the Poly-technical University of Catalonia you will take courses both in the Faculty of Computer Science and in the Faculty of Mathematics and Statistics.

This variety of participating faculties is another clear indication of the pronounced interdisciplinary nature of the master. In addition, we often count with guest lecturers from the larger Barcelona area like the *Autonomous University of Barcelona* or the *Spanish Academic Research Council.*

3 A short description of the compulsory courses

In this section we will briefly describe the five compulsory courses of our master. Each compulsory course is of 6 credits yielding a total of 30 credits in compulsory courses. A more detailed description can be found in Section A.

3.1 Basic Model Theory

ECTS: 6

Know fundamental theorem of model theory and know how to apply them such as Compactness and Löwenheim-Skolem, Lós for ultra-products, some of the preservation theorems. Understand the relation along saturation, homogeneity and universality concepts, back-and-forth system, the omission type and its all used withing numerable models of a theory. Appreciate what is a prime model, knowing the criteria to understand whether a model has one or not; along with Ryll-Nardewski theorem ad completeness-model and company-model; Fraissé constructions in order to obtain theories omega-categorical; and algebraic, and definable closure as their most important concepts.

3.2 Basic Set Theory

ECTS: 6

Recognize the ZFC axioms, knowing what results depend on which axioms, in particular, which choosing axiom, substitution axiom and foundation axiom, respectively. Appreciate the construction from the set theory perspective of Natural numbers, order in Rational numbers and the linear continuum; along with the theory of good-orders and transfinite ordinal numbers, with different forms of transfinite recursion and fundamental results of infinite cardinal arithmetic. Finally, express the structure of the universe, the Cumulative Hierarchy, definitions by recursion about the membership-relation.

3.3 Computability

ECTS: 5

Assimilate the standard model of computing and the meaning of Church's thesis. Recognize unsolvable problems and classify them by their degree of unsolvability in the arithmetic hierarchy with the phenomenon of incompleteness in sufficiently powerful theories of arithmetic.

3.4 Final Project

ECTS: 20

The master's thesis consists in a written presentation of a specialized topic of a subject similar to those of the master's program, chosen by the student in agreement with his or her thesis director.

The work should be a systematic, clear and rigorous explanation of the results found in several articles of the relevant scientific literature. It should serve for others to know the state of the art on the subject in a completely reliable way. If the chosen topic allows it, the student is also expected to contribute with some original research results, although of a modest nature.

3.5 Mathematical Logic

ECTS: 6

To know the most important classical meta-theoretical results on first-order logic (completeness, compactness and Löwenheim-Skolem). To know Gödel's incompleteness theorems and their corollaries and variants on aspects of incompleteness and undecidability. To know some decidable fragments of arithmetic.

3.6 Non-Classical Logics

ECTS: 5

Know the most representative non-classical logics, the different types of calculi: Hilbert type, Gentzen, Tableaux, Resolution, Natural Deduction, the most representative semantics, and understand the proofs of the different completeness theorems. Manipulate the most representative semantics and have the ability to relate semantics to calculi, how to differentiate and understand which problems are solved by the different non-classical logics and which properties are particular and which are general. Handle the different types of deductive calculi.

4 A short description of the optional courses

In this section we will briefly describe the optional courses from which you can choose up to a total of 40 credits. Of course, you can follow more credits if you like, but 40 of them is the minimum to obtain your final degree. A more detailed description of the optional courses can be found in Section A.

Moreover, students that enroll in an Erasmus exchange can follow courses at universities that have an exchange covenant with the University of Barcelona. You can read more about this in Section 7.2.

4.1 Abstract Algebraic Logic

ECTS: 5

Know the applications of generalized matrices for modeling sentential logics and Gentzen systems, the generalization of the Lindenbaum-Tarski method for generalized matrices, the main transfer theorems for full models, and the main classes of the Leibniz and Frege hierarchies, their characterizations, most outstanding properties, and their interrelationships.

4.2 Advanced Model Theory

ECTS: 5

Going into in-depth knowledge of the concepts, techniques and results of classical model theory (saturation, definability, prime models, indiscernible, etc.), having an introduction to the most elementary concepts and results of the stability theory. Recognize, at least in cases of medium difficulty, whether a first-order theory is complete, whether it has quantifier elimination, and in what cardinalities it is categorical, and classify a complete theory in the context of the hierarchy of stability and describe its models in cases of medium difficulty.

4.3 Algebraic Logic

ECTS: 5

Algebraic Logic is the discipline that studies *Bridge Theorems* that allow to cross the mirror between Logic and Algebra by associating a purely semantic interpretation (such as the amalgamation property) with a given metalogical property (such as the interpolation property). This allows to study metalogical phenomena through the lenses of their semantic counterparts, which are typically amenable to the methods of Universal Algebra, Lattice Theory and Category Theory. This course does this by presenting the basics of the theory of algebraizable logics, the Leibniz operator, and matrix semantics.

4.4 Algorithms for VSLI

ECTS: 5 Know the most important aspects of the use of algorithms for the automation of the design of logic circuits and the different techniques based on logic that are used for such automation, be able to design algorithms of different types for the design of logic circuits, be able to analyze, using the formal resources of logic, the design problems of logic circuits so that algorithms that allow automation can be designed.

4.5 Combinatorial Problem Solving

ECTS: 6

A combinatorial problem consists in, given a finite collection of objects and a set of constraints, finding an object of the collection that satisfies all constraints (and possibly that optimizes some objective function). Combinatorial problems are ubiquitous and have an enormous practical importance. In this course we will study three different general paradigms for solving combinatorial problems: linear programming, propositional satisfiability and constraint programming. For each of them, we will study the algorithmic foundations, as well as modelling techniques.

4.6 Combinatorial Set Theory

ECTS: 5

Know the main results on closed and co-final sets and on stationary sets of non-countable regular cardinals; build infinite trees with various properties (Aronszajn, Suslin, Kurepa trees), in ZFC or any of ZFC extensions; Ramsey's theorem and its generalizations to larger cardinals. Get started on some large cardinals, particularly Mahlo cardinals, weakly compact cardinals, and Ramsey cardinals, express the results of the subject with clarity and precision, know how to rigorously and elegantly demonstrate the main theorems studied in the subject. Know how to solve problems on the set concepts studied.

4.7 Computational complexity

ECTS: 6 Know the standard computing model learned in computability theory to take into account the use of resources such as time, space, randomness, or interaction, how to recognize NP-complete problems, and learn to appreciate the power of the probabilistically verifiable proof (PI) model compared to the classic deterministically verifiable proof (NP) model.

4.8 Development of Formal Logic

ECTS: 5 Acquire the necessary conceptual tools to critically analyze historiographical interpretations of basic issues that concern the historical development of logic. More specifically, acquire a better understanding of the process that led to the consolidation of First-Order Logic as a core system of contemporary logic and the role that historical and sociological facts, many of them contingent, played in its development.

Earn a robust understanding of the notions of logical form and logical validity, with a focus on the historical development of the notion of logical constant, from the classical, and profusely studied by medieval logicians, distinction between *categorematic* and *syncategorematic* signs to the seminal work by Gottlob Frege and Alfred Tarski and its consequences to contemporary logic.

4.9 Introduction to Mathematical Logic

ECTS: 5

Know the syntax and semantics of first-order logic. Know in depth at least one deductive calculation. Know the completeness theorem as well as its consequences: compactness and Löwenheim-Skolem. Learn how to manipulate formulas: equivalences, normal forms, Skolem forms. Understand the difference between a formula with free variables and a closed formula; formulas with free variables define sets and know how to express said sets using suitable formulas; how closed formulas define classes of structures and know how to express these classes with suitable formulas. Learn how to use the Homomorphism theorem to show limitations of first-order logic: certain properties cannot be expressed by means of formulas. Being able to make and manipulate formal deductions of reasonable complexity in a deductive calculation. Know how to use the compactness and Löwenheim-Skolem theorems to show the limitations of first-order logic at the level of closed formulas. Understand what it means for a theory to be complete and be able to use the Łos-Vaught test to show that a particular theory is complete (in simple examples).

4.10 Many-Valued Logics

ECTS: 5 Know the most representative many valued logics, the most representative semantics, understand the proofs of the different completeness theorems, how to relate logical properties to algebraic properties, how to generalize some results and proofs such as those of the completeness theorems.

4.11 Modal Logic

ECTS: 5 Know the basic concepts and results of contemporary modal logic, in particular, Kripke and algebraic semantics, model theory (bi-simulations), and modal aspects of the provability predicate for theories such as Peano Arithmetic.

4.12 Models of Set Theory

ECTS: 5

Know the main techniques for building models of Zermelo-Fraenkel set theory, especially constructability and forcing, know in detail the proofs of the results of Gödel and Cohen on the independence of the Axiom of Choice and the Generalized Continuum Hypothesis, the fundamental properties of Gödel's constructible universe (condensation, existence of a good global order, Jensen's diamond principle, etc.), and its generalizations, such as the L(R) model or the HOD model, the details of the "forcing" technique and learn how to apply it, the technique of iterated "forcing" and the demonstration of the consistency of the Suslin Hypothesis and Martin's Axiom.

Learn to prove results of consistency and independence in set theory and in mathematics in general, acquire skill in handling concepts of definability, as well as logical and metamathematical concepts such as consistency, independence, absoluteness with respect to different models, etc, acquire skill in the use of the "forcing" technique and be able to apply it to new problems, learn to solve advanced set theory problems.

4.13 Orders, Lattices and Boolean Algebras

ECTS: 5 Know the results of the theory of partial orders and adjoining pairs of monotone functions, the results of the lattice theory, in particular those referring to distributive lattices, the fundamental results of Boolean algebras, especially Stone's representation theorem, the relationship between Boolean algebras and classical logic, how to prove the theorems of medium difficulty of the subject, how to clearly expose the demonstrations, how to properly convey the basic theorems of lattice theory and Boolean algebras.

4.14 Proof Theory and Automated Theorem Proving

ECTS: 5

Know the theory of proof of propositional logic and first-order logic, the resolution method, basic methods of intelligent search, the execution mechanism in PROLOG, the basic ideas of ordinal analysis of formal theories. Know how to apply the Herbrand method, to apply the unification algorithm, to use the resolution method, to prove and apply cut elimination theorems

4.15 Universal Algebra

ECTS: 5 Know the basic tools of Universal Algebra for the study of abstract algebraic structures, the representability of algebras and the operators on classes of algebras, the equational and quasi-equational classes, as well as the equational and relative equational logics, and some problems related to the finite generation of algebra classes.

5 Application to the master

In this section we will describe the process of applying and getting accepted.

5.1 Criteria for admittance

The official undergraduate diplomas that automatically grant access to this master in pure and applied logic are:

1. Mathematics,
2. Computer science,
3. Philosophy,
4. Physics, or
5. Related subject matters.

Even though philosophy is among the official possible prerequisites, we tend to be very careful when admitting students that come with just a philosophy diploma. This is because the formal and mathematical level of our master is very high. Even though the main problems that are studied in the master are mostly rooted in deep philosophical questions, the methods employed to address these questions are mostly of formal or mathematical nature.

Thus, philosophy students can only be successful in our master if they have had already extensive training in formal reasoning. Students who do not have a proper background in formal/mathematical reasoning can be required to do preparatory courses before they are accepted to enrol.

To a lesser extend this may also hold for physics students, although experience has taught us that on average, physics students will do fine after some special attention in the first semester.

5.2 The admittance procedure

Typically, some time January/February prior to the start of a new edition of the master we open the so-called ***pre-registration***. You can find a link to the pre-registration process from any of the web-pages of the master:

`http://www.ub.edu/masterlogic/`

`https://www.ub.edu/portal/web/philosophy/university-master-s-degrees/-/ensenyament/detallEnsenyament/10456309`

It is in this process that the students express their interest and intention to officially enrol. This pre-registration comes with a minimal fee that cannot be retrieved if the student in the end decides not to enrol.

However, we normally take expressions of interest of students through the mail

masterlogic@ub.edu

before the pre-registration. It is here that we can provide feedback. We already can estimate the chances of getting admitted. What we wish to avoid is that students do the pre-registration while this is not really the master they want to study.

Some time after your pre-registration, you will receive notice whether you are accepted in the master or not. Acceptance is not automatic since the master can only allot 25 students. It is the *Coordinating Committee* of the master that decides on your admittance. Selection criteria comprise, but are not limited to, the quality of your academic track record, your motivation, possible references, and time of application (first to come, first to get).

If you read this text, you may feel that you should instantly file your application to get admitted. Of course, we are happy with timely (pre)-registrations but we find it even more important that you make a good and well-pondered decision. So, please do take a look at other masters and make a motivated decision. Over the past two editions we have not reached the full capacity of twenty-five students.

Once you get admitted, you can start the formal enrolment. During this process you will have to lay over official documents (diploma, if applicable, residence permit, etc.). This process is mainly through the administrative services of the university and the direction of the master only has minimal involvement here. Especially students from abroad are encouraged to formalise their enrolment timely since visa and/or residence permits may take much time. Often various documents need official translations (sworn translators with their officially registered stamps *apostille*, etc.). For European students the enrolments should be a pretty smooth and easy process.

