



# basic education

Department:  
Basic Education  
**REPUBLIC OF SOUTH AFRICA**

## NATIONAL SENIOR CERTIFICATE

**GRADE 11**

**MATHEMATICS P2**

**NOVEMBER 2019**

**MARKS: 150**

**TIME: 3 hours**

This question paper consists of 14 pages and a 21-page answer book.



**INSTRUCTIONS AND INFORMATION**

Read the following instructions carefully before answering the questions.

1. This question paper consists of ELEVEN questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you used to determine the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. Write neatly and legibly.



**QUESTION 1**

Mary wants to buy a car and visits a popular website. She finds a number of advertisements for the make of car that she would like to buy. She summarises the selling prices (in thousands of rands) of the cars on sale in the cumulative frequency table below.

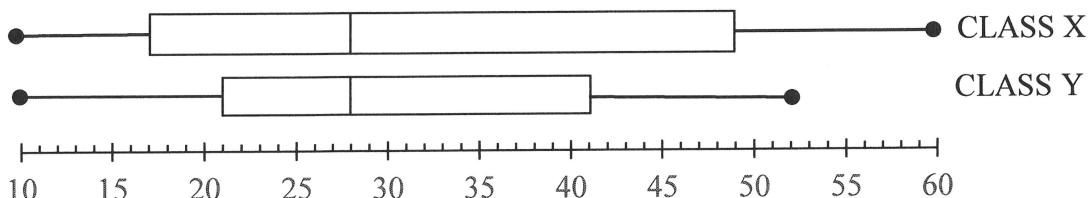
SELLING PRICE (IN THOUSANDS OF RANDS)	FREQUENCY	CUMULATIVE FREQUENCY
$50 \leq x < 60$	3	3
$60 \leq x < 70$	4	7
$70 \leq x < 80$	$a$	14
$80 \leq x < 90$	19	33
$90 \leq x < 100$	12	$b$
$100 \leq x < 110$	5	50

- 1.1 Write down the values of  $a$  and  $b$ . (2)
- 1.2 Draw a cumulative frequency graph (ogive) of the data on the grid provided in the ANSWER BOOK. (3)
- 1.3 Mary wants to spend a maximum of R95 000. Use the cumulative frequency graph to estimate the number of cars that are on sale in the price range that Mary can afford. (1)  
**[6]**



**QUESTION 2**

- 2.1 Two classes wrote a Mathematics test that had a maximum mark of 60. The results of each class are summarised in the box and whisker diagrams below.



- 2.1.1 Comment on the skewness of the results in class X. (1)
- 2.1.2 In which class is the standard deviation of the marks bigger? (1)
- 2.1.3 Comment on the average performance in the test of the two classes. Use relevant statistics to support your argument. (2)
- 2.2 The time, in minutes, that it took for the first goal to be scored in seven football games was recorded. The times, in ascending order, are represented by  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$ ,  $f$  and  $g$  in the table below.

$a$	$b$	$c$	$d$	$e$	$f$	$g$
-----	-----	-----	-----	-----	-----	-----

The following observations were made about the data:

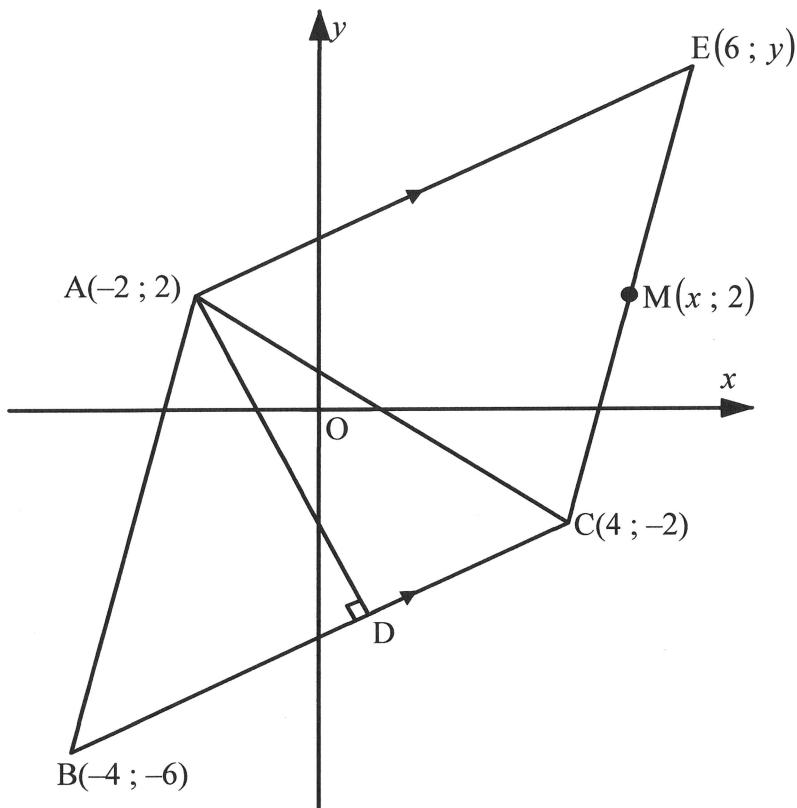
- All these goals were scored at different times.
- The minimum time for the first goal was 5 minutes.
- The range of the times was 48 minutes.
- The median time was 22 minutes.
- The difference between the time at the lower quartile and the minimum time was 7 minutes.
- The IQR of the times was 28 minutes.
- The mean time was 27 minutes.
- $e = 2c$

- 2.2.1 Determine the values of  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$ ,  $f$  and  $g$ . (8)
- 2.2.2 If the standard deviation of the data set is 15,87 minutes, how many goals were scored within ONE standard deviation of the mean time? (3)  
[15]



### QUESTION 3

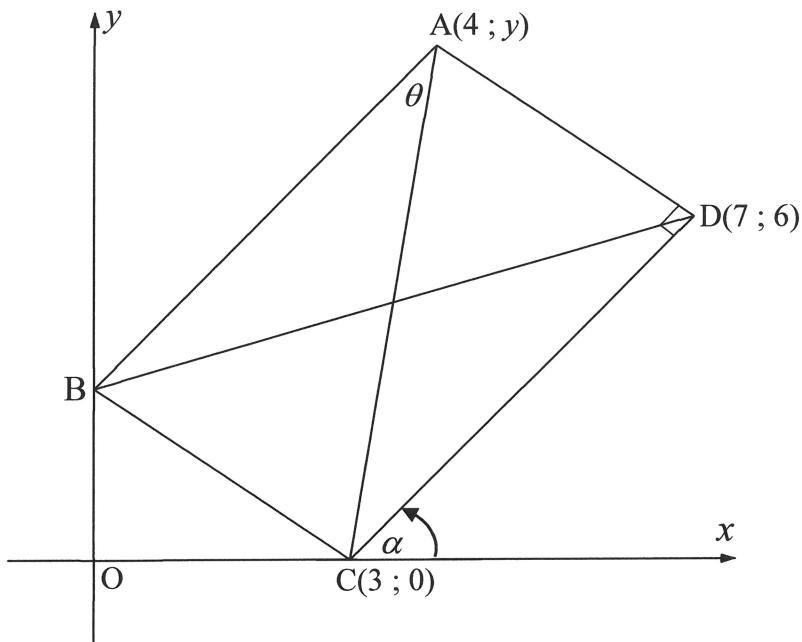
In the diagram,  $A(-2 ; 2)$ ,  $B(-4 ; -6)$ ,  $C(4 ; -2)$  and  $E(6 ; y)$  are the vertices of a quadrilateral having  $AE \parallel BC$ .  $D$  lies on  $BC$  such that  $AD \perp BC$  and  $AC$  is drawn.  $M(x ; 2)$  is a point on  $EC$ .



- 3.1 Calculate the gradient of  $BC$ . (3)
  - 3.2 If  $M$  is the midpoint of  $EC$ , determine the values of  $x$  and  $y$ . (3)
  - 3.3 Calculate the length of  $BC$ . (2)
  - 3.4 If it is further given that  $AE = \sqrt{80}$ , to which group of special quadrilaterals does  $ABCE$  belong? (1)
  - 3.5 Determine the equation of  $AD$  in the form  $y = mx + c$ . (3)
  - 3.6 Calculate the coordinates of  $D$ . (5)
  - 3.7 Determine the area of  $\Delta AEC$ . (3)
- [20]

**QUESTION 4**

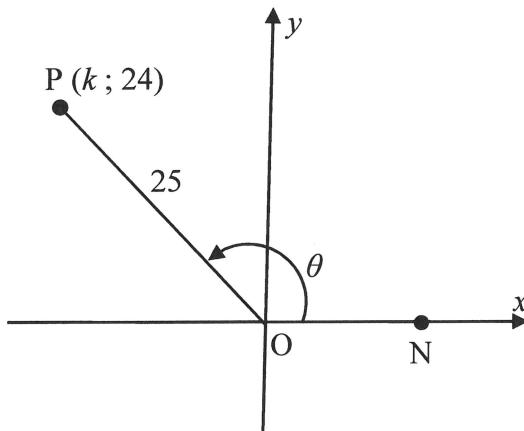
In the diagram, B is a point on the  $y$ -axis. A(4 ;  $y$ ), B, C(3 ; 0) and D(7 ; 6) are the vertices of rectangle ABCD. Diagonals BD and AC are drawn. The inclination of DC is  $\alpha$  and  $\hat{BAC} = \theta$ .



- 4.1 Determine the gradient of CD. (2)
  - 4.2 Calculate the size of  $\alpha$ . (2)
  - 4.3 Determine the value of  $y$ . (4)
  - 4.4 Calculate the size of  $\theta$ . (5)
- [13]

### QUESTION 5

- 5.1 In the diagram below,  $P(k ; 24)$  is a point in the second quadrant such that  $OP = 25$  units. N is a point on the positive  $x$ -axis and  $\hat{PON} = \theta$ .



WITHOUT calculating the size of  $\theta$ , determine the value of the following:

5.1.1  $k$  (2)

5.1.2  $\tan \theta$  (1)

5.1.3  $\sin \alpha$  if  $\theta + \alpha = 360^\circ$  (3)

5.1.4  $\cos^2 \theta - \sin^2 \alpha$  (3)

- 5.2 Simplify WITHOUT using a calculator:

$$\frac{\cos 210^\circ \cdot \tan 135^\circ}{\sin(-60^\circ) \cdot \cos 420^\circ} \quad (5)$$

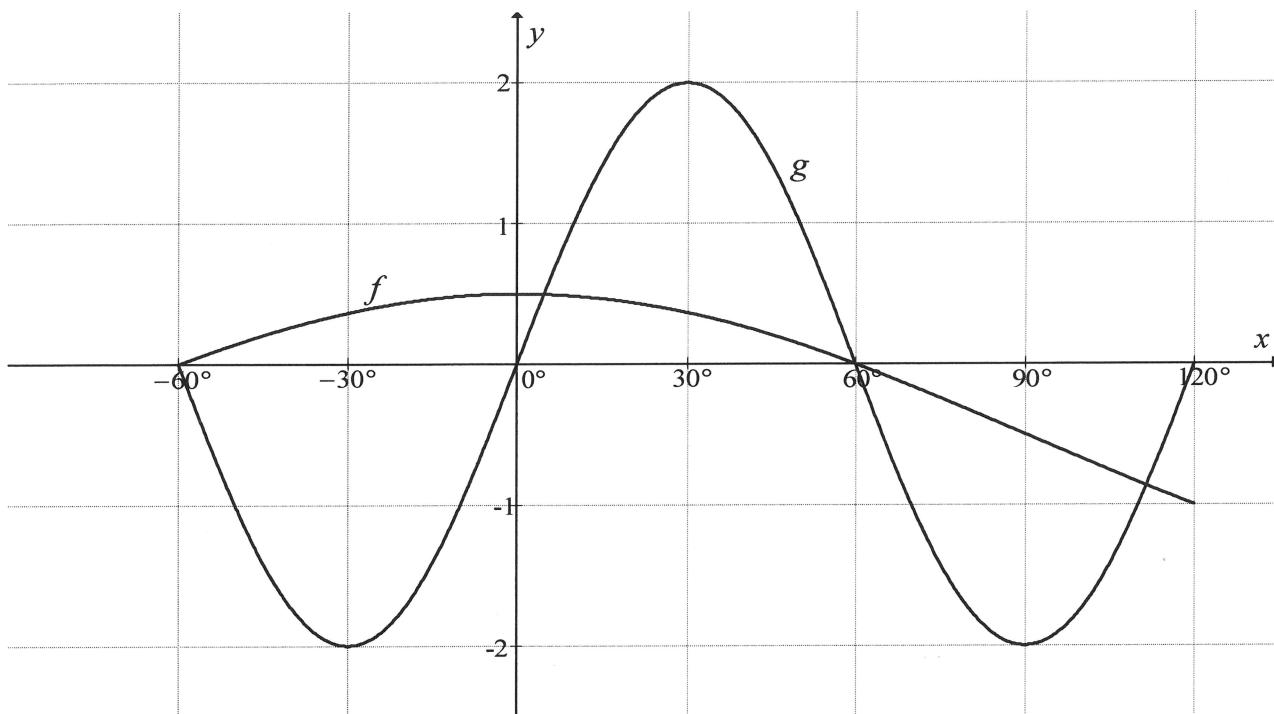
- 5.3 Prove the identity:

$$\frac{1}{\tan^2 x} - \cos^2 x = \frac{\cos^4 x}{\sin^2 x} \quad (4)$$

- 5.4 Determine the general solution of  $\sqrt{2} \sin x \cos x = \cos x$ . (6)  
[24]

**QUESTION 6**

Sketched below are the graphs of the functions  $f(x) = \cos x + q$  and  $g(x) = 2 \sin bx$  for  $x \in [-60^\circ; 120^\circ]$ .