To wrap the enrolment process up, here's a timetable that includes the steps and time-frames:

Procedure	When	Cost
Expression of interest	Any time of year	none
Pre-registration	January/February prior to start new edition	?
Admittance communication	Within not more than 3 weeks upon pre-registration	?
Registration	Until the end of August	?
Start courses	September	per credit

6 Financial aspects

In this section we will comment on important financial aspects related to the master enrollment.

6.1 Tuition fees

For students that come from European Member States, the price per credit is stipulated on a national Spanish level at 27.67 €. Thus, the tuition fees of the entire master will be

slightly less than 2500 € for the two years.

As such, the Barcelona master is one of the cheaper masters of its kind. The same prices apply to citizens from Iceland, Norway, Liechtenstein, Switzerland, Andorra, China or to citizens with a long-term residence permit.

For citizens from other countries a price of 82 € per credit applies. Given these discrepancies it may seem worth the effort to consider obtaining a long term residence permit for Spain in case of applying to the master. We are not really aware of the exact process details for each different country and conducting this process is responsibility of each individual student. Of we course, we will happily provide needed documents where we can, for example stating official conditional acceptance in the program, etc. In this context, it is always good to consult the official UB web pages since conditions may change over time:

`https://www.ub.edu/web/ub/en/estudis/oferta_formativa/master_universitari/matricula/matricula.html?`

At this page you may also find contact details on the prices and exact conditions.

6.2 Scholarships and fee waivers

Once enrolled, there is currently one regular funded scholarship provided by the Spanish ministry, known as the *Beca General* (`https://www.becaseducacion.gob.es/portada.html`). This scholarship requires residence permit or other assimilated preconditions.

For this and other opportunities, such as fee reductions and support, see the University of Barcelona scholarship portal (`www.ub.edu/beques/grausimasters/`) and the open funding calls (`https://www.ub.edu/portal/web/beques-monub/obertes`). A few of the opportunities are outlined below:

- Collaboration scholarships from the Spanish ministry (some calls require residence permit);
- Collaboration scholarships from the UB;
- Collaboration scholarships from the involved faculties;

These collaboration scholarships typically require you to perform some tasks (mostly research related to your master thesis and to a lesser extent administration related) of around two to three hours per day and they pay you a small amount in the order of three to four hundred euros per month. To apply to these scholarships one has to be enrolled and the outcome is not sure since some years the call is competitive (other years it is less so).

Furthermore there is:

- For each course, the best student(s) will get (provided some requirement on the amount of students who took the course) a flag *matrícula d'honor*. This implies that you get to enrol for free in a new course. In one edition of the master we had a student who was so outstanding that he simply paid the first semester and got all the other courses for free; We do not yet have an official fee-waiver mechanism but this is the closest thing we currently have. In one edition of the master we had a student who simply obtained *matrícula d'honor* in all courses so that effectively the due payment consisted of just around four courses and the final thesis writing.
- There is the *Formal Vindications Best Student Award*. The student with the best performance (according to the conditions specified in the call) in the first year wins a prize (previously 1000 €).
- Some times, there are projects under which you can perform some remunerated tasks related to the project (research and administration). For example, some times you can embed the writing of your Master Thesis into such a project.

Other opportunities

- Fundación Carolina scholarships for students from any country within the Ibero-American Community of Nations.

  https://www.fundacioncarolina.es/formacion/presentacion/

7 Institutional aspects of the master

As mentioned before, the master is hosted in two universities: the University of Barcelona (UB) and the Polytechnic University of Catalonia (UPC).

The master is coordinated through the Faculty of Philosophy of the University of Barcelona where also various courses are being taught. So, for various concrete administrative tasks, you will be dealing with the student administration of the Faculty of Philosophy.

At the University of Barcelona you will also follow courses in the Faculty of Mathematics and Computer Science.

At the Poly-technical University of Catalonia you will take courses both in the Faculty of Computer Science and in the Faculty of Mathematics and Statistics.

7.1 Important formal organs to the master

Apart from the various legal and organising university entities (*Quality Committee of the Faculty*, the *Faculty Board*, etc.) the main governing enitity that master students will be dealing with is the *Comissió Paritària de Coordinació* *del Màster*. It is this committee that is concerned with the daily management of the master and your main contact person will be the coordinator of the master.

The formal, leading and binding description of the committee and the functioning of the master can be found in the *Memòria de Titulació* which is available online:

`http://www.ub.edu/gestio-ensenyaments/Filosofia/MOCOD_M_memoria.pdf`

The Coordinating Committee is also responsible for the selection of new students to be admitted in the master. Further official documents that stipulate the institutional implementation of the master organisation can be found here: `http://www.ub.edu/gestio-ensenyaments/Filosofia.html`

Another important instrument to new students is the so-called *PAT* which stands for *Plá d'acció tutorial* and is some sort of Tutor action. The idea is that each student gets assigned a mentor/tutor who is available for help on various issues related to the master. Also, the PAT provides some actions both in the socialising as in the information providing sphere.

7.2 Erasmus Exchange

Master students are allowed to follow a part of their second year in another university that has an Erasmus covenant with the University of Barcelona. As such they will enroll and pay their credits in Barcelona but follow tuition in another country. It is no problem if the student follows a course that is not in the curriculum of our master since we can be flexible here.

Students that are interested in and Erasmus exchange should contact the director of the master a fair deal before the start of the second year.

8 Related research

When you choose a master you typically do this on the basis of various criteria. Do you know and like some of the professors? Is studying in Barcelona logistically feasible for you? Do you like the city and environment? Do like the academic and cultural life in the larger Barcelona area, etc.

One very important factor, we think, should be if the research conducted at the institute of your choice is of interest to you. Very often, master students get offered research positions during or upon finishing their master. These positions will naturally be within the research activities that are strongly represented through the master faculty staff.

With this context in mind, we have included a short section with some links for the potential student's perusal so to get an idea of the research interests and strongholds related to the master.

8.1 Research groups

Here we present a selection of represented research groups.

- ALBCOM - Algorithms, Computational Biology, Complexity and Formal Methods

  https://futur.upc.edu/ALBCOM?locale=en

- Barcelona Group on Pure and Applied Proof Theory

  https://www.ub.edu/prooftheory/

- Barcelona Institute of Analytic Philosophy

  http://www.ub.edu/biap/

- Barcelona Research Group on Set Theory

- Barcino (Barcelona Research Group in Non-Classical Logic)

  https://barcinologic.github.io/web/

- Mathematical Institute of the University of Barcelona

  http://www.imub.ub.edu/

- Research group in Analytic Philosophy

  http://www.ub.edu/grc_logos/project_card.php?idProj=86

- Research Group on Model Theory

  http://www.ub.edu/modeltheory/

8.2 Lecturers related to the master

To get an idea of active research you may also simply have a look at the scientific activities of active faculty members. Of course this is not exhausitve since there is also much related research activity going on in the larger Barcelona area that is not conducted by this master's teaching staff. Still, the research activity of the staff is a good indication and first approximation and as such we include it.

- Albert Atserias; Department of Computer Science (UPC); Computational Complexity

  https://www.cs.upc.edu/~atserias/

- Joan Baria; ICREA, Department of Mathematics and Computer Science (UB); Set Theory

  https://www.icrea.cat/Web/ScientificStaff/joan--bagaria-i-pigrau-119

- Jose Luis Balcázar; Department of Computer Science (UPC); Logic in data Mining

  https://www.cs.upc.edu/~balqui/

- Enrique Casanovas; Department of Mathematics and Computer Science (UB); Model Theory

  http://www.ub.edu/modeltheory/casanovas/e.html

- Rafel Farré; Department of Mathematics (UPC); Model Theory

  https://futur.upc.edu/RafaelFarreCirera

- Joan Gispert; Department of Mathematics and Computer Science (UB); Many-Valued Logics

  https://webgrec.ub.edu/webpages/000006/ang/jgispertb.ub.edu.html

- Joost J. Joosten; Department of Philosophy (UB); Proof Theory, Modal Logic, pure and applied

  http://www.joostjjoosten.nl//

- Juan Carlos Martinez; Department of Mathematics and Computer Science (UB); Boolean Algebras

  https://webgrec.ub.edu/webpages/000006/cas/jcmartinez.ub.edu.html

- Tommaso Moraschini; Department of Philosophy (UB); Algebraic Logic and Intuitionistic Logic

  https://moraschini.github.io/

- Sergi Oms; Department of Philosophy (UB); Philosophy of Logic

  https://webgrec.ub.edu/webpages/000015/ang/sergi.oms.ub.edu.html

We may also mention some scholars closely related to the master and active in the larger Barcelona Area, who however do not regularly teach in the master.

- Pilar Dellunde

  https://www.uab.cat/web/el-departament/pilar-dellunde-1260171823608.html

- Tommaso Flaminio;

  https://tomflaminio.wordpress.com/

- Genoveva Martí

  https://memoir.icrea.cat/researchers/marti-genoveva/

- José Martínez;

  http://www.ub.edu/grc_logos/jose-martinez

The list is incomplete and we encourage the interested student to investigate on herself to see which linguists, logicians, etc. work in the larger Barcelona Area.

9 Further reading

Some preliminary reads to get motivated/started before the master begins are:

- P. Pudlák Logical Foundations of Mathematics and Computational Complexity

  https://www.amazon.com/Logical-Foundations-Mathematics-Computational-Complexity-ebook/dp/B00CHGUL3Q

  https://users.math.cas.cz/~pudlak/preface_toc.pdf

- H. Enderton. A mathematical Introduction to Logic

  https://www.amazon.com/Mathematical-Introduction-Logic-Herbert-Enderton/dp/0122384520

- T. Franzén. Gödel's Theorem

  https://www.amazon.com/G%C3%B6dels-Theorem-Torkel-Franz%C3%A9n/dp/1568812388

- G. Boolos. Logic, Logic, and Logic

  https://www.amazon.es/Logic-George-Boolos/dp/067453767X

This list is biased and incomplete yet it may be a good starting point.

10 Alumni

Most of our students do end up in research or academica in general. Other students have found their way to industry, most notably software architecture and engineering. Shortly, a LinkedIn group is to be installed so that alumni of our master can stay in touch with each other.

A Full Teaching Plans per Course

In this appendix we will list all courses in alphabetical order and present its teaching plan in full.

A.1 Abstract Algebraic Logic

- **Course unit code:** 569073
- **Coordinator:** Tommaso Moraschini
- **Department:** Department of Philosophy
- **Credits:** 5
- **Single Program:** S
- **Estimated learning time:**

  Total number of hours: 125

  - **Face-to-face and/or online activities:** 42
    - **Lecture:** Face-to-face, 42
  - **Supervised project:** 40
  - **Independent learning:** 43
- **Recommendations:**

  It is recommended that the students have taken the following courses of the master program: Algebraic Logic and either Universal Algebra or Orders, Lattices and Boolean Algebras.
- **Competences to be gained during study:**
  - Being able to resolve the exercises in the area of the master program.
  - Being able to assemble, study and absorb the relevant scientific literature.
  - Having a working knowledge of the tools and techniques of the various branches of mathematical logic.
  - Developing a sufficiently vast knowledge of non-classical logic and algebraic logic.
  - Being able to connect syntactic and semantics properties of logics by means of bridge theorems.
  - Being able to expose in a clear, concise and precise way some of the recent trends in mathematical logic.
- **Learning objectives:**
  - **Referring to knowledge:**

    * Learn the core of the theory of matrix semantics for propositional logic
    * Absorb various generalizations of the Lindenbaum-Tarski process
    * Develop a detailed knowledge of the various classes of logics in the Leibniz hierarchy as well as their characterizations
    * Develop a detailed knowledge of the various classes of logics in the Frege hierarchy as well as their characterizations
    * Learn some prominent bridge theorems
  - **Referring to abilities, skills:**
    * Classify a propositional logic in the Leibniz and Frege hierarchies
    * Use this classification to derive a concrete description of its matrix semantics
    * Apply bridge theorems to test whether a concrete logic has a certain metalogical property
    * Being able to reproduce the proofs of the main theorems about the Leibniz and Frege hierarchies, as well as the proofs of some bridge theorems

- **Teaching blocks:**
  - **Matrix semantics:**
    * Matrices
    * Leibniz congruences
    * Reduced matrices
    * The class of matrix models of a logic
    * Czelakowski's Theorem
  - **Leibniz hierarchy:**
    * Protoalgebraic logics
    * Equivalential logics
    * Weakly algebraizable logics
    * Algebraizable logics
    * Truth-equational logics
  - **Full models and the isomorphism theorem:**
    * Full models
    * Isomorphism theorem
    * Full models and the Leibniz hierarchy
  - **Frege hierarchy:**
    * Selfextensional logics
    * Fully selfextensional logics
    * Fregean logics

* Fully Fregean logics

- **Teaching methods and general organization:**

  Taught classes that explain the content of the course and discuss motivating examples. The students will be asked to resolve weekly exercises.

- **Official assessment of learning outcomes:**

  This form of evaluation will be based on weekly exercises that will be corrected by the professor. If the students do not provide at least of 80% of exercises with a positive outcomes, then there will be a final exam.

  - **Examination-based assessment**
    Final exam

- **Reading and study resources:**

  Books:

  - Blok, W. and Pigozzi, D. Algebraizable logics. Memoirs of the American Mathematical Society. Providence (R.I.) American Mathematical Society, cop. 198, vol. 396.
  - Czelakowski, J. Protoalgebraic logics. Dordrecht [etc.] : Kluwer Academic, 2001.
  - Font, J.M. and Jansana, R. A general algebraic semantics for sentential logics. Berlin ; Barcelona [etc.] : Springer, cop. 1996.
  - Wójcicki, R. Theory of logical calculi: basic theory of consequence operations. Dordrecht [etc.] : Kluwer Academic, 1988.
  - Blok, W. and Pigozzi, D. Abstract algebraic logic and the deduction theorem. Downloadable at: `http://orion.math.iastate.edu/dpigozzi/`
  - Font, J.M., Jansana, R. and Pigozzi, D. A survey of abstract algebraic logic. In Studia Logica. Praha : ZakDad Narodowy im. Ossolin'skich, 2003, 74:13-97.
  - Font, J.M. Abstract algebraic logic - An introductory textbook, Vol. 60 of Studies in Logic - Mathematical Logic and Foundations, College publications 2016.

A.2 Advanced Model Theory

- **Course unit code:** 569075
- **Coordinator:** Enrique Casanovas Ruiz Fornells
- **Department:** Department of Mathematics and Computer Science
- **Credits:** 5
- **Single Program:** S

- **Estimated learning time:**

  Total number of hours: 125

  - **Face-to-face and/or online activities:** 42
    - **Lecture with practical component:** Face-to-face, 42
  - **Supervised project:** 18
  - **Independent learning:** 65

- **Recommendations:**

  It is very recommendable to have followed with success a basic model theory course.

- **Competences to be gained during study:**

  - To be able to carry out independent study of standard textbooks and journal articles on model theory.
  - To understand the interest and potential impact of model theory and its applications to mathematics.
  - To be able to handle standard model-theoretic problems, formulating them with accuracy and knowing the usual techniques to solve them.

- **Learning objectives:**

  - **Referring to knowledge:**
    - A deep knowledge of the basic notions, tools and results from model theory (saturation, definability, prime models, indiscernibles, etc).
    - Introduction to the elementary notions of stability theory (Morley and Baldwin-Lachlan theorems).
    - Familiarity with examples of complete first-order theories, particularly vector spaces and algebraically closed fields.
  - **Referring to abilities, skills:**
    - To be able to prove completeness, omega-categoricity and quantifier elimination for given examples of first-order theories.
    - To characterize definable sets in models of a given theory.
    - To classify the complexity of a theory from the point of view of stability theory.

- **Teaching blocks:**

  - **Preliminaries:**
    - Back and forth, saturation, the monster model, quantifier elimination, model-completeness, and omega-categoricity
  - **Definability:**

    * Definable, type-definable and invariant relations
    * Beth and Svenonious theorems
    * Definable and algebraic closure
    * Imaginaries
  - **Pregeometries:**
    * Closure operators with exchange
    * Independence, basis and dimension
    * Modularity
    * Vector spaces
  - **Ranks:**
    * Cantor-Bendixson rank
    * Morley rank
    * Omega-stability and superstability
    * Strongly minimal sets
    * Algebraically closed fields
  - **Prime models:**
    * Omitting types
    * Prime and atomic models
    * Indiscernibles
    * Two cardinal theorems
  - **Categoricity:**
    * Morley and Baldwin-Lachlan's theorems

- **Teaching methods and general organization:**
  - Detailed exposition in class, with examples and full proofs.
  - Asking questions and proposing problems to be solved by the student.
  - Class discussions on the main notions, examples, statements and relevant proofs.
  - Presentation of a topic by a student, previously discussed with the instructor.

- **Official assessment of learning outcomes:**
  - Weekly exercises. 40%
  - Oral presentation of a topic. 10%
  - Two partial exams. 25 % each.

  Examination based assessment:
  - Final examination

- **Reading and study resources: Books:**
  - E. Casanovas Advanced Model Theory.
    - These Lecture Notes will be delivered during the course.
  - E. Casanovas. Teoría de Modelos (Lecture Notes). 1999. Available at `http://www.ub.es/modeltheory/casanovas`
  - C.C. Chang and H.J. Keisler. Model Theory. North Holland PC, 3rd ed. 1990.
  - W. Hodges. Model Theory. Cambridge UP 1993.
  - D. Lascar. Stability in Model Theory. Longman Sci. and Tech. 1987.
  - D. Lascar. La théorie des modèles en peu de maux. Cassini, Paris 2009.
  - D. Marker. Model Theory: an introduction. Springer 2002.
  - D. Marker, M. Messmer and A. Pillay. Introduction to the Model Theory of Fields. Lecture Notes in Logic 5. Springer 1996.
  - A. Pillay. An introduction to Stability Theory. Oxford UP 1983.
  - A. Pillay. Geometrical Stability Theory. Oxford UP. 1996.
  - B. Poizat. Cours de théorie des modèles. Offilib. Villeurbanne 1985.
  - K. Tent and M. Ziegler. A course in Model Theory. Lecture Notes in Logic 40. Cambridge UP 2012.

A.3 Algebraic Logic

- **Course unit code:** 569066
- **Coordinator:** Ramon Jansana Ferrer
- **Department:** Department of Philosophy
- **Credits:** 5
- **Single Program:** S
- **Estimated learning time:**

  Total number of hours: 125
  - **Face-to-face and/or online activities:** 42
    - **Lecture:** Face-to-face, 42
  - **Supervised project:** 41
  - **Independent learning:** 42
- **Competences to be gained during study:**

  - At the end of the course the student will master the main methods and results of contemporary Algebraic Logic.
  - The student will acquire the capacity to solve problems of medium difficulty in the field of Algebraic Logic.
  - The student will be able to write in a correct style mathematical proof.
  - The student will be able to read standard literature in the field of Algebraic Logic.

- **Learning objectives:**
  - **Referring to knowledge:**
    * To understand the Lindenbaum-Tarski method and its several generalizations.
    * To understand and master the basic tools of contemporary Algebraic Logic.
    * To understand algebraic semantics, matrix semantics and generalized matrix semantics for sentential logics.
  - **Referring to abilities, skills:**
    * To solve problems in Algebraic Logic
    * To write proofs in a correct style

- **Teaching blocks:**
  - **Mathematical preliminaries:**
    * Algebraic languages
    * The notion of algebra
    * The algebra of terms
    * Homomorphisms
    * Congruences
    * Quotient algebras
    * Direct products
    * Varieties and quasivarieties
    * Categories
    * Closure operators, closure systems and consequence relations.
  - **The algebraization of Intuitionistic Logic. Heyting algebras.**
  - **The abstract concept of logic.**
    * Sentential logics
    * Ways of defining logics
    * Absolutely free algebras of arbitrary cardinalities
  - **The Lindenbaum-Tasrki method**

        - A first analysis of the Lindenbaum-Tarski method as applied to Intuitionistic Propositional Logic
        - Regularly algebraizable logics
        - Two modal logics
        - A second analysis of the Lindenbaum-Tarski method as applied to Intuitionistic Propositional Logic
        - Logical filters and Leibniz congruences
        - The class of algebras Alg*S of a logic S
        - Logical filters and congruences in equivalential and in regularly algebraizable logics
    - **The algebraization of some linear logics:**
        - Brief introduction to linear logic
        - The logics ILL$\hat{-}$ and ILL
        - Algebraic semantics for ILL$\hat{-}$ and ILL
        - Classical Linear Logic
        - Intutionistic linear logic without the multiplicative constant 1
    - **Generalizations and abstractions of the notion of algebraizability:**
    - **Algebraizable logics:**
        - Equational consequences and generalized quasivarieties
        - Transformers
        - Algebraizable logics
        - A syntactic characterization of algebraizable logics
        - Special classes of algebraizable logics
        - The transformers applied to arbitrary algebras
        - The isomorphism theorem
        - The Deduction Theorem

- **Teaching methods and general organization:**
    - Lectures
    - Problem solving, in the classroom and at home

  If online teaching is necessary because of the current health situation, the timetable of the course will be maintained and teaching will take place according to the guidelines of the academic authorities either completely or partly in non on-site form.

  In case of on-site teaching, the timetable will be maintained and all classes will be on-site.

- **Official assessment of learning outcomes:**

  - Weekly exercises. The final grade will be the mean of the grades of the different sets of exercises.
  - Depending on the evolution of the pandemic the partial exams (if any) and the final exam will be in person or by online means.

  Examination based assessment:

  - Final exam
    Depending on the evolution of the pandemic the exam will be in person or by online means.

- **Reading and study resources: Books:**
  - Janusz Czelakowski, Protoalgebraic logics, Kluwer, Dordrecht, 2001.
  - Wójcicki, R. Theory of logical calculi. Basic theory of consequence operations. Kluwer, Dordrecht, 1988.
  - Rasiowa, H. An algebraic approach to non-classical logics. North-Holland, Amsterdam, 1974.
  - Font, J.M.; Jansana, R. A general algebraic semantics for sentential logics. Second revised edition. Lecture Notes in Logic 7. Association of Symbolic Logic 2009. Electronic version at Project Euclid: `http://projecteuclid.org/euclid.lnl/1235416965`
  - Font, J.M. Abstract Algebraic Logic. An Introductory Textbook. College Publications, 2016.

  Chapter:

  - Jansana, R. Propositional consequence relations and algebraic logic. Stanford Encyclopedia of Philosophy, 2010. (`http://plato.stanford.edu/`)

A.4 Algorithms for VSLI

- **Course unit code:** –
- **Coordinator:** Jordi Cortadella Fortuny
- **Department:** Department of Philosophy
- **Credits:** 6
- **Single Program:** S
- **Estimated learning time:**
  Total number of hours: 126
  - **Face-to-face and/or online activities:** 45
    - **Lecture:** Face-to-face, 36

    - **Lecture with practical component:** Face-to-face, 18
  - **Independent learning:** 72

- **Competences to be gained during study:**

  Advanced computing

  - Capability to identify computational barriers and to analyze the complexity of computational problems in different areas of science and technology as well as to represent high complexity problems in mathematical structures which can be treated effectively with algorithmic schemes.
  - Capability to use a wide and varied spectrum of algorithmic resources to solve high difficulty algorithmic problems.
  - Capability to understand the computational requirements of problems from non-informatics disciplines and to make significant contributions in multidisciplinary teams that use computing.

  Generic

  - Capability to apply the scientific method to study and analyse of phenomena and systems in any area of Computer Science, and in the conception, design and implementation of innovative and original solutions.
  - Capacity for mathematical modeling, calculation and experimental designing in technology and companies engineering centers, particularly in research and innovation in all areas of Computer Science.

  Transversal Competences

  - Reasoning
    - Capacity for critical, logical and mathematical reasoning. Capability to solve problems in their area of study. Capacity for abstraction: the capability to create and use models that reflect real situations. Capability to design and implement simple experiments, and analyze and interpret their results. Capacity for analysis, synthesis and evaluation.
  - Basic
    - Ability to apply the acquired knowledge and capacity for solving problems in new or unknown environments within broader (or multidisciplinary) contexts related to their area of study.
    - Capability to communicate their conclusions, and the knowledge and rationale underpinning these, to both skilled and unskilled public in a clear and unambiguous way.

        * Possession of the learning skills that enable the students to continue studying in a way that will be mainly self-directed or autonomous.

- **Learning objectives:**
    - **Referring to knowledge:**
        * To know the most important aspects of the use of algorithms for the automation of logic circuit design and the different logic-based techniques used for such automation.
    - **Referring to abilities, skills:**
        * To be capable of designing algorithms of different types for logic circuit design.
        * To be capable of using formal logic techniques for analyzing design problems of logic circuits, in order to design algorithms that allow automation.
- **Teaching blocks:**

  Most of the design flow of an integrated circuit is automated, from specifications using hardware description languages to the physical layout. The flow goes through different phases of synthesis and analysis: behavioral synthesis, logic synthesis, floorplanning, placement, routing, timing analysis, formal verification, etc.

  In this course the most important algorithmic aspects of electronic circuit design automation are presented. An important part of the course is devoted to algorithms for Boolean function minimization and representation with logic gates. Algorithms for physical design (floorplanning, placement and routing) are mainly based on solving problems with graphical models.

  The course is organized in the following thematic teaching blocks:

    - **Introduction.** Integrated circuit fabrication. Layout layers and design rules. VLSI design flow. VLSI design styles.
    - **Two-level logic synthesis** Boolean Algebras. Representation of Boolean functions. Quine-McCluskey algorithm. Heuristic logic minimization: Espresso.
    - **Multi-level logic synthesis.** Kernel-based algebraic decomposition. AIG-based decomposition. Technology mapping for standard cells and FPGAs.
    - **Formal verification.** Binary Decision Diagrams. Combinational equivalence checking. Sequential equivalence checking. Model checking with temporal logic.
    - **Partitioning and Floorplanning.** Partitioning algorithms. Representation of floorplans. Slicing floorplans. Floorplanning algorithms.

- **Placement.** Optimization objectives. Algorithms for global placement. Algorithms for legalization and detailed placement.
- **Global routing.** Representation of routing regions. Algorithms for single-net and full-net routing.
- **Detailed routing.** Horizontal and vertical constraint graphs. Channel routing. Switchbox routing. Over-the-cell routing.