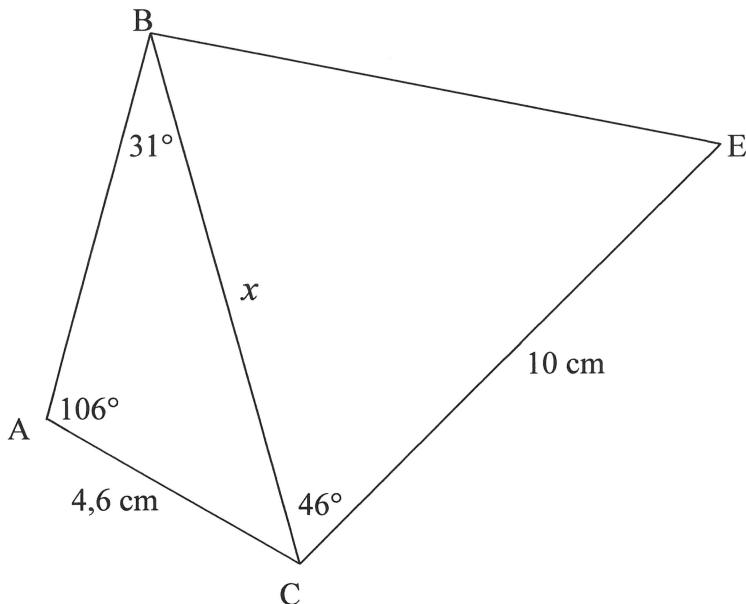


- 6.1 Write down the coordinates of the maximum turning point of  $g$  in the given interval. (1)
- 6.2 Determine the values of  $x$  where  $f$  is strictly increasing in the given interval. (2)
- 6.3 Determine the values of  $q$  and  $b$ . (2)
- 6.4 Use your graphs to determine the values of  $x$  for which  $2\cos x \sin 3x - \sin 3x \geq 0$ . (4)  
[9]

### QUESTION 7

7.1 In the diagram A, C, E and B are the vertices of a quadrilateral.

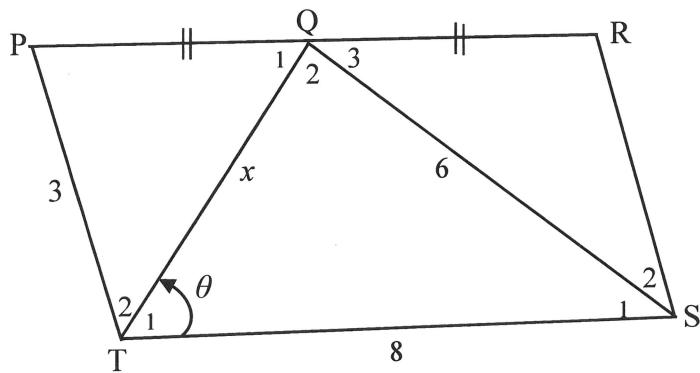
$$\hat{A}BC = 31^\circ, \hat{B}AC = 106^\circ, \hat{B}CE = 46^\circ, AC = 4,6 \text{ cm}, CE = 10 \text{ cm} \text{ and } BC = x.$$



7.1.1 Calculate the length of BC. (3)

7.1.2 Calculate the area of quadrilateral ACEB. (4)

7.2 In the diagram below, PTSR is a parallelogram. Q is the midpoint of PR. QS = 6 units, PT = 3 units, TS = 8 units, QT = x units and  $\hat{T}_1 = \theta$ .

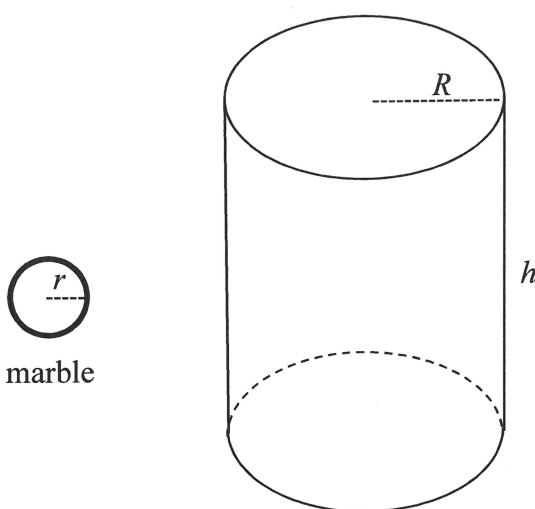


7.2.1 Show that  $\cos\theta = \frac{x^2 + 28}{16x}$  (3)

7.2.2 Hence, determine the length of QT. (6)  
[16]

**QUESTION 8**

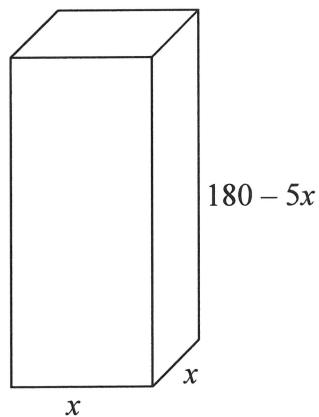
- 8.1 The cylindrical container shown below will be filled with spherical marbles. The container has a radius of  $R$  cm and a perpendicular height of  $h$  cm. The volume of the container is  $300 \text{ cm}^3$ . The radius of each marble is  $0,75 \text{ cm}$ .



$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

- 8.1.1 Show that the height of the container,  $h$ , is given by  $h = \frac{300}{\pi R^2}$  (2)
- 8.1.2 The container is filled with 100 marbles. Then water is poured into the container until the water reaches the top of the container.  
Calculate the volume of water that was poured into the container. (3)
- 8.2 In the diagram below, the right rectangular prism has a square base of  $x$  cm and a height of  $(180 - 5x)$  cm.



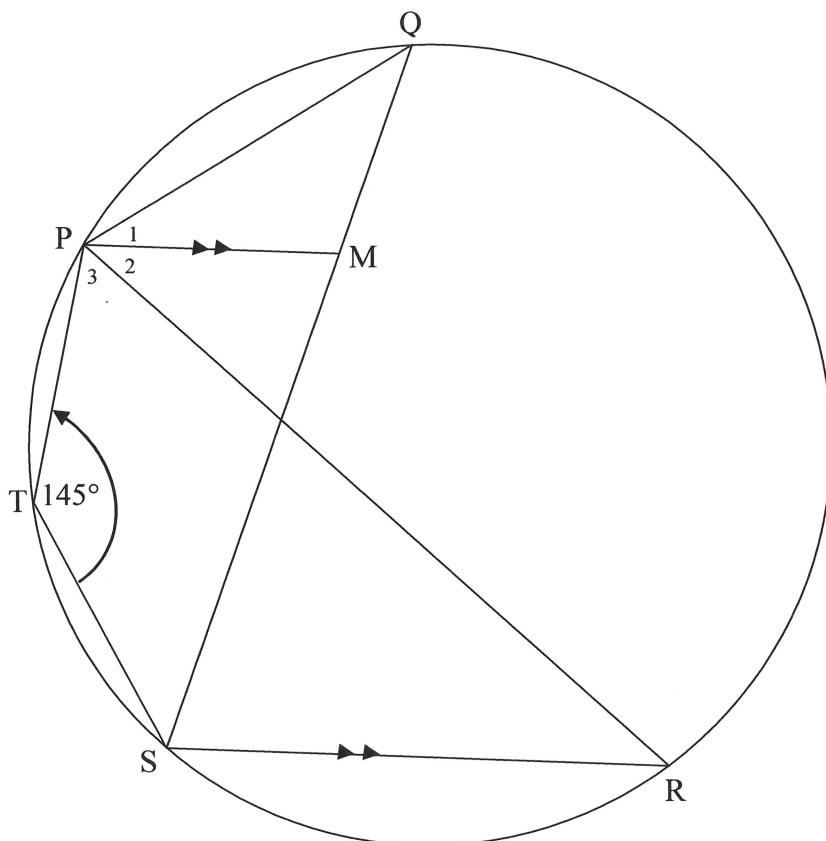
Calculate the maximum surface area of the prism.

(5)  
[10]

Give reasons for your statements and calculations in QUESTIONS 9, 10, and 11.

**QUESTION 9**

In the figure Q, P, T, S and R lie on the circle. M lies on QS such that  $PM \parallel SR$ .  
 $\hat{PTS} = 145^\circ$



9.1 Calculate the size of:

9.1.1  $\hat{Q}$  (2)

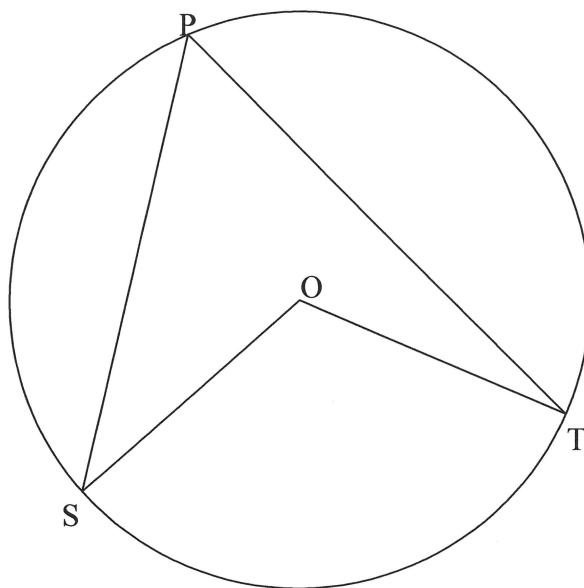
9.1.2  $\hat{R}$  (2)

9.1.3  $\hat{P}_2$  (1)

9.2 Why is PR a tangent to the circle passing through P, M and Q? (1)  
[6]

**QUESTION 10**

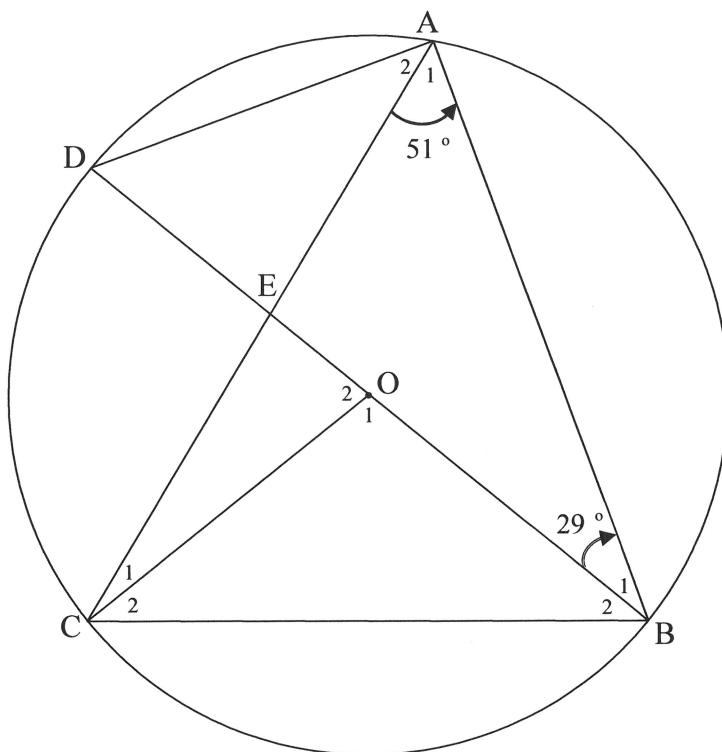
- 10.1 In the diagram below, O is the centre of the circle. P, T and S are points on the circumference of the circle. PS, PT and OT are drawn.