• **Teaching methods and general organization:**

- Lectures
- Problem solving, in the classroom and at home

If online teaching is necessary because of the current health situation, the timetable of the course will be maintained and teaching will take place according to the guidelines of the academic authorities either completely or partly in non on-site form.

In case of on-site teaching, the timetable will be maintained and all classes will be on-site.

• **Official assessment of learning outcomes:**

Grade = 40% FW + 30% FT + 20% EX + 10% SP

FW = Final Work (graded from 0 to 10) in which each participant is required to present a research paper or section of a book (previously assigned by the lecturer). The presentation consists of:

- 3-5 minutes background on the topic of the paper, a motivation.
- 1 minute overview of the key ideas of the paper.
- 15 minutes presentation with most important details.
- 5 minutes demo of a program that implements the ideas introduced in the paper.

FT = Final test graded from (0 to 10) including all the contents of course.

EX = Exercises assigned to the student and solved during the Autonomous Learning time

SP = Summaries and participation (graded from 0 to 10) in which each participant is required to deliver a summary (1 page extent) of each others presentation and to participate (with questions and comments).

• **Reading and study resources:**

- Logic synthesis and verification algorithms - Hachtel, G.D.; Somenzi, F, Kluwer Academic Publishers, 1996. ISBN: 0792397460 `http://cataleg.upc.edu/record=b1196653~S1*cat`

- Handbook of algorithms for physical design automation - Alpert, C.J.; Metha, D.P.; Sapatnekar, S.S. (eds.), CRC : Taylor Francis, 2009. ISBN: 9780849372421 `http://cataleg.upc.edu/record=b1431576~S1*cat`
- Electronic design automation: synthesis, verification, and test - Wang, L.-T.; Chang, Y.-W.; Cheng, K.-T. (eds.), Morgan Kaufmann Publishers/Elsevier, 2009. ISBN: 9780123743640 `http://cataleg.upc.edu/record=b1390006~S1*cat`

A.5 Basic Model Theory

- **Course unit code:** –
- **Coordinator:** Rafel Farré
- **Department:** UPC
- **Credits:** 6
- **Single Program:** S
- **Estimated learning time:**
  Total number of hours: 150
  - **Lecture:** 30
    - **Lecture with practical component:** Face-to-face, 15
  - **Supervised project:** 45
  - **Independent learning:** 60
- **Competences to be gained during study:**
  - Perform the usual type of demonstrations in Logic, exposing them in an understandable and elegant way.
  - To be able to present a presentation showing mastery of the fundamental concepts of mathematical logic.
  - To be able to solve the type of problems dealt with in the master's degree subjects.
  - Be able to apply the concepts of contemporary logic to the study of the history of logic.
  - To be able to understand the technical concepts necessary to study the arguments used in the philosophy of mathematics and logic.
  - To be able to understand articles in the relevant scientific literature.
  - Be able to ask relevant questions to guide research in logic.
  - To be able to begin to carry out original research in some subjects similar to those covered in the master's degree.

- To master the mathematical tools that apply to various specialized branches of mathematical logic.
- To be able to expose in an understandable way some current trends in research in mathematical logic.

- **Learning objectives:**
  - **Referring to knowledge:**
    - To know the theorems of Compacity and Löwenheim-Skolem.
    - To know the Lós Theorem for ultraproducts.
    - To know its use in the characterization of elementary classes.
    - To know some of the preservation theorems.
    - To know the relationship between the different concepts of saturation, homogeneity and universality.
    - To know what a Back and Forth system is and what it is used for.
    - To know the type omission theorem and its use in the study of the numerable models of a theory.
    - To know what a prime model is and to know the criterion to know if a theory has a prime model.
    - To know the Ryll-Nardewski Theorem. To know the concept of model-completeness and model-companion.
    - To know the Fraissé construction to obtain omega-categorical theories.
    - To know the concepts of algebraic closure and definable closure as well as their most important properties.
    - To know the imaginaries and some of their uses.
    - To know Vaught's two-cardinal theorem.
    - To know Morley's Range and its properties.
    - To know Morley's Theorem.
  - **Referring to abilities, skills:**
    - 1. To know how to apply the compactness and Löwenheim-Skolem Theorems.
    - 2. To know how to manipulate elementary chains of structures.
    - 3. To know how to manipulate ultraproducts.
    - 4. To be able to use the techniques of the previous sections to prove some of the preservation theorems.
    - 5. Understand what types are and know how to work with them.
    - 6. Master the language of elementary partial applications.
    - 7. Understand the concepts of saturation and homogeneity. To be able to prove results concerning the relationship between these concepts.

    * 8. Master the Back and Forth technique. Be able to use it to show the completeness and/or quantifier elimination of a relatively simple theory.
    * 9. Understand the type omission theorem and know how to apply it appropriately.
    * 10. Know how to obtain Fraissé's limit in examples of low complexity.
    * 11. To be able to work with imaginaries.
    * 12. Understand the meaning of indiscernibles and their use to build models with many automorphisms and few types.
    * 13. Understand and be able to work with Morley's Range.
    * 14. Understand the proof of Morley's theorem.
    * 15. Be able to reproduce demonstrations of medium complexity.

- **Thematic blocks:**
  - 1. Compacity and Löwenheim-Skolem Theorems.
  - 2. Elementary extensions and chains.
  - 3. Ultraproducts. Elementary classes.
  - 4. Preservation theorems.
  - 5. Types. Partial elementary applications. Saturation. Homogeneity.
  - 6. Back and Forth and partial isomorphism. Quantifier elimination.
  - 7. Type omission.
  - 8. Numerable models. Prime models. Omega-categoricity.
  - 9. Model-completeness.
  - 10. Fraissé amalgamation.
  - 11. Definability. Algebraic closure and definable closure.
  - 12. Imaginaries.
  - 13. Indiscernibles.
  - 14. Vaught's two cardinal theorem.

- **Teaching methodologies:**
  - Master classes: In master classes the contents of the subject are presented orally by a teacher without the active participation of the students.
  - Expository classes: In expository classes one or more students present orally a topic or work, previously prepared, in front of the rest of the group. Sometimes a previous written presentation may be encouraged.

- Written work: Activity consisting of the presentation of a written document.
- Problem solving: In the problem-solving activity, the teacher presents a complex question that the students must solve, either by students must solve, either working individually or in teams.

A.6 Basic Set Theory

- **Course unit code:** 569058
- **Coordinator:** Joan Bagaria Pigrau
- **Department:** Department of Mathematics and Computer Science
- **Credits:** 6
- **Single Program:** S
- **Estimated learning time:**

  Total number of hours: 150

  - **Face-to-face and/or online activities:** 45
    * **Lecture with practical component:** Face-to-face, 45
  - **Supervised project:** 25
  - **Independent learning:** 80
- **Competences to be gained during study:**
  - To have an acceptably clear idea of the two aspects of set theory: as the foundation of mathematics and as a specific mathematical theory.
  - To have a theoretical and practical mastery of the basic methods and results of set theory, that is, of the methods and results which, on the one hand, are presupposed in other mathematical disciplines and, on the other hand, form the indispensable basis for the deeper study of set theory.
- **Learning objectives:**
  - **Referring to knowledge:**
    * To understand the content of the axioms of ZFC, knowing which results depend on which axioms, in particular, which ones depend on the axiom of choice, the axiom of substitution, and the axiom of grounding.
    * To know the conjunctive construction of natural nouns, the order of rational nouns and the linear continuum.
    * To know the theory of good orders and transfinite ordinary names, with the different forms of transfinite recursion.

    * To know the fundamental results of infinite cardinal arithmetic.
    * To know the structure of the conjunctive universe (the cumulative hierarchy) and the definitions by recourse on the relationship of relevance.
  - **Referring to abilities, skills:**
    * Express the results clearly and accurately.
    * To demonstrate with rigour and elegance.

- **Thematic blocks:**
  - **Basic concepts:**
    * Set algebra
    * Ordered pairs
    * Relations
    * Functions
  - **Comparison of cardinality:**
    * The Theorem of Schröder-Bernstein
    * The Theorem of Cantor
    * Higher cardinality sets
  - **Zermelo-Fraenkel's basic axioms**
    * The language of set theory
    * The first ZFC axioms
    * Initial development
  - **Good orders:**
    * Comparability of good orders
    * The substitution axiom
    * Recursion to good orders
  - **Ordinal numbers:**
    * Definition and basic properties of ordinal numbers
    * The enumeration theorem
    * Transfinite recursion
    * Ordinal arithmetic
  - **Real numbers:**
    * The theorem of Cantor-Dedekind
    * Topology of the real line
    * The theorem of Cantor-Bendixson
    * Baire's category theorem
  - **Infinite cardinals:**
    * The axiom of choice

    * Equivalents of the choice axiom
    * Cardinal arithmetic
    * The theorem of König
    * Cofinality
    * Regular and singular cardinals
    * Inaccessible cardinals
  - **The cumulative hierarchy and the axiom of substantiation:**
    * Well-founded sets
    * The axiom of substantiation
    * The cumulative hierarchy
    * Recursion on the relationship of belonging

- **Reading and study resources:**
  - HRBACEK, K. i JECH, T. Introduction to Set Theory. New York: Marcel Dekker, 1999.
  - JECH, T. Set Theory: the third millennium edition, revised and expanded, Part I. Berlín : Springer, 2006.
  - JUST W. i WEESE, M. Discovering Modern Set Theory. Providence: American Mathematical Society, 1996.
  - MOSCHOVAKIS, Y. Notes on Set Theory. Springer, 1994.
  - SCHIMMERLING, E. A course on Set Theory, Cambridge University Press, 2011.

A.7 Combinatorial Problem Solving

- **Coordinator:** Enric Rodriguez Carbonell
- **Department:** Department of Computer Science
- **Credits:** 6
- **Single program:** S
- **More information:**

  `https://www.cs.upc.edu/~erodri/webpage/cps/cps.html`

  `https://www.fib.upc.edu/en/studies/masters/master-innovation-and-research-informatics/curriculum/syllabus/CPS-MIRI`
- **Estimated learning time:**

  Total number of hours: 10 (weekly)
  - **Theory:** 2
  - **Problems:** 0

- **Laboratory:** 1
- **Guided learning:** 0.6
- **Autonomous learning:** 6.4

- **Recommendations:**

  Requisites:

  - Basic knowledge on the Linux operating system and the C++ programming language.
  - Basic knowledge on linear algebra, graph algorithms and logics.

- **Competences to be gained during study:**

  - Capability to use a wide and varied spectrum of algorithmic resources to solve high difficulty algorithmic problems.
  - Capability to understand the computational requirements of problems from non-informatics disciplines and to make significant contributions in multidisciplinary teams that use computing.
  - Capability to apply the scientific method to study and analyse of phenomena and systems in any area of Computer Science, and in the conception, design and implementation of innovative and original solutions.
  - Capacity for mathematical modeling, calculation and experimental designing in technology and companies engineering centers, particularly in research and innovation in all areas of Computer Science.
  - Capacity for critical, logical and mathematical reasoning. Capability to solve problems in their area of study. Capacity for abstraction: the capability to create and use models that reflect real situations. Capability to design and implement simple experiments, and analyze and interpret their results. Capacity for analysis, synthesis and evaluation.
  - Ability to apply the acquired knowledge and capacity for solving problems in new or unknown environments within broader (or multidisciplinary) contexts related to their area of study.

- **Learning objectives:**

  - Modelling problems arising from computer science and other disciplines in the solving paradigms considered in the course: constraint programming, linear integer programming, propositional satisfiability.
  - Becoming familiar with state-of-the-art tools for the solving paradigms considered in the course: constraint programming, linear integer programming, propositional satisfiability.

- Understanding the algorithmic foundations of each of the solving paradigms considered in the course: constraint programming, linear integer programming, propositional satisfiability.

- **Teaching blocks:**
  - **Combinatorial Problems**
    * Informal definition
    * NP-complete problems vs. polynomial-time problems
    * Some examples and applications: propositional satisfiability, graph coloring, knapsack, bin packing, etc.
    * Approaches to problem solving
  - **Constraint Programming**
    * Basic definitions
    * Constraint Satisfaction Problems
    * Examples
    * Local consistency: arc consistency, directional arc consistency, bounds consistency
    * Constraint propagation for global constraints: all different
    * Search algorithms: basic backtracking, forward checking, partial/full lookahead
    * Variable and value ordering heuristics
    * Constraint Optimization Problems
    * Modeling and solving problems with CP
  - **Linear Programming**
    * Review of linear programming: The simplex algorithm
    * Duality and the dual simplex
    * Modelling and solving problems with linear programming
    * Mixed integer linear programming
    * Branch bound, cutting planes, branch cut
    * Totally unimodular matrices
    * Network simplex algorithm
    * Modelling and solving problems with mixed integer linear programming
  - **SAT solving and extensions**
    * Propositional logic
    * The satisfiability (SAT) problem
    * DPLL algorithm
    * Resolution
    * Conflict-Driven Clause Learning SAT solvers

    * Modeling and solving problems with SAT: cardinality constraints, pseudo-boolean constraints
    * Satisfiability Modulo Theories

- **Teaching methods and general organization:**

  The main feature of the teaching methodology is the use of materials accessible through the web, specifically designed for a self-learning course. These materials allow reformulating teaching in such a way that the traditional model of classes largely disappears. Thus:

  1. It regards the class as a baseline for work, which the student must continue and deepen on his/her own.
  2. It builds upon high quality materials (slides, lists of problems, solved problems, examples of laboratory practical work, LP/SAT/CP software, bibliographic references).
  3. It aims at motivating students, with examples, discussions, comments, etc... The intuitions behind the definitions, properties and techniques are discussed in group.