Prove the theorem which states that  $\hat{SOT} = 2 \times \hat{SPT}$ .

(5)

- 10.2 In the diagram, O is the centre of the circle.  
 Points A, B, C and D lie on the circumference of the circle. BOD is a diameter.  
 AC and BD intersect at E.  
 $\hat{A}_1 = 51^\circ$  and  $\hat{B}_1 = 29^\circ$ .



- 10.2.1 Determine the size of  $\hat{O}_1$ . (2)
- 10.2.2 Determine the size of  $\hat{A}_2$ . (2)
- 10.2.3 Determine the size of  $\hat{D}$ . (1)
- 10.2.4 Determine the size of  $A\hat{C}O$ . (3)

[13]

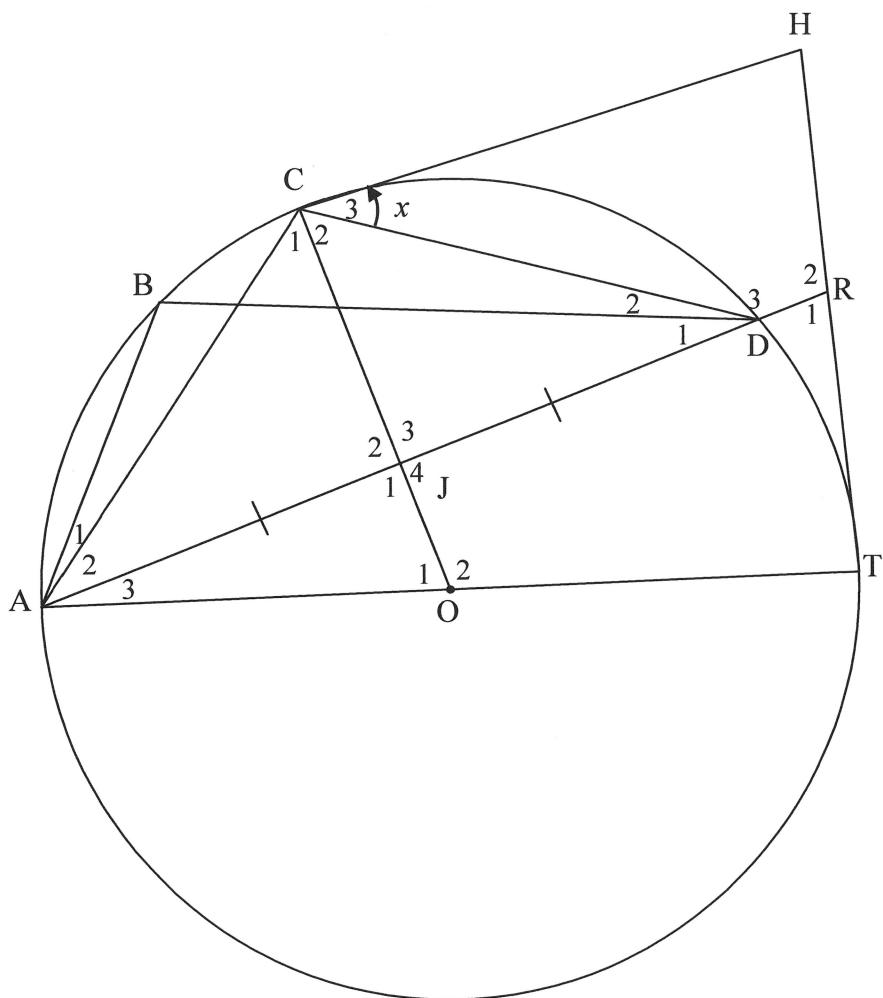
**QUESTION 11**

In the diagram, O is the centre of the circle through the points A, B, C, D and T. HC and HT are tangents to the circle at C and T respectively.

AD is produced to meet HT at R.

OC bisects AD at J.

Let  $\hat{C}_3 = x$ .



- 11.1 Write down, with a reason, another angle equal to  $\hat{C}_3$ . (2)
  - 11.2 Show that CHRJ is a trapezium. (5)
  - 11.3 Prove that OC bisects  $\hat{ACD}$ . (3)
  - 11.4 Write down, with a reason,  $\hat{ABD}$  in terms of  $x$ . (2)
  - 11.5 Determine  $\hat{R}_2$  in terms of  $x$ . (6)
- [18]

**TOTAL:** 150







NAME OF LEARNER: NAAM VAN LEERDER:	
CLASS: KLAS:	

# NATIONAL SENIOR CERTIFICATE *NASIONALE SENIOR SERTIFIKAAT*

MATHEMATICS P2/*WISKUNDE V2*

**GRADE/GRAAD 11**

**NOVEMBER 2019**

**SPECIAL ANSWER BOOK  
*SPESIALE ANTWOORDEBOEK***

QUESTION <i>VRAAG</i>	MARK <i>PUNT</i>			INITIAL <i>PARAAF</i>	MODERATION <i>MODERERING</i>	INITIAL <i>PARAAF</i>
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
<b>TOTAL <i>TOTAAL</i> (150)</b>						

This answer book consists of 21 pages.  
*Hierdie antwoordeboek bestaan uit 21 bladsye.*



\* I M A T A B 2 \*

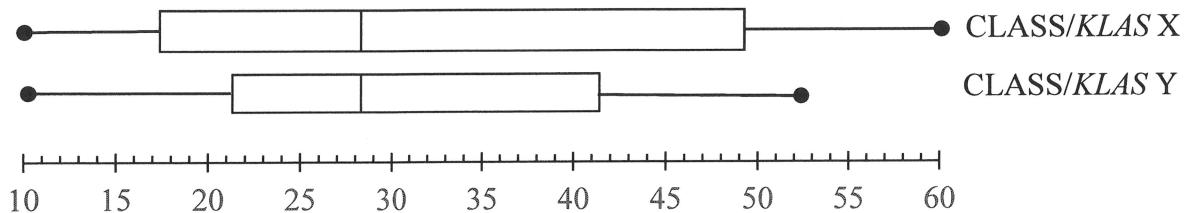


EASTERN CAPE

**QUESTION/VRAAG 1**

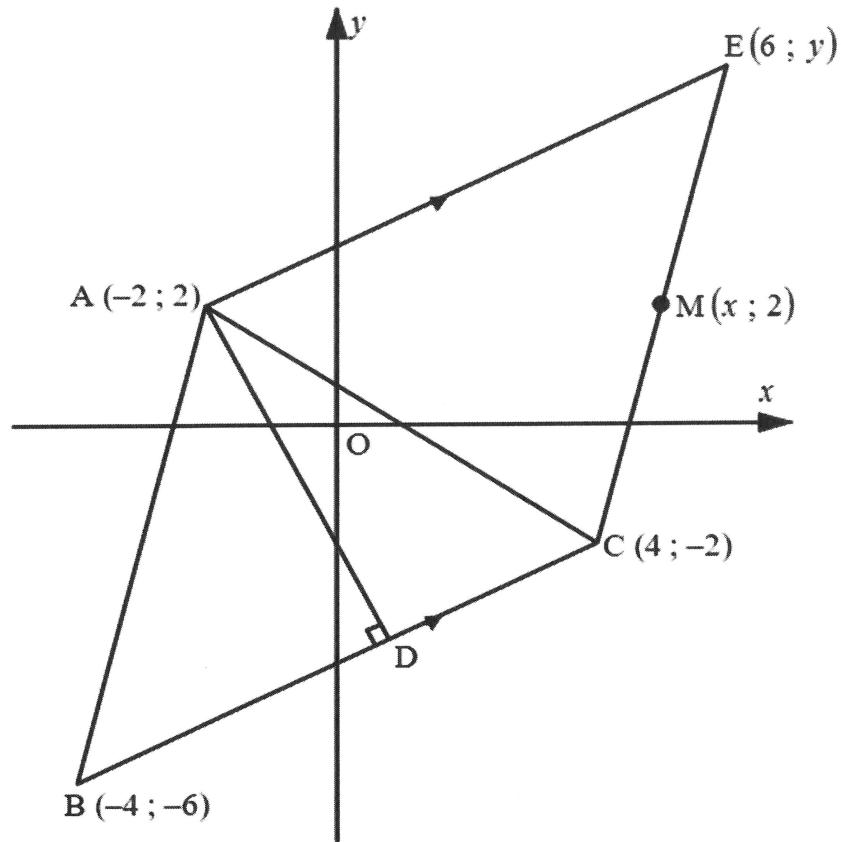
SELLING PRICE/ VERKOOPSPRYS (IN THOUSANDS OF RANDS/ IN DUISENDE RAND)	FREQUENCY/ FREKWENSIE	CUMULATIVE FREQUENCY/ KUMULATIEWE FREKWENSIE
$50 \leq x < 60$	3	3
$60 \leq x < 70$	4	7
$70 \leq x < 80$	$a$	14
$80 \leq x < 90$	19	33
$90 \leq x < 100$	12	$b$
$100 \leq x < 110$	5	50

	Solution/ <i>Oplossing</i>	Marks <i>Punte</i>
1.1		
1.2	<p style="text-align: center;"><b>Cumulative frequency graph on the price of cars/ Kumulatiewefrekwensie-grafiek oor die prys van motors</b></p>	(2)
1.3		(1) [6]

**QUESTION/VRAAG 2**

	<b>Solution/<i>Oplossing</i></b>	<b>Marks/ <i>Punte</i></b>
2.1.1		(1)
2.1.2		(1)
2.1.3		(2)
2.2.1		(8)
2.2.2		(3)
		[15]

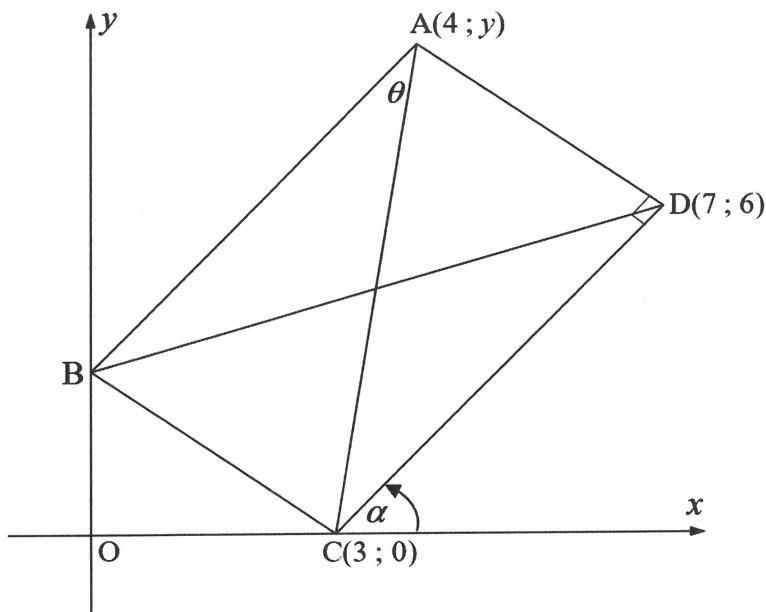


**QUESTION/VRAAG 3**

	<b>Solution/Oplossing</b>	<b>Marks/Punte</b>
3.1		(3)
3.2		(3)

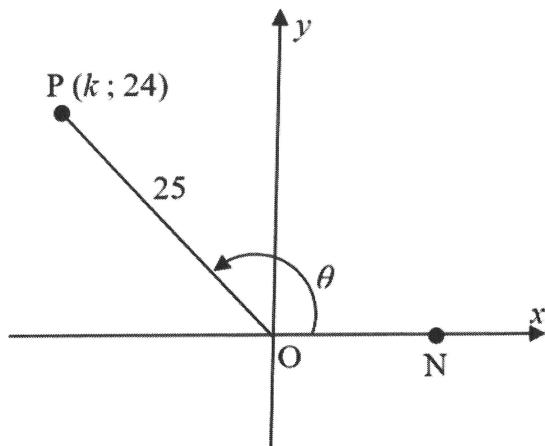


3.7		
	(3)	[20]

**QUESTION/VRAAG 4**

	<b>Solution/Oplossing</b>	<b>Marks/Punte</b>
4.1		(2)
4.2		(2)
4.3		(4)

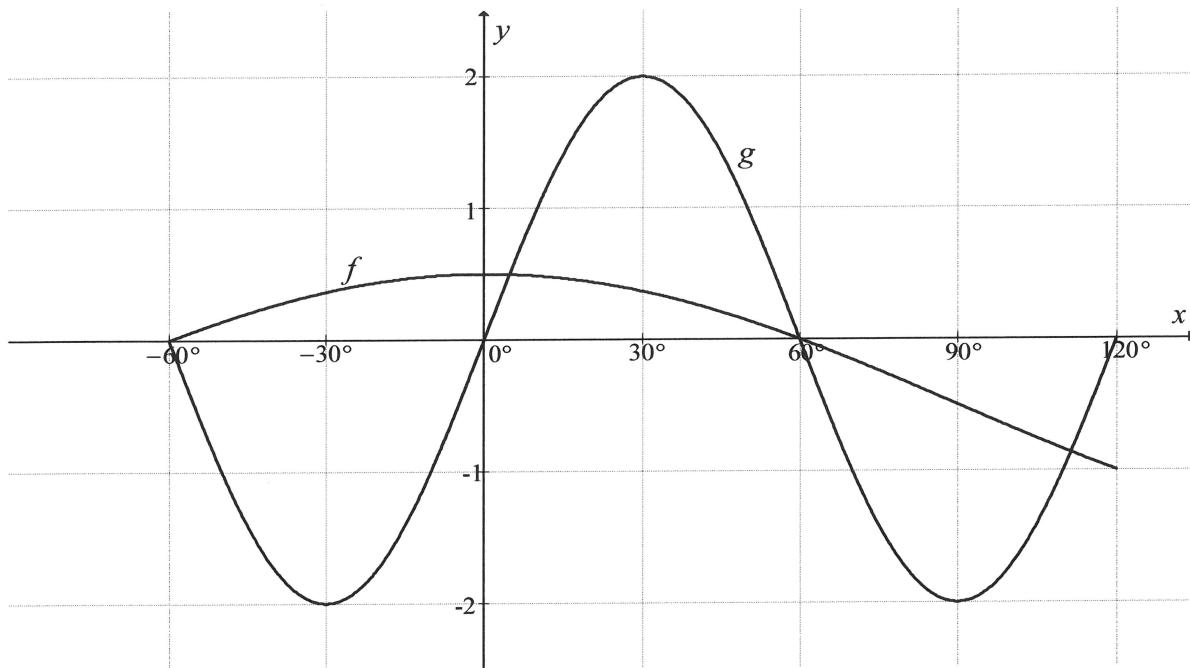
4.4		
		(5)
		[13]

**QUESTION/VRAAG 5**

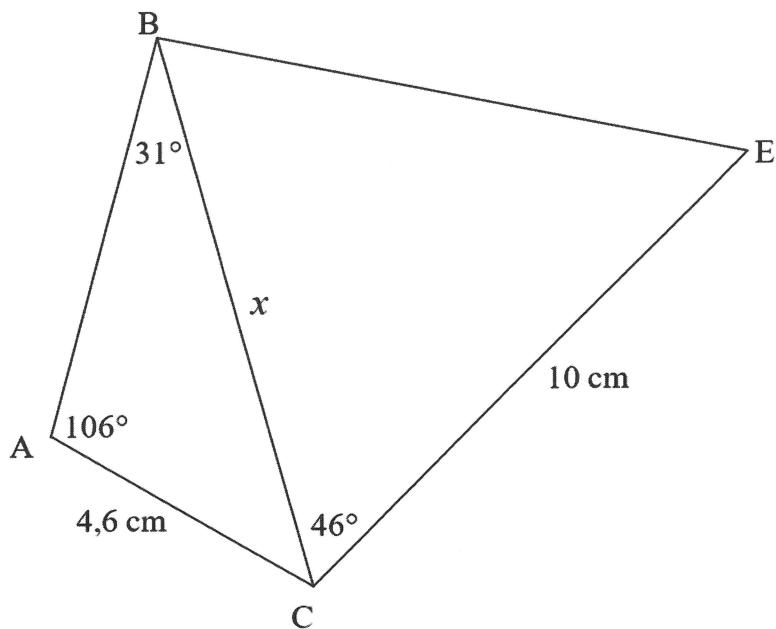
	<b>Solution/Oplossing</b>	<b>Marks/Punte</b>
5.1.1		
		(2)
5.1.2		
		(1)
5.1.3		
		(3)
5.1.4		
		(3)

5.2		
		(5)
5.3		
		(4)
5.4		
		(6)
		[24]

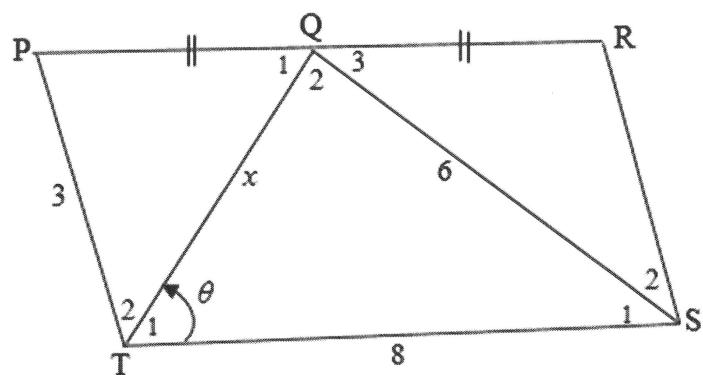


**QUESTION/VRAAG 6**

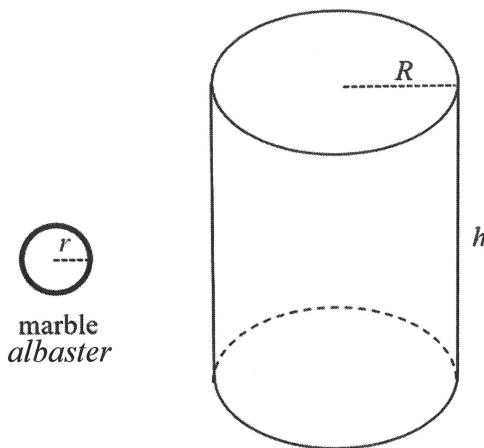
	<b>Solution/Oplossing</b>	<b>Marks/Punte</b>
6.1		(1)
6.2		(2)
6.3		(2)
6.4		(4)
		[9]

**QUESTION/VRAAG 7**

	<b>Solution/Oplossing</b>	<b>Marks/Punte</b>
7.1.1		(3)
7.1.2		(4)



7.2.1		(3)
7.2.2		(6)
		<b>[16]</b>

**QUESTION/VRAAG 8**

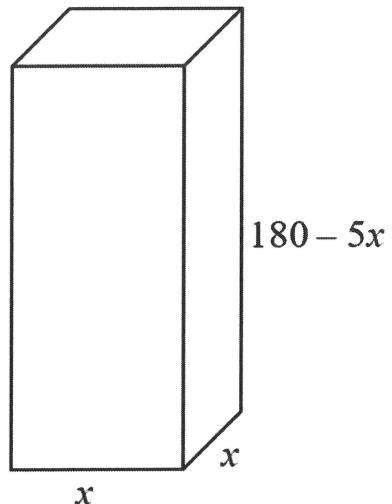
$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume van sfeer} = \frac{4}{3}\pi r^3$$

$$\text{Oppervlakte van 'n sfeer} = 4\pi r^2$$

	<b>Solution/<i>Oplossing</i></b>	<b>Marks <i>Punte</i></b>
8.1.1		(2)
8.1.2		(3)



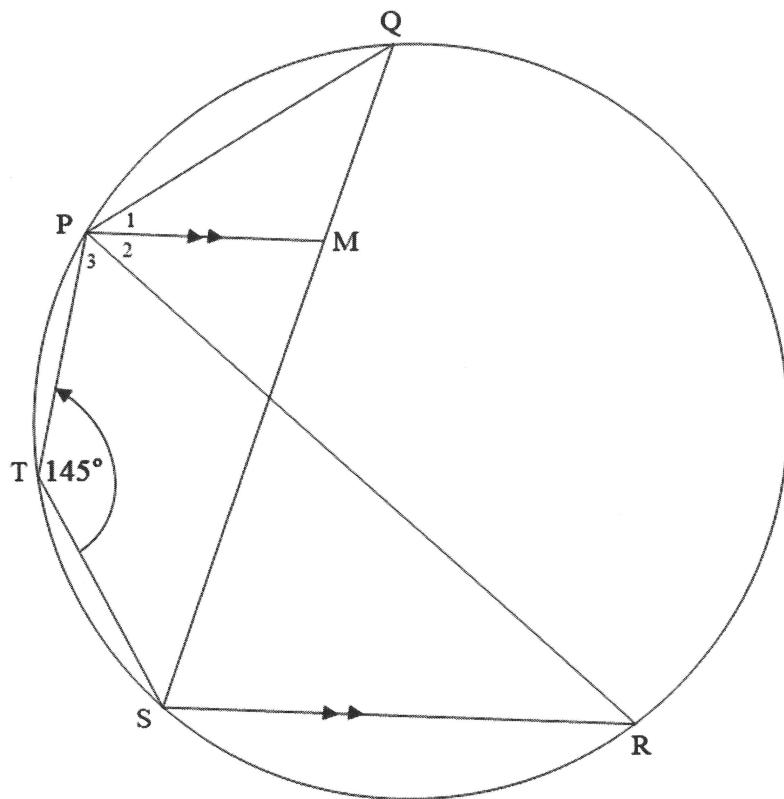
8.2

8.2		
		(5)
		[10]

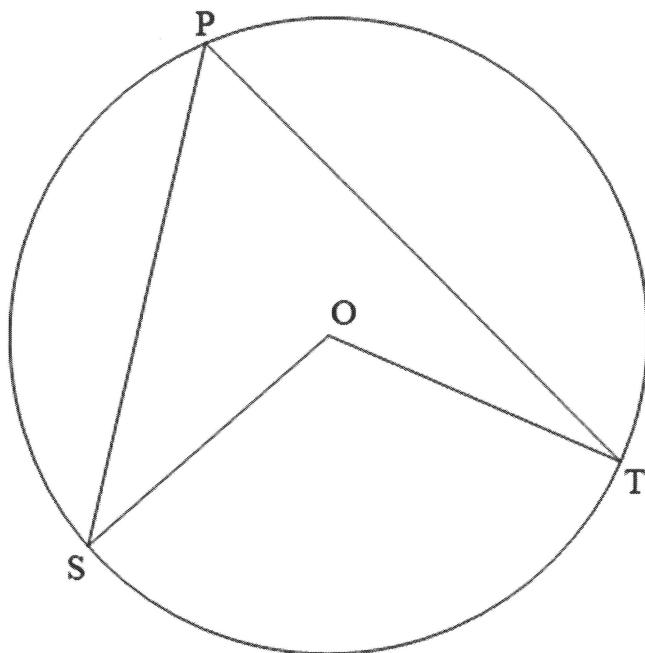
**Give reasons for your statements and calculations in QUESTIONS 9, 10 en 11.**