  The laboratory will encourage independent work by the students. The role of the teacher will be mainly to assist and evaluate the students, who should work mostly autonomously.

- **Official assessment of learning outcomes:**

  - 50% of the final grade corresponds to theory. This grade will be obtained by means of a written exam at the end of the course.
  - 50% of the final grade corresponds to laboratory. This grade will be obtained as the mean of three successive projects (one for CP, another one for LP, and another one for SAT) that the students will have to hand in.

- **Reading and study resources:**

  Books

  - Handbook of satisfiability - Biere, A. [et al.] (eds.), IOS Press, 2009. ISBN: 9781586039295
  - Introduction to algorithms - Cormen, T.H. [et al.], MIT Press, 2009. ISBN: 9780262033848
  - Handbook of constraint programming - Rossi, F.; Beek, P. van ; Walsh, T. (eds.), Elsevier, 2006. ISBN: 0444527264
  - Model building in mathematical programming - Williams, H.P, Wiley Sons, 2013. ISBN: 9781118443330
  - Computational techniques of the simplex method - Maros, I, Kluwer Academic Publishers , 2003. ISBN: 1402073321

A.8 Combinatorial Set Theory

- **Course unit code:** 569068
- **Coordinator:** Joan Bagaria Pigrau
- **Department:** Department of Mathematics and Computer Science
- **Credits:** 5
- **Single program:** S
- **Estimated learning time:**

  Total number of hours: 125

  - **Face-to-face and/or online activities:** 42
    - **Lecture:** face-to-face, 42
  - **Supervised project:** 18
  - **Independent learning:** 65

  Requisites:

  - 569058 - Basic Set Theory (Recommended)
- **Competences to be gained during study:**
  - To acquire a basic knowledge of some of the main topics in combinatorial set theory.
  - To learn to think and reason about sets and structures of uncountable size.
  - To be exposed to beautiful proofs and arguments involving infinity.
  - To train at solving problems and exercises.
- **Learning objectives:**
  - **Referring to knowledge:**

    To know some the main results in combinatorial set theory. The main topics are: closed unbounded sets and stationary sets of uncountable regular cardinals, infinite trees with various properties (Aronszajn trees, Suslin trees, Kurepa trees), infinite Ramsey theory, and basic concepts of the theory of large cardinals (inaccessible, Mahlo, weakly compact, Ramsey, measurable).
- **Teaching blocks:**
  - **Closed and unbounded sets and stationary subsets of uncountable regular cardinals**
  - **Trees: Aronszajn trees, Suslin trees, Kurepa trees**

- **An introduction to infinite Ramsey theory**
- **Some basic results in the theory of large cardinals**

- **Teaching methods and general organization:**
  - Lectures.
  - Problem solving and take-home weakly exercises.
- **Official assessment of learning outcomes:**

  Weakly problem set, to be handed in.

  - **Examination-based assessment** Final take-home exam.
- **Reading and study resources:**

  Books:

  - Kunen, K. 1980 Set Theory, An Introduction to Independence Proofs. North-Holland, Amsterdam.
  - Thomas Jech, Set theory. The Third Millenium Edition, Revised and Expanded, Springer-Verlag, 2002.

A.9 Computability

- **Course unit code:** –
- **Coordinator:** –
- **Department:** UPC
- **Credits:** 6
- **Single program:** S
- **More information:**
- **Estimated learning time:**

  Total number of hours: 150

  - **Face-to-face and/or online activities:** 50
    - **Lecture:** Face-to-face, 35
    - **Lecture with practical components:** Face-to-face, 15
  - **Supervised project:** 20
  - **Independent learning:** 80
- **Competences to be gained during study:**
  - To argue correctly according to standard criteria in current research.

- Present in a clear, concise and coherent way the solutions to the exercises and problems to be solved during the master's course in accordance with today's common standards in research.
-

- **Competences:**
  - –

- **Learning objectives:**
  - **Referring to knowledge:**
    * The student will have assimilated the standard model of computation and the meaning of Church's thesis.
  - **Referring to abilities, skills:**
    * The student will also be able to recognize unsolvable problems problems and classify them by their degree of unsolvability in the arithmetic hierarchy.
    * Familiarity with the phenomenon of incompleteness in sufficiently powerful theories.
  - **Referring to attitudes, values and norms:**
    * Lose fear of complex processes and acquire an attitude to start analyzing the most important (or understandable) parts of these processes.
    * Acquire the norm to place scientific activities and contributions within the literature landscape with due references to the main contributors.

- **Teaching blocks:**
  - **1. Turing computable functions**
  - **2. Mu-computable functions**
  - **3. Equivalence and Church-Turing thesis**
  - **4. Universal machine and undecidability of the halting problem**
  - **5. More undecidable problems**
  - **6. Decidability, semi-decidability, reducibility and completeness**
  - **7. Undecidability of first-order logic**
  - **8. Arithmetic hierarchy**
  - **9. Undefinability of truth**
  - **10. Incompleteness**

- **Teaching methods and general organization:**

  Lectures: In the lectures, the contents of the course are presented orally by a teacher without the active participation of the students.

- **Official assessment of learning outcomes:**

  A final exam and at-home exercises to turn in.

- **Reading and study resources:**

  Books:

  – –

A.10 Computational Complexity

- **Course unit code:** 569077
- **Coordinator:** Joost Johannes Joosten
- **Department:** Department of Philosophy
- **Credits:** 6
- **Single program:** S
- **More information:** `https://www.fib.upc.edu/en/studies/masters/master-innovation-and-research-informatics/curriculum/syllabus/CC-MIRI`
- **Estimated learning time:**

  Total number of hours: 150

  – **Face-to-face and/or online activities:** 56
    * **Lecture:** Face-to-face, 28
    * **Lecture with practical components:** Face-to-face, 14
    * **Problem-solving class:** Face-to-face, 14

  – **Supervised project:** 50 (eight times five homework sessions)

  – **Independent learning:** 54

- **Competences to be gained during study:**

  – Being able to understand and situate in the appropriate historical context of some of the main texts in the field of philosophy of modern logics and mathematics.

  – Being able to expose in a clear, concise, and precise way some of the main ideas present in the discussions concerning the historical development of logic.

- **Competences:**

- **Technical Competences of each Specialization**
  - **Advanced computing**
    - **CEE3.1 -** Capability to identify computational barriers and to analyze the complexity of computational problems in different areas of science and technology as well as to represent high complexity problems in mathematical structures which can be treated effectively with algorithmic schemes.
    - **CEE3.3 -** Capability to understand the computational requirements of problems from non-informatics disciplines and to make significant contributions in multidisciplinary teams that use computing.
- **Generic Technical Competences**
  - **Generic**
    - **CG1 -** Capability to apply the scientific method to study and analyse of phenomena and systems in any area of Computer Science, and in the conception, design and implementation of innovative and original solutions.
    - **CG3 -** Capacity for mathematical modeling, calculation and experimental designing in technology and companies engineering centers, particularly in research and innovation in all areas of Computer Science.
- **Transversal Competences**
  - **Reasoning**
    - **CTR6 -** Capacity for critical, logical and mathematical reasoning. Capability to solve problems in their area of study. Capacity for abstraction: the capability to create and use models that reflect real situations. Capability to design and implement simple experiments, and analyze and interpret their results. Capacity for analysis, synthesis and evaluation.
  - **Basic**
    - **CB8 -** Capability to communicate their conclusions, and the knowledge and rationale underpinning these, to both skilled and unskilled public in a clear and unambiguous way.
    - **CB9 -** Possession of the learning skills that enable the students to continue studying in a way that will be mainly self-directed or autonomous.

• **Learning objectives:**
- **Referring to knowledge:**
  - Understand different computational models and there interrelations.

    - Understand the concept of (space, time) complexity classes (deterministic, non-deterministic, probabilistic).
    - Understand Lower Bounds in Circuit Complexity and their importance.
    - Understand the impact of computational complexity, both to theoretical computer science and to applied computer science.
  - **Referring to abilities, skills:**
    - Be able to identify inherent computational barriers.
    - Be able to model computational processes and analyze the complexity of computational problems.
    - Know how to use the main pointers in the literature and be able to place a paper in the literature landscape.
    - Consolidate the previously defined competences and being able to apply them in various contexts both pure and applied.
  - **Referring to attitudes, values and norms:**
    - Lose fear of complex processes and acquire an attitude to start analyzing the most important (or understandable) parts of these processes.
    - Acquire the norm to place scientific activities and contributions within the literature landscape with due references to the main contributors.

- **Teaching blocks:**
  - **Computational Models and Complexity Measures**
    - Turing machine model. RAM model
    - Boolean circuit model
    - Time complexity
    - Space complexity
    - Circuit size
    - Circuit depth
    - Time and space hierarchy theorems
  - **P, NP and NP-completeness**
    - Polynomial time
    - Reducibilities
    - Non-deterministic algorithms and class NP
    - Cook-Levin Theorem
    - Many other NP-complete problems
  - **Polynomial-time Hierarchy and Alternations**
    - Oracle reducibility
    - NP and co-NP

        * Levels of the hierarchy
        * Quantifier alternations
        * Complete problems
    - **Space Complexity**
        * Polynomial space
        * Unbounded alternations
        * PSPACE-complete problems
        * Savitch Theorem
        * Immerman-Szcelepscenyi Theorem
        * Logarithmic space
        * NL-complete problems
    - **Randomized Computation**
        * Bounded-error and zero-error probabilistic polynomial time
        * Error-reduction
        * Randomized reductions
        * Valiant-Vazirani reduction to Unique SAT
    - **Counting and Enumeration**
        * Some examples: graph reliability, counting matchings and the permanent, partition functions
        * Counting computation paths in non-deterministic machines
        * Valiant's Theorem
        * Random self-reducibility of the permanent
    - **Probabilistic Proofs**
        * Interaction and randomness in proofs
        * Probabilistic proofs for graph non-isomorphism
        * Probabilistic proofs for P and Shamir's Theorem: IP = PSPACE
    - **Circuit Lower Bounds**
        * Monotone circuits
        * Lower bounds for clique and perfect matching
        * Bounded-depth circuits
        * Hastad's switching lemma
        * Approximation by polynomials

- **Teaching methods and general organization:**

  Blackboard lectures for theory classes and discussion sessions for the problem classes. The theory classes will follow the main textbook for the class [Arora and Barak] rather closely. Since we plan to cover more topics than is possible in the given time, students will be required to read the details in the textbook as homework (a draft of the book is available on-line for free). The aim of the discussion sessions is to solve some problems from that book and to discuss the reading material.

- **Official assessment of learning outcomes:**

  Students will be required to submit 5 problem/discussion sheets. Each will be given a grade in [0,1] (P1,...,P5).

  There will be a final exam graded in [0,10] (E).

  The final grade of the course will be MAX(P1+P2+P3+P4+P5+E/2, E).

  The problem/discussion sheets will consist of problems from the main textbook [Arora-Barak] and/or multiple choice questions that test if the student understood the material from the theory class (also covered in the main textbook).

  - **Examination-based assessment:**
    There will be a final exam graded in [0,10].

- **Reading and study resources:**

  Books:

  - **Computational complexity: a modern approach** - Arora, S.; Barak, B, Cambridge University Press, 2009. ISBN: 9780521424264
  - **Computational complexity** - Papadimitriou, C.H, Addison-Wesley , 1994. ISBN: 0201530821
  - **Computational complexity: a conceptual perspective** - Goldreich, O, Cambridge University Press , 2008. ISBN: 9780521884730

A.11 Development of Formal Logic

- **Course unit code:** 569064
- **Coordinator:** Sergi Oms Sardans
- **Department:** Department of Philosophy
- **Credits:** 5
- **Single program:** S
- **Estimated learning time:**

  Total number of hours: 125

  - **Face-to-face and/or online activities:** 42
    - **Lecture:** Face-to-face, 42
  - **Supervised project:** 18
  - **Independent learning:** 65
- **Competences to be gained during study:**

- Being able to understand and situate in the appropriate historical context of some of the main texts in the field of philosophy of modern logics and mathematics.
- Being able to expose in a clear, concise, and precise way some of the main ideas present in the discussions concerning the historical development of logic.

- **Learning objectives:**
  - **Referring to knowledge:**
    - Earn a robust understanding of the notions of logical form and logical validity, with a focus on the historical development of the notion of logical constant.
    - Acquire a basic acquaintance with some case studies related to the notions introduced during the course.
    - Acquire a better understanding of the process that led to the consolidation of First-Order Logic as a core system of contemporary logic.
  - **Referring to abilities, skills:**
    - Understand and critically analyze a complex text and identify its central ideas.
    - Write a short paper related to the subjects studied in the course in a clear, concise, and precise way.
    - Acquire the necessary conceptual tools to critically analyze historiographical interpretations of basic issues that concern the historical development of logic.