**Gee redes vir jou bewerings en berekeninge in VRAAG 9, 10 en 11.**

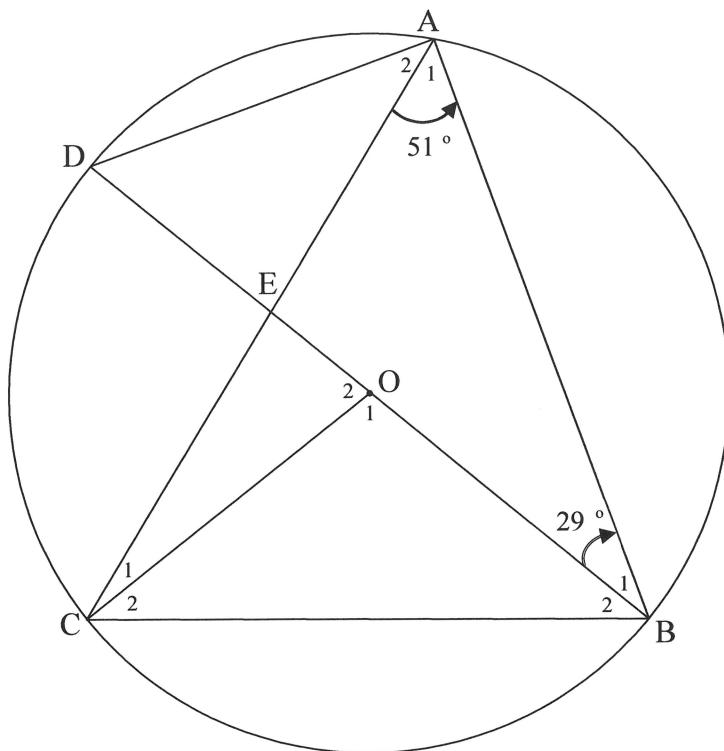
**QUESTION/VRAAG 9**



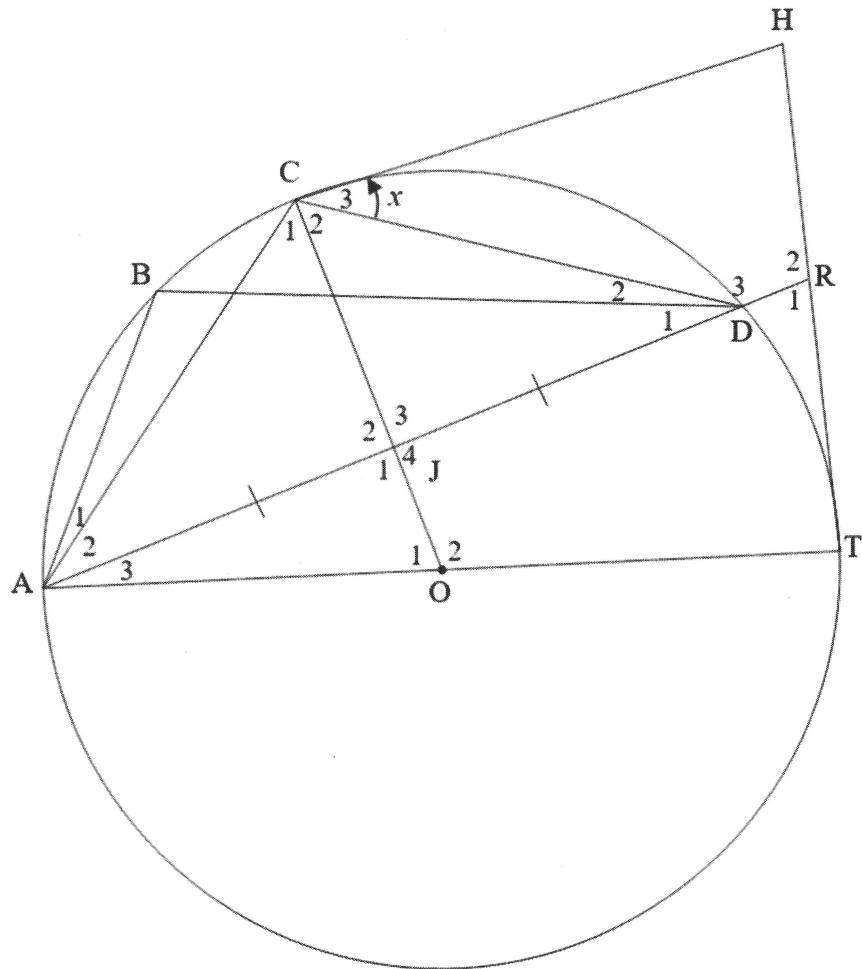
	<b>Solution/Oplossing</b>	<b>Marks Punte</b>
9.1.1		(2)
9.1.2		(2)
9.1.3		(1)
9.2		(1)
		[6]

**QUESTION/VRAAG 10**

	<b>Solution/<i>Oplossing</i></b>	<b>Marks/ <i>Punte</i></b>
10.1		(5)



10.2.1		
10.2.2		(2)
10.2.3		(2)
10.2.4		(1)
		(3)
		[13]

**QUESTION/VRAAG 11**

11.1		(2)
11.2		(5)

11.3								(3)
11.4								(2)
11.5								(6)
								[18]

**TOTAL/TOTAAL:**      **150**



**ADDITIONAL SPACE/BYKOMENDE RUIMTE**






# basic education

Department:  
Basic Education  
**REPUBLIC OF SOUTH AFRICA**

## NATIONAL SENIOR CERTIFICATE *NASIONALE SENIOR SERTIFIKAAT*

**GRADE/GRAAD 11**

**MATHEMATICS P2/WISKUNDE V2**

**NOVEMBER 2019**

**MARKING GUIDELINES/NASIENRIGLYNE**

**MARKS/PUNTE: 150**

These marking guidelines consist of 21 pages./  
*Hierdie nasienriglyne bestaan uit 21 bladsye.*

**NOTE:**

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking memorandum.
- Assuming values/answers in order to solve a problem is unacceptable.

**LET WEL:**

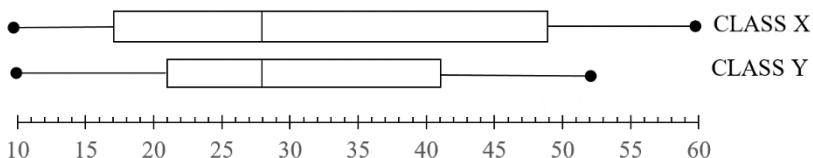
- As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.
- As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, sien die deurgehaalde antwoord na.
- Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing.
- Dit is onaanvaarbaar om waardes/antwoorde te veronderstel om 'n probleem op te los.

**QUESTION/VRAAG 1**

1.1	<b>SELLING PRICE/ VERKOOPSPRYS (IN THOUSANDS OF RANDS/ IN DUISENDE RAND)</b>	<b>FREQUENCY/ FREKWENSIE</b>	<b>CUMULATIVE FREQUENCY/ KUMULATIEWE FREKWENSIE</b>	
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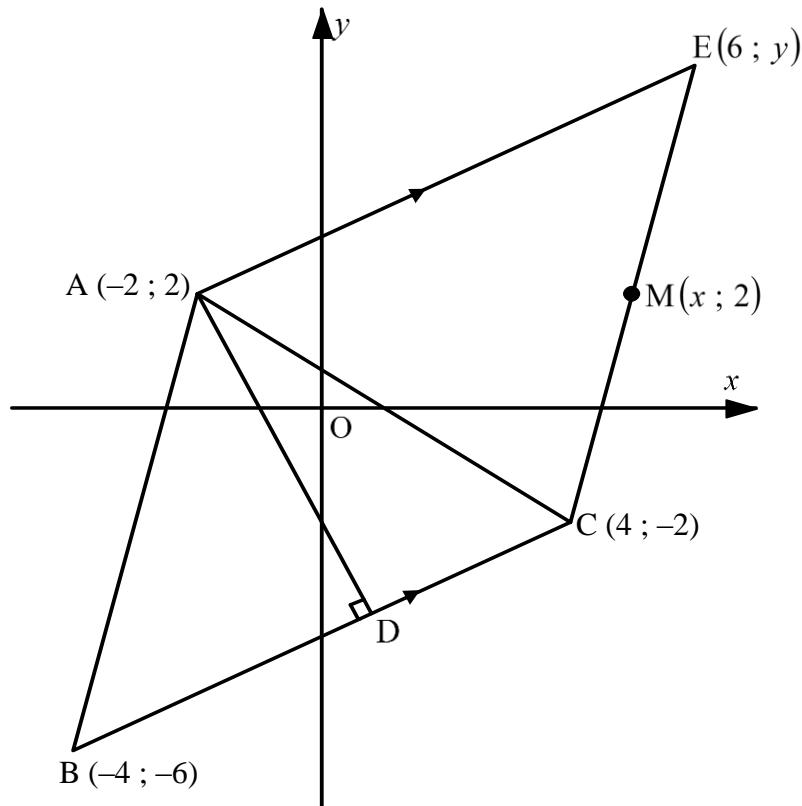
$\checkmark a = 7$   
 $\checkmark b = 45$   
(2)

1.2	<p style="text-align: center;"><b>Cumulative frequency graph on the price of cars</b></p> <table border="1"> <thead> <tr> <th>Price (in thousand rands)</th> <th>Cumulative Frequency</th> </tr> </thead> <tbody> <tr><td>50</td><td>0</td></tr> <tr><td>60</td><td>3</td></tr> <tr><td>70</td><td>7</td></tr> <tr><td>80</td><td>14</td></tr> <tr><td>95</td><td>33</td></tr> </tbody> </table>	Price (in thousand rands)	Cumulative Frequency	50	0	60	3	70	7	80	14	95	33	<ul style="list-style-type: none"> <li>✓ grounding/ geanker (50:0)</li> <li>✓ upper boundary/ boonste grens</li> <li>✓ smooth curve/ gladde kurwe</li> </ul> <p style="text-align: right;">(3)</p>
Price (in thousand rands)	Cumulative Frequency													
50	0													
60	3													
70	7													
80	14													
95	33													
1.3	<p>(95 000 ; 40) 40 cars to choose from/40 motors om vanuit te kies (Accept any answer between 37 and 43/ Aanvaar enige antwoord tussen 37 en 43)</p>	(1)												
		[6]												

**QUESTION/VRAAG 2**

2.1.1	Positively skewed/ <i>Positief skeef</i> or/of skewed to the right/ <i>skeef na regs</i>	✓ answer/ <i>antwoord</i> (1)
2.1.2	Class/ <i>Klas X</i>	✓ answer/ <i>antwoord</i> (1)
2.1.3	The average performance is the same./ <i>Die gemiddelde prestasie is dieselfde.</i> Both classes have the same median mark./ <i>Albei klasse het dieselfde mediaanpunt.</i>	✓ average performance is the same/ <i>gemiddelde prestasie dieselfde</i>  ✓ same median mark/ <i>dieselfde mediaanpunt</i> (2)
2.2.1	$a = 5$ $g = 5 + 48$ $= 53$ $d = 22$ $b = 5 + 7$ $= 12$  $f = 12 + 28$ $= 40$  $\frac{c + 2c + 132}{7} = 27$ $3c = 57$ $c = 19$ $e = 38$	✓ $a = 5$ ✓ $b = 12$ ✓ $c + 2c + 132 = 27$ ✓ $c = 19$ ✓ $d = 22$ ✓ $e = 38$ ✓ $f = 40$ ✓ $g = 53$ (8)
2.2.2	$(\bar{x} - \sigma; \bar{x} + \sigma)$ $(27 - 15,87; 27 + 15,87)$ $(11,13; 42,87)$  5 goals were scored within one standard deviation of the mean/ <i>5 doele is binne een standaardafwyking van die gemiddeld aangeteken</i>	✓✓ $(11,13; 42,87)$ ✓ answer/ <i>antwoord</i> [15]

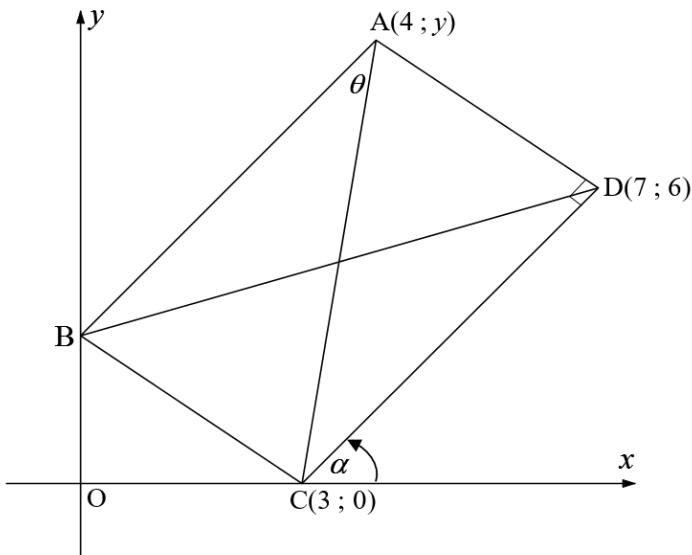
## QUESTION/VRAAG 3



3.1	$B(-4; -6) \quad C(4; -2)$ $m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-2 - (-6)}{4 - (-4)}$ $= \frac{4}{8}$ $= \frac{1}{2}$	$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-6 - (-2)}{-4 - 4}$ $= \frac{-4}{-8}$ $= \frac{1}{2}$	✓ gradient formula/ <i>gradiëntformule</i> ✓ subst into/ <i>vervanging in</i> gradient form./ <i>gradiëntform.</i>  ✓ answer/ <i>antwoord</i> (3)
3.2	$x = \frac{6+4}{2}$ $x = 5$ $\frac{y+(-2)}{2} = 2$ $y = 6$		✓ $x = 5$ ✓ $\frac{y+(-2)}{2} = 2$ ✓ $y = 6$ (3)

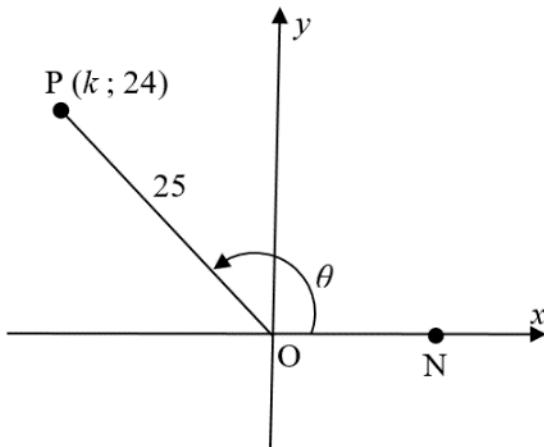
3.3	$\begin{aligned} BC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{((-4) - 4)^2 + ((-6) - (-2))^2} \\ &= \sqrt{80} \end{aligned}$	✓ subst. in distance formula/verv. in afstandformule  ✓ answer/antwoord (2)
3.4	Parallelogram	✓ answer/antwoord (1)
3.5	$m_{AD} = -2 \text{ (AD} \perp \text{BC)}$ $y = \frac{4}{3}x + c$ $2 = -2(-2) + c$ $-2 = c$ $\therefore y = -2x - 2$  <b>OR/OF</b>  $m_{AD} = -2 \text{ (AD} \perp \text{BC)}$ $y - 2 = -2(x - (-2))$ $y - 2 = -2x - 4$ $\therefore y = -2x - 2$	✓ $m_{AD} = -2$  ✓ subst. of/verv. m and point/en punt $(-2; 2)$ / ✓ answer/antwoord  ✓ $m_{AD} = -2$ ✓ subst. of/verv. m and point/en punt $(-2; 2)$ ✓ answer/antwoord (3)

<p>3.6</p> $m_{BC} = \frac{1}{2}$ <p>Equation of/Verg. van BC:</p> $y = \frac{1}{2}x + c$ $-2 = \frac{1}{2}(4) + c$ $-4 = c$ $\therefore y = \frac{1}{2}x - 4$ <p><b>OR/OF</b></p> $y - (-2) = \frac{1}{2}(x - 4)$ $y + 2 = \frac{1}{2}x - 2$ $\therefore y = \frac{1}{2}x - 4$	<p>✓ subst. of/verv. <math>m</math> and point/en punt <math>(4; -2)/</math></p> <p>✓ <math>y = \frac{1}{2}x - 4</math></p> <p><b>OR/OF</b></p> $-2x - 2 = \frac{1}{2}x - 4$ $-4x - 4 = x - 8$ $4 = 5x$ $x = \frac{4}{5}$ $y = (-2)\frac{4}{5} - 2$ $= \frac{-18}{5}$ $D\left(\frac{4}{5}; \frac{-18}{5}\right)$ $y = \frac{1}{2}\left(\frac{4}{5}\right) - 4$ $= \frac{-18}{5}$	<p>✓ equating both equations/vergelyking van beide vergelykings</p> <p>✓ <math>x</math>-value/-waarde</p> <p>✓ <math>y</math>-value/-waarde</p>
<p>3.7</p> $AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{\left(\left(\frac{4}{5}\right) - (-2)\right)^2 + \left(\left(\frac{-18}{5}\right) - 2\right)^2}$ $= \frac{14\sqrt{5}}{5}$ <p>Area of/Oppervlakte van <math>\Delta AEC = \frac{1}{2} AE \times AD</math></p> $= \frac{1}{2} \times \sqrt{80} \times \frac{14\sqrt{5}}{5}$ $= 28 \text{ units}^2 / \text{eenhede}^2$	<p>(5)</p> <p>✓ length of AD/ lengte van AD</p> <p>✓ subst into area formula</p> <p>✓ answer/antwoord</p>	<p>(3)</p> <p>[20]</p>

**QUESTION/VRAAG 4**

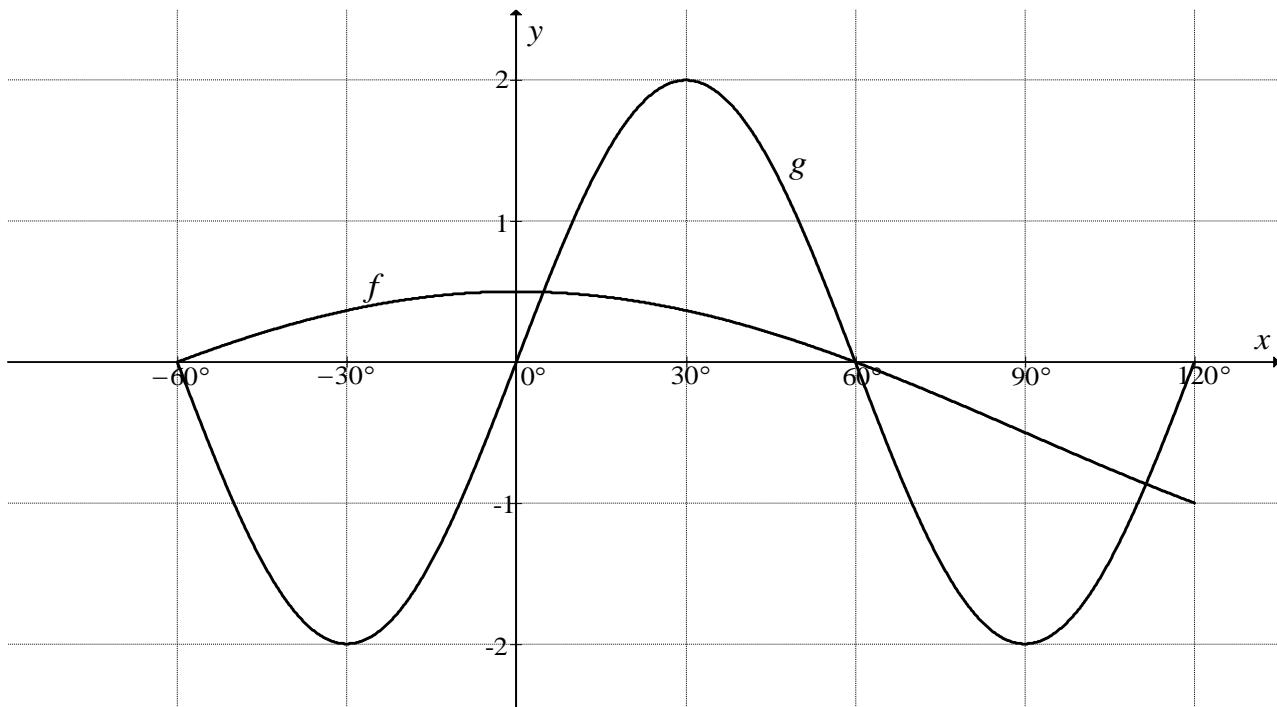
4.1	$m_{DC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{6-0}{7-3}$ $= \frac{3}{2}$	✓ substitution in gradient formula/ vervanging in gradiëntformule ✓ $m_{DC} = \frac{3}{2}$ (2)
4.2	$\tan \alpha = m_{CD}$ $\tan \alpha = \frac{3}{2}$ $\alpha = 56,31^\circ$	✓ $\tan \alpha = \frac{3}{2}$ ✓ $\alpha = 56,31^\circ$ (2)
4.3	$m_{CD} \times m_{AD} = -1$ $\frac{3}{2} \times \frac{y-6}{4-7} = -1$ $\frac{y-6}{-3} = -\frac{2}{3}$ $3y-18 = 6$ $y-6 = 2$ $y = 8$	✓✓ $\frac{y-6}{-3} = -\frac{2}{3}$ ✓ $3y-18 = 6$ ✓ $y = 8$ (4)

4.4	$m_{AC} = \frac{8-0}{4-3} = 8$ $\therefore \text{Inclination of AC} = \tan^{-1}(8)$ $= 82,87^\circ$ $\hat{A}CD = 82,87^\circ - 56,31^\circ$ $= 26,56^\circ$ $\therefore \theta = 26,56^\circ$	✓ $m_{AC} = 8$ ✓ $82,87^\circ$ ✓ $\hat{A}CD = 82,87^\circ - 56,31^\circ$ ✓ $\hat{A}CD = 26,56^\circ$ ✓ $\theta = 26,56^\circ$ (5)
		[13]

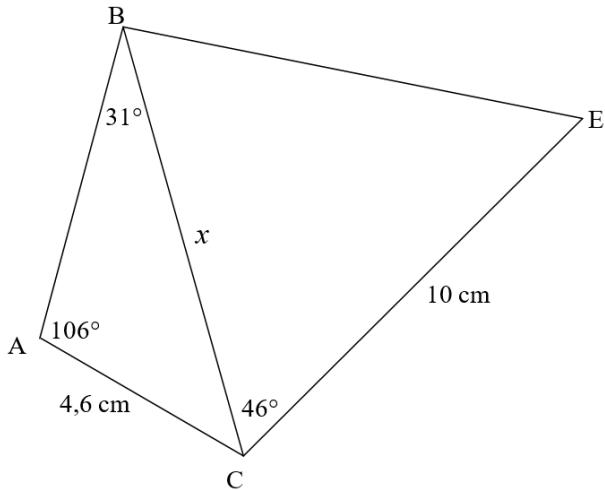
**QUESTION/VRAAG 5**

5.1.1	$x^2 + y^2 = r^2$ $(k)^2 + (24)^2 = 25^2$ $k^2 = 49$ $k = -7$	✓ subst./ vervanging ✓ answer/antwoord (2)
5.1.2	$\tan \theta = -\frac{24}{7}$	✓ answer/antwoord (1)
5.1.3	$\theta + \alpha = 360^\circ$ $\alpha = 360^\circ - \theta$ $\sin \alpha = \sin(360^\circ - \theta)$ $= -\sin \theta$ $= -\frac{24}{25}$	✓ $\sin \alpha = \sin(360^\circ - \theta)$ ✓ $-\sin \theta$ ✓ answer/antwoord (3)
5.1.4	$\cos^2 \theta - \sin^2 \alpha$ $= \left(\frac{-7}{25}\right)^2 - \left(-\frac{24}{25}\right)^2$ $= \frac{-527}{625}$	✓✓ substitution/ vervanging ✓ answer/antwoord (3)