- **Teaching blocks:**
  - **The road to contemporary logic: First-Order Logic**
  - **The notion of Logical Consequence**
    - Historical aspects
    - Formal and Material Consequence
  - **Logical form and logical constants**
    - The *Categorematical/Syncategorematical* distinction
    - The Grammatical criteria
    - Topic neutrality and permutation invariance
    - Pragmatic approaches
  - **A case study**
    - The formal structure of the semantic and logical paradoxes and its historical development

- **Teaching methods and general organization:**

  In some of the classes the lecturer will expose the theoretical material with a direct discussion of the relevant papers. There will also be presentations by the students on (parts of) some of the papers and books relevant for the course.

- **Official assessment of learning outcomes:**

  Presentations in class (40%), active participation in class (10%), and a final essay on some of the subjects of the course (50%)

  - **Examination-based assessment:**

    Essay on some of the subjects of the course.

- **Reading and study resources:**

  Books:

  - Frege, Gottlob, *Collected Papers on Mathematics, Logic, and Philosophy* (edited by Brian McGuiness). Oxford: Basil Blackwell. 1984.
  - Kneale, William and Martha Kneale. *The Development of Logic*. Oxford: Clarendon Press. 1962.
  - Quine, Willard Van Orman, *Mathematical Logic*. Harvard University Press. 1981.
  - Sher, Gila, *The Bounds of Logic. A Generalized Viewpoint*. Cambridge, Mass.: The MIT Press. 1991.
  - Tarski, Alfred, *Logic, Semantics, Metamathematics. 2nd Edition edited and introduced by John Corcoran*. Indianapolis: Hackett Publishing Company. 1983.

  Articles

  - Bonnay, Denis, "Logical Constants, or How to Use Invariance in Order to Complete the Explication of Logical Consequence", *Philosophy Compass*, 9: 54-65. 2014.
  - Etchemendy, John, "The Doctrine of Logic as Form", *Linguistics and Philosophy*, 6: 319-34. 1983.
  - Ferreirós, José, "The Road to Modern Logic-An Interpretation", *The Bulletin of Symbolic Logic*, 7: 441-84. 2001.
  - Frápolli, María José, "Qué son las constantes lógicas?", *Crítica*, 44: 65-99. 2012.
  - Gómez-Torrente, Mario, "The Problem of Logical Constants", *The Bulletin of Symbolic Logic*, 8: 1-37. 2002.
  - Oms, Sergi and Elia Zardini, "Inclosure and Intolerance", *Notre Dame Journal of Formal Logic*, 62: 201-20. 2021.

– Oms, Sergi, “The Sorites Paradox in Philosophy of Logic”, in S. Oms and E. Zardini (eds.), *The Sorites Paradox*. Cambridge: Cambridge University Press. 2019.
– Peacocke, Christopher, “What is a Logical Constant?”, *The Journal of Philosophy*, 73: 221-40. 1976.
– Priest, Graham, “The Structure of the Paradoxes of Self-Reference”, *Mind*, 103: 25-34. 1994.
– Read, Stephen, “Formal and Material Consequence”, *Journal of Philosophical Logic*, 23: 247-265. 1994.
– Tarski, Alfred and John Corcoran, “What Are Logical Notions?”, *History and Philosophy of Logic*, 7: 143-54. 1986.
– Warmbrod, Ken, “Logical Constants”, *Mind*, 108: 503-38. 1999.

Web resources

– Beall, Jc, Greg Restall, and Gil Sagi, ”Logical Consequence”, The Stanford Encyclopedia of Philosophy (Spring 2019 Edition), Edward N. Zalta (ed.), `https://plato.stanford.edu/archives/spr2019/entries/logical-consequence/`
– MacFarlane, John, ”Logical Constants”, The Stanford Encyclopedia of Philosophy (Winter 2017 Edition), Edward N. Zalta (ed.), `https://plato.stanford.edu/archives/win2017/entries/logical-constants/`

A.12 Final Project

- **Course unit code:** 569062
- **Coordinator:** Joost Johannes Joosten
- **Department:** Department of Philosophy
- **Credits:** 20
- **Single program:** S
- **Estimated learning time:**
  Total number of hours: 500
  - **Supervised project:** 50
  - **Independent learning:** 450
- **Competences to be gained during study:**
  - To be able to expound in a clear, economic and coherent way the solutions to the problems considered.
  - To be able to reason in a correct way according to the standards of nowadays research.

- To be able to develop a research Project at the master level.
- To be able to plan in an adequate and efficient way the work that will lead to the presentation of the solutions to the problems under consideration.
- To be able to plan in an adequate way, with respect to the time and space available, the public presentation of a research topic or a topic related with the research.
- To be able to use efficiently the computing and audiovisual technologies commonly used to present a research topic on mathematics; for example the use of the Latex Beamer package.
- To be able to perform the kind of proofs commonly found in Logic and to expound them in an understandable and elegant way.
- To command the central concepts of Mathematical Logic.
- To be able to understand the scientific papers relevant to the research in Logic.
- To be able to ask questions able to guide new research in Logic.
- To be able to start doing research in some areas similar to the ones covered by the master.
- To be in command of the main mathematical tools used in the different areas of Mathematical Logic.
- To be able to establish connections between the topics studied, with the mathematical tools needed for their study and with their philosophical implications.
- To be able to expound in an understandable way some of the present trends of the research on Mathematical Logic.

- **Learning objectives:**
  - **Referring to knowledge:**
    - To achieve a good knowledge of the topic chosen for the master dissertation, which has to be related to a subject similar to the ones studied in the master program.
    - To learn the central results obtained to the present on the topic chosen for the master dissertation.
  - **Referring to abilities, skills:**
    - To be able to start doing research.
    - To be able to write a research paper according to the standards required to publish in international Logic journals.
    - To be able to present in public a research work according to the standards required for contributed paper presentations and lectures on international meeting in Logic.

- **Teaching blocks:**
  - **Description:**

    The master dissertation consists in the written presentation of an specialized topic on a subject similar to the ones studied in the master. The topic will be chosen by the student together with the advisor.

    The master dissertation should be a systematization, clearly and rigorously expounded of results that are found in several papers of the scientific literature on the topic. It should be helpful for people to find the state of the art of the research on the topic, on a reliable manner.

    If the topic chosen is appropriate for it, it is advisable that the student present some original contribution.

- **Teaching methods and general organization:**
  1. The student together with the advisor choose the topic
  2. The student studies the relevant literature and discussed it regularly with the advisors (two or three times a month)
  3. The student writes a first draft of the master dissertation and discusses it with the advisor
  4. Discussion with the advisor of new drafts, if necessary, since the definite dissertation is completed

- **Official assessment of learning outcomes:**

  According to the regulations, there will be a public defence and an evaluation by a committee.

  In case the health situation requires it, the defence will be online using the means established by the university.

A.13 Introduction to Mathematical Logic

- **Course unit code:** –
- **Coordinator:** Rafel Farré
- **Department:** UPC
- **Credits:** 5
- **Single program:** S
- **Estimated learning time:**

  Estimated number of hours: 125
  - **Lectures:** 28
  - **Practical sessions:** 14

  - **Supervised learning:** 41
  - **Independent learning:** 42

- **Competences to be gained during study:**
  - Perform the usual type of demonstrations in Logic, exposing them in an understandable and elegant way.
  - To be able to present a presentation showing mastery of the fundamental concepts of mathematical logic.
  - To be able to solve the type of problems dealt with in the master's degree subjects.
  - Be able to understand relevant scientific literature articles.
  - Be able to ask pertinent questions to guide research in logic.
  - Master the mathematical tools that apply to various specialized branches of mathematical logic.
  - To be able to expose in an understandable way some current trends in research in mathematical logic.

- **Learning objectives:**
  - **Referring to knowledge:**
    * Know the syntax and semantics of first-order logic. To know in depth at least one deductive calculus.
    * To know the completeness theorem completeness theorem as well as its consequences: compactness and Löwenheim-Skolem.
    * To know the proofs of these results.
  - **Referring to abilities, skills:**
    * To know how to manipulate formulas: equivalences, normal forms, Skolem forms.
    * To understand the difference between a formula with free variables and a closed formula.
    * To understand that formulas with free variables define sets (express properties of individuals) and know how to express these sets by means of appropriate formulas.
    * To know how to use the homomorphism theorem to show limitations of first-order logic: certain properties cannot be expressed by formulas.
    * Be able to perform and manipulate formal deductions of reasonable complexity in a deductive calculus.
    * Know how to use compactness and Löwenheim-Skolem theories to show the limitations of first-order logic at the level of closed formulas.

    - Understand what it means for a theory to be complete and be able to use the Lós-Vaught test to show that a particular theory is complete (in simple examples).
    - Understand the meaning of the concepts and statements of the course.
    - To understand the proofs of the course.
    - To be able to replicate the proofs of the course of intermediate complexity.
    - Be able to sketch the outline of the proof of the main theorems of the course: completeness, compactness and Löwenheim-Skolem.

- **Teaching blocks:**
  - PROPOSITIONAL LOGIC
    - Syntax of formulas.
    - Principles of induction and recursion.
    - Interpretations.
    - Logical consequence and satisfiability.
    - Equivalence.
    - Normal forms.
  - FIRST ORDER LOGIC: SYNTAX
    - Syntax of formulas.
    - Principles of induction and recursion.
  - FIRST ORDER LOGIC: SEMANTICS
    - Structures and assignments.
    - Validity of a formula.
    - Logical consequence.
    - Satisfiability.
    - Equivalence.
    - Free and bound variables.
    - Closed formulas.
    - Matching lemma Substitution lemma.
    - Normal and Skolem forms.
    - Definability.
    - Homomorphism theorem.
  - FIRST ORDER LOGIC: DEDUCIVE CALCULUS
    - Deductions.
    - Completeness theorem.
    - Compactness theorem.
    - Löwenheim-Skolem theorem.

    * Elementary classes.
    * Completeness Theories.
    * Lós-Vaught test.

- **Teaching methods and general organization:**

  1. Lectures: In the lectures, the contents of the subject are presented orally by a teacher without the active participation of the students.
  2. Expository classes: In expository classes, one or more students orally present a previously prepared topic or work in front of the rest of the group. Sometimes a previous written presentation may be encouraged.
  3. Written work: Activity consisting of the presentation of a written document.
  4. Problem solving: In the problem solving activity, the teacher presents a complex question that the students must solve, either working individually, or in teams.

- **Official assessment of learning outcomes:**

  - Written exams account for 40% of the final grade.
  - An interview or oral presentation accounts for 30% of the final grade.
  - A written project or homework consisting of solving exercises account for 30% of the final grade.

A.14 Many-Valued Logics

- **Course unit code:** 569069
- **Coordinator:** Joan Gispert Brasó
- **Department:** Department of Mathematics and Computer Science
- **Credits:** 5
- **Single program:** S
- **Estimated learning time:**
  Total number of hours: 125
  - **Face-to-face and/or online activities:** 42
    * **Lecture with practical component:** Face-to-face, 42
  - **Supervised project:** 33
  - **Independent learning:** 50

- **Recommendations:**

  The course requires a basic knowledge on many-valued logics. It is recommended to previously take the couse "Non-classical logics" from the same master's degree course.

- **Competences to be gained during study:**

  - Manage logical matrices in order to generate or recognize new many-valued logics
  - Manage and understand the most significant algebraic semantics and beeing able to relate logical and algebraic properties.
  - Manage the basic literature as well as beeing able to read specialized papers.
  - Being able to expose in a clear, concise and precise way some of the recent trends in many-valued logic

- **Learning objectives:**

  - **Referring to knowledge:**
    - To know the most significant many-valued logics.
    - To know the most significant semantics.
    - To understand and to know some of the proofs of different completeness theorems
  - **Referring to abilities, skills:**
    - Relate and manage algebraic and logical properties.
    - Generalize some results and proofs such as the completeness theorems.

- **Teaching blocks:**

  - **Introduction:**
    - Historical perspective
    - The 3-valued Łukasiewicz logic. Other 3-valued logics. Matrix semantics.
  - **Many-valued logics as substructural logics**
    - Substructural logics, FL and FLew
    - Algebraization. FL-algebras and bounded commutative residuated lattices
    - Filters and congruences. Subdirect representations theorems
  - **Łukasiewicz logics:**
    - Finite valued semantics and calculi.
    - The infinite valued logic and its calculus. McNaughton theorem
    - MV-algebras, l-groups. Completeness theorems

    * Axiomatic and finitary extensions. Varieties and quasivarieties.

- **Product logic:**
    * Product logic and product algebras. Algebraic semantics, completeness theorem.
    * l-groups and product algebras. Standard completeness theorem.
    * Axiomatic and finitary extensions. Varieties and quasivarieties.
- **Gödel logics:**
    * Gödel logic and intuitionistic logic. Linearity.
    * Linear Heyting algebras. Completeness theorems
    * Axiomatic and finitary extensions. Varieties and quasivarieties.
- **Fragments and expansions**
    * Positive fragments and implicative fragments
    * Adding constants and other operators.

- **Teaching methods and general organization:**

  It is a 3 hour per week course. Usually the teacher will explain some theoretic results and he will give some examples to illustrate the new concept. Some exercises will be proposed to the students to be solved at home in order to help them in their learning. It is supposed that for each hour of face to face class the student will need between one and a half or two hours to study the new concepts and solve the proposed exercises. At mid course, every student must delve into some topic in order to give and oral and written presentation.

- **Official assessment of learning outcomes:** AC will have two parts:
    - Resolution of proposed exercises (25%)
    - Final exam (45%)
    - Oral and written presentation (30%)

    - **Examination-based assessment:**
      There will be a final exam (70%) and an oral and written presentation (30%) .

- **Reading and study resources:**

  Books:
    - CIGNOLI, R., D'OTTAVIANO, I. and MUNDICI, D. Algebraic foundations of many-valued reasoning, volume 7 of Trends in Logic, Kluwer, 2000
    - CINTULA, P., HAJEK, P. and NOGUERA, C (Editors) Handbook of Mathematical Fuzzy Logic. Volumes 1 and 2 (vol 37 and 38 of Studies in Logic) College Publications, December 21, 2011.

- GOTTWALD, S. A treatise on many-valued logics. Collection: Studies in logic and Computation. vol. 9. Baldock : Research Studies, 2001.
- HÁJEK, P. Metamathematics of Fuzzy Logic. Trends in Logic. Vol. 4. Dordrecht: Kluwer Academic, 1998.

Articles: During the course the teacher will supply references of papers in specialized journals.

A.15 Mathematical Logic

- **Course unit code:** 569060
- **Coordinator:** Enrique Casanovas Ruiz Fornells
- **Department:** Department of Mathematics and Computer Science
- **Credits:** 6
- **Single program:** S
- **Estimated learning time:**

  Total number of hours: 150

  - **Face-to-face and/or online activities:** 45
    - **Lecture:** Face-to-face, 45
  - **Supervised project:** 25
  - **Independent learning:** 80
- **Competences to be gained during study:**
  - To be able to independently study standard textbooks on mathematical logic and journal articles of medium difficulty level.
  - To be prepared to follow with success more specialized subjects of this master.
  - To understand the statements and proofs of Gödel's incompleteness theorems and other related results on incompleteness and undecidability.
- **Learning objectives:**
  - **Referring to knowledge:**
    - To know the standard classical metamathematical results on first-order logic: completeness, compactnes, Löwenheim-Skolem theorems, etc.
    - To know Gödel's incompleteness theorems as well as related results on logical undecidability and incompleteness.

        * To know some decidable fragments of arithmetic and first-order logic.

- **Teaching blocks:**
  - **Synthax of fits-order languages**
    * Free Algebras, induction and recursion
    * First-order languages
    * Terms
    * Formulas
    * Free and bound variables
    * Substitution
    * Subformulas
    * Renaming bound variables
  - **Semantics of first-order languages**
    * Structures, denotation and satisfaction
    * Coincidence and substitution lemmas
    * Logical consequence and satisfiability
    * Logical equivalence and logical validity
    * The propositional fragment of first-order logic
    * Reducts and expansions
    * Substructures and extensions
    * Homomorphisms and isomorphisms
    * Congruences
  - **Deductive calculus**
    * Propositional calculus
    * First-order calculus
    * First-order calculus with equality
  - **Basic notions of Model Theory**
    * Compactness and Löwenheim-Skolem theorems
    * Herbrand's theorem
    * Normal forms
    * Definability
    * Relativization and interpretability
    * Elimination of function symbols
    * Elimination of quantifiers
  - **Recursive functions and relations**
    * Natural numbers
    * Recursive and primitive recursive functions
    * Elimination of recursion

    * Recursively enumerable relations
    * Definability in arithmetic
    * Coding finite sequences of natural numbers
  - **Incompleteness and undecidability**
    * Recursive vocabularies and Gödel numberings
    * Arithmetizating deductions
    * Tarski's Theorem on the undefinability of arithmetical truth
    * The theories R, Q, and PA
    * Weak representability
    * Logic of finite structures
    * Representability
    * First incompleteness theorem
    * Second incompleteness theorem

- **Teaching methods and general organization:**

  Detailed exposition in class, with examples and full proofs.

  Asking questions and proposing problems to be solved by the student.

  Class discussions on the main notions and statements and relevant proofs.

  Presentation of a topic by the student, previously prepared with the teacher advice.

  In the case of virtual teaching required by the health situation, the time ranges will be maintained and teaching will be carried out as indicated by the university authorities in whole or partially in a non-contact format.

  In the case of face-to-face teaching, the time ranges will be maintained and all classes will be held in person.

- **Official assessment of learning outcomes:**

  Periodical exercises: 20%

  Two partial examinations: 35 % each.

  Short presentation of a topic in class: 10 %

  Depending on the health situation, the partial and final exams will be carried out in person or in a non-contact mode.

  - **Examination-based assessment:**

    Final examination.

    Depending on the health situation, the final exam will be carried out in person or in a non-contact mode.

- **Reading and study resources:**

  Books:

– Boolos, G.S., Burgess, J.P. and Jeffrey, R.C. *Computability and logic.* (Cambridge U.P. 1974.
– Ebbinghaus, H.D., Flum, J. and Thomas W. *Mathematical logic.* Springer 1994.
– Enderton, H.B. *A mathematical introduction to logic.* Academic Press 1972.
– Franzén, T. *Gödel's theorem: an incomplete guide to its use and abuse.* A.K. Peters Ltd 2005.
– Schoenfield, J.R. *Mathematical logic.* Addison-Wesley 1967.
– Tarski, A., Mostowski, A. and Robinson, R.M. *Undecidable theories.* North Holland PC, 1971.
– Ziegler, M. *Mathematische Logik.* Mathematik Kompakt, Birkhäuser 2010.

A.16 Modal Logic

- **Course unit code:** 569070
- **Coordinator:** Joost Johannes Joosten
- **Department:** Department of Philosophy
- **Credits:** 5
- **Single program:** S
- **More information:** `http://www.joostjjoosten.nl//`
- **Estimated learning time:**

  Total number of hours: 125
  - **Face-to-face and/or online activities:** 42
    * **Lecture:** 28
    * **Lecture with practical component:** Face-to-face, 14
  - **Supervised project:** 41
  - **Independent learning:** 42
- **Recommendations:**

  IMPORTANT: The course requires an advanced level of abstract/mathematical reasoning. Students that doubt if they qualify should contact Joost J. Joosten.
- **Competences to be gained during study:**
  - Write down the kind of proofs which are typical to the field of mathematical logic in a well-structured legible fashion.

  - Being able to understand cutting-edge research papers in the general field of modal logic.
  - Know a decent number of non-classical logics.
  - Being able to sketch in a succinct, clear and legible way certain tendencies in state-of-the-art research in mathematical logic.

- **Learning objectives:**
  - **Referring to knowledge:**
    * Basics of contemporary modal logic in general and in particular: abstract semantics (like Kripke models), basic model theory, arithmetical interpretations of provability logics and their applications.
  - **Referring to abilities, skills:**
    * Know to write proofs of intermediate complexity.
    * Being able to clearly expose the structure of an argument.
    * To be able to clearly sketch some main tracks in contemporary modal logic.

- **Teaching blocks:**
  - **Basic concepts:**
    * Modal languages
    * Abstract semantics
    * Relational semantics
    * Normal modal logics
    * Models and frames
  - **Basic model theory:**
    * Canonical models
    * Soundness theorems
    * Completeness results
    * Finite model property
    * Bisimulations
    * Definability results
  - **Provability logics:**
    * Gödel-Löb's provability logic
    * Arithmetical interpretations
    * Modal semantic arguments
    * Closed fragments
    * Applications to ordinal analysis and foundations of mathematics

- **Teaching methods and general organization:**

  There will be classes where the theoretical material is exposed by the lecturer. During theses classes interactivity is highly stimulated and new concepts will be discussed. Also, at some moments we will jointly address exercises in class. Apart from this, the student is required to make some exercises on his/her own that will then be discussed upon requested moments outside the regular classes at the lecturer's office. Exercises that are handed in will be corrected, evaluated and discussed.

- **Official assessment of learning outcomes:**

  On a regular basis the students are required to hand in sets of answers to exercises given out by the lecturer. These exercises will be corrected, evaluated and discussed. The exercises constitutes a significant part of the final score (tentatively: 60%). The remaining part will consists of a final exam (tentatively: 40%).

  - **Examination-based assessment** There will be a final exam.

- **Reading and study resources:**

  Books:

  - P. BLACKBURN i M. DE RIJKE i Y. VENEMA. Modal Logic. Cambridge: Cambridge University Press, 2001. Not mandatory
  - P. BLACKBURN i J. F. A. K. VAN BENTHEM i F. WOLTER. Handbook of Modal Logic, Elsevier, 2006. Not mandatory
  - G. BOOLOS. The logic of provability. Cambridge University Press, 1999. Highly recommended
  - R. JANSANA FERRER. Una introducción a la lógica modal. Madrid: Tecnos, 1990. Strongly recommended

  Article:

  - Reflection principles and provability algebras in formal arithmetic. L. D. Beklemishev

  Web page:

  - A reader will be made available at the course description which can be found at `http://www.phil.uu.nl/~jjoosten/`

A.17 Models of Set Theory

- **Course unit code:** 569074
- **Coordinator:** Joan Bagaria Pigrau
- **Department:** Department of Mathematics and Computer Science

- **Credits:** 5
- **Single program:** S
- **Estimated learning time:**
  Total number of hours: 125
  - **Face-to-face and/or online activities:** 42
    - **Lecture:** Face-to-face, 42
  - **Supervised project:** 18
  - **Independent learning:** 65
- **Competences to be gained during study:**
  - To be able to carry out independent study of standard textbooks and journal articles on advanced set theory.
  - To understand the proofs of some of the most important results in set theory, such as the independence of the Continuum Hypothesis and the Axiom of Choice from the standard set-theoretic axiom system (Zermeleo-Fraenkel).
  - To be able to analyze and solve standard problems in set theory by using standard model-theoretic and combinatorial techniques.
- **Learning objectives:**
  Referring to knowledge:
  - To introduce the student to some of the main ideas and techniques of advanced set theory, with emphasis on the metamathematical aspects. The student will acquire a basic knowledge of the main techniques involved in the construction of models of Zermelo-Fraenkel set theory.

  Referring to abilities, skills:
  - To acquire a working knowledge of constructibility and forcing techniques in order to apply them to prove consistency and independence results in set theory and other areas of mathematics.
- **Teaching blocks:**
  - **Constructibility**
    - We go in detail through the construction of Gödel's constructible universe L and we prove some of its properties. We show that the Generalized Continuum Hypothesis and the Axiom of Choice hold in L. We also prove that Jensen's diamond principle holds in L.
  - **Forcing:**

    * We develop the theory of forcing for building models of set theory.
  - **Applications of forcing:**
    * We give many applications of the forcing technique. For example, we build models where the Continuum Hypothesis fails, models where the Axiom of Choice fails, etc. The final application will be the construction of a model of Martin's axiom, using iterated forcing.

- **Teaching methods and general organization:**

  The course consists on lectures and exercises. The lectures, imparted by the instructor, consist on the presentation of the topics listed in the programme. A weekly set of problems to be solved is given to the students. The solutions have to be handed in the following week. The problems are reviewed and graded by the instructor and handed back to the students with comments.

- **Official assessment of learning outcomes:**

  Weakly set of exercises to be handed in. **Examination-based assessment:**

  - Mid-term exam, in class. Final take-home exam.

- **Reading and study resources:**

  Books

  - Jech, Thomas J. Set theory. Berlin [etc.]: Springer, 2003. The 3rd millennium ed., rev. and expanded.
  - Kunen, Kenneth. Set theory. London: College Publications, 2013. Rev. ed.

A.18 Non-Classical Logics

- **Course unit code:** 569057
- **Coordinator:** Joan Gispert Brasó
- **Department:** Department of Mathematics and Computer Science
- **Credits:** 6
- **Single program:** S
- **Estimated learning time:**

  Total number of hours: 150

  - **Face-to-face and/or online activities:** 45
    * **Lecture with practical component:** Face-to-face, 45

- **Supervised project:** 45
- **Independent learning:** 60

- **Teaching blocks:**
  - **Introduction:**
    * Classical logic: Hilbert calculus, Gentzen, Natural Deduction, Tableaux and Resolution
    * Truth tables.
  - **Intuitionistic Logic:**
    * Hilbert, and Gentzen Calculi, Tableaux
    * Kripke dematics and algebraic sematics: Heyting algebras
    * Completeness theorems
  - **Modal logic:**
    * Hilbert caluli and Kripke sematics
    * Basic modal logic: frames and models
    * Completeness theorems
    * Special cases: temporal logic and dynamic logic.
  - **Many valued logic and fuzzy logic**
    * Lukasiewicz, Product, Gödel, Basic and MTL
    * Hilbert calculi and algebraic semantics: MV-algebras, product algebras, Gödel algebras, BL algebras and MTL algebras

- **Teaching methods and general organization:**

  It is a 3 hour per week course. Usually the teacher will explain some theoretic results and he will give some examples to illustrate the new concept. Some exercises will be proposed to the students to be solved in class in order to help them in their learning. Every fifteen days approximately the teachers will give more complex exercises to be solve with the need of the whole material taught at this moment. It is supposed that for each hour of presential class the student will need between one and a half or two hours to study the new concepts and solve the proposed exercises.

- **Official assessment of learning outcomes:** AC will have two parts:
  - Resolution of proposed exercises (30%)
  - Two partial exams (70%)
  - **Examination-based assessment:**
    There will be a unique final exam.