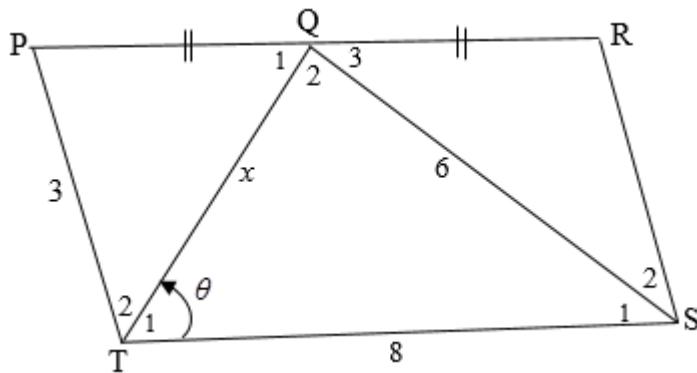
5.2	$  \begin{aligned}  & \frac{\cos 210^\circ \cdot \tan 135^\circ}{\sin(-60^\circ) \cdot \cos 420^\circ} \\  &= \frac{-\cos 30^\circ \cdot -\tan 45^\circ}{-\sin 60^\circ \cdot \cos 60^\circ} \\  &= \frac{\left(-\frac{\sqrt{3}}{2}\right)(-1)}{\left(-\frac{\sqrt{3}}{2}\right) \cdot \frac{1}{2}} \\  &= -2  \end{aligned}  $	$\checkmark -\cos 30^\circ \cdot -\tan 45^\circ$ $\checkmark -\sin 60^\circ \cdot \cos 60^\circ$ $\checkmark \left(-\frac{\sqrt{3}}{2}\right)(-1)$ $\checkmark \left(-\frac{\sqrt{3}}{2}\right) \cdot \frac{1}{2}$ $\checkmark \text{answer/antwoord}$
(5)		
5.3	$  \begin{aligned}  \text{LHS} &= \frac{1}{\tan^2 x} - \cos^2 x \\  &= \frac{1}{\frac{\sin^2 x}{\cos^2 x}} - \cos^2 x \\  &= \frac{\cos^2 x}{\sin^2 x} - \cos^2 x \\  &= \frac{\cos^2 x - \cos^2 x \sin^2 x}{\sin^2 x} \\  &= \frac{\cos^2 x(1 - \sin^2 x)}{\sin^2 x} \\  &= \frac{\cos^2 x(\cos^2 x)}{\sin^2 x} \\  &= \frac{\cos^4 x}{\sin^2 x} \\  &= \text{RHS}  \end{aligned}  $	$\checkmark \frac{\sin^2 x}{\cos^2 x}$ $\checkmark \text{common denominator/gemene noemer}$ $\checkmark \text{factors/faktore}$ $\checkmark 1 - \sin^2 x = \cos^2 x$
(4)		
5.4	$  \begin{aligned}  \sqrt{2} \sin x \cos x &= \cos x \\  \sqrt{2} \sin x \cos x - \cos x &= 0 \\  \cos x (\sqrt{2} \sin x - 1) &= 0 \\  \cos x &= 0 \\  x &= 90^\circ + 360^\circ k, \quad k \in \mathbb{Z} \\  \text{or} \\  x &= 270^\circ + 360^\circ k, \quad k \in \mathbb{Z} \quad \text{or/of} \quad \sin x = \frac{1}{\sqrt{2}} \\  &\quad \quad \quad x = 45^\circ + 360^\circ k, \quad k \in \mathbb{Z} \\  \text{OR} \\  x &= 90^\circ + 180^\circ k, \quad k \in \mathbb{Z} \quad \text{or} \quad x = 135^\circ + 360^\circ k, \quad k \in \mathbb{Z}  \end{aligned}  $	$\checkmark \text{standard form/stand.vorm}$ $\checkmark \text{factors/faktore}$ $\checkmark \text{both equations/beide vergelykings}$ $\checkmark$ $x = 90^\circ + 360^\circ k, \quad k \in \mathbb{Z}$ $\checkmark$ $x = 45^\circ + 360^\circ k, \quad k \in \mathbb{Z}$ $\checkmark$ $x = 135^\circ + 360^\circ k, \quad k \in \mathbb{Z}$
(6) [24]		

**QUESTION/VRAAG 6**

6.1	$(30^\circ; 2)$	✓✓ answer/antwoord (1)
6.2	$-60^\circ < x < 0^\circ$	✓ endpoints/eindpunte ✓ notation/notasie (2)
6.3	$q = -\frac{1}{2}$ $b = 3$	✓ $q = -\frac{1}{2}$ ✓ $b = 3$ (2)
6.4	$2\cos x \sin 3x - \sin 3x \geq 0$ $2\sin 3x \left( \cos x - \frac{1}{2} \right) \geq 0$ $g(x)f(x) \geq 0$ $0^\circ \leq x \leq 120^\circ$	✓✓ $2\sin 3x \left( \cos x - \frac{1}{2} \right) \geq 0$ ✓ endpoints/eindpunte ✓ notation/notasie (4)
		[9]

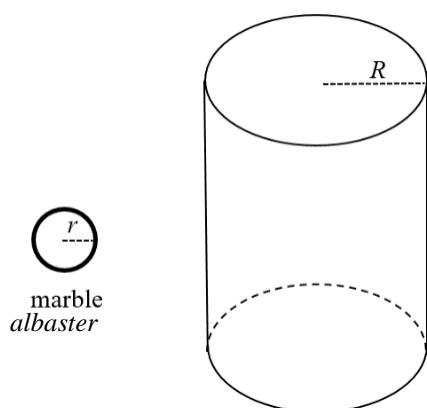
**QUESTION/VRAAG 7**

7.1.1	$\frac{BC}{\sin \hat{A}} = \frac{AC}{\sin \hat{B}}$ $\frac{x}{\sin 106^\circ} = \frac{4,6}{\sin 31^\circ}$ $x = \frac{4,6 \sin 106^\circ}{\sin 31^\circ}$ $x = 8,59 \text{ cm}$	✓ substitution into sine rule/vervanging in sinusreël ✓ $x = \frac{4,6 \sin 106^\circ}{\sin 31^\circ}$ ✓ answer/antwoord (3)
7.1.2	$\hat{A}CB = 43^\circ$ $\text{Area } ACEB = \left( \frac{1}{2} \times AC \times BC \times \sin \hat{A}CB \right) + \left( \frac{1}{2} \times BC \times CE \times \sin \hat{B}CE \right)$ $= \left( \frac{1}{2} \times 4,6 \times 8,59 \times \sin 43^\circ \right) + \left( \frac{1}{2} \times 8,59 \times 10 \times \sin 46^\circ \right)$ $= 44,37 \text{ cm}^2$	✓ $\hat{A}CB = 43^\circ$ ✓ ✓ substitution/vervanging ✓ answer/antwoord (4)



7.2.1	$\text{In } \triangle QTS$ $6^2 = 8^2 + x^2 - 2(8)(x)\cos \theta$ $16x\cos \theta = 8^2 + x^2 - 6^2$ $16x\cos \theta = x^2 + 28$ $\cos \theta = \frac{28+x^2}{16x}$	✓✓ substitution into cosine rule/ <i>vervanging in cosinusreël</i> ✓ $16x\cos \theta = x^2 + 28$ (3)
7.2.2	$\hat{Q}_1 = \theta$ and $PQ = 4$ $\text{In } \triangle QTP$ $3^2 = 4^2 + x^2 - 2(4)(x)\cos \theta$ $8x\cos \theta = 4^2 + x^2 - 3^2$ $8x\cos \theta = x^2 + 7$ $\cos \theta = \frac{7+x^2}{8x}$ $\frac{7+x^2}{8x} = \frac{28+x^2}{16x}$ $112x + 16x^3 = 224x + 8x^3$ $8x^3 = 112x$ $8x^3 - 112x = 0$ $8x(x^2 - 14) = 0$ $x \neq 0 \quad x = \sqrt{14}$	✓ ✓ $\hat{Q}_1 = \theta$ and $PQ = 4$ ✓ $3^2 = 4^2 + x^2 - 2(4)(x)\cos \theta$ ✓ $\cos \theta = \frac{7+x^2}{8x}$ ✓ equating/ <i>vergelyking</i> ✓ factors/faktore ✓ $x \neq 0 \quad x = \sqrt{14}$ (6)
		[16]

## QUESTION/VRAAG 8



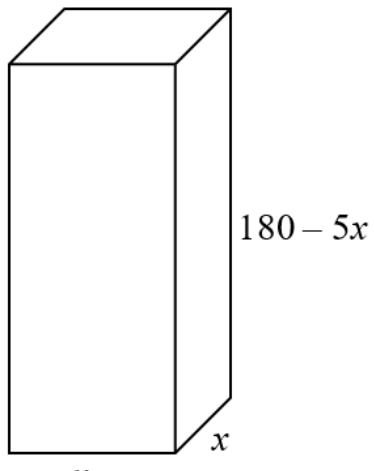
$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

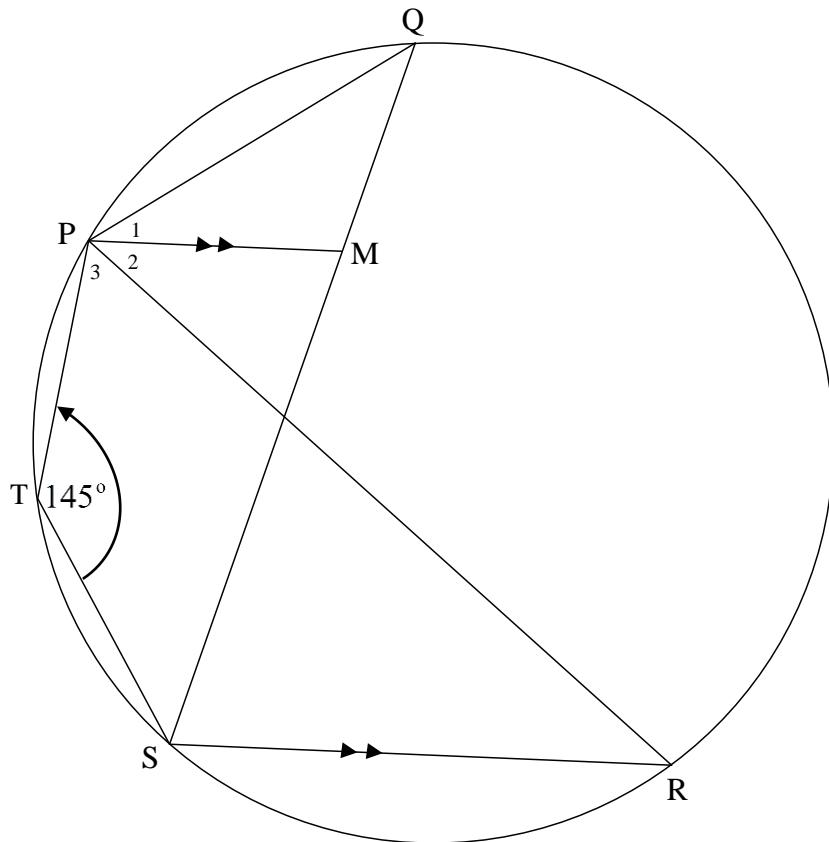
$$\text{Volume of sfeer} = \frac{4}{3}\pi r^3$$

$$\text{Oppervlakte van 'n sfeer} = 4\pi r^2$$

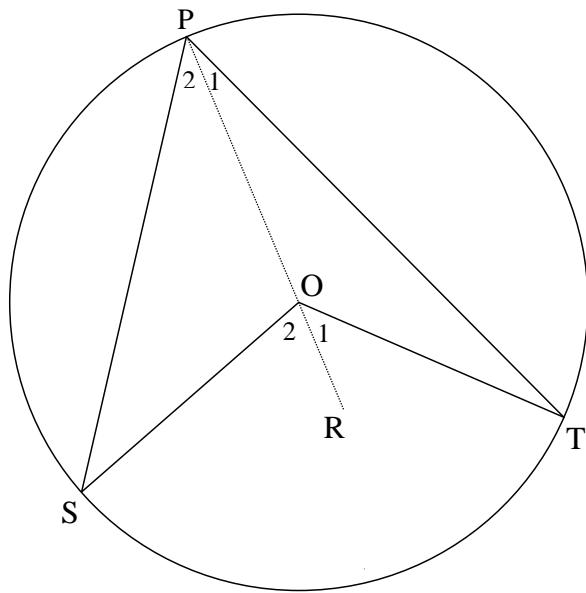
8.1.1	$V = \pi r^2 h$ $300 = \pi R^2 h$ $h = \frac{300}{\pi R^2}$	✓ substitution/ <i>vervanging</i> ✓ isolating $h$ / <i>isoleer h</i> (2)
8.1.2	Volume of marbles = $100\left(\frac{4}{3}\pi r^3\right)$ $= 100 \times \frac{4}{3}\pi(0.75)^3$ $= \frac{225\pi}{4}$ $= 176,71 \text{ cm}^3$  amount of water = $300 - 176,71$ $= 123,29 \text{ cm}^3$	✓ subst. into formula/ <i>vervanging</i> <i>in formule</i>  ✓ 176,71  ✓ 123,29 cm <sup>3</sup> (3)