- **Reading and study resources:**

  Books:

– ANDERSON, A. R. i DELNAP, N. D. Entailment. The logic of relevance. Vol I i II. Princeton (N.J.) : Princeton University Press, 1975.
– GOBLRE L. (Ed.). The Blackwell guide to philosophical Logic. Malden (Mass.): Blackwell, 2001.
– GOTTWALD, S. A treatise on many-valued logics. Col·lecció Studies in logic and Computation. vol. 9. Baldock : Research Studies, 2001.
– HÁJEK, P. Metamathematics of Fuzzy Logic. A Trends in Logica. Vol. 4. Dordrecht: Kluwer Academic, 1998.
– JANSANA, R. Una introducción a la lògica modal. Madrid: Tecnos, 1990.
– JAQUETTE, D. (Ed.) A companion to Philosophical Logic. Malden: Blackwell, 2002
– PALAU, G. Introducción filosófica a las lógicas no clásicas. Barcelona: Gedisa, 2002.
– PRIEST, G. An introduction to non-classical logics. Cambrige: Cambrige University Press, First Edition 2001, Second Edition 2008
– READ, S. Relevant logic. Oxford: Basil Blackwell, 1988.
– SCHECHTER, E. Classical and Non-classical Logics. Princeton (N.J.): Princeton University Press, 2005.

A.19 Orders, Lattices and Boolean Algebra

- **Course unit code:** 569067
- **Coordinator:** Tommaso Moraschini
- **Department:** Department of Philosophy
- **Credits:** 5
- **Single program:** S
- **Estimated learning time:**

  Total number of hours: 125

  - **Face-to-face and/or online activities:** 42
    - **Lecture with practical component:** 42
      - Lecture, face-to-face, 28
      - Lecture with practical component, face to face, 14
  - **Independent learning:** 60 (Includes all the time for independent study, problem solving and writing assignments)
- **Competences to be gained during study:**

- Correct argumentation, according to the standard of present-day research.
- To present, in a clear, concise and coherent way, the solutions to the proposed exercises.
- To plan in an adequate and efficient way the work towards the presentation of the solutions to the exercises.
- To be able to independently read and understand notes on the course written by another instructor.
- To be able to understand and reproduce proofs of intermedite difficulty in the field.

- **Learning objectives:**
  - **Referring to knowledge:**
    - To know the result of the theory of partial orders and of adjoint pairs of motonone functions.
    - To know the results of the theory of lattices, and in particular of the theory of distributive lattices.
    - To know the fundamental results of the theory of Boolean algebras, including Stone's representation theorem.
    - To understand the relation between Boolean algebras and classical logic.
    - To be able to expound which are the fundamental theorems of lattice theory and of the theory of Boolean algebras.
- **Teaching blocks:**
  - **Ordered sets**
    - Basic notions and distinguished elements
    - Order morphisms
    - Residuated functions, Galois connections and closure operators
    - Up-sets, down-sets, order ideals and order fitlers
  - **Lattices**
    - Closure operators and Galois connections
    - Semilattices
    - Lattices as ordered sets and as algebraic structures
    - Homomorphisms
    - Chain conditions and irreducible elements
    - Complete lattices and the fixpoint theorem
    - Modular and distributive lattices
    - Ideals and filters
    - Congruences

    * Completions of ordered sets, semilattices and lattices
    * The prime filter theorem and the representation of distributive lattices

- **Boolean algebras**
    * Boolean lattices, Boolean rings and Boolean algebras
    * Homomorphisms and subalgebras
    * Complete and atomic Boolean algebras
    * Ideals, filters and ultrafilters
    * Congruences
    * The Dedekind-MacNeille completion of a Boolean algebra
    * Rasiowa-Sikorski Theorem
    * Stone's representation of Boolean algebras
    * The Lindenbaum-Tarski algebras of classical logic
    * Completeness of first-order logic through Boolean algebras

- **Teaching methods and general organization:**

  Traditional expository sessions with handout of preliminary notes. Solving of exercises. Written presentation of solutions and other assignments.

  If online teaching will be necessary because of the current health situation, the timetable of the course will be maintained and teaching will take place according to the guidelines of the academic authorities either completely or partly in non on-site form. In case of on-site teaching, the timetable will be maintained and all classes will be on-site.

- **Official assessment of learning outcomes:** Every week, a set of exercises will be given to the students. These should be solved and returned to the teacher in written form. The final grade will be the mean of the grades of the different sets of exercises.

  - **Examination-based assessment:** Written examination.

    Depending on the health situation, the exam will be on-site or not.

- **Reading and study resources:**

  Books:

  - Balbes Dwinger. "Distributive lattices", University of Missouri Press, 1974.
  - Bergman. "Universal algebra: Fundamental and Selected Topics", Chapman and Hall/CRC, 2011.
  - Birkhoff. "Lattice Theory", 2nd edition, AMS Colloquium Publications, 1948.
  - Blyth. "Lattices and ordered algebraic structures", Springer, 2005.

- Davey Priestley. "Introduction to lattices and order", 2nd edition, Cambridge University Press, 2002.
- Givant Halmos. "Introduction to Boolean algebras", Springer, 2009.
- Grätzer. "Lattice Theory: Foundation", Birkhäuser-Springer, 2011.
- Roman. "Lattices and ordered sets", Springer, 2008.

A.20 Proof Theory and Automated Theorem Proving

- **Course unit code:** 569076
- **Coordinator:** Joost Johannes Joosten
- **Department:** Department of Philosophy
- **Credits:** 5
- **Single program:** S
- **More information:** `http://www.joostjjoosten.nl//Courses/2020MasterProofTheory/`
- **Estimated learning time:**
  Total number of hours: 125
  - **Face-to-face and/or online activities:** 42
    - **Lecture:** Face-to-face, 42
  - **Supervised project:** 18
  - **Independent learning:** 65
- **Competences to be gained during study:**
  - To be able to provide formal proofs in various proof systems for both propositional and predicate logic, including
    - Natural deduction;
    - Sequent calculus;
    - Resolution;
  - To be able to read state of art literature on proof-systems.
  - To understand the correctness of the above mentioned proof systems.
  - To understand the applications of proof theory to ordinal analysis and its repercussions on the foundations of mathematics.
  - To be able to write rudimentary prolog code.
- **Learning objectives:**

- **Referring to knowledge:**

  The main objective of this course is to obtain a thorough understanding of formal proof systems and their applications in mathematics and computer science. The student will get exposed to the theory but also be required to make formal derivations him/herself. By the end of the course we shall see how reasoning/theorem proving can be automated using PROLOG.

  We shall reflect on how proof theory provides us with insights about formalized epistemic processes when reasoning about mathematics (and possibly other fields). In particular we shall see how proof theory helps us understand the boundaries of such processes.

- **Referring to abilities, skills:**
  - To be able to reason within different kind of formal proof systems like Natural Deduction, Sequent Calculi and Frege/Hilbert Systems.
  - To normalize proofs and retrieve constructive information from such normalised proofs.
  - To solve problems in logic programming.

- **Teaching blocks:**
  - **Formal proof systems**
    - Rules
    - Axioms
    - Finitary versus non-finitary
    - Omega-rule
  - **Ordinal analysis**
    - Foundations of mathematics
    - Consistency proofs
  - **Automated reasoning in propositional logic**
    - Basic notions
    - The method of resolution
    - The method of Davis and Putnam
  - **Automated reasoning in predicate logic**
    - Skolem forms
    - Semantic trees/tableaux
    - Herbrand's method
    - The Unification Algorithm
    - The method of resolution
    - The PROLOG programming language

- **Teaching methods and general organization:**

  In the course we shall first provide a historically based overview of the main philosophical questions that lead to the discipline of proof theory. Next we shall see how epistemological processes can be formalized leading to the notions of formal proof and formal proof systems. We do this by lectures and historical examples. The proof systems will be exemplified using numerous exercises that have to be handed in on a weekly basis.

  At first our domain of discourse will be mainly propositional logic to readily switch to predicate logic.

- **Official assessment of learning outcomes:**

  The final grade is determined by

  - (A) Weekly homework questions; (40%)
  - (B) Midterm take-home exam; (30%)
  - (C) Final Exam; (30%)

  Of these parts Item (A) will be spread out over the whole duration of the course and Item (B) will take place around after two-and-a-half of the five credits.

  - **Examination-based assessment:** The "avaluació única" consists of a final exam over all the material discussed in class together with a set of homework exercises.

- **Reading and study resources:**

  Books:

  These are standard text-books that deal with matters of this course. We will announce what material shall eventually be used apart from the handouts.

  - Uwe Shöning, Logic for Computer Scientists.
  - C. L. Chang and R.C.T. Lee, Symbolic Logic and Mechanical Theorem Proving.
  - A. Troelstra and van D. van Dalen, Constructivism in Mathematics.
  - Troelstra, Proof theory.
  - M. Ben-Ari, "Mathematical logic for computer science", thrird edition, Springer, 2012.

A.21 Universal Algebra

- **Course unit code:** 569063
- **Coordinator:** Joan Gispert Brasó

- **Department:** Department of Mathematics and Computer Science
- **Credits:** 5
- **Single program:** S
- **Estimated learning time:**

  Total number of hours: 125

  - **Face-to-face and/or online activities:** 42
    - **Lecture with practical component:** Face-to-face, 42
  - **Supervised project:** 23
  - **Independent learning:** 60
- **Competences to be gained during study:**
  - Introduce the basic elements of Universal Algebra, the problems that it studies, and the tools used to solve them.
  - Manage and understand the concepts and the basic results , seeing its scope and limitations.
  - Be able to solve problems in the field of Universal Algebra, and write in a correct style mathematical proof.
  - Become familiar with techniques from universal algebra, as well as its use to study algebraic structures.
  - Manage the basic literature.
- **Learning objectives:**
  - **Referring to knowledge:**
    - Introduce the basic tools of universal algebra to the study algebraic structures.
    - Get the representability of algebras and operators on classes of algebras.
    - Introduce and study quasiequational and classes and equational and relative equational logics.
    - Study some problems related to the finite generation of classes of algebras.
  - **Referring to abilities, skills:**
    - Fluently use the results and notions introduced for the study of concretes algebras.
    - Find generators algebras concrete classes.
    - Identify properties of equational logic using them estructures partner, and vice versa.
- **Teaching blocks:**

- **Algebraic structures**
  - Sets, functions and operations. Algebras
  - Some common algebraic structures. Monoids, groups, rings,...
  - Order Relations. Semilattices, lattices and Boolean algebras.
- **Algebras and compatibility**
  - Subalgebras and homomorphisms
  - Equivalence Relations and Congruence relations.
  - Congruence relations and homomorphisms
- **Representations and operators**
  - Direct products and factor congruences
  - Subdirect products. Subdirectly irreducible algebras
  - Operators on classes. Varieties
- **Equational classes**
  - Free algebras
  - Birkhoff's Theorem
  - Mal'cev's Terms
- **Equational Logic**
  - Equational consequence. Fully invariant congruences
  - Equational deduction. Completeness
  - The lattice of subvarieties of a variety
- **Ultraproducts and quasivarieties**
  - Reducet products and Ultraproducts
  - Quasiequational classes and Quasivarieties
  - Subdirect products representations. Jónsson's Lema
- **Relative Equational Logic**
  - Reltive equatonal logic and quasivarieties
  - Fully invariant families of congruences
- **Some additional topics**
  - Discriminator term, discriminator varieties. Quasiprimal and primal algebras
  - Partial Algebras. Finite model and finite embedding properties
  - Application to the study of particular classes of algebras: Lattices, residuated lattices, BCK-algebras, groups,...
  - Boolean representations.

- **Oficial assessment of learning outcomes:** Will take into account the exercises done in class among the proposed, and supervised work. If necessary, there will also be an examen.

- **Examination-based assessment:** An exam consisting in solving some exercises and the development of some questions.

- **Reading and study resources:**

  Books:

  - Balbes R. and Dwinger P., Distributive lattices. University of Missouri Press. Pricenton. 1974.
  - BellJ. and Slomson A.B., Models an ultraproducts. North-Holland. Amsterdam. 1971
  - Bergman, C. Universal Algebra. Pure and applied Mathematics 301.CRC Press. Boca Raton 2012
  - Birkhoff G., Lattice Theory. A.M.S. Colloquium Publications vol. XXV. Providence 1973.
  - Burris S. and Sankappanavar H.P. A course in Universal Algebra. GTM 78, Springer Verlag. 1981. Link: `http://www.math.uwaterloo.ca/~snburris/htdocs/ualg.html`
  - Chang C.C. and Keisler ,J. Model Theory. 3a edició. North-Holland. Amsterdam 1990.
  - Czelakowski J. Protoalgebraic Logics . Kluwer. Academic Publishers. 2001.
  - Deneke K., Wismath S.L., Universal Algebra and applications in Theoretical Computer Science. ChapmanHall/CRC.. 2002
  - Grätzer G., Universal Algebra. Sª Edición. Springer Verlag. 1979.
  - Grätzer G., General Lattice Theory. 2ª Edició. Birkkhäuser. Berlin 1998.
  - Kaarli K. and Pixley A.F., Polynomials completeness in Algebraic Systems. Chapman Hall/CRC. 2001.