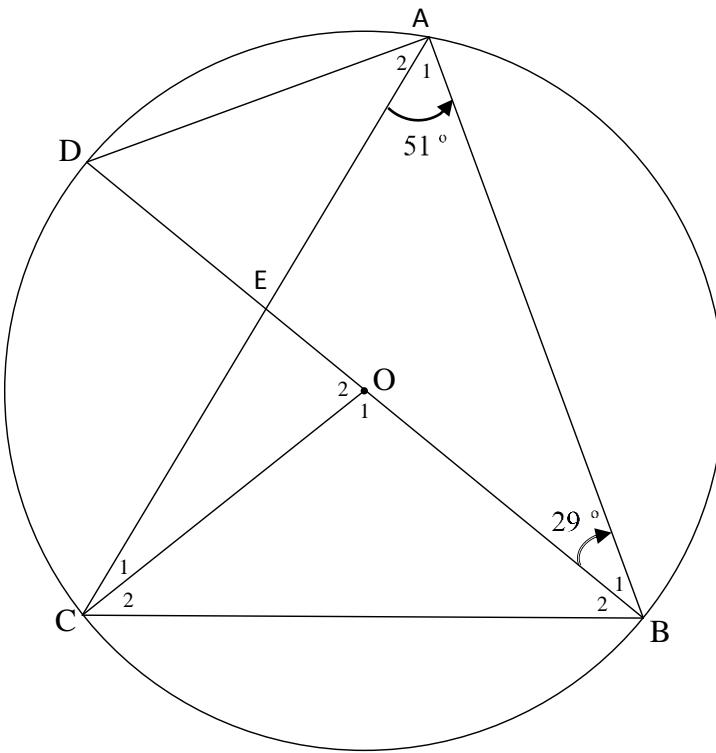
8.2	$\text{TSA} = 2 \times \text{area of the base} + (\text{perimeter} \times \text{height})$ $\text{TSA} = 2 \times \text{oppervlakte van basis} \pm (\text{omtrek} \times \text{hoogte})$ $= 2(x^2) + 4x(180 - 5x)$ $= 2x^2 + 720x - 20x^2$ $= -18x^2 + 720x$ <p>Surface area will be maximum when</p> <p><i>Oppervlakte sal op maksimum wees wanneer</i></p> $x = \frac{-b}{2a} = \frac{-720}{2(-18)} = 20$ $\text{max. surface area/maks. oppervlakte} = -18(20)^2 + 720(20)$ $= 7200 \text{ cm}^2$	<ul style="list-style-type: none"> <li>✓ subst. into formula/vervanging in formule</li> <li>✓ simplification/vereenvoudiging</li>   <li>✓ value of/waarde van <math>x</math></li> <li>✓ subst./verv <math>x = 20</math></li>   <li>✓ answer/antwoord (5)</li> </ul> <p>[10]</p>
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**QUESTION/VRAAG 9**

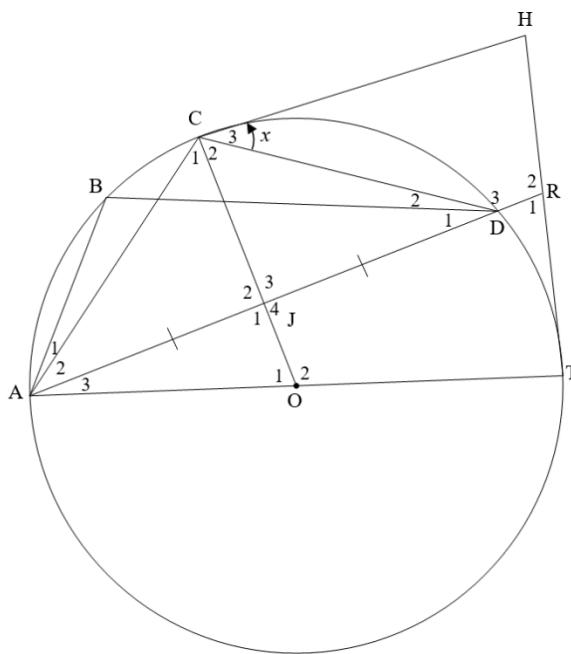
9.1.1	$\hat{Q} = 180^\circ - 145^\circ$ [opp $\angle^s$ of cyclic quad ] $= 35^\circ$	$\checkmark S \checkmark R$ (2)
9.1.2	$\hat{R} = 180^\circ - 145^\circ$ [opp $\angle^s$ of cyclic quad ] or [ $\angle^s$ in the same segment ] $= 35^\circ$	$\checkmark S \checkmark R$ (2)
9.1.3	$\hat{P}_2 = \hat{R} = 35^\circ$ [alt $\angle^s$ , $PM \parallel SR$ ]	$\checkmark S / R$ (1)
9.2	$\therefore \hat{P}_2 = \hat{Q}$ $\therefore PR$ is a tangent to circle $PMQ$ [converse tan - chord theorem]	$\checkmark S / R$ (1)
		<b>[6]</b>

**QUESTION/VRAAG 10**

10.1	<p>Construction/<i>Konstruksie</i>: Draw/Trek POR</p> <p>Let <math>\hat{P}_2 = x</math>  <math>\hat{P}_2 = \hat{S}</math> [angles opp = sides]  <math>\hat{O}_2 = 2x</math> [exterior <math>\angle</math> of <math>\Delta</math>]</p> <p>Let <math>\hat{P}_1 = y</math>  <math>\hat{P}_1 = \hat{T}</math> [angles opp = sides]  <math>\hat{O}_1 = 2y</math> [exterior <math>\angle</math> of <math>\Delta</math>]</p> $\hat{O}_1 + \hat{O}_2 = 2x + 2y = 2(x + y)$ $\hat{P}_1 + \hat{P}_2 = x + y$ $\therefore \text{S}\hat{\text{O}}\text{T} = 2 \times \text{S}\hat{\text{P}}\text{T}$	<p>✓ Construction/ <i>Konstruksie</i></p> <p>✓ S/R ✓ S/R</p> <p>✓ S</p> <p>✓ S</p>	(5)
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10.2.1	$\hat{O}_1 = 102^\circ$ [angle at centre = 2 times angle at circumference/ <i>middelpuntshoek = 2 keer omtrekshoek</i> ]	✓ S   ✓ R (2)
10.2.2	$\hat{A} = 90^\circ$ [ $\angle$ in a semi – circle] $\hat{A}_2 = 39^\circ$	✓ S   ✓ R (2)
10.2.3.	$\hat{D} = 61^\circ$ [sum of int $\angle^s$ of $\Delta$ ]	✓ S (1)
10.2.4	$\hat{ACB} = \hat{D} = 61^\circ$ [ $\angle^s$ in the same segment ] $\hat{C}_2 = \frac{180^\circ - 102^\circ}{2}$ [sum of int $\angle^s$ of $\Delta$ ] $= 39^\circ$ $\hat{ACO} = 61^\circ - 39^\circ$ $= 22^\circ$	✓ S /R ✓ S ✓ S (3)
		[13]

**QUESTION/VRAAG 11**

11.1	$\hat{C}_3 = \hat{A}_2 = x$ [tan - chord]	$\checkmark S$ $\checkmark R$ (2)
11.2	$\hat{J}_3 = 90^\circ$ [line from centre to midpt of chord] $O\hat{C}H = 90^\circ$ [tan $\perp$ rad] $CH \parallel JR$ [co-interior $\angle = 180^\circ$ ] $\therefore CHRJ$ is a trapezium [one propssides $\parallel$ ]	$\checkmark S$ $\checkmark R$ $\checkmark S$ $\checkmark R$ $\checkmark S$ <b>OR/OF</b> $\hat{J}_4 = 90^\circ$ [line from centre to midpt of chord] $O\hat{C}H = 90^\circ$ [tan $\perp$ rad] $CH \parallel JR$ [corresp $\angle =$ ] $\therefore CHRJ$ is a trapezium [one propssides $\parallel$ ]
11.3	In $\Delta CJA$ and $\Delta CJD$ $\hat{J}_2 = \hat{J}_3$ [line from centre to midpt of chord] $AJ = JD$ [given] $CJ = CJ$ [common side] $\therefore \Delta CJA \equiv \Delta CJD$ [SAS] $\hat{C}_1 = \hat{C}_2$ [ $\Delta CJA \equiv \Delta CJD$ ] $OC$ bisects $\hat{ACD}$	$\checkmark S$ $\checkmark S$ $\checkmark S$ <b>OR/OF</b> $\hat{J}_4 = 90^\circ$ [line from centre to midpt of chord] $O\hat{C}H = 90^\circ$ [tan $\perp$ rad] $CH \parallel JR$ [corresp $\angle =$ ] $\therefore CHRJ$ is a trapezium [one propssides $\parallel$ ]

11.4	$\hat{C}_2 = 90^\circ - x \quad [\tan \perp \text{rad}]$ $\hat{B} = \hat{C}_1 + \hat{C}_2 \quad [\angle^s \text{ in the same segment}]$ $= 90^\circ - x + 90^\circ - x \quad [\hat{C}_1 = \hat{C}_2]$ $= 180^\circ - 2x$ <p><b>OR/OF</b></p> $\hat{A}DC = x \quad [\text{alt } \angle =, CH \parallel JR]$ $\hat{C}_1 + \hat{C}_2 = 180^\circ - 2x \quad [\text{sum of int } \angle^s \text{ of } \Delta]$ $\hat{A}BD = 180^\circ - 2x \quad [\angle^s \text{ in the same segment}]$	<span style="display: inline-block; width: 15px; height: 15px; border: 1px solid black; border-radius: 50%; background-color: #ccc;"></span> S <span style="display: inline-block; width: 15px; height: 15px; border: 1px solid black; border-radius: 50%; background-color: #ccc;"></span> S <span style="display: inline-block; width: 15px; height: 15px; border: 1px solid black; border-radius: 50%; background-color: #ccc;"></span> S <span style="display: inline-block; width: 15px; height: 15px; border: 1px solid black; border-radius: 50%; background-color: #ccc;"></span> S	(2)
11.5	$\hat{T} = 90^\circ \quad [\tan \perp \text{rad}]$ $\hat{C}AO = 90^\circ - x \quad [\angle^s \text{ opp sides}]$ $x + \hat{A}_3 = 90^\circ - x$ $\hat{A}_3 = 90^\circ - 2x$ $\therefore \hat{R}_2 = 90^\circ + 90^\circ - 2x \quad [\text{ext } \angle^s \text{ of } \Delta]$ $= 180^\circ - 2x$	<span style="display: inline-block; width: 15px; height: 15px; border: 1px solid black; border-radius: 50%; background-color: #ccc;"></span> S <span style="display: inline-block; width: 15px; height: 15px; border: 1px solid black; border-radius: 50%; background-color: #ccc;"></span> R <span style="display: inline-block; width: 15px; height: 15px; border: 1px solid black; border-radius: 50%; background-color: #ccc;"></span> S/ R <span style="display: inline-block; width: 15px; height: 15px; border: 1px solid black; border-radius: 50%; background-color: #ccc;"></span> S <span style="display: inline-block; width: 15px; height: 15px; border: 1px solid black; border-radius: 50%; background-color: #ccc;"></span> S <span style="display: inline-block; width: 15px; height: 15px; border: 1px solid black; border-radius: 50%; background-color: #ccc;"></span> S	(6) [18]

**TOTAAL: 150**